Inverse Trigonometric
EXERCISE # 1

Questions based on **Principal Value**

Q.1 $\cos^{-1}(-1) =$

(A) $\frac{\pi}{2}$ (B) 0 (C) π (D) 2π

Sol. **[C]** $\cos^{-1}(-1) = \pi$

Q.2 $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to-

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$

(C) $\frac{3\pi}{4}$ (D) $-\frac{3\pi}{4}$

Sol. **[A]** $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Q.3 $\cos^{-1}(\cos 5\pi/4)$ is given by -

(A) $5\pi/4$ (B) $3\pi/4$

(C) $-\pi/4$ (D) None of these

Sol. **[B]** $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Questions based on **Properties I to IV**

Q.4 $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$ is equal to -

(A) $-\frac{17\pi}{15}$ (B) $\frac{17\pi}{15}$

(C) $\frac{2\pi}{15}$ (D) $\frac{13\pi}{15}$

Sol.[D] $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{17}{15}\pi\right)\right]$

$= \cos^{-1}\left[\cos\left(2\pi - \frac{13}{15}\pi\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{13}{15}\pi\right)\right] = \frac{13}{15}\pi$

Q.5 The value of $\sin\left[\text{arc}\cos\left(-\frac{1}{2}\right)\right]$ is-

(A) $\frac{1}{\sqrt{2}}$ (B) 1

(C) $\frac{\sqrt{3}}{2}$ (D) none of these

Sol. **[C]**

$\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\pi - \cos^{-1}\frac{1}{2}\right]$

$= \sin\left(\cos^{-1}\frac{1}{2}\right) = \sin^{-1}\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Q.6 If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then

$\cos^{-1}x + \cos^{-1}y =$

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) π

Sol.[B] $\because$ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

So $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

Q.7 $\tan^{-1}[\tan(-6)]$ is equal to

(A) $2\pi - 6$ (B) $2\pi + 6$

(C) $-2\pi + 6$ (D) -6

Sol.[A] $\because$ $-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

$\Rightarrow \tan^{-1}[\tan(2\pi - 6)] = 2\pi - 6$

Q.8 $\cos^{-1}(\cos 10)$ is equal to

(A) $4\pi + 10$ (B) $4\pi - 10$

(C) $-4\pi + 10$ (D) 10

Sol.[B] $\cos^{-1}(\cos 10)$

$\because \cos^{-1}\cos\theta = \theta$ if $0 \le \theta \le \pi$

$\Rightarrow \cos^{-1}(\cos(4\pi - 10)) = 4\pi - 10$

Q.9 The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is -

(A) $\frac{3\pi}{5}$ (B) $\frac{7\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $-\frac{\pi}{10}$

Sol. **[D]**

We have

$$\sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right] \Rightarrow \sin^{-1}\left[\cos\frac{3\pi}{5}\right]$$

$$\Rightarrow \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \sin^{-1}\sin\left(-\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

Q.10 If $\pi \le x \le 2\pi$, then $\cos^{-1}(\cos x)$ is equal to-

(A) x (B) –x

(C) $2\pi + x$ (D) $2\pi - x$

Sol. **[D]**

$\cos^{-1}(\cos x)$ when $\pi \le x \le 2\pi$

then $\cos^{-1}\cos(2\pi - x) = 2\pi - x$

Questions based on

Properties V and VI

Q.11 If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ then x =

(A) –1 (B) $\frac{1}{6}$

(C) $-1, \frac{1}{6}$ (D) None of these

Sol.[B] $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (6x + 1)(x - 1) = 0$

$x = 1,\ x = -\frac{1}{6}$

But $x \ne 1$ so $x = -\frac{1}{6}$

Q.12 $\cot\left[\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right] =$

(A) 1 (B) –1 (C) $\sqrt{2}$ (D) $-\sqrt{2}$

Sol.[A] $\cot\left[\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right]$

$$= \cot\left[\tan^{-1}\frac{\frac{1}{2}+\frac{1}{5}+\frac{1}{8}-\frac{1}{2}\cdot\frac{1}{5}\cdot\frac{1}{8}}{1-\frac{1}{10}-\frac{1}{40}-\frac{1}{16}}\right]$$

$$= \cot\left[\tan^{-1}\frac{40+16+10-1}{80-8-2-5}\right]$$

$$= \cot[\tan^{-1} 1] = \cot\frac{\pi}{4} = 1$$

Q.13 If $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then x =

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\sqrt{\frac{5}{2}}$ (D) $\pm\sqrt{\frac{5}{2}}$

Sol.[D] $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{\frac{x-1}{x+2}+\frac{x+1}{x+2}}{1-\frac{x^2-1}{(x+2)^2}} = \frac{\pi}{4}$$

$$\Rightarrow \frac{2x(x+2)}{(x+2)^2 - x^2 + 1} = 1$$

$\Rightarrow 2x^2 + 4x = 4x + 4 + 1$

$\Rightarrow 2x^2 = 5$

$$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$$

Questions based on

Properties VII and VIII

Q.14 If $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$, then x =

(A) 0 (B) 1 (C) 1/2 (D) ¼

Sol.[B] $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$

$\Rightarrow \sin^{-1}\frac{4}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$

$\Rightarrow \cos^{-1}x = 0 \Rightarrow x = \cos 0 = 1$

Q.15 If $\sin^{-1} x/5 + \mathrm{cosec}^{-1} 5/4 = \pi/2$, then x =

(A) 4 (B) 5
(C) 1 (D) 3

Sol.[D] $\sin^{-1}\frac{x}{5} + \mathrm{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{\pi}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$\Rightarrow x = 3$

Q.16 If $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1}x$, then x =

(A) 1, –1 (B) 1, 0
(C) 0, 1/2 (D) None

Sol.[C] $\Theta\ \sin^{-1}\left(x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2}\right) = \cos^{-1}x$

$\Rightarrow \sin^{-1}\left(x\sqrt{2x-x^2} + (1-x)\sqrt{1-x^2}\right) = \sin^{-1}\sqrt{1-x^2}$

$\Rightarrow x\sqrt{2x-x^2} + (1-x)\sqrt{1-x^2} = \sqrt{1-x^2}$

$\Rightarrow x\sqrt{2x-x^2} = x\sqrt{1-x^2}$

$\Rightarrow x^2(2x - x^2) = x^2(1 - x^2)$

$\Rightarrow x^2(2x - x^2 - 1 + x^2) = 0$

$\Rightarrow x^2(2x - 1) = 0$

$\Rightarrow x = 0,\ x = \frac{1}{2}$

Questions based on **Properties IX , X & XI**

Q.17 $1 + \cot^2(\sin^{-1}x) =$

(A) $\frac{1}{2x}$ (B) x^2 (C) $\frac{1}{x^2}$ (D) $\frac{2}{x}$

Sol.[C] $1 + \cot^2(\sin^{-1}x)$

$= \mathrm{cosec}^2(\sin^{-1}x)$

$= [\mathrm{cosec}(\mathrm{cosec}^{-1}\frac{1}{x})]^2 = \frac{1}{x^2}$

Q.18 $\sin[\cot^{-1}\cos\tan^{-1}x]$ is equal to -

(A) $\sqrt{\left(\frac{x^2-1}{x^2+2}\right)}$ (B) $\sqrt{\left(\frac{x-2}{x^2+1}\right)}$

(C) $\sqrt{\left(\frac{x^2+1}{x^2+2}\right)}$ (D) None of these

Sol.[C] $\sin[\cot^{-1}\cos\tan^{-1}x]$

$= \sin\left[\cot^{-1}\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

$= \sin\left[\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right] = \sin\left[\sin^{-1}\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right]$

$= \sqrt{\frac{x^2+1}{x^2+2}}$

Q.19 If the sum of the acute angles $\tan^{-1}x$ and $\tan^{-1}\left(\frac{1}{3}\right)$ is 45º, then the value of x is -

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{2}$

Sol.[D] $\Theta\ \tan^{-1}x + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{x+\frac{1}{3}}{1-\frac{x}{3}} = \frac{\pi}{4}$

$\Rightarrow \frac{3x+1}{3-x} = 1$

$\Rightarrow 3x + 1 = 3 - x$

$\Rightarrow 4x = 2 \Rightarrow x = \frac{1}{2}$

Q.20 If $x = \sin(2\tan^{-1}2)$, $y = \sin(\frac{1}{2}\tan^{-1}\frac{4}{3})$.

Then

(A) $x = y^2$ (B) $y^2 = 1 - x$

(B) $x^2 = \frac{y}{2}$ (D) $y^2 = 1 + x$

Sol.[B] $x = \sin(2\tan^{-1}2)$, $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

$\Theta\ x = \sin\left(\pi + \tan^{-1}\frac{4}{1-4}\right)$

$= \sin\left(\pi - \tan^{-1}\frac{4}{3}\right) = \sin\left(\tan^{-1}\frac{4}{3}\right)$

$x = \sin\left(\sin^{-1}\frac{4}{5}\right) = \frac{4}{5}$

$y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

Let $\tan^{-1}\frac{4}{3} = \theta$

$\Rightarrow \tan\theta = \frac{4}{3} \Rightarrow \cos\theta = \frac{3}{5}$

$y = \sin\frac{\theta}{2}$

$\Rightarrow y^2 = \frac{1-\cos\theta}{2} = \frac{1-\frac{3}{5}}{2} = \frac{1}{5}$

Clearly $y^2 = 1 - x$

True or False type Questions

Q.21 If $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$ $(0 \le x \le 1)$, then the interval in which θ lies is given by $\pi/4 \le \theta \le \pi/2$

Sol. **True**

$\theta = \frac{\pi}{2} - \tan^{-1} x$

$\Rightarrow \theta = \cot^{-1} x$

$0 \le x \le 1$

$\Rightarrow$ clearly $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ True

Q.22 $\cos 2\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}\right) = \frac{3}{5}$

Sol. **True**

$\cos 2\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}\right)$

$= \cos 2\left(\tan^{-1}\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}.\frac{2}{9}}\right)$

$= \cos 2\tan^{-1}\frac{1}{2}$

$= \cos\left(\cos^{-1}\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right)$

$= \cos\cos^{-1}\frac{3}{5} = \frac{3}{5}$ R.H.S.

