HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. D **2.** D **3.** D **4.** D **5.** C

6. $I_1 \propto a_1^2, I_2 \propto a_2^2$

$I_{resultant} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

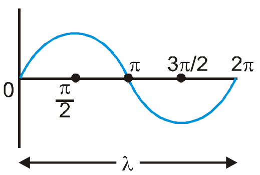
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, When $\cos\phi = 1, \phi = 0, 2\pi, ...$

7. A

8. C

9. $y_1 = a \sin\omega t = a \cos(\omega t - \pi/2)$,
$y_2 = a \cos \omega t$

10.



When path difference is λ then phase difference is 2π

When path difference is 1 then phase difference is $\frac{2\pi}{\lambda}$

When path difference is x then phase difference is $\frac{2\pi}{\lambda}x$

11. $y_1 = a \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = a \sin \omega t$

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$ where $\phi = \frac{\pi}{3}$

$= \sqrt{a^2 + a^2 + 2aa\cos\frac{\pi}{3}} = \sqrt{3}a$

12. In interference pattern we can see that resultant amplitude of super imposed wave depends on the phase difference of waves so it varies from maximum to minimum amplitude by redistributing of energy but total energy remains conserved.

13. C

14. Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

ϕ – Angle between two waves or phase difference. So A depends on both amplitude & phase difference.

15. Sustained interference means a interference pattern (arrangement of fringes) can not be change with time. It is only possible when the phase difference between waves at a point does not change with time.

16. From the definition of coherent source.

17. Given $\frac{I_1}{I_2} = 4$ $\therefore I_1 = 4I$ (let the $I_2 = I$)

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (2\sqrt{I} + \sqrt{I})^2 = 9I$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(2\sqrt{I} - \sqrt{I}\right)^2 = I$

$\therefore \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{9I - I}{9I + I} = \frac{8I}{10I} = \frac{4}{5}$

18. For minima $y_n = (2n-1)\frac{\lambda D}{2d}$ where n is the no. of minimum.

For maxima $y_n = \frac{n\lambda D}{d}$

$y_{5^{th}\,dark} = \frac{9\lambda D}{2d}$; $y_{1^{st}\,maxima} = \frac{\lambda D}{d}$

By Equation $y_{5^{th}\,min} - y_{1^{st}\,max} = 7 \times 10^{-3}$

$\frac{9\lambda D}{2d} - \frac{\lambda D}{d} = 7 \times 10^{-3}$

$\Rightarrow \lambda = \frac{7 \times 10^{-3} \times 15 \times 10^{-5} \times 2}{7 \times 50 \times 10^{-2}}$ $\therefore \lambda = 600$ nm

19. B

20. On a given screen width nβ= car tan, Here n is number of fringes and $\beta = \frac{\lambda d}{D}$ is the fringe width

$n_1\beta_1 = n_2\beta_2 \Rightarrow n_2\lambda_1 = n_2\lambda_2$

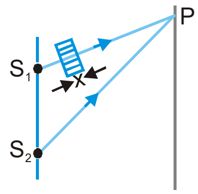
$\Rightarrow n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{92 \times 5898}{5461} \cong 99.360$

Number of fringes are integers so $n_2 = 99$

21. Fringe width $\beta = \frac{\lambda D}{d}$;

$\beta' = \frac{\lambda' D'}{d'} = \frac{\lambda \times 2D}{d/2}$, $\beta' = 4\frac{\lambda D}{d} = 4\beta$

22. Mica sheet of thickness 't'



Refractive index 'μ'

$\Delta X_P = S_2P - [(S_1P - t)_{air} + t_{sheet}]$
$= (S_2P - S_1P) - (\mu t - t)$
$= d\sin\theta - (\mu - 1)t$

Additional path difference

Position of n^{th} maxima

$$n\lambda = \frac{dy_n}{D} - (\mu - 1)t \Rightarrow y_n = \frac{nD\lambda}{d} + \frac{D}{d}(\mu - 1)t$$

Shift of interference pattern $= \frac{D}{d}(\mu - 1)t$

23. D

24. Central maxima i.e.

$$I_0 = (\sqrt{I_1} + \sqrt{I_2})^2 \Rightarrow I_0 = (2\sqrt{I})^2 = 4I$$

$$\therefore I = \frac{I_0}{4}$$

[Intensity due to single slit]

25. Central fringe is that fringe where the path difference $\Delta x = 0$, for all wavelengths. So the central fringe is white followed by the some coloured fringes and after that there is very much overlapping in fringes so equal illumination exists.

26. For maxima, $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

When $I_1 = I_2$ then $I_{max} = 4I$

For minima $I_{min} = \left(\sqrt{I_1} : \sqrt{I_2}\right)^2$ $I_{min} = 0$

$$I_2 = \frac{I_1}{2}, \text{ then } I'_{max} = \left(\sqrt{\frac{I_1}{2}} + \sqrt{I_1}\right)^2 < 4I_1$$

$$I'_{min} = \left(\sqrt{\frac{I_1}{2}} : I_1\right)^2 > I_{min}$$

27. Fringe width remains unchanged $\beta = \frac{\lambda d}{D}$

But central bright fringe (zero path difference)

Shift downwards therefore whole fringe pattern shifts downwards.

28. $\lambda = 200\text{nm}, d = 700\text{ nm}$

$$\text{No. of maxima} = \frac{2d}{\lambda} = \frac{2 \times 700\text{nm}}{200\text{nm}} = 7$$

29. A

30. When $\mu = 1$ i.e. no change in path difference due to optical path difference so the mid point of the screen is again the central bright fringe.

But when $\mu > 1$ the central bright fringe will shift according to $\frac{D}{d}(\mu - 1)t$

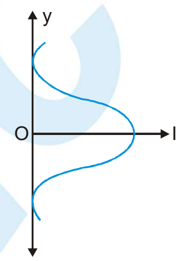
As μ increases, CBF will shift upwards from midpoint i.e. at mid point. Less bright fringe appears and when the shift is equal to value of fringe width, the dark fringe (zero intensity) will appear.

OR

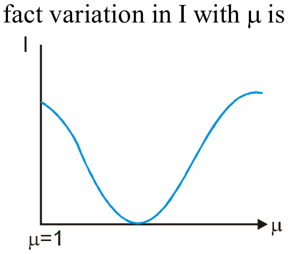
❖ Intensity variation

O ≡ central bright fringe

y ≡ distance from 'O' screen



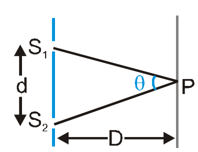
❖ When μ increases, the fringe patterns shift towards the slit when sheet introduced shift $\frac{D(\mu - 1)}{t}$ from the above two fact variation in I with μ is



31. C

32. There is a phase change of π when the ray enters from rarer medium into denser medium, the boundary is called rigid boundary and also no change in phase when it enters into rarer medium.

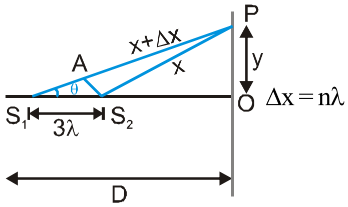
33. As distance between S_1 & S_2 is very much less and equal to 'd' so the angle made at 'P' is very much small



$$\therefore \text{Angle} = \frac{\text{Arc}}{D} \Rightarrow \theta = \frac{d}{D}$$

$$\therefore \text{Fringe width} = \frac{\lambda D}{d} = \frac{\lambda}{\theta} \quad \left(Q\ \frac{d}{D} = \theta\right)$$

34.



in this case path difference is d cos θ.

So $n\lambda = \lambda = 3\lambda\cos\theta \Rightarrow \cos\theta = \frac{1}{3}$

$\Rightarrow \frac{D}{\sqrt{D^2+y^2}} = \frac{1}{3} \Rightarrow 3D = \sqrt{D^2+y^2}$

$\Rightarrow y = \sqrt{8}D = 2\sqrt{2}D$

35. Path difference for points A & C is O so constructive interference take place.

Path difference for B & D = $5\mu m = \frac{5\lambda}{2}$

So distructive interference take place.

36. At the central Maxima $\Delta x = 0$

But upward shift $= \frac{(2\mu-1)tD}{d}$ and

Downward shift $= \frac{(\mu-1)2tD}{d}$

So net shift $y = \frac{tD}{d}[2\mu-1-2\mu+2] \Rightarrow y = \frac{tD}{d}$

37. Let t be the thickness so corresponding

$\Delta x = \mu t = \frac{3}{2}t$. Also $I_{max} = 4I$ So $I' = 2I$

We know $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\Rightarrow 2I = I + I + 2\sqrt{I^2}\cos\left(\frac{3\pi t}{\lambda}\right) \Rightarrow \cos\left(\frac{3\pi t}{\lambda}\right) = 0$

$\Rightarrow \cos\left(\frac{3\pi t}{\lambda}\right) = \cos\frac{\pi}{2}$ or $\cos\frac{3\pi}{2}$ or $\frac{5\pi}{2}$

$\Rightarrow \frac{3\pi t}{\lambda} = \frac{\pi}{2} \Rightarrow t = \frac{\lambda}{6}$; $\frac{3\pi t}{\lambda} = \frac{3\pi}{2} \Rightarrow t = \frac{\lambda}{2}$

and $\frac{3\pi t}{\lambda} = \frac{5\pi}{2} \Rightarrow t = \frac{5\lambda}{6}$

38. B

39. $I' = \frac{3}{4}(4I) = 3I$.

So $4I = I + I + 2I\cos\phi \Rightarrow \cos\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

But value should lie between 3π & 6π.

So it cannot be $\frac{\pi}{3}$

For second minima $\phi = 3\pi$

For third maxima $\phi = 6\pi$

40.	A	**41.**	D	**43.**	C	**44.**	C
45.	B	**46.**	B	**47.**	D	**48.**	D
49.	C	**50.**	B	**51.**	A	**52.**	D
53.	B	**54.**	B	**55.**	D	**56.**	A
57.	A	**58.**	C	**59.**	C	**60.**	A
61.	A	**62.**	B	**63.**	D	**64.**	C
65.	B	**66.**	D	**67.**	C	**68.**	D
69.	A	**70.**	B	**71.**	C	**72.**	A
73.	C	**74.**	D	**75.**	D	**76.**	A
77.	B	**78.**	B	**79.**	A	**80.**	C
81.	A	**82.**	C	**83.**	B	**84.**	C
85.	C	**86.**	A	**87.**	A		

88. We know $I \propto A^2$.

$\Rightarrow \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \Rightarrow \sqrt{\frac{4}{1}} = \frac{A_1}{A_2} \Rightarrow A_1 : A_2 = 2:1$

89. A

90. Contrast indicates the ratio of maximum possible intensity on screen to the minimum possible intensity.

As $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$

so it only depends on the source intensity.

91. C

92. As $\lambda << d$; we can we $\beta = \frac{\lambda D}{d}$

we get $\beta = \frac{500\times10^{-9}\times1}{10^{-3}} = 0.5$ mm.

As β is not very small; hence it might so happen that till 1000[th] maxima, we no longer can apply

$y' = 1000 \times \beta$.

Lets see if we can apply:

At 1000[th] maxima. Path difference is 1000 λ.

$\Rightarrow\ 1000\lambda = d\sin\theta = \dfrac{d\times y}{\sqrt{D^2+y^2}}$

$\Rightarrow\ (5\times10^{-4})^2 = \dfrac{(10^{-3}\,m)^2\times y^2}{D^2+y^2}$

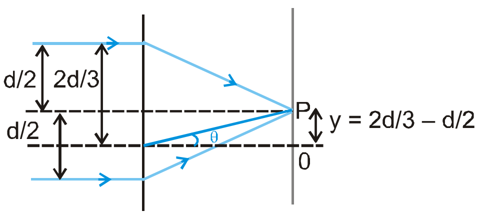
$\Rightarrow\ 0.25\,D^2 = y^2(1-0.25) \Rightarrow y = \left(\dfrac{0.25}{0.75}\right)^{\frac{1}{2}}\times D$

$y = \dfrac{D}{\sqrt{3}} = 0.577$ m.

As 0.577 m. and 0.5 m. are quite distant, so we could not use y' = 1000 β for such a high maxima.

93. D **94.** A

95.

