# HINTS & SOLUTIONS

## EXERCISE - 1

### Single Choice

**2.** $\dfrac{1^2+2^2+3^2+---+n^2}{n} = \dfrac{46n}{11}$

$\Rightarrow \dfrac{n(n+1)(2n+1)}{6n} = \dfrac{46n}{11}$

$\Rightarrow 11(n+1)(2n+1) = 276n$

$\Rightarrow 22n^2 - 243n + 11 = 0$

$\Rightarrow (n-11)(22n-1) = 0$

$\therefore n = 11$ ✈ $n \neq \dfrac{1}{22}$

**3.** $\dfrac{\sum_{i=1}^{11} x_i}{11} = \dfrac{15+\sum_{i=1}^{11} x_i}{12} \Rightarrow \sum_{i=1}^{11} x_i = 165$

$\Rightarrow \bar{x} = 15$

**8.** Req. mean

$= \dfrac{a+(a+d)+(a+2d)+\ldots\ldots+\{a+(n-1)d\}}{n}$

$= \dfrac{\frac{n}{2}[a+a+(n-1)d]}{n} = a + \dfrac{(n-1)d}{2}$

**11.** First we changed the classes into continuous form, we have

| Class | 0.5 - 10.5 | 10.5 - 20.5 | 20.5 - 30.5 | 30.5 - 40.5 | 40.5 - 50.5 |
|---|---|---|---|---|---|
| fi | 5 | 7 | 8 | 6 | 4 |

Here model class is 20.5 – 30.5

$\therefore$ Mode = ● + $\dfrac{(f_0 - f_1)}{(2f_0 - f_1 - f_2)} \times h$

$= 20.5 + \dfrac{(8-7)}{(16-7-6)} \times 10 = 23.83$

**12.** Here number of terms = n + 1 (odd)

$\therefore$ Median = $\left(\dfrac{n+2}{2}\right)$ th term = $\left(\dfrac{n}{2}+1\right)$ th term $= {}^{2n}C_{n/2}$

**13.** $n = 88$

Median $= \dfrac{44^{th}\text{ value} + 45^{th}\text{ value}}{2} = \dfrac{56+57}{2} = 56.5$

M.D.(median)

$= \dfrac{\sum_{i=1}^{88}|x_i - 56.5|}{88} = \dfrac{43.5+42.5+\ldots.+0.5+0.5+\ldots.+43.5}{88}$

$= \dfrac{1+3+5+\ldots.+85+87}{88} = 22$

**14.** ✈ S.D. $= \sqrt{\dfrac{1}{n}\Sigma x_i^2 - \left(\dfrac{1}{n}\Sigma x_i\right)^2}$

So, S.D. of first n natural numbers

$= \sqrt{\dfrac{1}{n}\Sigma n^2 - \left(\dfrac{1}{n}\Sigma n\right)^2}$

$= \sqrt{\dfrac{1}{n}.\dfrac{n(n+1)(2n+1)}{6} - \left\{\dfrac{1}{n}.\dfrac{n(n+1)}{2}\right\}^2}$

$= \sqrt{\dfrac{(n+1)(2n+1)}{6} - \dfrac{(n+1)^2}{4}} = \sqrt{\dfrac{(n+1)(n-1)}{12}}$

$= \sqrt{\dfrac{n^2-1}{12}}$

**16.** $T_n = (2n-1)(2n+1)(2n+3)$

Req. A.M. $= \dfrac{S_n}{n} = \dfrac{\Sigma T_n}{n} = \dfrac{1}{n}[\Sigma(8n^3+12n^2-2n-3)]$

$= \dfrac{1}{n}[8\Sigma n^3 + 12\Sigma n^2 - 2\Sigma n - \Sigma 3]$

$= 2n^3 + 8n^2 + 7n - 2$

**19.** ✈ $\sigma = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2} = \sqrt{\dfrac{\Sigma x_i^2}{n} - \bar{x}^2}$

$\Rightarrow \Sigma x_i^2 = n(\sigma^2 + \bar{x}^2)$

**21.** Here $N = 10,\ u_i = \dfrac{x_i - 25}{10}$

$\therefore\ \Sigma f_i u_i = 1 \times (-2) + 3 \times (-1) + 4 \times 0 + 2 \times 1 = -3$

and, $\Sigma f_i u_i^2 = 1 \times 4 + 3 \times 1 + 4 \times 0 + 2 \times 1 = 9$

$$\sigma = h\sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} = 10\sqrt{\frac{9}{10} - \left(\frac{-3}{10}\right)^2} = 9$$