True

Q.23 $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\cos\alpha\sin\beta}{\sin\alpha+\cos\beta}$

Sol. **False**

Taking L.H.S.

$2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}$

$= \tan^{-1}\frac{2\tan\frac{\alpha}{2}\left(\frac{1-\tan\beta/2}{1+\tan\beta/2}\right)}{1-\tan^2\frac{\alpha}{2}\left(\frac{1-\tan\beta/2}{1+\tan\beta/2}\right)^2}$

so change $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and solving , we get

$= \tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta} \ne$ R.H.S. False

Fill in The Blanks type Questions

Q.24 If $y = 2\tan^{-1} x + \sin^{-1}[2x/(1+x^2)]$ for all x, then ——— $\le y \le$ ———

Sol. $y = 2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2}$ $\forall\, x\in R$

$y = 2\tan^{-1}x + 2\tan^{-1}x$ $\forall\, x\in R$

$y = 4\tan^{-1}x$ $\forall\, x\in R$

Since $\frac{-\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

$\therefore\ 4 \times \frac{-\pi}{2} < 4\tan^{-1}x < 4 \times \frac{\pi}{2}$

$\Rightarrow -2\pi < 4\tan^{-1}x < 2\pi$

$\therefore -2\pi < y < 2\pi$

Q.25 If $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$,

then x =......

Sol. $\tan^{-1}\frac{b}{x} = \frac{\pi}{2} - \tan^{-1}\frac{a}{x}$

$\Rightarrow \tan^{-1}\frac{b}{x} = \cos^{-1}\frac{a}{x} \Rightarrow \tan^{-1}\frac{b}{x} = \tan^{-1}\frac{x}{a}$

Q.26 If $\tan^{-1}\frac{1}{a-1} = \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{a^2-x+1}$,

then x = ...

Sol. $\tan^{-1}\frac{1}{a-1} = \tan^{-1}\dfrac{\frac{1}{x}+\frac{1}{a^2-x+1}}{1-\frac{1}{x}\left(\frac{1}{a^2-x+1}\right)}$

$\Rightarrow \frac{1}{a-1} = \frac{a^2+1}{(a^2+1)x-x^2-1}$

$\Rightarrow x^2 - x(a^2+1) + 1 + (a-1)(a^2+1) = 0$

$\Rightarrow (x^2 - a^2) - (a^2+1)(x-a) = 0$

$\Rightarrow (x-a)(x+a-a^2-1) = 0$

$\Rightarrow x = a$ or $x = a^2 - a + 1$

Q.27 If $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1}x$,

then x =.......

Sol. $\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right) = \sin^{-1}x$

$\Rightarrow 3x\sqrt{25-16x^2} + 4x\sqrt{25-9x^2} = 25x$

one root is x = 0, other roots are given by the equation

$3\sqrt{25-16x^2} + 4\sqrt{25-9x^2} = 25$

$\Rightarrow 3\sqrt{25-16x^2} = 25 - 4\sqrt{25-9x^2}$

On squaring and solving, we get

$4 = \sqrt{25-9x^2}$

again squaring and solving, we obtain

$x = \pm 1$ (satisfy the equation)

$\Rightarrow x = 0, 1, -1$ are the solution

EXERCISE # 2

Part-A Only Single Correct Answer type Questions

Q.1 If $\sin^{-1}x = \theta + \beta$ and $\sin^{-1}y = \theta - \beta$, then $1 + xy =$

(A) $\sin^2\theta + \sin^2\beta$ (B) $\sin^2\theta + \cos^2\beta$

(C) $\cos^2\theta + \cos^2\beta$ (D) $\cos^2\theta + \sin^2\beta$

Sol. **[B]**

$\sin^{-1}x = \theta + \beta$ and $\sin^{-1} y = \theta - \beta$

$\Rightarrow x = \sin(\theta + \beta)$ and $y = \sin(\theta - \beta)$

put the value and solving we get

$1 + xy = \sin^2\theta + \cos^2\beta$

Q.2 The value of

$$\left[\cot\left\{\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right) + \cos^{-1}\left(\frac{\sqrt{12}}{2}\right) + \sec^{-1}\sqrt{2}\right\}\right]$$

is equal to -

(A) 0 (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

Sol. **[A]**

$$\sin^{-1}\left[\cot\left\{\sin\left(\frac{\sqrt{2-\sqrt{3}}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\frac{1}{\sqrt{2}}\right\}\right]$$

$$\Rightarrow \sin^{-1}\left[\cot\left\{\tan^{-1}\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \frac{\pi}{6} + \frac{\pi}{4}\right\}\right]$$

$$\Rightarrow \sin^{-1}\left[\cot\left\{\tan^{-1}(2-\sqrt{3}) + \frac{5\pi}{12}\right\}\right]$$

$$\Rightarrow \sin^{-1}\left[\cot\left\{\frac{\pi}{12} + \frac{5\pi}{12}\right\}\right]$$

$$\sin^{-1}\left[\cot\frac{\pi}{2}\right] = 0$$

$= \sin^{-1} 0 = 0$

Q.3 The number of positive integral solutions of $\cos^{-1}\left(4x^2 - 8x + \frac{7}{2}\right) = \frac{\pi}{3}$ are

(A) one (B) two

(C) three (D) none of these

Sol. **[D]**

$4x^2 - 8x + \frac{7}{2} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow 4x^2 - 8x + 3 = 0 \Rightarrow 4x^2 - 6x - 2x + 3 = 0$

$\Rightarrow (2x - 1)(2x - 3) = 0$; $x = \frac{1}{2}$, $x = \frac{3}{2}$

No integral solution.

Q.4 If we consider only the principal values of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is-

(A) $\frac{\sqrt{29}}{3}$ (B) $\frac{29}{3}$

(C) $\sqrt{3}/29$ (D) None of these

Sol. **[D]**

We have

$\cos^{-1}\frac{1}{5\sqrt{2}} = \tan^{-1} 7$, $\sin^{-1}\frac{4}{\sqrt{17}} = \tan^{-1} 4$

$\Rightarrow \tan\left(\tan^{-1}\frac{7-4}{1+28}\right) = \tan\left(\tan^{-1}\frac{3}{29}\right) = \frac{3}{29}$

Q.5 If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x equals-

(A) –1 (B) 1

(C) 0 (D) None of these

Sol. **[A]**

$(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$

put $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$ and $\tan^{-1} x = y$

$y^2 + \left(\frac{\pi}{2} - y\right)^2 = \frac{5\pi^2}{8}$

Solving we get $y = -\frac{\pi}{4}, \frac{3\pi}{4}$; $x = -1$

Q.6 $\cos^{-1}\left[\sin\left(-\frac{\pi}{9}\right)\right] + \sin^{-1}\left[\cos\frac{33\pi}{9}\right] =$

(A) $\frac{30\pi}{22}$ (B) $\frac{7\pi}{9}$

(C) $\frac{36\pi}{89}$ (D) None of these

Sol. **[B]**

$$\cos^{-1}\left(-\sin\frac{\pi}{9}\right)+\sin^{-1}\left[\cos\left(4\pi-\frac{3\pi}{9}\right)\right]$$

$$\Rightarrow \pi-\cos^{-1}\left(\sin\frac{\pi}{9}\right)+\sin^{-1}\left(\cos\frac{3\pi}{9}\right)$$

$$\Rightarrow \pi-\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{9}\right)\right]+\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{9}\right)\right]$$

$$\Rightarrow \pi-\frac{\pi}{2}+\frac{\pi}{9}+\frac{\pi}{2}-\frac{3\pi}{9} \quad \Rightarrow \pi-\frac{2\pi}{9}=\frac{7\pi}{9}$$

Q.7 The value of $\sin^{-1}(\sin 10)$ is -

(A) 10 (B) $10-3\pi$

(C) $3\pi-10$ (D) None of these

Sol. **[C]**

Since $3\pi < 10 < 3\pi+\frac{\pi}{2}$

$0 < 10-3\pi < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < 3\pi-10 < 0 \Rightarrow \sin^{-1}(\sin 10) = 3\pi-10$

Q.8 If $x = \tan^{-1}(1/7)$ and $y = \tan^{-1}(1/3)$, then -

(A) $\cos 2x = \sin 4y$ (B) $\cos 4y = \cos 2x$

(C) $\cos 2y = \sin 4x$ (D) None of these

Sol. **[A]**

$\because x = \tan^{-1}\left(\frac{1}{7}\right) \Rightarrow \tan x = \frac{1}{7}$

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-\frac{1}{49}}{1+\frac{1}{49}} = \frac{48}{50} = \frac{24}{25}$$

and $\because y = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan y = \frac{1}{3}$

$$\Rightarrow \sin 2y = \frac{2\tan y}{1+\tan^2 y} = \frac{2\left(\frac{1}{3}\right)}{1+\frac{1}{9}} = \frac{3}{5}$$

$$\text{and } \cos 2y = \frac{1-\tan^2 y}{1+\tan^2 y} = \frac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{4}{5}$$

$$\Rightarrow \sin 4y = 2\sin 2y\cos 2y = 2.\frac{3}{5}.\frac{4}{5} = \frac{24}{25}$$