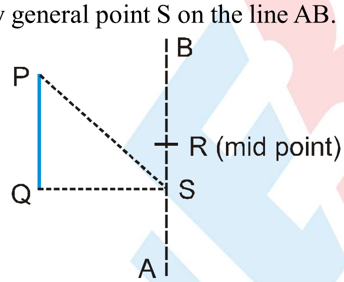


we know that P will be the central maxima (at which path difference is zero)

Now $OP = \dfrac{d}{2}-\dfrac{d}{3}=\dfrac{d}{6}$

96. C

97. Lets take any general point S on the line AB.



Clearly: for any position of S on line AB; we have for Δ PQS:

PQ + QS > PS {in any triangle sum of 2 sides is more then the third side}

$\Rightarrow$ PS – QS < 3λ.

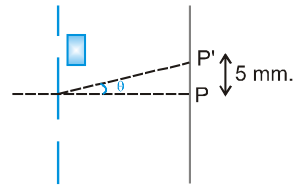
As PS – QS represents the path difference at any point on AB $\Rightarrow$ it can never be more than 3λ. Now minimas occur at.

$\dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}$ only.

so 3 minimas below R (mid point of AB)

and 3 also above R.

98.



Clearly the central maxima at P(initially) shifts to P' where PP' = 5 mm.

So now, path difference at P' must be zero.

$\Rightarrow\ d\sin\theta = (\mu-1)t \Rightarrow d\tan\theta = (\mu-1)t$

$\mu = 1 + \dfrac{d.(PP')}{Dt}$; get $\mu = 1.2$

99. D **100.** D

101. For strong reflection.

$2\mu t = \dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}\ ......\Rightarrow \lambda = 4\mu t, \dfrac{4\mu t}{3}, \dfrac{4\mu t}{5}, \dfrac{4\mu t}{7}\ ......$

$\Rightarrow$ 3000 nm. 1000 nm, 600 nm, 430 nm, 333 nm.

$\Rightarrow$ only option is 600 nm.

102. $\dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2-\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2+\left(\sqrt{I_2}-\sqrt{I_2}\right)^2}$

$= \dfrac{I_1}{I_1}\times\dfrac{\left(1+\sqrt{\dfrac{I_2}{I_1}}\right)^2-\left(1-\sqrt{\dfrac{I_2}{I_1}}\right)^2}{\left(1+\sqrt{\dfrac{I_2}{I_1}}\right)^2+\left(1-\sqrt{\dfrac{I_2}{I_1}}\right)^2}$

$= \dfrac{(1+2)^2-(1-2)^2}{(1+2)^2+(1-2)^2} = \dfrac{8}{10} = \dfrac{4}{5}$

103. Let us say, n^{th} minima of 400 nm coincides with m^{th} minima of 600 nm.

$\Rightarrow \left(n+\dfrac{1}{2}\right)\ 400\times\dfrac{D}{d} = \left(m+\dfrac{1}{2}\right).\ 600\times\dfrac{D}{d}$

$\Rightarrow$ 400 n = 600m + 100.

$\Rightarrow\ n = \dfrac{6m+1}{4}$

= (some integer or non-integer) + 0.25

Hence n can never be an integer. So no minima of 600 nm coincides with any minima of 400 nm.

104. C **105.** B

106. At B; DP = 4l (maxima) (mfPp"B)

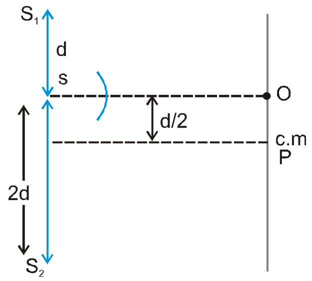
At $x = \infty$; $\Delta P = 0$ (maxima)

Hence in between; the point at which path difference is either λ, or 2λ or 3λ $\rightarrow$ they will be maximas. Hence 3 maximas.

107. A

108. The 2 sources are.

As O is a maxima,



Hence OP = β.

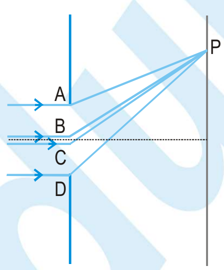
$$\Rightarrow \frac{d}{2} = \frac{\lambda.D}{(3d)};$$

$$\text{get } \lambda = \frac{3d^2}{2D}$$

109. C

110. B

111. $\left.\begin{matrix}\phi_A - \phi_B = \pi \\ \phi_C - \phi_D = \pi\end{matrix}\right\}$



for first dark fringe at P

also $\phi_C \sim \phi_B$

$\phi_A - \phi_D = 2\pi$

EXERCISE - 2

Part # I : Multiple Choice

1. A,C

2. $$\frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{9}{1}$$

by checking the options : $I_1 = 4$ unit.

$I_2 = 1$ unit.

and $\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = 2.$

3. If it is performed with white light. the central point will have maxima of all the colours, hence it will look white.

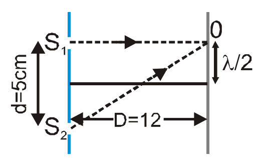
$\beta = \frac{\lambda D}{d}$; as λ_v is minimum

so first maxima after white as will be that of violet.

So there will be no dark fringe as it is not possible be have minimas of all the colors at the same point.

4. By Geometry, path difference at '0' for minima should be

$(2n-1)\frac{\lambda}{2}$



$\therefore S_2O - S_1O = (2n-1)\frac{\lambda}{2}$

$\Rightarrow \sqrt{D^2 + d^2} - D = (2n-1)\frac{\lambda}{2}$

$\Rightarrow (13-12)\text{cm} = (2n-1)\frac{\lambda}{2}$

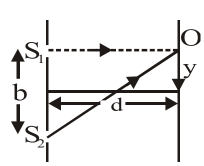
For n=1, 2,3 $\Rightarrow \lambda = 2\text{cm}, \frac{2}{3}$ cm,

5. A,C

6. Seperation between slits =b, screen distance d (>>b)

Path difference at 'O' must be odd multiple of $\frac{\lambda}{2}$ for

missing wavelengths $S_2O - S_1O = \frac{n\lambda}{2}$



$\Rightarrow \sqrt{d^2+b^2} - d = \frac{n\lambda}{2}$

$\Rightarrow d^2 + b^2 = d^2 + \frac{n^2\lambda^2}{4} + 2 \times \frac{n\lambda}{2} \times d$

$\Rightarrow \lambda = \frac{b^2}{nd}$ (n=1,3,5)

7. The intensity of light is $I(\theta) = I_0\cos^2\left(\frac{\delta}{2}\right)$

where $\delta = \frac{2\pi}{\lambda}(\Delta x) = \left(\frac{2\pi}{\lambda}\right)(d\sin\theta)$

(i) For $\theta = 30°$

$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{10^6} = 300$ m and $d = 150$ m

$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2} \quad \therefore \frac{\delta}{2} = \frac{\pi}{4}$

$\therefore I(\theta) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$

(ii) For $\theta = 90°$

$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi \Rightarrow \frac{\delta}{2} = \frac{\pi}{2}$ and $I(\theta) = 0$

8. As path difference $d\sin\theta = n\lambda \Rightarrow d = \frac{n\lambda}{\sin\theta}$

As $\sin\theta < 1$ so $d > n\lambda$, where n is an integer

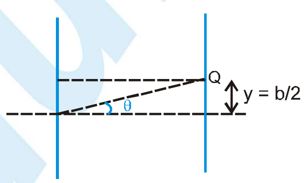
Therefore $d \neq \lambda$ and $d \neq \frac{\lambda}{2}$

9. A,C,D

10. Clearly at Q, path difference = d sinθ

$\Rightarrow b\sin\theta \approx b\tan\theta \approx \frac{b.y}{d} = \frac{b^2}{2d}$

Now whenever $\frac{b^2}{2d}$ will be odd multiple of $\frac{\lambda}{2}$, those λ's will be having minima at point Q.



$\Rightarrow \frac{b^2}{2d} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ ....

$\Rightarrow \lambda = \frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}$ .....

Part # II : Assertion & Reason

1.	E	**2.**	A	**3.**	B	**4.**	A	**5.**	B
6.	D	**7.**	A	**8.**	A	**9.**	D	**10.**	C

EXERCISE - 3

Part # I : Matrix Match Type

1. Initially at a distance x from central maxima on screen is

$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi x}{\beta}$,

where $\beta = \frac{D\lambda}{d}$ $I_{max} = 9I_0$ and $I_{min} = I_0$

(A) At points where intensity is $\frac{1}{9}$ th of maximum intensity, minima is formed

$\therefore$ Distance between such points is β, 2β, 3β, 4β, ...

(B) At points where intensity is $\frac{3}{9}$ th of maximum intensity, $\cos\frac{2\pi x}{\beta} = -\frac{1}{2}$ or $x = \frac{\beta}{3}$.

$\therefore$ Distance between such points is

$\frac{\beta}{3}, \frac{2\beta}{3}, \beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta$, ...

(C) $\cos\frac{2\pi x}{\beta} = 0$ or $x = \frac{\beta}{4}$.

$\therefore$ Distance between such points is

$\frac{\beta}{2}, \beta, \beta + \frac{\beta}{2}, 2\beta$, ...

(D) $\cos\frac{2\pi x}{\beta} = \frac{1}{2}$ or $x = \frac{\beta}{6}$.

$\therefore$ Distance between such points is $\frac{\beta}{3}, \frac{2\beta}{3}$,

$\beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta$, ...

2. Additional path difference by introducing the thin sheet is given by (μ–1)t

Resultant path difference at P = Geometrical path difference + Optical path difference

3. A → R,S ; B → P,Q,S ; C → P,Q,S ; D → R,S

Part # II : Comprehension

Comprehension # 1

1. C **2.** D **3.** A

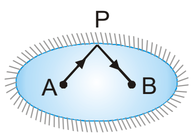
Comprehension # 2:

1. Optical path length

$$= \int \mu dx = \int_0^1 \left(1 + x^2\right) dx = \left(x + \frac{x^3}{3}\right)_0^1 = \frac{4}{3} m$$

2. Optical path length must be optimum i.e. minimum.

3.



For any point AP + PB = constant

= 2 (semi–major axis of ellipse)

Comprehension # 3 :

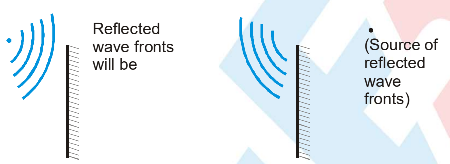
1. D **2.** C **3.** B

Comprehension # 4 :

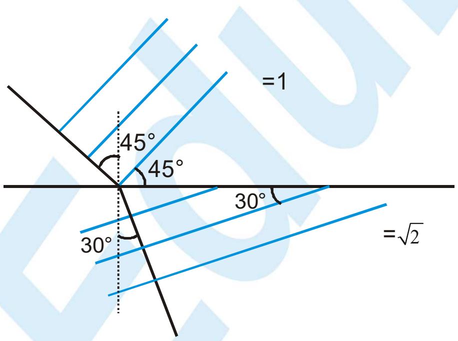
1. Wave front of point source is spherical. The point source is at origin and distance travelled by wave in 't' time with a speed of light 'c' is 'ct'. Hence radius of wave front is 'ct'.

$\therefore$ Equation of sphere is $x^2 + y^2 + z^2 = (ct)^2$

2.

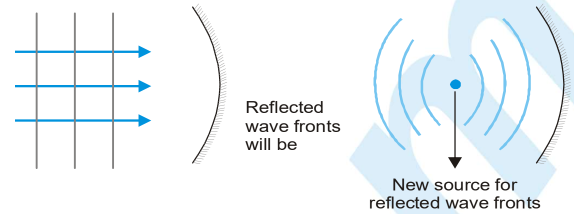


3.

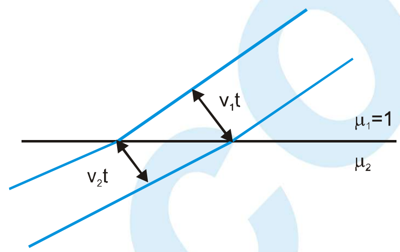


$\mu_1 \sin 45° = m_2 \sin\theta \Rightarrow \theta = 30°$

4.