**23.** First we arranged the observations in ascending order 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

Here $n = 10$ (even)

$\therefore$ Median (M)

$$= \frac{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}}{2}$$

$$= \frac{46 + 48}{2} = 47$$

$\Sigma|x_i - M| = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86$

$\therefore$ Mean deviation from Median

$$= \frac{\Sigma|x_i - M|}{n} = 8.6$$

**24.** Let the number of terms of two series are $n_1$ and $n_2$ whose means are $\bar{x}_1$ and $\bar{x}_2$ resp.

$$\because\ \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Now $\bar{x} - \bar{x}_1 = \dfrac{n_2(\bar{x}_2 - \bar{x}_1)}{(n_1 + n_2)} > 0 \quad (\because\ \bar{x}_2 > \bar{x}_1)$

$\Rightarrow\ \bar{x} > \bar{x}_1$ ...(i)

Again $\bar{x} - \bar{x}_2 = \dfrac{n_1(\bar{x}_1 - \bar{x}_2)}{(n_1 + n_2)} < 0 \quad (\because\ \bar{x}_1 < \bar{x}_2)$

$\Rightarrow\ \bar{x} < \bar{x}_2$ ...(ii)

by (i) & (ii) $\bar{x}_1 < \bar{x} < \bar{x}_2$

**26.** Arrange the given observations in ascending order

40, 54, 62, 68, 76, 90

no. of terms (n) = 6 (even)

$\therefore$ Median (M)

$$= \frac{\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}}{2} = \frac{62 + 68}{2} = 65$$

$\Sigma|x_i - M| = 25 + 11 + 3 + 3 + 11 + 25 = 78$

M.D. from median $= \dfrac{\Sigma|x_i - M|}{n} = \dfrac{78}{6} = 13$

$\therefore$ Coefficient of M.D. $= \dfrac{\text{M.D.}}{\text{median}} = \dfrac{13}{65} = 0.2$

**31.** $\sigma_x = 3 \quad \Rightarrow \quad \dfrac{\sum x_i^2}{100} - (\bar{x})^2 = 9$

$\Rightarrow\ \sum x_i^2 = 23400$

$\sum z_i = 250 \times 15.6 = 3900$

$\therefore\ \Sigma y_i = \Sigma z_i - \Sigma x_i = 3900 - 1500 = 2400$

$\sigma_z^2 = 13.44 \quad \Rightarrow \quad \dfrac{\sum x_i^2 + \sum y_i^2}{250} - (15.6)^2 = 13.44$

$\Rightarrow\ \sum y_i^2 = 40800$

$$\Rightarrow\ \sigma_y = \sqrt{\frac{\sum y_i^2}{150} - \left(\frac{\sum y_i}{150}\right)^2} = \sqrt{\frac{40800}{150} - \left(\frac{2400}{150}\right)^2} = 4$$

**32.** Let the no. of boys and girls are $n_1$ and $n_2$ resp., then

$$\frac{65n_1 + 55n_2}{n_1 + n_2} = 61$$

$\Rightarrow\ 4n_1 = 6n_2 \quad \Rightarrow\ n_1 : n_2 = 3 : 2$

**36.** Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100$

**45.** $\because\ \dfrac{x_1 + x_2 + \ldots + x_{10}}{10} = 20$ ...(1)

Req. mean $= \dfrac{(x_1 + 4) + (x_2 + 8) + \ldots + (x_{10} + 40)}{10}$

$= \dfrac{(x_1 + x_2 + \ldots + x_{10})}{10} + \dfrac{4(1 + 2 + \ldots + 10)}{10}$

$= 20 + 22 = 42$ [by eq. (1)]