Hence $\cos 2x = \sin 4y$

Q.9 If $\theta = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$, then $\sin\theta$ =

(A) $\tan\frac{1}{2}x$ (B) $\tan^2(x/2)$

(C) $\frac{1}{2}\tan^{-1}(x/2)$ (D) None of these

Sol. **[B]**

$$\theta = \tan^{-1}\frac{1}{\sqrt{\cos x}} - \tan^{-1}\sqrt{\cos x}$$

$$= \tan^{-1}\frac{1-\cos x}{2\sqrt{\cos x}} \Rightarrow \tan\theta = \frac{1-\cos x}{2\sqrt{\cos x}}$$

$$\Rightarrow \sin\theta = \frac{1-\cos x}{1+\cos x} = \tan^2\frac{x}{2}$$

Q.10 If a, b, c be positive real numbers and the value of $\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ then $\tan\theta$ is equal to -

(A) 0 (B) 1

(C) $\frac{a+b+c}{abc}$ (D) None of these

Sol. **[A]**

Let $a+b+c = x$

then $\theta = \tan^{-1}\sqrt{\frac{ax}{bc}} + \tan^{-1}\sqrt{\frac{bx}{ca}} + \tan^{-1}\sqrt{\frac{cx}{ab}}$

taking R.H.S. we have

$$\Rightarrow \tan^{-1}\frac{\sqrt{\frac{ax}{bc}}+\sqrt{\frac{bx}{ca}}}{1-\sqrt{\frac{ax}{bc}}\sqrt{\frac{bx}{ca}}} + \tan^{-1}\sqrt{\frac{cx}{ab}}$$

$$\Rightarrow \tan^{-1}\frac{\sqrt{\frac{x}{abc}}(a+b)}{1-\frac{x}{c}} + \tan^{-1}\sqrt{\frac{cx}{ab}}$$

$$\Rightarrow \tan^{-1}\frac{\sqrt{\frac{cx}{ab}}(a+b)}{-(a+b)} + \tan^{-1}\sqrt{\frac{cx}{ab}}$$

$\because \tan^{-1}(-\theta) = \pi - \tan^{-1}\theta$

$$\Rightarrow \pi - \tan^{-1}\sqrt{\frac{cx}{ab}} + \tan^{-1}\sqrt{\frac{cx}{ab}} = \pi \Rightarrow \theta = \pi$$

$\Rightarrow \tan\theta = \tan\pi = 0$

Q.11 $\tan^{-1}2x + \tan^{-1}3x = n\pi + \frac{3\pi}{4}$, then x =

(A) $1, -\frac{1}{6}$ (B) $\frac{1}{2}, \frac{1}{3}$

(C) 4, 5 (D) None of these

Sol. **[A]**

$\tan^{-1} 2x + \tan^{-1} 3x = n\pi + \frac{3\pi}{4}$

$\Rightarrow \tan^{-1} \frac{2x+3x}{1-6x^2} = n\pi + \frac{3\pi}{4}$

$\Rightarrow \frac{2x+3x}{1-6x^2} = -1$

$\Rightarrow 6x^2 - 5x - 1 = 0$

$\Rightarrow (6x + 1)(x - 1) = 0$

$\Rightarrow x = 1, -\frac{1}{6}$

Q.12 If $\tan(x + y) = 33$ and $x = \tan^{-1} 3$, then y will be-

(A) 0.3 (B) $\tan^{-1}(1.3)$

(C) $\tan^{-1}(0.3)$ (D) $\tan^{-1}\left(\frac{1}{18}\right)$

Sol. **[C]**

$\tan(x + y) = 33$ and $x = \tan^{-1} 3$

$\Rightarrow \frac{\tan x + \tan y}{1 - \tan x \tan y} = 33$

put tan x = 3 and solving we get

$\tan y = .3 \Rightarrow y = \tan^{-1}(.3)$

Q.13 The value of $2\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ is equal to-

(A) $\cot^{-1} x$ (B) $\sec^{-1} x$

(C) $\tan^{-1} x$ (D) None of these

Sol. **[C]**

$2\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta$ and solve we have

$2\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = 2\tan^{-1} \tan \frac{\theta}{2}$

$\theta = \tan^{-1} x$

Q.14 In a ΔABC, if $A = \tan^{-1} 2$ and $B = \tan^{-1} 3$, then C is equal to-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) none of these

Sol. **[B]**

$A = \tan^{-1} 2$ $B = \tan^{-1} 3$

In any Δ, we have $A + B + C = \pi$

$\tan^{-1} 2 + \tan^{-1} 3 + c = \pi$

$\Rightarrow \tan^{-1} \frac{2+3}{1-6} + c = \pi \Rightarrow \tan^{-1}(-1) + c = \pi$

$\Rightarrow \pi - \frac{\pi}{4} + c = \pi \Rightarrow c = \frac{\pi}{4}$

Q.15 If $\tan^{-1} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \alpha$, then x^2 is equal to-

(A) $\sin 2\alpha$ (B) $\sin\alpha$ (C) $\cos 2\alpha$ (D) $\cos\alpha$

Sol. **[A]**

$\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} = \frac{\sin x}{\cos x}$

using componendo and dividendo, we get

$\frac{2\sqrt{1+x^2}}{2\sqrt{1-x^2}} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$

Squaring, we get

$\frac{1+x^2}{1-x^2} = \frac{1+\sin 2\alpha}{1-\sin 2\alpha}$

from above $x^2 = \sin 2\alpha$

Q.16 The value of $\cos(2\cos^{-1} x + \sin^{-1} x)$ at $x = \frac{1}{5}$ is -

(A) $\frac{2\sqrt{6}}{5}$ (B) $-\frac{2\sqrt{6}}{5}$

(C) $\frac{3\sqrt{6}}{5}$ (D) none of these

Sol. **[B]**

$\cos(2\cos^{-1}x + \sin^{-1}x)$

$= \cos(\cos^{-1} x + \cos^{-1} x + \sin^{-1} x)$

$= \cos\left(\frac{\pi}{2} + \cos^{-1} x\right) = -\sin(\cos^{-1} x)$

$= -\sin(\sin^{-1}\sqrt{1-x^2}) = -\sqrt{1-x^2}$

at $x = \frac{1}{5}$, we get

$= \sqrt{1-\frac{1}{25}} = -\frac{2\sqrt{6}}{5}$

Q.17 If $\cos^{-1}\frac{p}{a} + \cos^{-1}\frac{q}{b} = \alpha$, then

$\frac{p^2}{a^2} - \frac{2pq}{ab}\cos\alpha + \frac{q^2}{b^2}$ equals-

(A) $\sin^2\alpha$ (B) $\cos^2\alpha$

(C) $\tan^2\alpha$ (D) $\cot^2\alpha$

Sol. **[A]**

$\cos^{-1}\frac{p}{a} + \cos^{-1}\frac{q}{b} = \alpha$

$\Rightarrow \cos^{-1}\left\{\frac{p}{a}.\frac{q}{b} - \sqrt{1-\frac{p^2}{a^2}}\sqrt{1-\frac{q^2}{b^2}}\right\} = \alpha$

$\Rightarrow \sqrt{1-\frac{p^2}{a^2}}\sqrt{1-\frac{q^2}{b^2}} = \frac{pq}{ab} - \cos\alpha$

Squaring both the side

$\left(1-\frac{p^2}{a^2}\right)\left(1-\frac{q^2}{b^2}\right) = \frac{p^2q^2}{a^2b^2} - 2\frac{pq}{ab}\cos\alpha + \cos^2\alpha$

$1-\frac{p^2}{a^2} - \frac{q^2}{b^2} = \cos^2\alpha - 2\frac{pq}{ab}\cos\alpha$

$\Rightarrow \frac{p^2}{a^2} - 2\frac{pq}{ab}\cos\alpha + \frac{q^2}{b^2} = \sin^2\alpha$

Q.18 $\tan\left[\frac{1}{2}\sin^{-1}\frac{2a}{1+a^2} + \frac{1}{2}\cos^{-1}\frac{1-a^2}{1+a^2}\right] =$

(A) $\frac{2a}{1+a^2}$ (B) $\frac{a}{1+a^2}$

(C) $\frac{2a}{1-a^2}$ (D) $\frac{2}{1+a^2}$

Sol. **[C]**

$\tan\left[\frac{1}{2}\sin^{-1}\frac{2a}{1+a^2} + \frac{1}{2}\cos^{-1}\frac{1-a^2}{1+a^2}\right]$

$\Rightarrow \tan\left[\frac{1}{2}.2\tan^{-1}a + \frac{1}{2}.2\tan^{-1}a\right]$

$\Rightarrow \tan[2\tan^{-1}a]$

$\Rightarrow \tan\left(\tan^{-1}\frac{2a}{1-a^2}\right) = \frac{2a}{1-a^2}$

Q.19 Solution of the equation
$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x)$ is-

(A) $x = 2n\pi + \frac{\pi}{4}$ (B) $x = n\pi + \frac{\pi}{4}$

(C) $x = n\pi + \frac{\pi}{2}$ (D) none of these

Sol. **[B]**

$2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x)$

$\Rightarrow \tan^{-1}\frac{2\cos x}{1-\cos^2 x} = \tan^{-1}(2\,\text{cosec}\,x)$

$\Rightarrow 2.\cot x.\,\text{cosec}\,x = 2\,\text{cosec}\,x$

$\Rightarrow \text{cosec}\,x(\cot x - 1) = 0$

$\Rightarrow \cot x - 1 = 0 \Rightarrow \cot x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4}$ $\quad\because \text{cosec}\,x \neq 0$

Q.20 The value of $\cot^{-1}\left[\frac{\sqrt{(1-\sin x)}+\sqrt{(1+\sin x)}}{\sqrt{(1-\sin x)}-\sqrt{(1+\sin x)}}\right]$ is $\forall x \in (0, \pi/2]$