5.



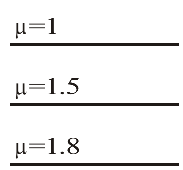
$$\frac{v_1 t}{v_2 t} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{2}{1} = \frac{\mu_2}{1} \Rightarrow \mu_2 = 2$$

6. Angle made by the direction of light with the y–axis is $\cos\beta = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{14}}$

Comprehension # 5 :

1. For strongly reflect light path difference

$$\left(2\mu t + \frac{\lambda}{2}\right) - \frac{\lambda}{2} = \lambda \text{ (for minimum thickness)}$$



$$2\mu t = \lambda \quad \therefore t = \frac{600 nm}{2 \times 1.5} = 200 nm$$

2. Again as previous question path difference for n,t reflect light must be the odd multiple of $\frac{\lambda}{2}$

$$\therefore 2\mu t = \frac{n\lambda}{2}$$

$$\therefore t = \frac{n \times 640}{2 \times 2 \times 1.33} nm = n \times 120 \text{ (n=1,3,5)}$$

Hence = 3 × 120 = 360 nm

3. Path difference $\Rightarrow 2\mu t - \frac{\lambda}{2} = 0$

$$2\mu t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2 \times 2 \times \mu} = \frac{\lambda}{4\mu}$$

4. $t = 350\text{nm}, n = 1.35$

$$2nt - \frac{\lambda}{2} = \frac{m\lambda}{2} \quad (m = 1,3,5,...)$$

$$t = \frac{(m+1)\frac{\lambda}{2}}{2 \times 1.35} = 350\text{nm}$$

$$\lambda = \frac{350 \times 2.7 \times 2}{(m+1)} = \frac{945 \times 2}{m+1}$$

For $m = 1,3,5, \lambda = 945, 473...$

5. $t = 1\ \mu m, n = 1.35, \lambda = 600$ nm

$$\text{Path difference} = \left|2 \times 1.35 \times 10^{-6} - \frac{\lambda}{2}\right|$$

$2.7 \times 10^{-6} - 300 \times 10^{-9} = 2.4\ \mu m$

Comprehension # 6 :

1. A,C,D

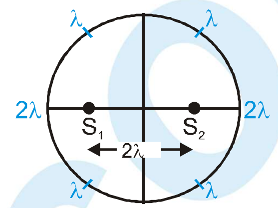
2. A,B,D

3. B,D

EXERCISE - 4

Subjective Type

1. The position of maxima where the path difference between two ways is integral multiple of λ.
And positions of minima where the path difference between two ways is odd multipleof $\lambda/2$.



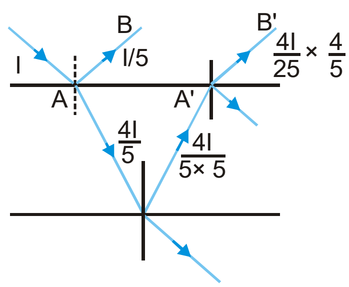
2. **(i)** $I_{result} = I + 4I + 2 \times \sqrt{I} \times \sqrt{4I} \cos\frac{\pi}{4}$

$$= 5I + 2\sqrt{2}\ I = 7.8I$$

(ii) $I_{result} = I + 4I + 2\sqrt{I} \times \sqrt{4I} \times \cos\pi = 5I - 4I = I$

(iii) $I_{result} = I + 4I + 2\sqrt{I} \times \sqrt{4I}\ \cos 4\pi = 5I + 4I = 9I$

3. As given reflection coefficient = 20% $\therefore$ AB = $\frac{I}{5}$



$$A'B' = \frac{16I}{125}$$

If AB and A'B' interference than

$$I_{max} = \left(\sqrt{\frac{I}{5}} + \sqrt{\frac{16I}{125}}\right)^2 = \frac{I}{5} \times \frac{81}{25}$$

$$I_{min} = \left(\sqrt{\frac{I}{5}} - \sqrt{\frac{16I}{125}}\right)^2 = \frac{I}{5} \times \frac{1}{25}$$

$\therefore I_{max}/I_{min} = 81 : 1$

4. Shifting of fringe pattern due to plate is given by

$\frac{D}{d}(\mu-1)t$ [Towards the side of plate]

Due to two plates introducing infront of slits then shifting is resultant of both

$$\frac{D}{d}[(1.7-1)2t-(1.4-1)t] = \frac{5\lambda D}{d}$$

(Position of 5th bright fringe)

(t – thickness of one plate)

$t = 5\lambda = 5 \times 4800$ Å $= 2.4$ μm

∴ Thickness of second $= 2t = 4.8$ μm

5. $d = 0.2$ cm, $\lambda = 5896$ Å, $D = 1$m

$$\text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{5896 \times 10^{-10} \times 1}{0.2 \times 10^{-2}} = 0.3 \text{ mm}$$

If system is immersed in water (μ=1.33), then the fringe width becomes

$$\beta' = \frac{\beta}{\mu} = \frac{0.3}{1.3}\text{mm} = 0.225 \text{ mm}$$

6. Due to the introduction of sheet in front of one slit (thickness t and refrective index μ) the shift $= \frac{D}{d}(\mu-1)t$

i.e. the path difference becomes $(\mu-1)t$ instead of zero at centre of screen $(\Delta x \neq 0)$

$$\therefore \text{ Phase difference} = \frac{2\pi}{\lambda} \times (\mu-1)t$$

∴ Resultant intensity

$$I_0 = \frac{I}{4} + \frac{I}{4} + 2\sqrt{\frac{I}{4}}\sqrt{\frac{I}{4}}\cos\left[\frac{2\pi}{\lambda}(\mu-1)t\right]$$

Intensity at centre I = 4I'

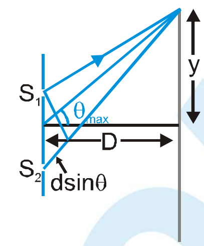
$$\Rightarrow I' = \frac{I}{4}$$

[I' – Intensity due to one slit]

$$I_0 = \frac{2I}{4}\left[1 + \cos\left[\frac{2\pi}{\lambda}(\mu-1)t\right]\right]$$

$$I_0 = I\cos^2\left[\frac{2\pi}{\lambda}\frac{(\mu-1t)}{2}\right] = I\cos^2\left(\frac{\pi(\mu-1)t}{\lambda}\right)$$

7. The length of the screen for the fringe pattern $= 2y$



$$d\sin\theta_{max} = \frac{y}{D} = 1;\ y = D$$

∴ No. of maxima

$$= \frac{2 \times D}{\text{fringe width}} = \frac{2D}{\lambda D/d}$$

$$= \frac{2d}{\lambda} = \frac{2 \times 5\text{cm}}{3\text{cm}} = 3.3 \text{ (say 3)}$$

8. Fringe width $\beta = \frac{\lambda D}{d}$; $\beta' = \frac{\lambda(D - 5 \times 10^{-2})}{d}$

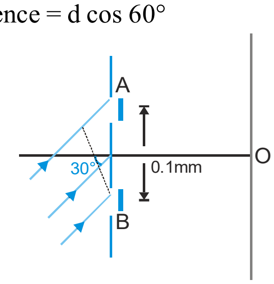
$$\text{Given } |\beta' - \beta| = \left|\frac{\lambda(D-5\times10^{-2})}{d} - \frac{\lambda D}{d}\right|$$

$$\therefore 3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{d}$$

$$\therefore \lambda = \frac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}} = 6000 \text{ Å}$$

9. 1.98×10^{-2} mm

10. Path difference = d cos 60°



As given at 0 the Intensity = 3I

$$3I = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\phi$$

$$\phi = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda}\left[\frac{d}{2} + (1.5-1)\times 20.4 \times 10^{-6} - \frac{1}{2}t \times 10^{-6}\right] = \frac{2\pi}{3}$$

$\Rightarrow 0.1 \times 10^{-3} + (20.4 - t)10^{-6}$
$= 2/3 \times 6000 \times 10^{-10} = 4 \times 10^{-7}$

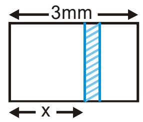
$\Rightarrow t = 20.4 - 0.4 + 100 = 120\ \mu m$

11. $I = 10^{-15}\ W/m^2,\ \lambda = 4000\sqrt{3}$ Å, t = 3mm.

The path difference due to glass plate 3mm

Path difference= $\int ds = \int_0^{3\,mm} (n-1)dx$

$\int_0^3 (1 + \sqrt{x} - 1)dx = \frac{2}{3}x^{3/2} = 2\sqrt{3}\,mm$



$\therefore$ Phase difference

$= \frac{2\pi}{4000\sqrt{3} \times 10^{-10}} \times 2\sqrt{3} \times 10^{-3} = \pi \times 10^{7-3}$

$\therefore 2\pi n = 10^4 \times \pi$

At point I there is point of maxima $n = 5 \times 10^3$

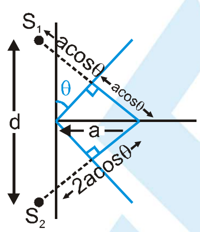
$\therefore$ Intensity $= I + I + 2\sqrt{I} \times \sqrt{I} \cos 10^4 \pi$
$= 4I = 4 \times 10^{-15}\ W/m^2$

12. Fringe pattern forms on a screen the distance of the nth maxima in x–direction i.e. x cordinates is $n\lambda D'$ and y–position is decided by the SHM of spring.

$$D' = D + \frac{Mg}{k}(1 - \cos\omega t)$$

13. Distance between two sources S_1 and S_2

$d = 2 \times 2a\cos\theta\sin\theta = 2a\sin 2\theta$



Screen distance $D = b + 2a\cos^2\theta$

$$\beta = \frac{\lambda D}{d} = \frac{\lambda(b + 2a\cos^2\theta)}{2a\sin 2\theta} = \frac{\lambda(b+2a)}{4a\theta}$$

(if θ is very much small)

14. Fringe width

$$\beta = \frac{\lambda D}{d} = \frac{(5000 \times 10^{-10})(80 \times 10^{-2})}{\frac{4}{3} \times 2 \times 10^{-3}} m = 150\ \mu m$$

Net upward shift

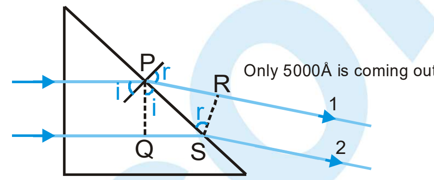
$$= \frac{D}{d}\left({}_w\mu_g - 1\right)t_1 - \frac{D}{d}\left({}_w\mu_y - 1\right)t_2 = 25\,\mu m$$

Phase difference at point C

$$\Delta\phi = 2\pi\left(\frac{25\,\mu m}{150\,\mu m}\right) = \frac{\pi}{3}$$

$$I_C = I_{max}\cos^2\frac{\Delta\phi}{2} = I_{max}\left(\frac{3}{4}\right) \Rightarrow \frac{I_C}{I_{max}} = \frac{3}{4}$$

15. Path difference between rays 1 and 2 :



$\Delta x = \mu(QS) - PR$ ...(i)

Further $\frac{QS}{PS} = \sin i$; $\frac{PR}{PS} = \sin r$

$\therefore \frac{PR/PS}{QS/PS} = \frac{\sin r}{\sin i} = \mu \therefore \mu(QS) = PR$

Substituting in equation (i), we get Δx=0

$\therefore$ Phase difference between rays 1 and 2 will be 0 or these two rays will interfere constructively.

So maximum intensity will be obtained from their interference or

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{4I} + \sqrt{I}\right)^2 = 9I$$

16. (i) For the lens, u = –0.15 m; f = +0.10 m

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)} \text{ or } v = 0.3\ m$$

Linear magnification , $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optic axis (m= –2). Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, d = distance between S_1 and S_2 = 1.5 mm

$\Delta = 1.30 - 0.30 = 1.0\ m = 10^3\ mm$

$\lambda = 500\ nm = 5 \times 10^{-4}\ mm$

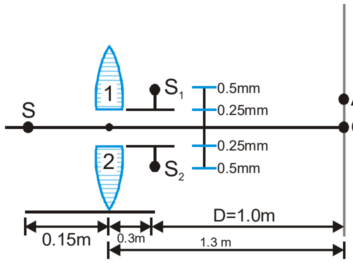
Therefore, fringe width,

$$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} mm = \frac{1}{3} mm$$

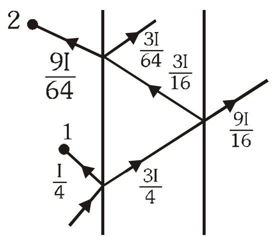
Now, as the point A is at the third maxima

OA = 3ω = 3(1/3) mm or OA = 1mm

(ii) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width ω will increase or the distance OA will increase.