**49.** $\text{W.M.} = \dfrac{0\times 1 + 1\times {}^nC_1 + 2\times {}^nC_2 + \ldots\ldots + n\times {}^nC_n}{1 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n}$

$$= \frac{1\times {}^nC_1 + 2\times {}^nC_2 + \ldots\ldots + n\times {}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots\ldots + {}^nC_n} = \frac{\sum_{r=1}^{n} r.{}^nC_r}{2^n}$$

$$= \frac{\sum_{r=1}^{n} r.\frac{n}{r}.{}^{n-1}C_{r-1}}{2^n} = \frac{n\sum_{r=1}^{n} .{}^{n-1}C_{r-1}}{2^n} = \frac{n.2^{n-1}}{2^n} = \frac{n}{2}$$

**51.** Since mean deviation is minimum when it is taken by median, so here K is median of given observations.

$$K = \text{median} = \left(\frac{n+1}{2}\right) \text{th observation}$$

$= 51^{th}$ observation

$\therefore \quad K = x_{51}$

**52.** $\text{Average speed} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{d+d+d}{t_1+t_2+t_3}$

$$= \frac{3d}{\frac{d}{v_1}+\frac{d}{v_2}+\frac{d}{v_3}} = \frac{3}{\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}} = \text{H.M. of } v_1, v_2, v_3$$

**54.** On arranging the values in the ascending order

$\alpha - \frac{7}{2},\ \alpha - 3,\ \alpha - \frac{5}{2},\ \alpha - 2,\ \alpha - \frac{1}{2},\ \alpha + \frac{1}{2},$ $\alpha + 4, \alpha + 5$ ($\because$ $\alpha > 0$)

Here number of observations n = 8 (even)

$$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)\text{th obser.} + \left(\frac{n}{2}+1\right)\text{th obser.}\right]$$

$$= \frac{1}{2}\left[(\alpha - 2) + \left(\alpha - \frac{1}{2}\right)\right] = \alpha - \frac{5}{4}$$

**56.** For S.D. of first n odd natural numbers

$1, 3, 5, \ldots, (2n-1)$

$\Sigma x_i = 1 + 3 + 5 + \ldots + (2n-1) = n^2$

$\Sigma x_i^2 = 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2$

$= [1^2 + 2^2 + 3^2 + \ldots + (2n)^2] - [2^2 + 4^2 + \ldots + (2n)^2]$

$= [1 + 2^2 + 3^2 + \ldots + (2n)^2] - 2^2[1^2 + 2^2 + \ldots + n^2]$

$$= \frac{2n(2n+1)(4n+1)}{6} - 4 . \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n+1)}{3}[(4n+1) - 2(n+1)]$$

$$= \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}$$

$$\therefore \text{Req. S.D.} = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{4n^2-1}{3} - n^2} = \sqrt{\frac{n^2-1}{3}}$$

**58.** 3 occurs maximum number of times so mode is 3.

**59.** $\text{S.D.} = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n}}$ and $\text{M.D.} = \dfrac{\Sigma|x_i - \bar{x}|}{n}$

let $|x_i - \bar{x}| = y_i$

i.e. $(x_i - \bar{x})^2 = |x_i - \bar{x}|^2 = y_i^2$

Now $\quad (\text{S.D.})^2 - (\text{M.D.})^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n} - \left(\dfrac{\Sigma|x_i - \bar{x}|}{n}\right)^2$

$$= \frac{\Sigma y_i^2}{n} - \left(\frac{\Sigma y_i}{n}\right)^2 = \sigma_y^2 > 0$$

($\because$ $\sigma^2$ is non negative and all values are not same)

$\Rightarrow$ $(\text{S.D.})^2 - (\text{M.D.})^2 > 0$

$\Rightarrow$ $\text{S.D.} > \text{M.D.}$

**61.** Let $x_1, x_2, \text{----}\ x_n$ are n positive numbers such that

$x_1 \times x_2 \times \text{----} \times x_n = 1 \qquad \ldots(1)$

$\because$ A.M. $\geq$ G.M.