(A) $\pi - x$ (B) $2\pi - x$

(C) $x/2$ (D) $\pi - x/2$

Sol. **[D]**

$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$

$= \cot^{-1}\left[\frac{\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}-\cos\frac{x}{2}-\sin\frac{x}{2}}\right]$

$= \cot^{-1}\left(-\cot\frac{x}{2}\right)$

$= \cot^{-1}\left\{\cot\left(\pi-\frac{x}{2}\right)\right\} = \pi - \frac{x}{2}$

Q.21 $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] =$

(A) $\frac{3-\sqrt{5}}{2}$ (B) $\frac{3+\sqrt{5}}{2}$

(C) $\frac{2}{3-\sqrt{5}}$ (D) $\frac{4}{3+\sqrt{5}}$

Sol.[A] Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta$

$\Rightarrow \cos\theta = \frac{\sqrt{5}}{3}$ and $\sin\theta = \frac{2}{3}$

there fore

$\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right] = \tan\frac{\theta}{2}$

$\Rightarrow \tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \Rightarrow \tan\frac{\theta}{2} = \frac{1-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{3-\sqrt{5}}{2}$

Q.22 The number of real solution of

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is -

(A) Zero (B) One
(C) Two (D) Infinite

Sol. **[C]**

Given equation exist only when

$x(x+1) \ge 0$, $x^2 + x + 1 \ge 0$ but ≤ 1

$\Rightarrow 0 \le x^2 + x + 1 \le 1$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x(x+1) = 0$

$\Rightarrow x = -1, 0$

Part-B One or More Than one correct Answer Type Questions

Q.23 One of the values of x satisfying

$\tan(\sec^{-1}x) = \sin\cos^{-1}\frac{1}{\sqrt{5}}$ is-

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{3}{\sqrt{5}}$

(C) $-\frac{\sqrt{5}}{3}$ (D) $-\frac{3}{\sqrt{5}}$

Sol. **[B, D]**

$\tan(\sec^{-1}x) = \sin\cos^{-1}\frac{1}{\sqrt{5}}$

$\Rightarrow \tan(\tan^{-1}\sqrt{x^2-1}) = \sin\sin^{-1}\frac{2}{\sqrt{5}}$

$\Rightarrow \sqrt{x^2-1} = \frac{2}{\sqrt{5}}$

$\Rightarrow x^2 = \frac{9}{5} \Rightarrow x = \pm\frac{3}{\sqrt{5}}$

Option (B), (D) are correct.

Q.24 Let $\theta = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\phi = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$ then-

(A) $\theta > \phi$ (B) $4\theta - 3\phi = 0$

(C) $\theta + \phi = \frac{7\pi}{12}$ (D) None of these

Sol. **[B, C]**

$\theta = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$= \tan^{-1}\left(\tan\left(\pi+\frac{\pi}{4}\right)\right) = \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$ .. (i)

and $\phi = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$

$= \tan^{-1}\left(-\tan\left(\pi-\frac{\pi}{3}\right)\right) = \tan^{-1}\tan\frac{\pi}{3} = \frac{\pi}{3}$

$\Rightarrow \phi = \frac{\pi}{3}$ ... (ii)

from option

clearly $\phi > \theta$ and $4\theta - 3\phi = \pi - \pi = 0$

$\theta + \phi = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

$\Rightarrow$ option (B) & (C) are correct.

Q.25 If $\frac{1}{2} < |x| < 1$ then which of the following are real-

(A) $\sin^{-1}x$ (B) $\tan^{-1}x$
(C) $\sec^{-1}x$ (D) $\cos^{-1}x$

Sol. **[A, B, D]**

Given $\frac{1}{2} < |x| < 1$

from option, we know that

$\sin^{-1}x$ is real for $-1 \le x \le 1$

$\tan^{-1}x$ is real for $-\infty < x < \infty$

$\sec^{-1}x$ is real for $x \le -1, x \ge 1$

$\cos^{-1}x$ is real for $-1 \le x \le 1$

$\Rightarrow$ option (A), (B) , (D) are correct.

Q.26 Let $f(x) = \sin^{-1}x + \cos^{-1}x$. Then $\frac{\pi}{2}$ is equal to-

(A) $f\left(-\frac{1}{2}\right)$

(B) $f(k^2 - 2k + 3)$, $k \in R$

(C) $f\left(\frac{1}{1+k^2}\right)$, $k \in R$

(D) $f(-2)$

Sol. **[A, C]**

If $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Then we must have $-1 \le x \le 1$

clearly $f\left(-\frac{1}{2}\right)$ and $f\left(\frac{1}{1+k^2}\right)$ gives correct values

so option (A), (C) are correct.

Q.27 The greatest and least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are-

(A) $\frac{\pi^3}{32}$ (B) $-\frac{\pi^3}{8}$

(C) $\frac{7\pi^3}{8}$ (D) $\frac{\pi}{2}$

Sol. **[A, C]**

$(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$= (\sin^{-1} x + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x (\sin^{-1}x + \cos^{-1} x)$

$= \frac{\pi^3}{8} - \frac{3\pi}{8} \sin^{-1} x \cos^{-1} x$

Let $\sin^{-1} x = t \Rightarrow \cos^{-1} x = \frac{\pi}{2} - t$

Put the value, we get

$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(t - \frac{\pi}{4}\right)^2$ ... (i)

minimum value $= \frac{\pi^3}{32}$

Again $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ or $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4} \le \left(t - \frac{\pi}{4}\right) \le \frac{\pi}{4}$

$\Rightarrow \frac{\pi^2}{16} \le \left(t - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$

so from (i)

$\Rightarrow \frac{\pi^3}{32} + \frac{3\pi}{2} \cdot \frac{9\pi^2}{16} = \frac{28\pi^3}{32} = \frac{7\pi^3}{8}$

maximum value $= \frac{7\pi^3}{8}$

$\Rightarrow$ greatest value $= \frac{7\pi^3}{8}$ and least value $= \frac{\pi^3}{32}$.

Q.28 $\sin^{-1} x > \cos^{-1} x$ holds for-

(A) all values of x (B) $x \in (0, 1/\sqrt{2})$

(C) $x \in (1/\sqrt{2}, 1)$ (D) $x = 0.75$.

Sol. **[C, D]**

$\sin^{-1} x > \cos^{-1} x$

$\Rightarrow 2\sin^{-1} x > \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x > \frac{\pi}{4}$

$\Rightarrow \sin(\sin^{-1} x) > \sin\frac{\pi}{4}$

$\Rightarrow x > \frac{1}{\sqrt{2}}$

But $-1 \le x \le 1 \Rightarrow x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ and $\frac{1}{\sqrt{2}} < .75 < 1$

$\Rightarrow$ option (C) and (D) are correct.

Q.29 α, β and γ are three angles given by

$\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(-\frac{1}{2}\right)$

and $\gamma = \cos^{-1}\frac{1}{3}$. Then-

(A) $\alpha > \beta$ (B) $\beta > \gamma$

(C) $\alpha < \gamma$ (D) none of these

Sol. **[B, C]**

$\alpha = \tan^{-1}\frac{2(\sqrt{2}-1)}{1-(3-2\sqrt{2})} = \tan^{-1} 1 = \frac{\pi}{4} = 45^o$

$\beta = 3.\frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12} = 105^o$

$\gamma = \cos^{-1}\frac{1}{3} = \tan^{-1} 2\sqrt{2} < \frac{\pi}{2}$

clearly $\alpha < \gamma < \beta$

Option (B), (C) are correct.

Part-C Assertion Reason type Questions

The following questions 30 to 31 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q.30 **Assertion (A) :** The solution of

$\sin^{-1}(6x) + \sin^{-1}(6\sqrt{3})x = \frac{-\pi}{2}$ is $x = \pm 1/12$.

Reason (R): As, $\sin^{-1}x$ is defined for $|x| \le 1$.

Sol. **[D]**

$\frac{\pi}{2} + \sin^{-1} 6\sqrt{3}\,x = -\sin^{-1} 6x$

Taking sine both side

$\cos(\sin^{-1} 6\sqrt{3}\,x) = -6x \Rightarrow \sqrt{1-108x^2} = -6x$

$\Rightarrow 1 - 108x^2 = 36x^2 \Rightarrow x^2 = \frac{1}{144} = \pm\frac{1}{12}$

But $x = \frac{1}{12}$ does not satisfy the equation

But $x = -\frac{1}{12}$ is satisfy

$\Rightarrow x = -\frac{1}{12}$ is the solution and $\sin^{-1} x$ defined for

$|x| \le$ is true.

$\Rightarrow$ Assertion is false but reason is true.

Q.31 **Assertion (A) :** The solution set for $\cos^{-1}(\cos 4) > 3x^2 - 4x$ is ϕ.

Reason (R) : The value of $\cos^{-1}(\cos x) = 2\pi - x$, for $x \in (\pi, 2\pi)$.

Sol. **[D]**

Θ $\cos^{-1}(\cos x) = 2\pi - x$ for $x \in (\pi, 2\pi)$

R is true

Θ $\cos^{-1}(\cos 4) > 3x^2 - 4x$

$\Rightarrow 2\pi - 4 > 3x^2 - 4x$

$2\pi > 3x^2 - 4x + 4$

Θ minimum of

$3x^2 - 4x + 4$ is $-\frac{16-48}{12} = \frac{8}{3}$

But $2\pi = 6.28$ Apr.