17. Each plate reflects 25% and transmits 75°. Incident beam has an intensity I. This beam undergoes multiple reflections and refractions. The corresponding intensity after each reflection and refraction (transmission) are shown in figure.



Interference pattern is to take place between rays 1 and 2.
$I_1 = I/4$ and $I_2 = 9I/64$

$$\therefore \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{1}{49}$$

18. 2

19. We know that $I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\phi)$

(a) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos(0) = 9I.$

(b) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos\left(\frac{\pi}{2}\right) = 5I.$

(c) $I_{res} = I + 4I + 2\sqrt{I} \times 2\sqrt{I}\cos(180) = I.$

20. 1.625 mm

21. We know $\beta = \frac{\lambda D}{d}$

(a) if D is increased ⇒ β increases.
so the width of interference fringes increases.

(b) λ decreases ⇒ β decrease again.

(c) d is increased ⇒ β decreases.
If the source slit is moved nearer. no effects on λ, D or d, so β remains same.

(d) By slightly increasing the width of the slits ,we are only increasing the intensity of incident beam.Againno change in λ, D, d. so β unchanged but sharpness of the fringe increase.

22. $\frac{180}{\pi} \times 2 \times 10^{-4}$ degree $= 0.011^{\circ}$

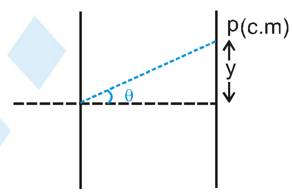
23. AS $\beta = \frac{\lambda D}{d} = 0.4$ mm

⇒ by changing μ; λ becomes $\frac{\lambda}{\mu}$

$\Rightarrow \beta' = \frac{3}{4} \times 0.4 = 0.3$ mm.

24. **(a)** By inserting the strips, β does not change.

$$\beta = \frac{\lambda D}{d} = \frac{480 \times 10^{-9} \times 1\,m}{0.12 \times 10^{-2}} = 4.0 \times 10^{-4}\,m.$$



(b) Lets try to find out the point at which path difference is zero. (central maxima).
So at P; path difference due to extra travelling (i.e. dsinθ) must be equal to the optical path difference.
at centre point path difference $= (\mu_2 - \mu_1)t = 83.33\lambda$
so as we move up path difference will decrease so at height $\frac{\beta}{3}$ we will get maxima and at a distance $\frac{2\beta}{3}$ on lower side maxima will formed.

25. As $\lambda << d$

$\Rightarrow \beta_1 = \frac{\lambda_1 D}{d} \qquad \beta_2 = \frac{\lambda_2 D}{d}$

So, $\beta_2 - \beta_1 = \frac{(\lambda_2 - \lambda_1)D}{d}$

$$= \frac{(700 - 580) \times 10^{-9} \times 1.5}{0.20 \times 10^{-3}} = 0.9 \text{ mm.}$$

26. $\frac{\lambda}{4(\mu - 1)}$

27. The path difference is $2\mu t$.

Now for destructive interface it can be

$$2\mu t = \frac{\lambda}{2} \text{ or } \frac{3\lambda}{2} \text{ or } \frac{5\lambda}{2} \text{ and so on ....}$$

$$\mu = \frac{\lambda}{4t}, \frac{3\lambda}{4t}, \frac{5\lambda}{4t} \ .....$$

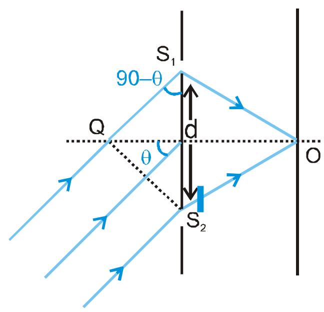
$$= \frac{580\times10^{-9}}{4\times0.3\times10^{-6}}, \frac{3\times580\times10^{-9}}{4\times0.3\times10^{-6}} \ ....$$

$= 0.4833, 3 \times 0.4833$.............

so only $\mu = 3 \times 0.4833 = 1.45$ is the answer,

$\{1.3 < \mu < 1.5\}$

28.



clearly, at O, no difference of path due to inside motion of rays.

only path difference = $S_1Q = d\cos(90–\theta) = d\sin\theta$.

$$= d \times \frac{\lambda}{2d} = \frac{\lambda}{2}$$

Net phase difference

So $$\Delta\phi = \frac{\lambda}{2} \times \frac{2\pi}{\lambda} \pm (\mu - 1)t.\frac{2\pi}{\lambda}$$

$$= \pi \pm (\mu - 1)\frac{\lambda}{2(\mu-1)}\frac{2\pi}{\lambda} = 2\pi, 0$$

$\Rightarrow$ constructive interference and hence

I_{res} = maximum

29. $\lambda = 6000\ \text{A}^o = 6000 \times 10^{-10}$m

$\theta_1 = 30^o, m = 1$

(i) For first maximum

$$\sin\theta_m = \frac{\left(m + \frac{1}{2}\right)\lambda}{a} \Rightarrow \sin\theta_1 = \frac{3\lambda}{2a}$$

or $$a = \frac{3\lambda}{2\sin\theta_1} = \frac{3\times6\times10^{-7}}{2\times\sin30^o}$$

(ii) For first minimum

$$\sin\theta_m = \frac{m\lambda}{a}$$

or $$\sin\theta_1 = \frac{\lambda}{a} \Rightarrow a = \frac{\lambda}{\sin\theta_1}$$

$$a = \frac{6\times10^{-7}}{\sin30^o}\text{A} = 1.2 \times 10^{-6}\text{m}$$

30. 100 nm

31. (a) Clearly 3^{rd} bright fringe will be at $y = 3\beta$.

$$= \frac{3\times\lambda D}{d} = \frac{3\times(6500\times10^{-10})\times1.2}{2\times10^{-3}} = 0.117 \text{ cm.}$$

$= 1.17$ mm.

(b) Say m^{th} bright fringe of 6500 Å coincides with n^{th} bright fringe of 5200 Å .

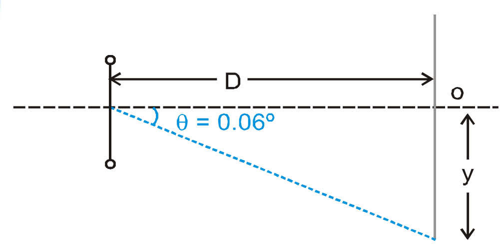
$\Rightarrow n\beta_1 = m\beta_2$

$$\Rightarrow \frac{n}{m} = \frac{5}{4}$$

Hence, for least distance, 5^{th} bright fringe of 5200 Å coincides with 4^{th} bright fringe of 6500 Å.

$$\Rightarrow y' = 4\beta_1 = \frac{4\times(6500\times10^{-10})\times1.2}{2\times10^{-3}} = 0.156 \text{ cm} = 1.56 \text{ mm}$$

32.



Say 'n' fringes are present in the region shown by 'y'

$$\Rightarrow y = n\beta = \frac{n.\lambda D}{d} \Rightarrow$$

$$\frac{y}{D} \approx \tan(0.06^o) \approx \frac{0.06\times\pi}{180} = \frac{n\lambda}{d}$$

$$\Rightarrow n = \frac{10^3\times\pi}{180}\times0.06 = \frac{\pi}{3} > 1.$$

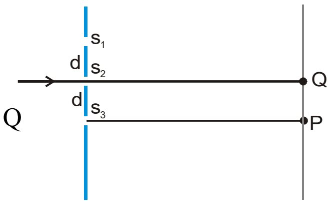
Hence; only one maxima above and below point O. So total 3 bright spots will be present (including point 'O' i.e. the central maxima).

33. **(a)** 428 nm, 600 nm, **(b)** 500 nm

34. Q is equidistant from s_1 and s_3

Resultant amplitude due to light coming from S_1 and S_3 is

$A_R = 2A$



Resultatnt amplitude at Q

$$A_R = \sqrt{3}A$$

$$\left(\sqrt{3}A\right)^2 = (2A)^2 + A^2 + 2\,2A \times A\cos\phi$$

$$3A^2 = 5A^2 + 4A^2\cos\phi$$

$$-\frac{1}{2} = \cos\phi\,,\ \phi = \frac{2\pi}{3}\,,\ \frac{4\pi}{3}$$

$$\Delta x = \sqrt{D^2 + d^2} - D$$

$$= D\left[1 + \frac{d^2}{2D^2} - 1\right] = \frac{d^2}{2D} \Rightarrow \quad \frac{d^2}{2D} \times \frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$$\text{for } \lambda_{max} \ \Rightarrow \ \lambda = \frac{3d^2}{2D}$$

Path difference between S_2 and S_3 at Q

$$\Delta p = \sqrt{D^2 + d^2} - D = \frac{d^2}{2D}$$

$$\text{Phase difference } \Delta\phi = \frac{2\pi}{r}, \Delta p = \frac{2\pi}{3d^2}\,.\,2D\,.\,\frac{d^2}{2D}$$

$$\Delta\phi = \frac{2\pi}{3},$$

Phase differance between S_1 and S_3

$$= \left(\sqrt{D^2 + 4d^2} - D\right)\frac{2\pi}{3d^2} \times 2D$$

$$D\left[1 + \frac{4d^2}{2D^2} - 1\right]\frac{2\pi \times 2D}{3d^2}$$

$$\frac{4d^2 \times \pi \times 2}{3d^2} = \frac{8\pi}{3} = 2\pi + \frac{2\pi}{3} = \frac{2\pi}{3}$$

$(A_R)_P = A\sqrt{3}$

$(I)_P = 3I$

35. **(a)** $\Delta\phi = \left(\frac{1}{1} + \frac{\mu}{D}\right)\frac{\pi d^2}{\lambda}$

(b) $\Delta\phi = \left(\frac{\mu}{1} + \frac{1}{D}\right)\frac{\pi d^2}{\lambda}$; $D_{min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$

36. Clearly : $\beta_{initial} = \frac{\lambda D}{d_i}$

$$\Rightarrow \quad 0.25 \times 10^{-3} = \frac{\lambda}{d} \times 1\text{ m}$$

$$\Rightarrow \quad \frac{\lambda}{d} = 2.5 \times 10^{-4} \qquad \text{.............(1)}$$

$$\text{Afterwards} \quad : \quad \frac{2}{3}\beta_i = \frac{\lambda D}{(d + \Delta d)}$$

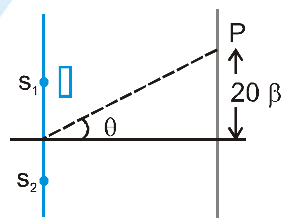
$$\Rightarrow \quad \frac{\lambda}{d + \Delta d} = \frac{2}{3} \times \frac{2.5 \times 10^{-4}}{1} \qquad \text{.............(2)}$$

Dividing (1) and (2)

$$\frac{d + \Delta d}{d} = \frac{3}{2} \quad \Rightarrow \quad d = 2\,(\Delta d) = 2.4\text{ mm.}$$

$$\&\ \lambda = 2.4 \times 2.5 \times 10^{-7}\text{ m} = 600\text{ nm.}$$

Now



Now P becomes central maxima.