So $\quad \dfrac{x_1 + x_2 + \text{----} + x_n}{n} \geq (x_1 \times x_2 \times \text{----} \times x_n)^{1/n}$

$\Rightarrow$ $x_1 + x_2 + \ldots\ldots + x_n \geq n \qquad$ by (1)

**63.** Here $u = \frac{x}{h} - \frac{a}{h}$

→ S.D. is not depend on change of origin but it is depend on change of scale.

$\therefore \quad \sigma_u = \frac{\sigma_x}{h}$

**64.** Let $\bar{x} = \frac{x_1 + x_2 + ..... + x_n}{n}$

New Mean $= \frac{(x_1 + 1) + (x_2 + 2) + ..... + (x_n + n)}{n}$

$= \frac{(x_1 + x_2 + ..... + x_n)}{n} + \frac{(1 + 2 + ..... + n)}{n} = \bar{x} + \frac{n+1}{2}$

**65.** $\sum_{i=1}^{10}(x_i - 15) = 7 \Rightarrow \sum x_i - 150 = 7 \Rightarrow \frac{\sum x_i}{10} = 15.7$

**66.** $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

Let $y_i = \frac{ax_i + b}{c} = \left(\frac{a}{c}\right)x_i + \frac{b}{c}$

$\Rightarrow \bar{y} = \left(\frac{a}{c}\right)\bar{x} + \frac{b}{c} \Rightarrow y_1 - \bar{y} = \frac{a}{c}(x_i - \bar{x})$

$\Rightarrow$ Req. S.D. $= \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n}}$

$= \sqrt{\frac{\Sigma\left[\frac{a^2}{c^2}(x_i - \bar{x})^2\right]}{n}} = \left|\frac{a}{c}\right|\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \left|\frac{a}{c}\right|\sigma$

## EXERCISE - 2

### Part # I : Matrix Match Type

**1.** **(A)** Due to low value 1, mean is not preferred

**(B)** Mean, Median, Mode and S.D. are dependent on change of scale.

**(C)** S.D. is independent of change of origin.

**(D)** Range is always greater than or equal to S.D.

### Part # II : Comprehension

**Comprehension # 2**

**2.** As data has outliners at 90,000 and 95,000 we should use median in place of mode

& Median $= \frac{10^{th}\text{ value} + 11^{th}\text{ value}}{2} = 12000$

**3.** For a normally distributed data, we many use either mean or median. However, mean is preferred as it include all the values in the data set for its calculation and any change in any of the scores will affect the value of the mean which is not the case with median or mode.

## EXERCISE - 3

### Part # I : AIEEE/JEE-MAIN

**2.** Given $n = 15, \Sigma x = 170, \Sigma x^2 = 2830$

Since one observation 20 was found be wrong and it replaced by its correct value 30

correct sum of observation = 170 – 20 + 30 = 180

correct sum of squares of observations

$$= 2830 - 20^2 + 30^2 = 3330$$

The correct variance $= \frac{3330}{15} - \left(\frac{180}{15}\right)^2 = 78$

**6.** Number of observations = 2n

and observations are

a, a, ----- n times, –a, –a, ---- n times

Here $\bar{x} = \frac{na + (-na)}{2n} = 0$

$\because$ S.D.$= \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x_i^2}{2n}}$ $(\because \bar{x} = 0)$

$2 = \sqrt{\frac{2na^2}{2n}} = |a|$ $(\because$ S.D. = 2)

**10.** Population A has 100 obser 101, 102, ..... 200 and variance $V_A$.

Population B has 100 obser. 151, 152, ..... 250

i.e. (101 + 50), (102 + 50), ..... (200 + 50) and variance $V_B$

$\because$ Variance is independent of change of origin.

i.e. $V_B = V_A \Rightarrow \frac{V_A}{V_B} = 1$

**11.** Let number of boys and girls are $n_1$ and $n_2$ resp.

$\because$ $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 50 = \frac{52n_1 + 42n_2}{n_1 + n_2}$

$\Rightarrow n_1 = 4n_2$ ...(1)

$\therefore$ Percentage of boys $= \frac{n_1}{n_1 + n_2} \times 100 = 80$ [using (1)]