$\Rightarrow$ A is false

Part-D Column Matching type Questions

Q.32

Column I	**Column II**
(A) $\cos^{-1}\lambda + \cos^{-1}\mu + \cos^{-1}\nu = 3\pi$ then $\lambda\mu + \mu\nu + \nu\lambda$ is	(P) 2n
(B) If $\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$, then $2\sqrt{5}\,(2x^2 + 1)$ is	(Q) $\sin^{-1}x - \frac{\pi}{6}$
(C) $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$ then $\sum_{i=1}^{2n} x_i$ is	(R) 10
(D) $f(x) = \sin^{-1}\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right\}$, $-\frac{1}{2} \le x \le 1$ is	(S) 3

Sol. **A → S, ; B → R ; C → P ; D → Q**

(A) $\cos^{-1}\lambda + \cos^{-1}\mu + \cos^{-1}\nu = 3\pi$

$-1 \le \cos^{-1} x \le 1 \Rightarrow \cos^{-1}x = \pi$

$\Rightarrow \lambda = -1, \mu = -1, \nu = -1$

$\Rightarrow \lambda\mu + \mu\nu + \nu\lambda = 3$

(B) $\sin^{-1}x + \tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x = \cos^{-1}x$

$\Rightarrow x = \frac{\sqrt{1-x^2}}{x} \Rightarrow x^4 = 1 - x^2$

$\Rightarrow \left(x^2 + \frac{1}{2}\right)^2 = \frac{5}{4} \Rightarrow (2x^2 + 1) = \sqrt{5}$

$2\sqrt{5}\,(2x^2 + 1) = 10$

(C) $-\frac{\pi}{2} \le \sin^{-1}x_i \le \frac{\pi}{2}$ $\therefore \sin^{-1} x_i = \frac{\pi}{2}$

Θ $1 \le i \le 2n \Rightarrow x_i = 1,\ i \in [1, 2n]$

$\sum_{i=1}^{2n} x_i = 2n$

(D) Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) = \sin^{-1}\sin\left(\theta - \frac{\pi}{6}\right)$

$= \theta - \frac{\pi}{6} = \sin^{-1}x - \frac{\pi}{6}$

Q.33 Let $f(x) = \sin^{-1}x$, $g(x) = \cos^{-1}x$ and $h(x) = \tan^{-1}x$. For what interval of variation of x the following are true.

Column I	**Column II**
(A) $f(\sqrt{x}) + g(\sqrt{x}) = \pi/2$	(P) $[0, \infty)$
(B) $f(x) + g\left(\sqrt{1-x^2}\right) = 0$	(Q) $[0, 1]$
(C) $g\left(\frac{1-x^2}{1+x^2}\right) = 2h(x)$	(R) $(-\infty, 1)$
(D) $h(x) + h(1) = h\left(\frac{1+x}{1-x}\right)$	(S) $[-1, 0]$

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EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Solve for x, $\sin\left[\left(\frac{1}{5}\right)\cos^{-1}x\right] = 1$.

Sol. Let $\frac{1}{5}\cos^{-1}x = \alpha$

$\because 0 \le \cos^{-1}x \le \pi$

$\therefore 0 \le \frac{1}{5}\cos^{-1}x \le \frac{\pi}{5} \Rightarrow 0 \le \alpha \le \frac{\pi}{5}$

But in the interval $\left[0, \frac{\pi}{5}\right]$

$\sin\alpha \ne 1$

Clearly given equation has no solution.

Q.2 Solve for x, $\cot^{-1}(x) - \cot^{-1}(x+2) = 15^\circ$.

Sol. $\cot^{-1}x - \cot^{-1}(x+2) = 15^\circ$

$\Rightarrow \cot^{-1}\frac{x(x+2)+1}{x+2-x} = 15^\circ$

$\Rightarrow \frac{x(x+2)+1}{2} = \cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow x^2 + 2x + 1 = (\sqrt{3}+1)^2 \Rightarrow (x+1)^2 = (\sqrt{3}+1)^2$

$\Rightarrow (x+1) = \pm(\sqrt{3}+1) \Rightarrow x = \sqrt{3}$ or $x = -(\sqrt{3}+2)$

Q.3 Let x, y be positive integers. Then find the pairs (x, y) satisfying the equation

$\tan^{-1}x + \cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$.

Sol. Given equation can be written as

$\tan^{-1}x + \tan^{-1}\frac{1}{y} = \tan^{-1}3$

$\Rightarrow \tan^{-1}\frac{x+\frac{1}{y}}{1-\frac{x}{y}} = \tan^{-1}3$

$\Rightarrow \frac{xy+1}{y-x} = 3$

$\Rightarrow xy + 1 = 3y - 3x$

$\Rightarrow y = \frac{1+3x}{3-x}$

We have to find only +ve and integral solutions. This is possible only when x = 1 and 2

then x = 1; y = 2

and x = 2 ; y = 7

Hence (1, 2) and (2, 7) are only two pair.

Q.4 Find the sum of the series

$\tan^{-1}(1/3) + \tan^{-1}(2/9) + ...$

$+ \tan^{-1}\left[\frac{2^{n-1}}{1+2^{2n-1}}\right] + ...\infty$.

Sol. $\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) = \sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n.2^{n-1}}\right)$

$= \sum_{n=1}^{\infty}(\tan^{-1}2^n - \tan^{-1}2^{n-1})$

$= \tan^{-1}2 - \tan^{-1}1 + \tan^{-1}4 - \tan^{-1}2 + ... + \tan^{-1}\infty$

$= -\tan^{-1}1 + \tan^{-1}\infty = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

Q.5 Find the sum of the series

$\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + ...$

$+ \sin^{-1}\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{[n(n+1)]}} + ...\infty$

Sol. $T_n = \sin^{-1}\left[\frac{1}{\sqrt{n}}.\sqrt{\frac{n}{n+1}} - \sqrt{\frac{n-1}{n}}.\frac{1}{\sqrt{n+1}}\right]$

$= \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{1-\frac{1}{n+1}} - \sqrt{1-\frac{1}{n}}.\frac{1}{\sqrt{n+1}}\right]$ ..(i)

Let if $\sin\theta = \frac{1}{\sqrt{n}}$ then $\cos\theta = \sqrt{1-\frac{1}{n}}$

and $\sin\phi = \frac{1}{\sqrt{n+1}}$ then $\cos\phi = \sqrt{1-\frac{1}{n+1}}$

Put the values in (i), we obtain

$= \sin^{-1}(\sin\theta\cos\phi - \cos\theta\sin\phi)$

$= \sin^{-1}(\sin(\theta-\phi)) = \theta - \phi$

$\Rightarrow \sin^{-1}\frac{1}{\sqrt{n}} - \sin^{-1}\frac{1}{\sqrt{n+1}}$

Put n = 1, 2, 3, ... and adding, we have

$S_n = \sin^{-1}1 - \sin^{-1}\frac{1}{\sqrt{n+1}}$

$\therefore S_\infty = \sin^{-1} 1 = \frac{\pi}{2}$

Q.6 If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, prove that
$2(x^2 - x^2y^2 + y^2) = 1 + x^4 + y^4$.

Sol. $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} (x\sqrt{1-y^2} + y\sqrt{1-x^2}) = \frac{\pi}{2}$

$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = 1$

squaring both the side we have

$x^2(1-y^2) + y^2(1-x^2) + 2xy\sqrt{1-x^2}\sqrt{1-y^2} = 1$

$\Rightarrow 2xy\sqrt{1-x^2}\sqrt{1-y^2} = 1 - x^2 - y^2 + 2x^2y^2$

again squaring and solving we get
$1 + x^4 + y^4 = 2(x^2 + y^2 - xy)$

Q.7 Prove that,

$\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2}) = \frac{14}{15}$.

Sol. Taking L.H.S. we have

$\sin\left(\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}\right) + \cos(\tan^{-1}2\sqrt{2})$

$\Rightarrow \sin\left(\tan^{-1}\frac{3}{4}\right) + \cos(\tan^{-1}2\sqrt{2})$

$\Rightarrow \sin^{-1}\left(\sin\frac{3}{5}\right) + \cos\left(\cos^{-1}\frac{1}{3}\right)$

$\Rightarrow \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$ R.H.S.

Q.8 Find the value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$.

Sol. $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \Rightarrow \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \cos\theta = \frac{1}{8}$

We have to find the value of $\sin\frac{\theta}{4}$

$\because \cos\theta = \frac{1}{8}$

$\Rightarrow \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \frac{\sqrt{7}}{4} \Rightarrow \cos\frac{\theta}{2} = \frac{3}{4}$

$\therefore \sin\frac{\theta}{4} = \sqrt{\frac{1-\cos\theta/2}{2}} = \sqrt{\frac{1-\frac{3}{4}}{2}}$

$\Rightarrow \sin\frac{\theta}{4} = \frac{1}{2\sqrt{2}}$

Q.9 Prove that,

$2\tan^{-1}(-3) = -\cos^{-1}\left(-\frac{4}{5}\right)$

$= -\pi + \cos^{-1}\left(\frac{4}{5}\right) = -\frac{1}{2}\pi + \tan^{-1}\left(-\frac{4}{3}\right)$.