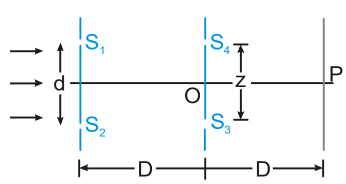
$\Rightarrow$ for point P : $d\sin\theta = (\mu - 1)\,t \Rightarrow d.\tan\theta \approx (\mu - 1)t$

$$\Rightarrow \quad d\,\frac{20\beta}{D} = (\mu - 1)t \ \Rightarrow \ \frac{d}{D} \times \frac{20 \times \lambda D}{d} = (1.5 - 1)t$$

$$\Rightarrow \quad t = \frac{20\lambda}{0.5} = \frac{20 \times 600 \times 10^{-9}}{0.5} = 24\ \mu\text{m.}$$

37. 0, 1.5 mm

38. Where $Z = \frac{D\lambda}{2d} = \frac{\beta}{2} \Rightarrow OS_4 = \frac{\beta}{4}$ as shown.



If intensity at 'p' is I then intensity of light at S_3 and S_4 is I/4 & I/4

✈ Path difference $S_4P - S_3P = 0$

So, intensity of slits S_1 and S_2

$\Delta\phi$ at S_4

$$\Delta p = \frac{yd}{D} = \frac{d}{D}\left(\frac{z}{2}\right) = \frac{\lambda}{4}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\frac{I}{4} = I_R = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\frac{\pi}{2}$$

$I_1 = I_2 = \frac{I}{8}$ intensity of S_1 and S_2.

(a) If $z = \frac{3D\lambda}{d} = 3\beta$

For intensity at S_3 and S_4

Path difference at S_3, $\Delta p = \frac{yd}{D}$

$$\Delta p = \frac{d}{D}\cdot\left(\frac{z}{2}\right) = \frac{d}{D}\cdot\left(\frac{3D\lambda}{d}\right)\times\frac{1}{2}$$

$$\Delta p = \frac{3\lambda}{2}\times\frac{2\pi}{\lambda} = 3\pi$$

$$I_3 = \frac{I}{8} + \frac{I}{8} + 2\sqrt{\frac{I}{8}\cdot\frac{I}{8}}\cos(3\pi)$$

$I_3 = I_4 = 0$

✈ Path difference between light coming from S_3 and S_4 at 'p' is zero.

∴ Intensity at P = 0.

(b) If $z = \frac{D\lambda}{3d} = \frac{\beta}{3}$

$$\Delta p = \frac{yd}{D} = \frac{d}{D}\cdot\frac{z}{2} = \frac{\lambda}{6}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{6} = \frac{\pi}{3}$$

$$I_3 = I_4 = \frac{I}{8} + \frac{I}{8} + \frac{2I}{8}\cos\frac{\pi}{3}$$

$$I_3 = I_4 = \frac{3I}{8}$$

Hence, intensity at 'p' is

$$I_p = \frac{3I}{8} + \frac{3I}{8} + 2\frac{3I}{8}\cos 0^\circ$$

$$I_p = \frac{3I}{2}$$

(c) If $z = 4.\frac{\lambda D}{d} = 4\beta$

$$\Delta p = \frac{yd}{D} = \frac{z}{2}\cdot\frac{d}{D} = \left(\frac{4\lambda D}{d}\right)\times\frac{1}{2}\times\frac{d}{D} = 2\lambda$$

$$\Delta\phi = \frac{2\pi}{\lambda}.2d = 4\pi$$

$$I_3 = I_4 = \frac{I}{8} + \frac{I}{8} + \frac{2I}{8}\cos 4\pi = \frac{I}{2}$$

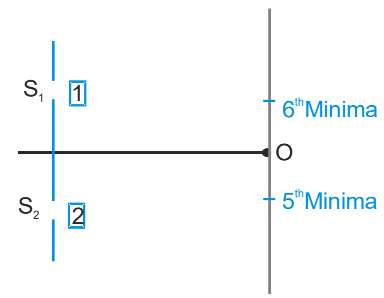
at 'p' $I_p = \frac{I}{2} + \frac{I}{2} + 2\sqrt{\frac{I}{2}\cdot\frac{I}{2}}\cos 0^\circ$

$I_p = 2I.$

39. **(a)** $t_B = 120\ \mu m$ **(b)** $\beta = 6$ mm ; $I_{max} = 9I$, $I_{min} = I$
(c) $\beta/6 = 1$ mm **(d)** I (at 5 cm above O),
I (at 5 cm below O) = 3 I

40. $\mu_1 = 1.4$, $\mu_2 = 1.7$ and let t be the thickness of each glass plates.

Path difference at O, due to insertion of glass plates will be -



$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t = 0.3t$ ....**(1)**

Now since 5^{th} maxima (earlier) lies below O and 6^{th} minima lies above O.

This path difference should lie between 5λ and $5\lambda + \lambda/2$

So let $\Delta x = 5\lambda + \Delta$ ....**(2)**

where $\Delta < \lambda\ /2$

Due to the path difference Δ x, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta) = 10\pi + \frac{2\pi}{\lambda}.\Delta \quad ....(3)$$

Intensity at O is given $\frac{3}{4}I_{max}$ and since

$$I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right) \quad \therefore \quad \frac{3}{4}I_{max} = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

or $\frac{3}{4} = \cos^2\left(\frac{\phi}{2}\right)$ ....(4)

From equations (3) and (4), we find that

$\Delta = \lambda/6$

i.e. $\Delta x = 5\lambda + \lambda/6 = (31\lambda/6)\lambda = 0.3\,t$

$$\therefore \quad t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400\times10^{-10})}{1.8}\ \text{m}$$

or $t = 9.3\times10^{-6}$ m $= 9.3$ µm **Ans. (14)**

41. **(a)** 80 cm behind the lens
(b) 4 mm (c) $\beta = 60$ µm

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Resolving power of an optical instrument is inversely proportional to λ i.e., R.P. $\propto \frac{1}{\lambda}$

$$\therefore \quad \frac{\text{Resolving power at }\lambda_1}{\text{Resolving power at }\lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = 5:4$$

4. For possible interference maxima on the screen, the condition is

$d\sin\theta = n\lambda$ ....(i)

Given : d = slit width $= 2\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$

$\Rightarrow \quad 2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence, $n = 2\times1 = 2$

Thus, equation (i) must be satisfied by 3 integer values i.e. $-1, 0, 1$. Hence, the maximum number of possible interference maxima is 3.

6. Since width is double hence intensity will double.

7. $I = I_0\cos^2\theta$

Intensity of polarized light

$= \frac{I_0}{2}$

$\therefore$ Intensity of untransmitted light

$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$

8. Phase difference $= \frac{2\pi}{\lambda}\times$ path difference

i.e $\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \frac{\pi}{3}$

As $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$$

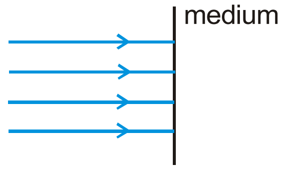
$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right)$$

$$\frac{I}{I_0} = \frac{3}{4}$$

Direction : Questions number 56 – 58 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

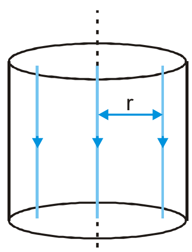
9.



Beam is incident normally so does not diverge or converge so it travel as a cylindrical beam.

10. Beam does not converge or diverge so shape of wave front remain planar.

11. $\mu = \mu_0 + \mu_2 (I)$



as it is given that intensity of beam is decreasing with increasing radius, and as I decreases μ also decreases.

Now by $V = \frac{C}{\mu}$

speed of light is minimum at axis of cylinder.

12. $\Delta x_1 = 0$

$\Delta\phi = 0^o$

$I_1 = I_0 + I_0 + 2I_0 \cos 0^o = 4I_0$

$\Delta x_2 = \frac{\lambda}{4}$

$\Delta\theta = \frac{2\pi}{\lambda}.\frac{\lambda}{4} = \left(\frac{\pi}{2}\right)$

$I_2 = I_0 + I_0 + 2I_0 \cos\frac{\pi}{2} = 2I_0$

$\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}.$

13. For coherent sources :

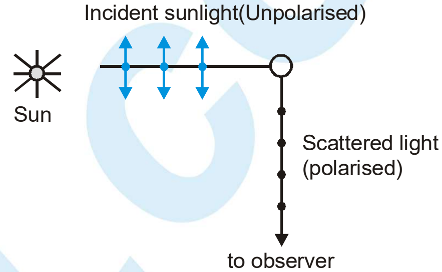
$I_1 = 4I_0$

For incoherent sources

$I_2 = 2I_0$

$\frac{I_0}{I_2} = \frac{2}{1}.$

14. The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated.



15. **S_1 :** When light reflects from denser med. (Glass) a phase dift of π is generated.

S_2 : Centre maxima or minima depends on thickness of the lens.

16. It will be concentric circles **Ans (4)**

17. **4**

Part # II : IIT-JEE ADVANCED

1. **(a)** Shape of the interference fringes will be circular.

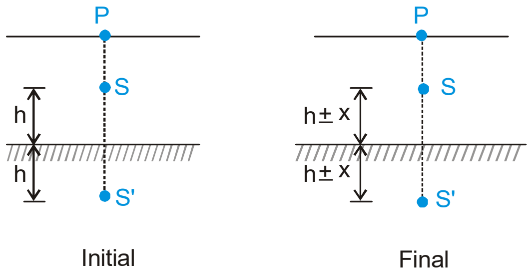
(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36\, I_0$

$\therefore \frac{I_1}{I_2} = \frac{I_0}{0.36 I_0} = \frac{1}{0.36}$ or $\sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$

$$\therefore \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$ **Ans.**

(c) Initially path difference at P between two waves reaching from S and S′ is

Therefore for maximum intensity at P :



$$2h=\left(n-\frac{1}{2}\right)\lambda \qquad \text{......(1)}$$

Now let the source S is displaced by x (away or towards mirror) then new path difference will be 2h + 2x or 2h – 2x. So for maximum intensity at P.

$$2h+2x=\left[n+1-\frac{1}{2}\right]\lambda \qquad \text{.....(2)}$$

or

$$2h-2x=\left[n-1-\frac{1}{2}\right]\lambda \qquad \text{......(3)}$$

Solving **(1)** and **(2)** or **(1)** and **(3)** we get

$$x=\frac{\lambda}{2}=\frac{600}{2}=300\text{ nm} \qquad \textbf{Ans.}$$

Note : Here we have taken the condition of maximum intensity at P as :

$$\text{Path difference } \Delta x=\left(n-\frac{1}{2}\right)\lambda$$

because the reflected beam suffers a phase difference of π.

2. Path difference due to slab should be integral multiple of λ or $\Delta x = n\lambda$

or $(\mu-1)\,t=n\lambda \qquad n = 1, 2, 3, ...$

or $t=\dfrac{n\lambda}{\mu-1}$

For minimum value of t, n = 1

$$\therefore\ t=\frac{n\lambda}{\mu-1}=\frac{\lambda}{1.5-1}=2\lambda$$

3. PR = d

PO = d secθ

and CO = PO cos 2θ = d secθ cos 2θ

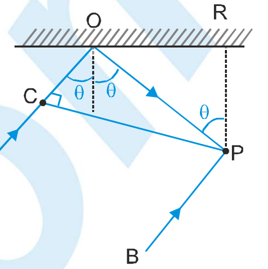
path difference between the two rays is,

$\Delta p = CO + OP + \lambda/2 = d\sec\theta\cos 2\theta + d\sec\theta + \lambda/2$

(one is reflected, while another is direct)

Therefore condition for constructive interference path difference should be

$\Delta p=\lambda, 2\lambda, 3\lambda$ ....

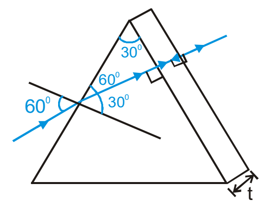


or $d\sec\theta\,(1+\cos 2\theta)+\dfrac{\lambda}{2}=\lambda$

or $\left(\dfrac{d}{\cos\theta}\right)(2\cos^2\theta)=\dfrac{\lambda}{2}$

or $\cos\theta=\dfrac{\lambda}{4d}$

4. $\sin i_1 = m\sin r_1$



or $\sin 60^0=\sqrt{3}\ \sin r_1$

$\therefore\ \sin r_1=\dfrac{1}{2}$ or $r_1=30^0$

Now $r_1+r_2=A$

$\therefore\ r_2=A-r_1=30^0-30^0=0^0$

Therefore, ray of light falls normally on the face AC and angle of emergence $i_2=0^0$.

(b) Multiple reflection occurs between surfaces of film. Intensity will be maximum if destructive interference takes place in the transmitted wave.