**12.** $\because$ $\frac{\Sigma x_i}{n} = \bar{x}$

$\Rightarrow \frac{a + b + 8 + 5 + 10}{5} = 6 \Rightarrow a + b = 7$ ...(1)

and $\frac{\Sigma x_i^2}{n} - \bar{x}^2 = \sigma^2$

$\Rightarrow \frac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 = 6.8$

$\Rightarrow a^2 + b^2 = 25$ ...(2)

On solving equation (1) & (2) a = 4, b = 3 or a = 3, b = 4

**13.** Sum of first n even natural numbers

$\Sigma x_i = 2 + 4 + 6 + ..... + 2n$

$= 2[1 + 2 + 3 + .... + n] = n(n + 1)$

Sum of square of first n even natural numbers

$\Sigma x_i^2 = 2^2 + 4^2 + 6^2 + ....... + (2n)^2$

$= 2^2[1^2 + 2^2 + 3^2 + .... + n^2]$

$= \frac{2n(n + 1)(2n + 1)}{3}$

Variance of first n even natural numbers

$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$= \frac{2(n + 1)(2n + 1)}{3} - (n + 1)^2 = \frac{n^2 - 1}{3}$

So, statement-1 is false and statement-2 is true.

**14.** $\because$ M.D. of A.P. a, a + d, a + 2d, .... a + 2nd about mean is

M.D. $= \frac{n(n + 1)}{2n + 1}|d|$

given numbers 1, 1 + d, 1 + 2d, .... 1 + 100d are in A.P. and its M.D. is 255.

Here a = 1, 2nd = 100 d i.e. n = 50

$\therefore \frac{50 \times 51}{101}|d| = 255 \Rightarrow d = 10.1$

**15.** Here $n_1 = n_2 = 5, \sigma_1^2 = 4, \sigma_2^2 = 5, \bar{x}_1 = 2, \bar{x}_2 = 4$

Variance of combined set

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2$$

$$= \frac{5\times4 + 5\times5}{(5+5)} + \frac{5\times5}{(5+5)^2}(2-4)^2 = \frac{9}{2} + 1 = \frac{11}{2}$$

**16.** Mediam = 25.5 a

$x_i$ : a  2a  3a ............ 50 a

$|x_i - M|$ : 24.5a  23.5a .................. 24.5a

$$\sum|x_i - M| = [24.5a + 23.5a + ...... + 0.5a + 0.5a + ...... + 24.5a]$$

$$= 2a[0.5 + 1.5 + ...... + 24.5]$$

$$\sum|x_i - M| = 25\times25a$$

$$\text{M. D.} = \frac{25\times25a}{50}$$

$$50 = \frac{25\times25a}{50} \Rightarrow a = 4$$

**17.** $\bar{x} = \text{mean} = \frac{\Sigma x_i}{n}$

mean of $x_i$ is given as 30 gm

If each data is increased by some number (i.e. 2) the mean is also increased by 2.

i.e. corrected mean = 30 + 2 = 32 gm

$$\sigma = \text{standard deviation} = \sqrt{\frac{\Sigma|x_i - \bar{x}|^2}{n}}$$

standard deviation does not depend on change of origin so if every data is increased by same number (i.e. 2) then standard deviation remains same.

So the corrected standard deviation = 2gm.

**21.** Variance

$$= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{4+9+a^2+121}{4} - \left(\frac{16+a}{4}\right)^2$$

$$= \frac{4(134+a^2) - 256 - a^2 - 32a}{16}$$

$$3a^2 - 32a + 280 = 16.\left(\frac{7}{2}\right)^2 = 4\times49$$

$$3a^2 - 32a + 84 = 0$$

## MOCK TEST

**1.** Assume that x is not even (i.e. it is odd) so that $x^2$ is not even (i.e., it is odd) x is odd so that you can write $x = 2\times k + 1$ for some integer k then

$x^2 = 4k^2 + 4k + 1$

$= 2\times(2k^2 + 2k) + 1$

Which is clearly odd. And its Convers is also rule.