Sol. Let $\tan^{-1}(-3) = \alpha \Rightarrow \tan\alpha = -3$

and $-\frac{\pi}{2} < \alpha < 0$

$\Rightarrow -\pi < 2\alpha < 0$

$\Rightarrow \cos(-2\alpha) = \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{1-9}{1+9} = -\frac{4}{5}$

$-2\alpha = \cos^{-1}\left(-\frac{4}{5}\right)$

$\Rightarrow 2\tan^{-1}(-3) = -\cos^{-1}\left(-\frac{4}{5}\right)$

Again $-\pi < 2\alpha < 0 \Rightarrow 0 < 2\alpha + \pi < \pi$

so $\cos(\pi + 2\alpha) = -\cos 2\alpha = \frac{4}{5}$

$\pi + 2\alpha = \cos^{-1}\frac{4}{5}$

$\Rightarrow 2\tan^{-1}(-3) = -\pi + \cos^{-1}\frac{4}{5}$

Again $-\pi < 2\alpha < 0$

$-\frac{\pi}{2} < 2\alpha + \frac{\pi}{2} < \frac{\pi}{2}$

so $\tan\left(\frac{\pi}{2} + 2\alpha\right) = -\cot 2\alpha = \frac{-1}{\tan 2\alpha}$

$= \frac{\tan^2\alpha - 1}{2\tan\alpha} = \frac{9-1}{2(-3)} = -\frac{4}{3}$

$\Rightarrow 2\tan^{-1}(-3) = -\dfrac{\pi}{2} + \tan^{-1}\left(-\dfrac{4}{3}\right)$

Q.10 If $\alpha = 2 \text{ arc tan}\left(\dfrac{1+x}{1-x}\right)$ and

$\beta = \text{arc sin}\left[\dfrac{1-x^2}{1+x^2}\right]$ for $0 < x < 1$

then prove that $\alpha + \beta = \pi$.

Sol. $\alpha = 2\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$; $\beta = \sin^{-1}\dfrac{1-x^2}{1+x^2}$

$\alpha + \beta = 2\tan^{-1}\left(\dfrac{1+x}{1-x}\right) + \sin^{-1}\dfrac{1-x^2}{1+x^2}$

$\because \dfrac{1+x}{1-x} > 1$

$\Rightarrow \pi + \tan^{-1}\dfrac{2\left(\dfrac{1+x}{1-x}\right)}{1-\left(\dfrac{1+x}{1-x}\right)^2} + \sin^{-1}\dfrac{1-x^2}{1+x^2}$

$\Rightarrow \pi + \tan^{-1}\dfrac{1-x^2}{-2x} + \tan^{-1}\dfrac{1-x^2}{2x}$

$= \pi - \tan^{-1}\dfrac{1-x^2}{2x} + \tan^{-1}\dfrac{1-x^2}{2x} = \pi$

Part-B Passage based objective questions

Passage # 1 (Q. 11 to 13)

The domain and range of inverse circular function are defined as follows :

	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right] - \{\pi\}$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\dfrac{\pi}{2}\right\}$

Q.11 $\sin^{-1}x < \dfrac{3\pi}{4}$ then solution set of x is-

(A) $\left(\dfrac{1}{\sqrt{2}}, 1\right]$ (B) $\left(-\dfrac{1}{\sqrt{2}}, -1\right]$

(C) $\left[-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right]$ (D) none of these

Sol.[A] Clearly $\dfrac{\pi}{2} \le \sin^{-1}x < \dfrac{3\pi}{4} \Rightarrow x \in \left(\dfrac{1}{\sqrt{2}}, 1\right]$

Q.12 $\sin^{-1}x + \text{cosec}^{-1}x$ at $x = -1$ is -

(A) π (B) 2π (C) 3π (D) $-\pi$

Sol.[C] $\sin^{-1}x + \text{cosec}^{-1}x$ at $x = -1$

$\Rightarrow \dfrac{3\pi}{2} + \dfrac{3\pi}{2} = 3\pi$

Q.13 If $x \in [-1, 1]$, then range of $\tan^{-1}(-x)$ is

(A) $\left[\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right]$ (B) $\left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$

(C) $[-\pi, 0]$ (D) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

Sol.[B] $x \in [-1, 1]$ then $\tan^{-1}(-x)$ is

$\pi - \dfrac{\pi}{4} \le \tan^{-1}(-x) \le \pi + \dfrac{\pi}{4}$

Range $\left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$

Passage # 2 (Q. 14 to 16)

Let us define here two functions in the domain of $[-1, 1]$, $f(x) = \sin^{-1}x$, $g(x) = \cos^{-1}x$

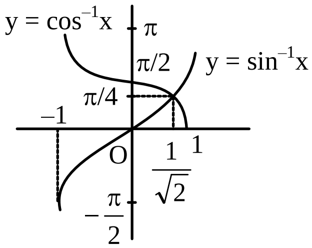
Now another function

$I(x) = \max.\{f(x), g(x)\}, -1 \le x \le 1$

$J(x) = \min\{f(x), g(x)\}, -1 \le x \le 1$

On the basis of above passage, answer the following questions :

Sol. Graph is



Q.14 The range of J(x) is-

(A) $\left[-\frac{\pi}{2}, 0\right]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) none of these

Sol. **[B]**

$$J(x) = \begin{cases} \sin^{-1} x & -1 \le x \le \frac{1}{\sqrt{2}} \\ \cos^{-1} x & \frac{1}{\sqrt{2}} < x \le 1 \end{cases}$$

Range is $\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$

Q.15 The number of points where I(x) is not differential are -

(A) 3 (B) 1

(C) 0 (D) Infinite

Sol. **[A]**

$$J(x) = \begin{cases} \cos^{-1} x & -1 \le x \le \frac{1}{\sqrt{2}} \\ \sin^{-1} x & \frac{1}{\sqrt{2}} < x \le 1 \end{cases}$$

Clearly I(x) is not differentiable at $\frac{1}{\sqrt{2}}$ $\Rightarrow$ one point

Q.16 The set of x such that J(x) is increases, is -

(A) (–1, 0) (B) (–1, 1/√2)

(C) (–1, 1) (D) none of these

Sol. **[B]**

Clearly J(x) is increases

in $\left(-1, \frac{1}{\sqrt{2}}\right)$

EXERCISE # 4

Old IIT-JEE Questions

Q.1 If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - ....\right)$

$+ \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - ....\right) = \frac{\pi}{2}$

for $0 < |x| < \sqrt{2}$, then x equals **[IIT 2001]**

(A) $\frac{1}{2}$ (B) 1

(C) $-\frac{1}{2}$ (D) – 1

Sol. **[B]**

Given $\sin^{-1} A + \cos^{-1} B = \frac{\pi}{2}$

But $\sin^{-1} A + \cos^{-1} A = \frac{\pi}{2}$

$\Rightarrow x - \frac{x^2}{2} + \frac{x^3}{4} - .... = x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....$

$\Rightarrow \frac{x}{1+\frac{x}{2}} = \frac{x^2}{1+\frac{x^2}{2}}$ from G.P.

$\Rightarrow x(2 + x^2) = x^2(2 + x) \Rightarrow 2x(x - 1) = 0$

$\Rightarrow x = 1, 0$ but $x \neq 0 \Rightarrow x = 1$

Q.2 $\cos \tan^{-1} \sin \cot^{-1} x =$ **[IIT-2002]**

(A) $\sqrt{\frac{x^2+1}{x^2+2}}$ (B) $\sqrt{\frac{x^2-1}{x^2+2}}$

(C) $\sqrt{\frac{x^2+1}{x^2-2}}$ (D) $\sqrt{\frac{x^2-1}{x^2-2}}$

Sol. Let $\cot^{-1} x = \theta$

$\Rightarrow \cot \theta = x$

$\Rightarrow \sin \theta = \frac{1}{\sqrt{x^2+1}}$

$\Rightarrow \cos \tan^{-1} \frac{1}{\sqrt{x^2+1}}$

Again let $\tan^{-1} \frac{1}{\sqrt{x^2+1}} = t$

$\Rightarrow \tan t = \frac{1}{\sqrt{x^2+1}}$

$\Rightarrow \cos t = \sqrt{\frac{x^2+1}{x^2+2}}$

Q.3 For which value of x,

$\sin(\cot^{-1}(x + 1)) = \cos(\tan^{-1} x)$ **[IIT Scr. 04]**

(A) 1/2 (B) 0

(C) 1 (D) – 1/2

Sol. **[D]**

$\sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right)$

$= \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right)$

$\Rightarrow \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{x^2+1}}$

solving we get $x = -\frac{1}{2}$

Q.4 If $0 < x < 1$, then

$\sqrt{1+x^2}\ [\{x \cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2}$

is equal to **[IIT- 2008]**

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Sol.[C] $\sqrt{1+x^2}\ [\{x \cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$

$= \sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{1/2}$

$= \sqrt{1+x^2}\,[x^2+1-1]^{1/2}$

$= x\sqrt{1+x^2}$

Q.5 Let (x, y) be such that

$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$

Match the statement in Column I with statements in Column II. **[IIT- 2007]**

Column I

(A) If a = 1 and b = 0, then (x, y)

(B) If a = 1 and b = 1, then (x, y)

(C) If a = 1 and b = 2, then (x, y)

(D) If a = 2 and b = 2, then (x, y)

Column II

(P) lies on the circle $x^2 + y^2 = 1$

(Q) lies on $(x^2 - 1)(y^2 - 1) = 0$

(R) lies on y = x

(S) lies on $(4x^2 - 1)(y^2 - 1) = 0$

Sol. **A → P ; B → Q ; C → Q ; D → S**

(A) a = 1 and b = 0, we have

$\sin^{-1}x + \cos^{-1}y = 0$

$\Rightarrow \cos^{-1}y = -\sin^{-1}x \Rightarrow \sqrt{1-y^2} = -x$

$\Rightarrow x^2 + y^2 = 1$

(B) a = 1 & b = 1, we have

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}xy = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}y + \cos^{-1}xy = \cos^{-1}x$

$\Rightarrow xy^2 - \sqrt{1-y^2}\,\sqrt{1-x^2y^2} = x$

$\Rightarrow (1-y^2)(1-x^2y^2) = x^2(1-y^2)^2$

$\Rightarrow (1-y^2)(1-x^2y^2-x^2+x^2y^2) = 0$

$\Rightarrow (1-y^2)(1-x^2) = 0 \Rightarrow (x^2-1)(y^2-1) = 0$

(C) a = 1; b = 2 we have

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}2xy = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}y + \cos^{-1}2xy = \cos^{-1}x$