For maximum thickness

$\Delta x=2\mu t=\lambda \qquad$ (t = thickness)

$$\therefore\ t=\frac{\lambda}{2\mu}=\frac{6600}{2\times 2.2}=1500\text{ Å}=150\text{ nm} \quad \textbf{Ans.}$$

5. Let nth minima of 400 nm coincides with m th minima of 560 nm, then

$$(2n-1)\left(\frac{400}{2}\right)=(2m-1)\left(\frac{560}{2}\right) \quad \text{or}$$

$$\frac{2n-1}{2m-1}=\frac{7}{5}=\frac{14}{10}=.........$$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is,

$$Y_1=\frac{(2\times4-1)(1000)(400\times10^{-6})}{2\times0.4}=14\text{ mm}$$

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is,

$$Y_2 = \frac{(2\times 11-1)(1000)(400\times 10^{-6})}{2\times 0.1} = 42 \text{ mm}$$

$\therefore$ Required distance $= Y_2 - Y_1 = 28$ mm

Hence, the correct option is **(D)**.

6. Let n_1 bright fringe corresponding to wavelength $\lambda_1 = 500$ nm coincides with n_2 bright fringe corresponding to wavelength $\lambda_2 = 700$ nm.

$$\therefore \quad n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7th maxima of λ_1 coincides with 5th maxima of λ_2. Similarly 14th maxima of λ_1 will coincide with 10th maxima of λ_2 and so on.

$$\therefore \quad \text{Minimum distance} = \frac{n_1\lambda_1 D}{d} = 7\times 5\times 10^{-7}\times 10^{3}$$

$$= 3.5\times 10^{-3}\text{ m} = 3.5\text{ mm}.$$

7. On increasing speed of electron, de Broglie wavelength associated with it will decrease $\left(\lambda = \frac{h}{mv}\right)$.

Since fringe width $\beta = \frac{D\lambda}{d}$, it will decrease.

8. Intensity of one slit $= \frac{I}{4}$

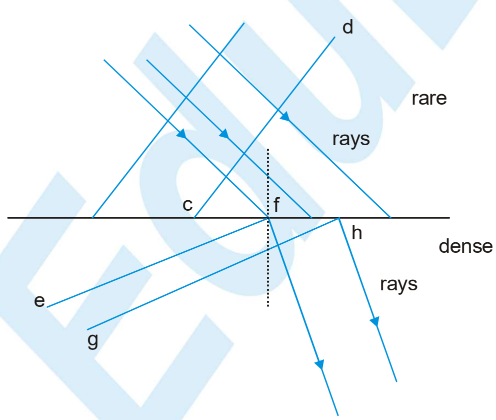
$$\therefore \quad \frac{I}{4} = \frac{I}{4} + \frac{I}{4} + 2\frac{I}{4}\cos\phi \Rightarrow \cos\phi = -\frac{1}{2}$$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

$$\text{Also} \quad \frac{\phi}{2\pi} = \frac{\Delta}{\lambda} \Rightarrow \Delta = \frac{2\pi}{3\times 2\pi}\times\lambda = \frac{\lambda}{3}$$

$$\therefore \quad d\sin\theta = \frac{\lambda}{3} \Rightarrow \sin\theta = \frac{\lambda}{3d} \Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

9.



Wavefronts (Normal to rays) are parallel. So rays should also be parallel.

10. c and d are at same wavefronts

So, $\phi_C = \phi_d$ and $\phi_e = \phi_f$

$\therefore \quad \phi_d - \phi_f = \phi_c - \phi_f$

D is incorrect

$\phi_d - \phi_c = 0 \qquad \phi_f - \phi_e = 0$

So both should be equal.

11. Ray is bending towards normal.

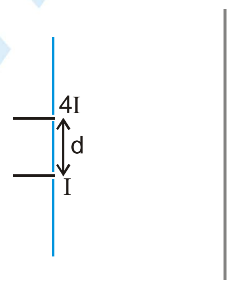
So, medium 1 should be rarer and medium 2 should be denser.

So, $V_1 > V_2$.

12. If d = λ, then maximum path difference will be less than λ. So there will be only central maximum on the screen.

If $\lambda < d < 2\lambda$, then the maximum path difference will be less than 2λ.

So there will be two more maximum on screen corresponding to path difference $\Delta x = \lambda$



So **(A)** and **(B)** are correct.

Intensity of dark fringe becomes zero when intensities of two slits are equal. Initial intensity at both the slits are unequal so there will some brightness at dark fringe. Hence when intensity of both slits is made same the intensity at dark on screen shall decrease to zero. So both **(C)** and **(D)** are false.

13. **(A)** $I(P_1) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{4}\right)$

$$= I_0 + I_0 + 2I_0 \cdot \frac{1}{\sqrt{2}} = (2+\sqrt{2})I_0$$

$$I(P_2) = I_1 + I_2 + 2I_0 \cdot \frac{1}{\sqrt{2}} = 2\sqrt{I_1 I_2}\ .\cos\frac{2\pi}{3}$$

$$= I_0 + I_0 + 2I_0 \cdot \left(-\frac{1}{2}\right) = I_0$$

$\therefore \quad I(P_1) > I(P_2)$

(B) $\delta(P_0) = (\mu - 1)t \cdot \frac{2\pi}{\lambda} = \frac{\lambda}{4} \cdot \frac{2\pi}{\lambda} = \frac{\pi}{2}$

$\delta(P_1) = \left[(\mu-1)t - \frac{\lambda}{4}\right] \cdot \frac{2\pi}{\lambda} = 0$

$\delta(P_2) = \left[(\mu-1)t - \frac{\lambda}{3}\right] \cdot \frac{2\pi}{\lambda} = \frac{-\pi}{6}$

$I(P_0) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos\delta(P_0) \quad = I_0 + I_0 + 2I_0 .$

$\cos\frac{\pi}{2} \; = \; 2I_0$

$I(P_0) = 4I_0$

$I(P_2) = I_0 + I_0 + 2I_0 \cos\left(-\frac{\pi}{6}\right) \quad = (2 + \sqrt{3})I_0$

(C). $\delta(P_0) = (\mu - 1)t \cdot \frac{2\pi}{\lambda} = \frac{\lambda}{2} \cdot \frac{2\pi}{\lambda} = \pi$

$\delta(P_1) = \left((\mu-1)t \cdot \frac{2\pi}{\lambda} - \frac{\lambda}{4}\right)\frac{2\pi}{\lambda} = \frac{\pi}{2}$

$\delta(P_2) = \left[(\mu-1)t - \frac{\lambda}{3}\right] \cdot \frac{2\pi}{\lambda} = \frac{-\pi}{6}$

$I(P_0) = \left(\frac{\lambda}{2} - \frac{\lambda}{3}\right) \cdot \frac{2\pi}{\lambda} = \frac{\pi}{6} \times \frac{2\pi}{\lambda} = \frac{\pi}{3}$

(D) $\delta(P_0) = \frac{3\pi}{4} \times \frac{2\pi}{\lambda} = \frac{3\pi}{2}$

$I(P_1) = 0$

$I(P_2) > 0,\ I(P_0) > 0$

14. $\beta = \frac{\lambda D}{d}$

VIBGYOR λ increase

$\lambda_R > \lambda_G > \lambda_B$

So $\beta_R > \beta_G > \beta_B$

15. For half of maximum intensity

$2I_0 = I_0 + I_0 + 2I_0 \cos\theta$

θ (Phase difference) $= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.........

Path difference is $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.........$\left(\frac{2n+1}{4}\lambda\right)$

16. $2d\sin\theta = \lambda$

$d = \frac{\lambda}{2\sin\theta}$

differntiate

$\partial(d) = \frac{\lambda}{2}\,\partial(\text{cosec}\theta)$

$\partial(d) = \frac{\lambda}{2}(-\text{cosec}\theta\cot\theta)\partial\theta$

$\partial(d) = \frac{-\lambda\cos\theta}{2\sin^2\theta}\partial\theta$

as θ = increases , $\frac{\lambda\cos\theta}{2\sin^2\theta}$ decreases

Alternate solution

Sol. $d = \frac{\lambda}{2\sin\theta}$

$\ln d = \ln\lambda - \ln 2 - \ln\sin\theta$

$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta} \times \cos\theta(\Delta\theta)$

Fractional error $|+(d)| = |\cot\theta\,\Delta\theta|$

Absoulute error $\Delta d = (d\cot\theta)\,\Delta\theta$

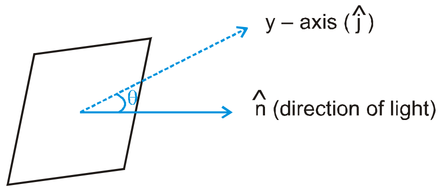
$\frac{d}{2\sin\theta} \times \frac{\cos\theta}{\sin\theta}$

$\Delta d = \frac{\cos\theta}{\sin^2\theta}$

MOCK TEST

1. $I \propto A^2 \quad \therefore \quad \frac{I_1}{I_2} = \frac{2^2}{3^2} = 4/9$

2. $x + 2y + 3z = c$ represents a plane.



Now angle θ is given by :

$$\cos\theta = \frac{\hat{n}\cdot\hat{j}}{|\hat{n}||\hat{j}|} = \frac{b}{\sqrt{a^2+b^2+c^2}} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

where $\hat{n} = a\hat{i} + b\hat{j} + c\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$.

3. $$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 - \left(\sqrt{I_1}-\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 + \left(\sqrt{I_2}-\sqrt{I_2}\right)^2}$$

$$= \frac{I_1}{I_1} \times \frac{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 - \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2}{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 + \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2}$$

$$= \frac{(1+2)^2 - (1-2)^2}{(1+2)^2 + (1-2)^2} = \frac{8}{10} = \frac{4}{5}$$

4. in cases I, II, III, IV the path differences are respectively

$\frac{\lambda}{2}, \lambda, \frac{\lambda}{4}$ and $\frac{3\lambda}{4}$

$\Rightarrow$ phase differences are respectively

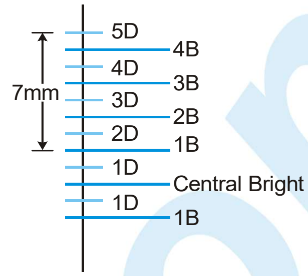
$\pi, 2\pi, \pi/2, 3\pi/2$

and $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$

$\therefore$ the intensity in the four cases are

$0, I_0, \frac{I_0}{2}, \frac{I_0}{2}$ respetively.

5. There are three and a half fringes from first maxima to fifth minima as shown.



$\Rightarrow \beta = \frac{7mm}{3.5} = 2mm \Rightarrow \lambda = \frac{\beta D}{d} = 600$ nm.

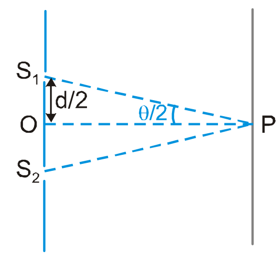
6. For 100th max.

$d \sin\theta = 100\lambda$

$$\sin\theta = \frac{100 \times 5000 \times 10^{-9}}{1 \times 10^{-3}} = \frac{5 \times 10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$$

$\therefore y = D\tan\theta = 1 \times \tan 30 = \frac{1}{\sqrt{3}}$

7. In ΔS_1PO



$\tan\frac{\theta}{2} = \frac{d/2}{D}$

As $D >> d$

$\therefore$ θ is very small.

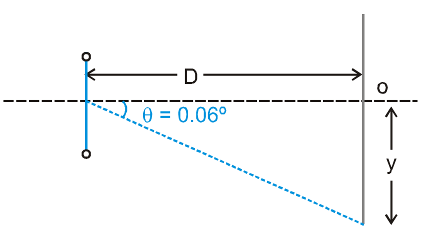
$\Rightarrow \tan\frac{\theta}{2} \approx \frac{\theta}{2}$

$\Rightarrow \frac{\theta}{2} = \frac{d}{2D} \Rightarrow \frac{D}{d} = \frac{1}{\theta} \Rightarrow$ Fringe width $= \frac{\lambda D}{d} = \frac{\lambda}{\theta}$

8. Say 'n' fringes are present in the region shown by 'y'

$\Rightarrow y = n\beta = \frac{n.\lambda D}{d}$

$\Rightarrow \frac{y}{D} \approx \tan(0.06°) \approx \frac{0.06 \times \pi}{180} = \frac{n\lambda}{d}$



$\Rightarrow$ $n = \frac{10^3 \times \pi}{180} \times 0.06 = \frac{\pi}{3} > 1.$

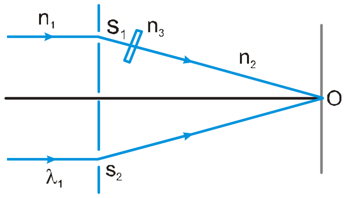
Hence; only one maxima above and below point O. So total 3 bright spots will be present (including point 'O' i.e. the central maxima).