**2.** Each of (a), (b), (c) is equivalent to $p \leftrightarrow q$.

**3.** Negation of p v q is $(\sim p) \wedge (\sim q)$.

**4.** $p \to (q \to p) \equiv \sim p \vee (q \to p)$

$\equiv (\sim p) \vee (\sim q \vee p)$

$\equiv (\sim q) \vee (p \vee \sim p)$

$\equiv (\sim q) \vee T = T$

$\therefore p \to (q \to p)$ is a tautology.

Also $p \to (p \vee q) \equiv \sim p \vee (p \vee q)$

$\equiv (\sim p \vee p) \vee q \equiv T \vee q = T$

$\therefore p \to (p \vee q)$ is also a tautology.

Thus, $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

**5.** $p \to (q \vee r) \equiv (\sim p) \vee (q \vee r)$

$\equiv (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv (p \to q) \vee (p \to r)$

Also, $p \to (q \vee r) \equiv (\sim p) \vee (q \vee r) \equiv (\sim p \vee q) \vee r$

$\equiv \sim(p \wedge (\sim q)) \vee r \equiv p \wedge (\sim q) \to r$

Interchanging the roles of q and r in the above paragraph, we find

$p \to (p \vee r) \equiv p \wedge (\sim q) \to r \equiv p \wedge (\sim r) \to q$

For p = T, q = F, r = F, $p \to (q \vee r)$ is F, but $(p \wedge q) \to (p \vee r) \vee (q \wedge r)$ is T.

$\therefore p \to (q \vee r)$

and $p \wedge q \to (p \wedge r) \vee (q \wedge r)$

Are not equivalent

**6.** $\sim(p \to q) \equiv \sim(\sim p \vee q) \equiv \sim(\sim p) \wedge (\sim q)$ [De Morgan's Laws]

$\therefore \sim(p \to q) \equiv p \wedge (\sim q)$

**7.** Let p : I become a teacher

q : I will open a school

Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

**8.** Contrapositive of $p \to (q \to r)$ is

$\sim(q \to r) \to \sim p$

$\equiv \sim[\sim(q \to r)] \vee (\sim p)$

$\equiv (q \to r) \vee (\sim p) \equiv (\sim p) \vee (q \to r) \equiv p \to (q \to r)$

**9.** $p \to r = p \to (p \to q)$

$\equiv (\sim p) \vee (p \to q) \equiv (\sim p) \vee [(\sim p) \vee q]$

$\equiv [(\sim p) \vee (\sim p)] \vee q$

$\equiv (\sim p) \vee q \equiv p \to q = r$

Thus, statement-I is not a tautology.

**10.**

| P | ~ p | p ⇒ ~ p | ~ p ⇒ p | (p ⇒ ~ p) ∧ (~ p ⇒ p) |
|---|---|---|---|---|
| T | F | F | T | F |
| F | T | T | F | F |

Clearly, (p ⇒ ~ p) ∧ (~ p ⇒ p) is a contradiction.

**11.** The event follows binomial distribution with
$n = 5, = 3/6 = 1/2.$
$q = 1 - p = 1/2;$
$\therefore$ Variance = npq = 5/4

**12.** $\sum x = 170, \sum x^2 = 2830$
$\sum x^1 = 170 - 20 + 30 = 180$
Increase in $\sum x^2 = 900 - 400 = 500$ then
$\sum x^2 = 2830 - 400 + 900 = 3330$

$$\text{Variance} = \frac{1}{2}\sum x^{1^2} - \left[\frac{1}{2}\sum x^1\right]^2$$

$$= \frac{1}{15}\times 3330 - \left(\frac{1}{15}\times 180\right)^2$$

$= 222 - 144 = 78.$

**13.** (1) np = 4
npq = 2

$$q = \frac{1}{2},\ p = \frac{1}{2}$$

n = 8

$$p(x=1) = {}^8C_1\left(\frac{1}{2}\right)^8 = \frac{1}{32}$$

**14.** (C) $x_i = a$ for $i = 1, 2, ...., n$ and

$x_i = -a$ for $i = n, ......, 2n$

$$\text{S.D.} = \sqrt{\frac{1}{2n}\sum_{i=1}^{2n}(x_i - \bar{x})^2}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{1}{2n}\sum_{i=1}^{2n} x_i^2} \qquad \left(\text{Since} \sum_{i=1}^{2n} x_i = 0\right)$$