$\Rightarrow 2xy^2 - \sqrt{1-y^2}\,\sqrt{1-4x^2y^2} = x$

$\Rightarrow (1-y^2)(1-4x^2y^2) = x^2(1-2y^2)^2$

$\Rightarrow 1 - 4x^2y^2 - y^2 + 4x^2y^4 = x^2 - 4x^2y^2 + 4x^2y^4$

$\Rightarrow x^2 + y^2 = 1$

(D) a = 2 and b = 2, we have

$\cos^{-1}y + \cos^{-1}2xy = \cos^{-1}2x$

$\Rightarrow 2xy^2 - \sqrt{1-y^2}\,\sqrt{1-4x^2y^2} = 2x$

$\Rightarrow (1-y^2)(1-4x^2y^2) = 4x^2(1-y^2)^2$

$\Rightarrow (1-y^2)(1-4x^2y^2-4x^2+4x^2y^2) = 0$

$\Rightarrow (1-y^2)(1-4x^2) = 0$

$\Rightarrow (4x^2-1)(y^2-1) = 0$

Q.6 Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d(f(\theta))}{d(\tan\theta)}$ is **[IIT-2011]**

Sol. $\because \tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = \sin^{-1}\tan\theta$

so $f(\theta) = \sin(\sin^{-1}\tan\theta) = \tan\theta$

$\because \frac{d(f(\theta))}{d(\tan\theta)} = \frac{d(\tan\theta)}{d(\tan\theta)} = 1$

EXERCISE # 5

Q.1 Find the following

$\tan^{-1}\left[\dfrac{3\sin 2\alpha}{5+3\cos 2\alpha}\right] + \tan^{-1}\left[\dfrac{\tan\alpha}{4}\right]$

where $-\pi/2 < \alpha < \pi/2$

Sol. Let $\tan \alpha = t$

Then $\sin 2\alpha = \dfrac{2t}{1+t^2}$ and $\cos \alpha = \dfrac{1-t^2}{1+t^2}$

Put the values and solving, we get

$\tan^{-1}\dfrac{3t}{4+t^2} + \tan^{-1}\dfrac{t}{4} \Rightarrow \tan^{-1}\dfrac{\frac{3t}{4+t^2}+\frac{t}{4}}{1-\frac{3t}{4+t^2}.\frac{t}{4}}$

$\Rightarrow \tan^{-1}(t) = \tan^{-1}\tan\alpha = \alpha$

Q.2 Prove that,

$2\cot^{-1}5 + \cot^{-1}7 + 2\cot^{-1}8 = \dfrac{\pi}{4}$

Sol. Taking L.H.S. we have

$2\left(\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{8}\right) + \tan^{-1}\dfrac{1}{7}$

$\Rightarrow 2\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{7}$

$\Rightarrow \tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{1}{7}$

$\Rightarrow \tan^{-1}\dfrac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{28}} = \tan^{-1}\dfrac{25}{25}$

$\Rightarrow \tan^{-1}1 = \dfrac{\pi}{4}$

Q.3 If $\dfrac{m\tan(\alpha-\theta)}{\cos^2\theta} = \dfrac{n\tan\theta}{\cos^2(\alpha-\theta)}$, then prove that

$\theta = \dfrac{1}{2}\left[\alpha - \tan^{-1}\left(\dfrac{n-m}{n+m}\tan\alpha\right)\right]$

Sol. From the given relation

$\dfrac{m}{n} = \dfrac{\cos^2\theta\tan\theta}{\tan(\alpha-\theta)\cos^2(\alpha-\theta)}$

$= \dfrac{2\cos\theta\sin\theta}{2\sin(\alpha-\beta)\cos(\alpha-\beta)} = \dfrac{\sin 2\theta}{\sin(2\alpha-2\theta)}$

using componendo and dividendo, we get

$\dfrac{n+m}{n-m} = \dfrac{\sin(2\alpha-2\theta)+\sin 2\theta}{\sin(2\alpha-2\theta)-\sin 2\theta}$

$= \dfrac{2\sin\alpha\cos(\alpha-2\theta)}{2\cos\alpha\sin(\alpha-2\theta)}$

$\Rightarrow \tan(\alpha-2\theta) = \dfrac{n-m}{n+m}\tan\alpha$

$\Rightarrow \theta = \dfrac{1}{2}\left[\alpha - \tan^{-1}\left(\dfrac{n-m}{n+m}\tan\alpha\right)\right]$

Q.4 Find the sum of the series

$\cot^{-1}\left(2^2+\dfrac{1}{2}\right) + \cot^{-1}\left(2^3+\dfrac{1}{2^2}\right)$

$+ \cot^{-1}\left(2^4+\dfrac{1}{2^3}\right) + \ldots\ldots\ldots\ldots\ldots \infty$

Sol. Given series is

$\sum_{n=1}^{\infty}\cot^{-1}\left(2^{n+1}+\dfrac{1}{2^n}\right)$

$= \sum_{n=1}^{\infty}\tan^{-1}\dfrac{2^n}{1+2^n.2^{n+1}}$

$= \sum_{n=1}^{\infty}\tan^{-1}\dfrac{2^{n+1}-2^n}{1+2^{n+1}2^n}$

$= \sum_{n=1}^{\infty}(\tan^{-1}(2^{n+1})-\tan^{-1}2^n)$

$= \tan^{-1}2^2 - \tan^{-1}2 + \tan^{-1}2^3 - \tan^{-1}2^2 + \ldots.\tan^{-1}\infty$

$= -\tan^{-1}2 + \dfrac{\pi}{2} = \cot^{-1}2 = \tan^{-1}\dfrac{1}{2}$

Q.5 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that

$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

Sol. $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$

$\Rightarrow \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) = \sin^{-1}z$

$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$

squaring, we get

$x^2(1-y^2) + y^2(1-x^2) + 2xy\sqrt{1-x^2}\sqrt{1-y^2} = z^2$

$\Rightarrow 2xy\sqrt{1-x^2} + y\sqrt{1-y^2} = z^2 - x^2 - y^2 + 2x^2y^2$

Again squaring and solving we obtain

$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

Q.6 Solve for x,

$$\sin^{-1}(6x) + \sin^{-1}(6\sqrt{3}\,x) = -\frac{\pi}{2}$$

Sol. $\frac{\pi}{2} + \sin^{-1}(6\sqrt{3}\,x) = -\sin^{-1} 6x$

taking sine, we obtain

$\cos(\sin^{-1} 6\sqrt{3}\,x) = -6x$

$\Rightarrow \sqrt{1-108x^2} = -6x$

squaring, we get

$1 - 108\,x^2 = 36\,x^2$

$\Rightarrow x^2 = \frac{1}{144} \Rightarrow x = \frac{1}{12}$ or $-\frac{1}{12}$

But $x = \frac{1}{12}$ does not satisfy the given equation

So $x = -\frac{1}{12}$ is the solution.

Q.7 Prove that, $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$

Sol. Taking L.H.S.

$$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \cos\left(\tan^{-1}\frac{\frac{2}{7}}{1-\frac{1}{49}}\right)$$

$$= \cos\left(\tan^{-1}\frac{7}{24}\right)$$

$= \cos\cos^{-1}\frac{24}{25} = \frac{24}{25}$ ... (i)

from R.H.S. we obtain

$$\sin\left(4\tan^{-1}\frac{1}{3}\right) = \sin\left(2\tan^{-1}\frac{1}{3}\right)$$

$$= \sin\left[2\left(\tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}\right)\right] = \sin\left(2\tan^{-1}\frac{3}{4}\right)$$

$$= \sin\left(\tan^{-1}\frac{\frac{6}{4}}{1-\frac{9}{16}}\right) = \sin\left(\tan^{-1}\frac{24}{7}\right)$$

$= \sin\left(\sin^{-1}\frac{24}{25}\right) = \frac{24}{25}$ ... (ii)

From (i) and (ii), we get

L.H.S. = R.H.S.

Q.8 Prove that,

$$2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan(x/2)\right] = \cos^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right]$$

Sol. Let $\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan(x/2)\right] = \theta$

$\Rightarrow \tan\theta = \sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}$

$\because \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

Put the value of tan θ and solving we get

$\cos 2\theta = \frac{b+a\cos x}{a+b\cos x}$

$\Rightarrow 2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right] = \cos^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right]$

Hence proved.