9. At path difference $\frac{\lambda}{6}$, phase difference is $\frac{\pi}{3}$

$$I = I_O + I_O + 2I_O \cos\frac{\pi}{3} = 3\,I_O \qquad I_{max} = 4I_0$$

So the required ratio is $\frac{3I_0}{4I_0} = 0.75$

10.



light wavelength in medium n_1 is $= \lambda_1$

$\Rightarrow$ Wavelength in vaccum $= \lambda_0 = n_1\lambda_1$

The path difference between the lights waves reaching point O $= (n_3 - n_2)\,t$ = extra path which the light from S_1 travelled compared to the path from S_2.

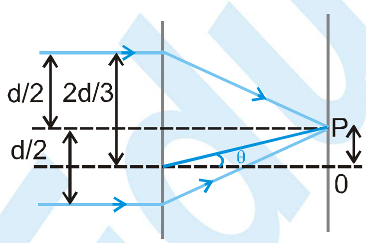
Corresponding phase difference $= \frac{2\pi}{\lambda_0}$ (path difference)

$$= \frac{2\pi}{n_1\lambda_1}(n_3 - n_2)\,t$$

11. Here path difference will be :

$\Delta x = (\mu_2 - \mu_1)\,t \;\Rightarrow\; \delta = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)\,t$

12.



we know that P will be the central maxima (at which path difference is zero)

Now $OP = \frac{d}{2} - \frac{d}{3} = \frac{d}{6}$

13. When light passes through a medium of refractive index μ, the optical path it travels is (μt).

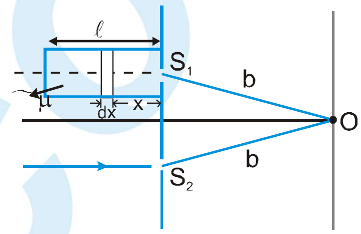
Therefore, before reaching O light through S_1 travels $(\mu l + b)$ distance while that through S_2 travels a distance $(l + b)$

i.e. : path difference $= (\mu l + b) - (l + b) = (\mu - 1)\,l$.

For a small element 'dx' path difference $\Delta x = [(1 + ax) - 1]\,dx = ax\,dx$

For the whole length ;

$$\Delta x = \int_0^l ax\,dx = \frac{al^2}{2}$$



For a minima to be at 'O'.

$$\Delta x = (2n + 1)\frac{\lambda}{2}$$

i.e. : $\frac{al^2}{2} = (2n + 1)\frac{\lambda}{2}$.

For minimum 'a'; $n = 0$

$\Rightarrow \frac{al^2}{2} = \frac{\lambda}{2} \Rightarrow a = \frac{\lambda}{l^2}$ **Ans.**

14. Shift of fringe pattern $= (\mu - 1)\frac{tD}{d}$

$$\therefore \frac{30\,D\,(4800 \times 10^{-10})}{d} = (0.6)\,t\,\frac{D}{d}$$

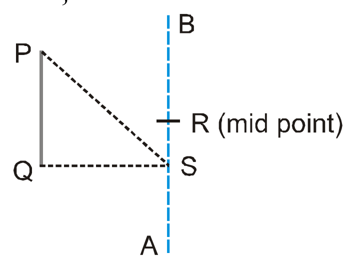
$30 \times 4800 \times 10^{-10} = 0.6\,t$

$$t = \frac{30 \times 4800 \times 10^{-10}}{0.6} = \frac{1.44 \times 10^{-5}}{0.6} = 24 \times 10^{-6}$$

15. Lets take any general point S on the line AB.

Clearly: for any position of S on line AB; we have for Δ PQS:

PQ + QS > PS {in any triangle sum of 2 sides is more then the third side}



$\Rightarrow$ PS – QS < 3λ.

As PS – QS represents the path difference at any point on AB

$\Rightarrow$ it can never be more than 3λ. Now minimas occur at.

$\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ only.

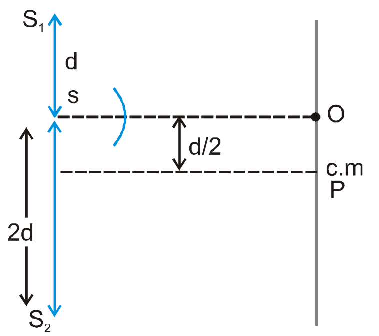
So 3 minimas below R (mid point of AB) and 3 also above R.

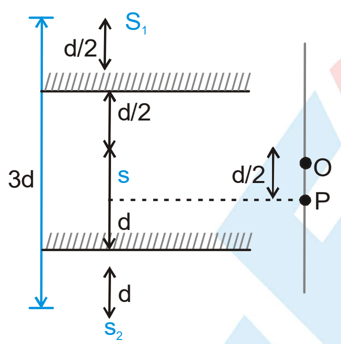
16. The 2 sources are.

As O is a maxima, Hence OP = β.

$$\Rightarrow \frac{d}{2} = \frac{\lambda . D}{(3d)} \text{ ; get } \lambda = \frac{3d^2}{2D}$$

$$= \frac{3d.d}{2D} = n\lambda \Rightarrow \lambda = \frac{3d^2}{2nD} \Rightarrow \lambda = \frac{3d^2}{2d}, \frac{3d^2}{4d}$$





17. Ray N undergoes reflection at surface II with phase change of π

$\Rightarrow$ $n_3 > n_2$

Ray Q undergoes a phase-change of π at II, but there is no phase change when it is reflected from surface I.

$\Rightarrow$ $n_1 < n_2$

18. Eq. of path diff. for maxima in tansmission (or weak reflection);

$$\Delta P_{opt} = 2n_2 L = \frac{\lambda_{vacuum}}{2}, \frac{3\lambda_{vacuum}}{2} \text{ ......}$$

$$\Rightarrow 2\left(\frac{n_2}{n_1}\right) L = \frac{\lambda}{2}, \frac{3\lambda}{2}, \text{......} \Rightarrow L = \frac{\lambda n_1}{4n_2}.$$

(notice that λ = wavelength in medium is related to λ_{vacuum} as, $\lambda_{vacuum} = n_1 \lambda$)

19. Constructive interference happens when $2t = (m - 1/2)\lambda$. The minimum value of m is m = 1; the maximum value is

the integer portion of $\frac{2t}{\lambda} + \frac{1}{2} = \frac{2 \times 0.034 \times 10^{-3}}{680 \times 10^{-9}} + \frac{1}{2}$

$= 100.5$

$m_{max} = 100$

20. Path difference $= \sqrt{D^2 + d^2} - D = 1$ cm

Also ; $\left[\sqrt{D^2 + d^2} - D\right] = (2n - 1)\frac{\lambda}{2} \Rightarrow \lambda = \frac{2(1)}{2n - 1}$

For n = 1,2,3,............

$\lambda = 2\text{cm}, \frac{2}{3}\text{ cm}, \frac{2}{5}\text{ cm}, \text{.............}$

21. $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = \left(\sqrt{I} + \sqrt{\frac{I}{2}}\right)^2 < 4I$

$I_{min} = \left(\sqrt{I} - \sqrt{\frac{I}{2}}\right)^2 > 0 \Rightarrow \beta = \frac{\lambda D}{d}$

becuase λ D.d. are unchanged so β also remain unchanged.

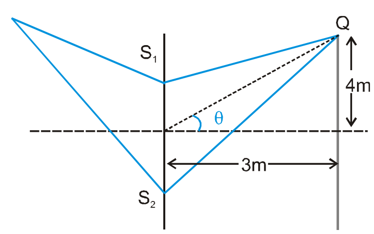
22. As d << D, $\Rightarrow$ path difference = d sinθ (at 0) = 1mm × sin 30° = 0.5 mm

if it is a maxima $\Rightarrow 10^{-3} \times 0.5 = (5000 \times 10^{-10})\text{m} \times (n)$

n must be integer. get n = 1000.

Hence O is a maxima of intensity $4I_0$

Now



Now path difference at Q = d sinθ only $QS_1 \approx QS_2$.

d sin θ = 1 × 1/2 = 0.5 mm = integer multiple of λ.

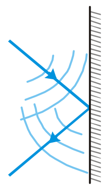
Hence maxima.

23. If maximum intensity is observed at P then for maximum intensity to be also observed at Q, S_1 and S_2 must have phase difference of $2m\pi$ (where m is an integer).

24. Statement 1 is false because constructive interference can be obtained if phase difference of sources is 2π, 4π, 6π, etc.

25. Wave fronts are spherical in shape of radius ct.

26.



The wave fronts are always perpendicular to the light rays.

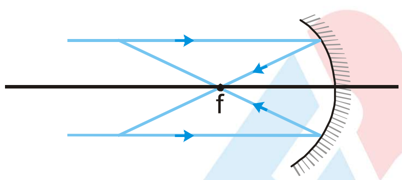
27. Using snell's law ;

$$\frac{\sin(45^0)}{\sin r} = \frac{\sqrt{2}}{1} \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^0$$

Hence, **(B)** is correct.

Note : The shown lines are wavefronts and not rays.

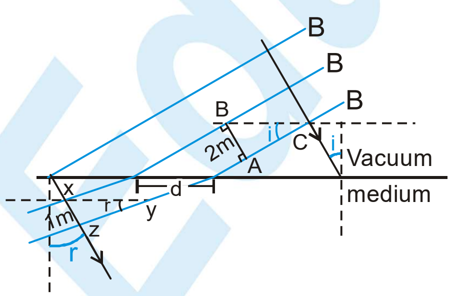
28. After reflection by mirror the parallel rays concentrate at the focus.



Hence the plane wave front becomes spherical concentrated at the focus.

29. In ΔABC ; $\sin(i) = \frac{2}{d}$ In Δxyz ; $\sin(r) = \frac{1}{d}$

$$\Rightarrow \frac{\sin i}{\sin r} = 2 = \mu .$$



30. Order of the fringe can be counted on either side of the central maximum. For example fringe no. 3 is first order bright fringe.

31. Since, 2nd fringe represent central bright fringe hence, 4th fringe results from a phase difference of 4π between the light waves incidenting from two slits .

32. $\Delta X_C = \lambda$; $\Delta X_A = \frac{\lambda}{2}$

$$\Delta X_C - \Delta X_A = \frac{\lambda}{2} = 300 \text{ nm}.$$

33. By using $(\mu - 1)t = n\lambda$, we can find value of n, that is order of the fringe produced at P, if that particular strip has been placed over any of the slit. If two strips are used in conjuction (over each other), path difference due to each is added to get net path difference created. If two strips are used over different slits, their path differences are subtracted to get net path difference.

Now, $n_1 = \frac{(\mu_1 - 1) t_1}{\lambda} = 5$

$n_2 = 4.5$

and $n_3 = 0.5$

For (a), order of the fringe is 4.5 i.e. 5th dark.

for (b), net order is $5 - 0.5 = 4.5$

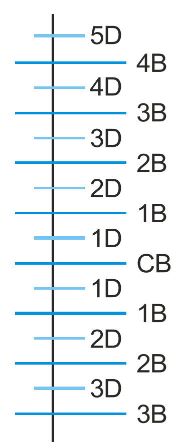
i.e. fifth dark.

for (c) net order is $5 - (0.5 + 4.5) = 0$

i.e. it is central bright again at P.

for (d) net order is $(5 + 0.5) - (4.5) = 1$

i.e. first bright



34. Initially at a distance x from central maxima on screen is

$$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos\frac{2\pi x}{\beta}, \text{ where } \beta = \frac{D\lambda}{d}$$

$I_{max} = 9I_0$ and $I_{min} = I_0$

(A) At points where intensity is $\frac{1}{9}$ th of maximum intensity, minima is formed

$\therefore$ Distance between such points is $\beta, 2\beta, 3\beta, 4\beta, ...$

(B) At points where intensity is $\frac{3}{9}$ th of maximum intensity, $\cos\frac{2\pi x}{\beta} = -\frac{1}{2}$ or $x = \frac{\beta}{3}$.