$$\Rightarrow 2 = \sqrt{\frac{1}{2n}2na^2} \qquad \Rightarrow |a| = 2$$

**15.** (D) Mode + 2Mean = 3Median
$\Rightarrow$ Mode = 3 × 22 – 2 × 21 = 66 – 42 = 24

**16.** (A) $\sigma_x^2 = \dfrac{\sum d_i^2}{n}$ (Here deviations are taken from the mean)
Since A and B both has 100 consecutive integers, therefore both have same standard deviation and hence the variance.

$$\therefore \frac{V_A}{V_B} = 1 \qquad \left(\text{As } \sum d_i^2 \text{ is same in both the cases}\right)$$

**17.** (D)
Mean of a, b, 8, 5, 10 is 6

$$\Rightarrow \frac{a+b+8+5+10}{5} = 6 \Rightarrow a + b = 7 ... (1)$$

Given that variance is 6.8

$$\therefore \text{Variance} = \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$$

$\Rightarrow a^2 + b^2 = 25$
$a^2 + (7 - a)^2 = 25$ (using (1))
$\Rightarrow a^2 - 7a + 12 = 0$
$\therefore$ a = 4.3 and b = 3.4

**18.** (D) Statement-II is true
Statement-I :
Sum of even natural numbers = n(n + 1)

$$\text{Mean } (x) = \frac{n(n+1)}{n} = n + 1$$

$$\text{Variance} = \left[\frac{1}{n}\sum (x_i)^2\right] - (\bar{x})^2$$

$$= \frac{1}{n}\left[2^2 + 4^2 + .... + (2n)^2\right] - (n+1)^2$$

$$= \frac{1}{n}2^2[1^2 + 2^2 + .... + n^2] - (n+1)^2$$

$$= \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{(n+1)[2(2n+1) - 3(n+1)]}{3}$$

$$= \frac{(n+1)[4n + 2 - 3n - 3]}{3} = \frac{(n+1)(n-1)}{3} = \frac{n^2 - 1}{3}$$

$\therefore$ Statement 1 false

**19.** (C) Mean $(\overline{x}) = \dfrac{\text{sum of quantities}}{n} = \dfrac{\frac{n}{2}(a+1)}{n}$

$$=\frac{1}{2}[1+1+100d]=1+50d$$

$$\text{M.D} = \frac{1}{n}\sum|x_i - \overline{x}|$$

$$\Rightarrow 255 = \frac{1}{101}[50d+49d+.....d+0+d+......+50d]$$

$$= \frac{2d}{101}\left[\frac{50\times 51}{2}\right]$$

$$\Rightarrow d = \frac{255\times 101}{50\times 51} = 10.1$$

**20.** (A) $\sigma_x^2 = 4;\ \sigma_y^2 = 5 \qquad \overline{x} = 2;\ \overline{y} = 4$

$$\frac{\sum x_i}{5} = 2, \sum x_i = 10, \sum y_i = 20,$$

$$\sigma_x^2 = (\frac{1}{2}\sum x_i^2) - (\overline{x})^2 = \frac{1}{5}(\sum y_i^2) - 16$$

$$\sum x_i^2 = 40; \sum y_i^2 = 105$$

$$\sigma_i^2 = \frac{1}{10}(\Sigma x_i^2 + \Sigma y_i^2) - \left(\frac{\overline{x}+\overline{y}}{2}\right)^2$$

$$= \frac{1}{10}(40+105) - 9 = \frac{145-90}{10} = \frac{55}{10} = \frac{11}{2}$$

**21.** (B) $a_1 = \sqrt{7} < 7$ Let $a_m < 7$

then $a_{m+1} = \sqrt{7 + a_m}$

$$\Rightarrow a_{m+1}^2 = 7 + a_m < 7 + 7 < 14.$$

$$\Rightarrow a_{m+1} < \sqrt{14} < 7;$$

So $a_n < 7 \forall n \therefore a_n > 3.$

**22.** (D) $S(k) = 1 + 3 + 5 + ..... + (2k-1) = 3 + k^2$

$S(K+1) = 1 + 3 + 5 + ...... + (2k-1) + (2k+1)$

$= (3 + k^2) + 2k + 1 = k^2 + 2k + 4$ [ from $S(k) = 3 + k^2$]

$= 3 + (k^2 + 2k + 1) = 3\ (k+1)^2 = S(k+1).$

Although S(k) in itself is not true but it considered true will always imply towards S(k + 1).