Q.9 The greater of the two angles

$A = 2\tan^{-1}(2\sqrt{2} - 1)$ and

$B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$ is........ **[IIT 1989]**

Sol. $A = 2\tan^{-1}(2\sqrt{2} - 1)$

and $B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

Here, $A = 2\tan^{-1}(2\sqrt{2} - 1)$

$= 2\tan^{-1}(2 \times 1.414 - 1) = 2\tan^{-1}(1.828)$

$\therefore A > 2\tan^{-1}(\sqrt{3}) = 2.\frac{\pi}{3} = \frac{2\pi}{3}$

To find the value of B, we first say

$\sin^{-1}\frac{1}{3} < \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ so that $0 < 3\sin^{-1}\frac{1}{3} < \frac{\pi}{2}$

Now, $3\sin^{-1}\frac{1}{3} = \sin^{-1}\left(3.\frac{1}{3} - 4.\frac{1}{27}\right) = \sin^{-1}\left(\frac{23}{27}\right)$

$= \sin^{-1}(0.851) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(0.6) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\therefore B < \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Thus $A > \frac{2\pi}{3}$ and $B < \frac{2\pi}{3}$

Hence, A > B

Q.10 Solve for θ,

$\theta = \tan^{-1}(2\tan^2\theta) - \frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)$

[REE 97]

Sol. Let tan θ = t

$\Rightarrow \sin 2\theta = \frac{2t^2}{1+t^2}$ and $\cos 2\theta = \frac{1-t^2}{1+t^2}$

Now,

$\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)$

$= \frac{1}{2}\sin^{-1}\frac{3.2t}{5(1+t^2)+4(1-t^2)}$

$= \frac{1}{2}\sin^{-1}\frac{6t}{9+t^2} = \frac{1}{2}\sin^{-1}\frac{2.t/3}{1+\left(\frac{t}{3}\right)^2}$

$= \frac{1}{2} . 2\tan^{-1}\frac{t}{3} = \tan^{-1}\frac{t}{3}$

Now $\theta = \tan^{-1} 2t^2 - \tan^{-1}\frac{t}{3}$

$= \tan^{-1}\frac{2t^2 - \frac{t}{3}}{1+2t^2.\frac{t}{3}}$

$\Rightarrow \tan\theta = \frac{t(6t-1)}{3+2t^3}$

$\Rightarrow t(3 + 2t^3) = t(6t-1)$

$\Rightarrow t(2t^3 - 6t + 4) = 0$

$\Rightarrow t = 0$ or $2t^3 - 6t + 4 = 0$

$\Rightarrow t^3 - 3t + 2 = 0$

$\Rightarrow (t-1)^2 (t+2) = 0$

$\Rightarrow t = 1, -2$ or $t = 0$

$\therefore \theta = \pi, \frac{\pi}{4}, \pi - \tan^{-1} 2$

Q.11 Using the principal values, express the following as a single angle :

$3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \sin^{-1}\frac{142}{65\sqrt{5}}$

[REE 99]

Sol. $\therefore 3\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{3.\frac{1}{2}-\frac{1}{8}}{1-3.\frac{1}{4}} = \tan^{-1}\frac{11}{2}$

and $2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}} = \tan^{-1}\frac{5}{12}$

and $\sin^{-1}\frac{142}{65\sqrt{5}} = \tan^{-1}\frac{142}{31}$

But $\frac{11}{2} . \frac{5}{12} > 1$

$\Rightarrow \pi + \tan^{-1}\frac{\frac{11}{2}+\frac{5}{12}}{1-\frac{55}{24}} + \tan^{-1}\frac{142}{31}$

$\Rightarrow \pi + \tan^{-1}\left(\frac{142}{-31}\right) + \tan^{-1}\frac{142}{31}$

$\Rightarrow \pi - \tan^{-1}\frac{142}{31} + \tan^{-1}\frac{142}{31} = \pi$

Q.12 Solve for x,

$\sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c} = \sin^{-1}x,$

where $a^2 + b^2 = c^2$, $c \neq 0$ **[REE 2000]**

Sol. $\sin^{-1}\left(\frac{ax}{c}\sqrt{1-\frac{b^2x^2}{c^2}} + \frac{bx}{c}\sqrt{1-\frac{a^2x^2}{c^2}}\right) = \sin^{-1}x$

$\Rightarrow \frac{ax}{c}\sqrt{1-\frac{b^2x^2}{c^2}} + \frac{bx}{c}\sqrt{1-\frac{a^2x^2}{c^2}} = x$

Squaring we get

$a^2\left(1-\frac{b^2x^2}{c^2}\right)+b^2\left(1-\frac{b^2x^2}{c^2}\right)$

$+ 2ab\sqrt{1-\frac{b^2x^2}{c^2}}\sqrt{1-\frac{a^2x^2}{c^2}} = c^2$

$\because a^2 + b^2 = c^2$

$\Rightarrow \sqrt{1-\frac{b^2x^2}{c^2}}\sqrt{1-\frac{a^2x^2}{c^2}} = \frac{abx^2}{c^2}$

$\Rightarrow (c^2 - b^2x^2)(c^2 - a^2x^2) = a^2b^2x^4$

$\Rightarrow c^4 - b^2c^2x^2 - a^2c^2x^2 = 0$

$\Rightarrow x^2 = \frac{c^2}{a^2+b^2} = 1$

$\Rightarrow x = \pm 1$

Q.13 Solve for x, $\cos^{-1} x\sqrt{3} + \cos^{-1}x = \frac{\pi}{2}$

Sol. $\cos^{-1} x\sqrt{3} = \frac{\pi}{2} - \cos^{-1} x$

Taking cos both side

$\Rightarrow x\sqrt{3} = \sin(\cos^{-1}x)$

$\Rightarrow x\sqrt{3} = \sqrt{1-x^2}$

$\Rightarrow 3x^2 = 1 - x^2 \quad \Rightarrow x = \pm\frac{1}{2}$

But $-\frac{1}{2}$ does not satisfy the equation

so $x = \frac{1}{2}$ is the solution.

Q.14 Prove that,

$\cos^{-1}\sqrt{\frac{a-x}{a-b}} = \sin^{-1}\sqrt{\frac{x-b}{a-b}}$

$= \cot^{-1}\sqrt{\frac{a-x}{x-b}} = \frac{1}{2}\sin^{-1}\left[\frac{2\sqrt{(a-x)(x-b)}}{a-b}\right]$

Sol. Let $\cos^{-1}\sqrt{\frac{a-x}{a-b}} = \theta \Rightarrow \cos\theta = \sqrt{\frac{a-x}{a-b}}$

$\because \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{a-x}{a-b}} = \sqrt{\frac{x-b}{a-b}}$

$\Rightarrow \theta = \sin^{-1}\sqrt{\frac{x-b}{a-b}}$

$\Rightarrow \cos^{-1}\sqrt{\frac{a-x}{a-b}} = \sin^{-1}\sqrt{\frac{x-b}{a-b}}$

$\because \cot\theta = \frac{\cos\theta}{\sin\theta} = \sqrt{\frac{a-x}{x-b}}$

$\Rightarrow \theta = \cot^{-1}\sqrt{\frac{a-x}{x-b}}$

$\Rightarrow \cos^{-1}\sqrt{\frac{a-x}{a-b}} = \cot^{-1}\sqrt{\frac{a-x}{x-b}}$

Again

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\sqrt{\frac{x-b}{a-b}}\cdot\sqrt{\frac{a-x}{a-b}}$

$\Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{2\sqrt{(x-b)(a-x)}}{a-b}\right)$

$\Rightarrow \cos^{-1}\sqrt{\frac{a-x}{a-b}} = \frac{1}{2}\sin^{-1}\left(\frac{2\sqrt{(a-x)(x-b)}}{a-b}\right)$

Q.15 If $X = \text{cosec}(\tan^{-1}(\cos(\cot^{-1}(\sec(\sin^{-1}a)))))$ and $Y = \sec(\cot^{-1}(\sin(\tan^{-1}(\text{cosec}(\cos^{-1}a)))))$ where $0 \le a \le 1$. Find the relation between X and Y. Express them in terms of 'a'.

Sol. $X = \text{cosec.}\tan^{-1}.\cos.\cot^{-1}.\sec.\sin^{-1}a$ ..(i)

$Y = \sec\cot^{-1}.\sin\tan^{-1}.\text{cosec}.\cos^{-1}a$ ..(ii)

From (i)

$X = \text{cosec}\tan^{-1}\cos\cot^{-1}\sec\sec^{-1}\frac{1}{\sqrt{1-a^2}}$

$= \text{cosec}\tan^{-1}\cos\cot^{-1}\frac{1}{\sqrt{1-a^2}}$

$= \text{cosec}\tan^{-1}\cos\cos^{-1}\frac{1}{\sqrt{2-a^2}}$

$= \text{cosec}\tan^{-1}\frac{1}{\sqrt{2-a^2}} = \text{cosec}\,\text{cosec}^{-1}\sqrt{3-a^2}$

$X = \sqrt{3-a^2}$

Solve (ii) same as above

we get $Y = \sqrt{3-a^2} \Rightarrow X = Y = \sqrt{3-a^2}$

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	B	D	C	B	A	B	D	D	B	A	C	B	D
Q.No.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	C	C	D	B										

21. True **22.** True **23.** False **24.** $-\pi, \pi$ **25.** $\sqrt{ab}$

26. $a, a^2 - a + 1$ **27.** $0, \pm 1$

EXERCISE # 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	D	D	A	B	C	A	B	A	A	C	C	B	A
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**								
Ans.	B	A	C	B	D	A	C								

PART-B

Q.No.	23	24	25	26	27	28	29
Ans.	B,D	B,C	A,B,D	A,C	A,C	C,D	B,C

PART-C

Q.No.	30	31
Ans.	D	D

PART-D

32. $A \to S, B \to R, C \to P, D \to Q$ **33.** $A \to Q, B \to S, C \to P,Q\ D \to R,S$

EXERCISE # 3

1. No solution **2.** $x = \sqrt{3}$ or $x = -\left(2+\sqrt{3}\right)$ **3.** (1, 2); (2, 7) **4.** $\pi/4$ **8.** $\frac{1}{2\sqrt{2}}$

11. (A) **12.** (C) **13.** (B) **14.** (B) **15.** (A) **16.** (B)

EXERCISE # 4

Q.No.	1	2	3	4
Ans.	B	A	D	C

5. $\mathbf{A \to P,\ B \to Q, C \to P,\ D \to S}$ **6.** 1

EXERCISE # 5

1. α **4.** $\tan^{-1}(1/2)$ **6.** $-\frac{1}{12}$ **9.** $A > B$ **10.** $0, \pi/4, \pi - \tan^{-1} 2$

11. π **12.** $x = \pm 1$ **13.** $x = \frac{1}{2}$ **15.** $X = Y = \sqrt{(3-a^2)}$