$\therefore$ Distance between such points is $\frac{\beta}{3}, \frac{2\beta}{3}, \beta,$

$\beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta, ...$

(C) $\cos\frac{2\pi x}{\beta} = 0$ or $x = \frac{\beta}{4}$.

$\therefore$ Distance between such points is $\frac{\beta}{2}, \beta, \beta + \frac{\beta}{2},$ $2\beta, ...$

(D) $\cos\frac{2\pi x}{\beta} = \frac{1}{2}$ or $x = \frac{\beta}{6}$.

$\therefore$ Distance between such points is $\frac{\beta}{3}, \frac{2\beta}{3}, \beta,$

$\beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta, ...$

35. (A) When d = 99.4 λ, 398 points of maximum intensity are formed on periphery of circle and 396points of minimum intensity are formed on periphery of circle

(B) When d = 99.6 λ, 398 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

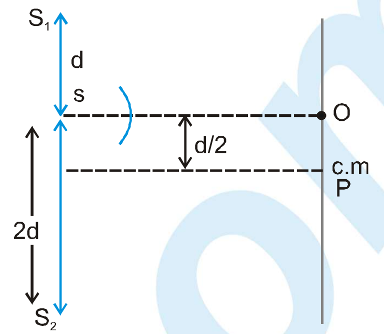
(C) When d = 100 λ, 400 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

(D) When d = 100.4 λ, 402 points of maximum intensity are formed on periphery of circle and 400 points of minimum intensity are formed on periphery of circle

36. $\beta = \frac{\lambda(a+b)}{2(\mu-1)\alpha} = \frac{\lambda}{\mu'}\frac{(a+b}{\left(\frac{\mu}{\mu'}-1\right)\alpha}$

$\beta \propto \frac{1}{\mu-\mu'}\quad \frac{\beta}{4} = \frac{1.5-1}{1.5-4/3} = \frac{\frac{0.5}{0.5}}{3} = 3 \quad \beta = 12$ mm. **Ans.**

37. y-coordinate of 3[rd] order maxima $= \frac{3\lambda D}{d}$

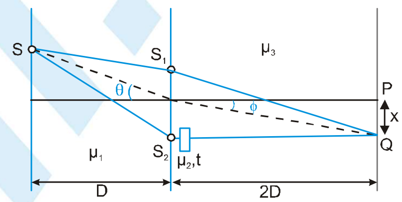


where d is the distance between both the sources at any time t

$d = 4a - 2x = 4a - 2a\sin\omega t = 2a(2-\sin\omega t)$

$\therefore \quad y = \frac{3\lambda D}{2a(2-\sin\omega t)}$

38. (i) For the central order bright to be formed at Q



$(SS_1)\mu_1 + (S_1Q)\mu_3 = (SS_2)\mu_1 + (S_2Q - t)\mu_3 + \mu_2 t$

or $(S_1Q - S_2Q)\mu_3 = (SS_2 - SS_1)\mu_1 + (\mu_2 - \mu_3)t$

$d\sin\phi\,\mu_3 = (d\sin\theta)\mu_1 + (\mu_2-\mu_3)t$ ....**(1)**

$= \frac{d^2}{D}\mu_1 + (\mu_2 - \mu_3)t$

$= \frac{1^2}{10^3} \times \frac{4}{3} + \left(\frac{3}{2} - \frac{9}{5}\right)\frac{4}{9} \times 10^{-2} = 0$

$\therefore \phi = 0$ or the central order bright is formed at P only.

(ii) In absence of slab from equation (1) ; t = 0

$d\sin\phi\,\mu_3 = (d\sin\theta)\mu_1$

$\frac{x}{2D}\mu_3 = \frac{d}{D}\mu_1$

or $\quad x = \frac{2\mu_1 d}{\mu_3} = \frac{40}{27}$ mm.

Ans. **(i)** 0 **(ii)** $\frac{40}{27}$ **mm downwards**

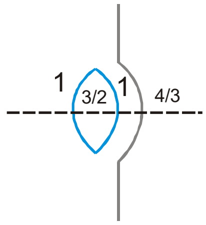
39. **(a)** 80 cm behind the lens **(b)** 4 mm **(c)** $\beta = 60\ \mu m$

Lets find out the radius of curvature of equi. convex lens.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\Rightarrow \frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right)$$

$\Rightarrow$ R = 10 cm.



Now for lens :

$$\frac{1}{V} - \frac{1}{-20} = \frac{1}{10} \quad \Rightarrow \quad \frac{1}{V} = \frac{1}{20}$$

$\Rightarrow$ for surface of tube (of R = 10 cm.)

$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{4/3}{V} - \frac{1}{+20} = \frac{4/3 - 1}{-10}$$

$\Rightarrow$ V = + 80 cm.

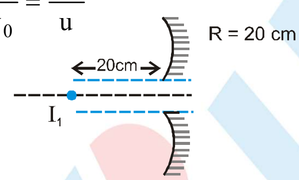
(b) Now for mirrors.

As the object for the mirrors is at 20 cm so the image will be at 20 cm only

$\therefore$ u = – 2 f $\Rightarrow$ v = – 2f also.

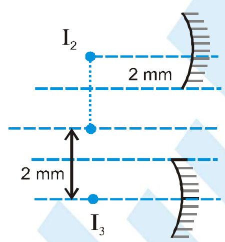
$$\Rightarrow \text{magnification} = m = \frac{y_I}{y_0} = \frac{-v}{u}$$

$$\Rightarrow \frac{y_I}{-(1\,mm)} = -\left(\frac{-20}{-20}\right)$$



$\Rightarrow$ y_I = + (1 mm)

so the final images are like.



so the distance between the images is 4 mm.

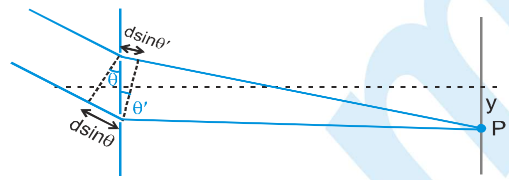
(c) Now, these I_2 and I_4 behave as the 2 sources for fringe pattern.

$$\Rightarrow \beta = \frac{\lambda D}{d} = \frac{vD}{fd} = \frac{(c/\mu)D}{fd}$$

$$= \left(\frac{3\times 10^8}{\frac{4}{3}}\right) \times \frac{0.8}{\frac{3}{4}\times 10^{15} \times (4\times 10^{-3})} = 60\ \mu m.$$

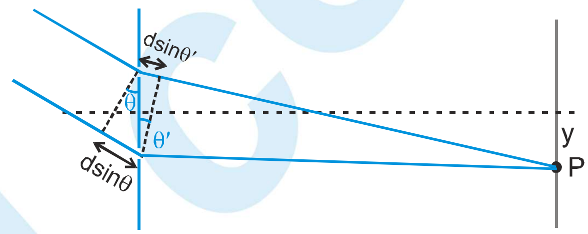
40. If phase difference at point P is zero then

$n_1 d \sin\theta = n_2 d \sin\theta' \Rightarrow \theta' = 37°$



and as $\tan\theta' = \frac{y}{D} \Rightarrow y = -\frac{3}{4}$ m

It is negative because upper path in medium n_2 is longer than lower path in the same medium.



$\Rightarrow \theta' = 37°$

$\tan\theta' = \frac{y}{D} \Rightarrow y = -\frac{3}{4}$ m **Ans.** I_0

(iii) y-coordinate of the nearest maxima above 'O' is $\frac{x}{\sqrt{154}}$ cm. then x is **Ans.** 150

As we go up from point O, path difference will increase. At O, phase difference is $3\pi + \frac{\pi}{3}$ and when it becomes 4π, there will be maximum. Extra path difference created in medium 2 must lead to $\frac{2\pi}{3}$ phase difference.

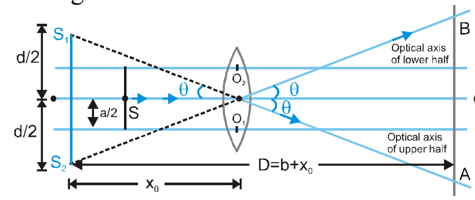
$$\frac{2\pi}{\lambda_a} \cdot d\sin\theta_1 \cdot n_2 = \frac{2\pi}{3}$$

Using values $\sin\theta_1 = \frac{3}{25}$

$$\Rightarrow \tan\theta_1 = \frac{3}{\sqrt{616}} = \frac{y}{D}$$

$$y = \frac{300}{2\sqrt{154}}\ \text{cm} = \frac{150}{\sqrt{154}}\ \text{cm}$$

41. From Fig.



$\tan\theta = \theta = \dfrac{d}{2x_0}$ $\therefore\ d = 2x_0\theta$

Since, the line joining S_1 and S_2 is parallel to screen.

$$\therefore\ \Delta x = \frac{\Delta}{d}\lambda = \left(\frac{b+x_0}{2x_0\theta}\right)\lambda = \left(\frac{b+x_0}{x_0}\right)\frac{\lambda}{2\theta} = \left(1+\frac{b}{x_0}\right)\frac{\lambda}{2\theta}$$

But source S is situated in focal plane. So, images S_1 and S_2 are situated at infinitely large distance i.e., $x_0 \to \infty$.

$$\Delta x = \left(1+\frac{b}{\infty}\right)\frac{\lambda}{2\theta} = \frac{\lambda}{2\theta} = \frac{\lambda}{2\left(\frac{a}{2f}\right)} = \frac{f\lambda}{a}$$

Here, $f = 25$ c, $\lambda = 0.6\times10^{-6}$ m, $a = 1$ mm

Putting the values, $\Delta x = 0.15$ mm

In Fig. the fringe pattern is observed between points A and B.

Since, the arrangement is similar to Young's experiment.

So, number of fringes $= 2\,[n_1] + 1$

where n_1 = number of fringes on either side of central point C of screen.

From Fig. $\tan\theta = \dfrac{BC}{OC} = \dfrac{BC}{b} = \dfrac{1}{b}$

$$\therefore\ \bullet = b\tan\theta = b\,\theta = \frac{ba}{2f}$$

$$\therefore\ n_1 = \frac{1}{\Delta x} = \frac{ba}{2f\times\frac{f\lambda}{a}}$$

$$\therefore\ n_1 = \frac{a^2 b}{2f^2\lambda}$$

$$\Rightarrow\ n_1 = \frac{(10^{-3})^2(50\times10^{-2})}{2\times(25\times10^{-2})^2\times0.6\times10^{-6}}$$

$= 6.67$ $\therefore\ n = 2\,[n_1] + 1 = 2\times6+1 = 13$

42. **(i)** Velocity and acceleration of central maximum

= velocity and acceleration of screen

[$\because$ it does not move to the left or right on the screen.]

$v_{screen} = 0 + gt = 100$ m/s

$\therefore\ \vec{v}_s = 100\,\hat{j}$ m/s (in vector form) and $\vec{a}_s = 10\,\hat{j}$

(ii) $D = D_0 + \dfrac{1}{2}gt^2$

[where D in distance of screen at a time t along y axis]

Position vector of 1[st] maximum is given by

$$\vec{r} = \pm\frac{\lambda D}{d}\hat{i} + D\hat{j}$$

Differentiating with respect to time,

$$\vec{V} = \frac{dD}{dt}\hat{j} \pm \frac{d}{dt}\left(\frac{D\lambda}{d}\right)\hat{i}$$

$$= 100\,\hat{j} \pm \frac{5\times10^{-7}}{10^{-3}}(100)\,\hat{i}$$

$$= \pm 5\times10^{-2}\,\hat{i} + 100\,\hat{j}$$

$$\vec{a} = \frac{d^2D}{dt^2}\hat{j} \pm \frac{\lambda}{d}\frac{d^2D}{dt^2}\hat{i} = 10\,\hat{j} \pm 5\times10^{-3}\,\hat{i}$$

Ans. **(i)** 100 **(ii)** 10 **(iii)** 75