**23.** (A) By the principle of mathematical induction (A) is true.

**24.** (C) $P(n) = \dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + ... + \dfrac{1}{\sqrt{n}}$

$$P(2) = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$$

Let us assume that

$P(k) = \dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + ... + \dfrac{1}{\sqrt{k}} > \sqrt{k}$ is true

$\therefore P(k+1) = \dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + ... + \dfrac{1}{\sqrt{k}} + \dfrac{1}{\sqrt{k+1}} > \sqrt{k-1}$

has to be true

$$\text{L.H.S.} > \sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k(1+k)+1}}{\sqrt{k+1}}$$

Since $\sqrt{k(k+1)} > k \qquad (\forall\ k \geq 0)$

$$\therefore \frac{\sqrt{k(1+k)+1}}{\sqrt{k+1}} > \frac{k+1}{\sqrt{k+1}} = \sqrt{k+1}$$

Let $P(n) = \sqrt{n(n+1)} < n+1$

Statement-I is correct $P(2) = \sqrt{2\times 3} < 3$

If $P(k) = \sqrt{k(k+1)} < (k+1)$ is true

Now $P(k+1) = \sqrt{(k+1)(k+2)} < k+2$

Since $(k+1) =< k+2$

$$\therefore \sqrt{(k+1)(k+2)} < k+2$$

Hence statement- II is not a correct explanation of Statement - I

**25.** (B) $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots + (361 + 380 + 400)$

$$= \sum \frac{[r^2 + r(r+1) + (r+1)^2](r+1-r)}{(r+1)-r}$$

$$= 1 + \sum_{r=1}^{19} \frac{(r+1)^3 - r^3}{1}$$

$= (1 + 2^3 + 3^3 + 4^3 + \ldots + 20^3) - (1^3 + 2^3 + 3^3 + \ldots 19^3)$

$$= \frac{20^2(21)^2}{4} - \frac{19^2(20)^2}{4}$$

$$= \frac{(20)^2}{4}[441 - 361] = 8000$$

**26.** (C) For $n = 2$, $a_2 = 2^{2^2} + 1 = 17 = 10 + 7$

let $a_k = 2^{2^k} + 1 = 10m + 7$ be true where $k > 1$, $m \in N$ ..... (1)

Now

$$a_{k+1} - 2^{2^{k+1}} + 1 = \left(2^{2^k}\right)^2 + 1 = (10m+6)^2 - 1$$

(by (1))

$- 10(10m^2 + 12m + 3) + 7$

$\therefore$ Digit of one's place of $a_n$ is 7.

**27.** For $n = 1$, by $p(n): \cos\theta \cos 2\theta \cos 4\theta \ldots \cos[(2^{n-1})\theta]$

$\therefore$ P(1) : $\cos\theta$ in option (a) $n = 1$ we get $\cos\theta$.

$\therefore$ Ans. (a) $\sin 2^n\theta / 2^n \sin\theta$

**28.** (B) For $n = 1$,
$1/(1.2.3) = 1/6$
Now, for $n = 1$, value of only option (b)
$n(n+3)/4(n+1)(n+2)$ is $1/6$
$\therefore$ $n(n+3)/4(n+1)(n+2)$

**29.** (C) For every $n \in N$, $P(n) : a^n - b^n$
$P(1) = a - b$ and $P(2) = a^2 - b^2 = (a - b)(a + b)$
$\therefore$ $a - b$

**30.** $p(n) : n^2 + n + 1 = n(n+1) + 1$
P (1) : 3 which is true.
$p(n) : n^2 + n + 1 = n(n+1) + 1$
which is always odd number $\therefore$ $\forall\, n \in N$