INDEFINITE INTEGRATION

EXERCISE # 1

Questions based on Standard Integral

Q.1 $\int \frac{4+5\sin x}{\cos^2 x}$ dx equals -

(A) 4 tan x – sec x + c

(B) 4 tan x + 5 sec x + c

(C) 9 tan x + c

(D) None of these

Sol.[B] I = $\int (4\sec^2 x + 5\sec x \tan x)\,dx$

= 4 tan x + 5 sec x + c

Q.2 $\int \frac{(a^x - b^x)^2}{a^x b^x}$ dx equals -

(A) $(a/b)^x + 2x + c$ (B) $(b/a)^x + 2x + c$

(C) $(a/b)^x - 2x + c$ (D) None of these

Sol. [D] I = $\int \frac{a^{2x} + b^{2x} - 2a^x b^x}{a^x b^x}$

$= \int \left[\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x - 2\right] dx$

$= \left(\frac{a}{b}\right)^x \lambda n\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)^x \lambda n\left(\frac{b}{a}\right) - 2x + c$

Questions based on Integration by substitution

Q.3 $\int \frac{dx}{e^x + e^{-x}}$ equals -

(A) $\log(e^x + e^{-x}) + c$ (B) $\log(e^x - e^{-x}) + c$

(C) $\tan^{-1}(e^x) + c$ (D) $\tan^{-1}(e^{-x}) + c$

Sol. [C] I = $\int \frac{e^x}{e^{2x}+1}$ dx

Let $e^x = t \Rightarrow e^x\,dx = dt$

I = $\int \frac{1}{t^2+1}$ dt = $\tan^{-1}t + c$

= $\tan^{-1} e^x + c$

Q.4 $\int \sqrt{\frac{a+x}{a-x}}$ dx is equal to -

(A) $\sin^{-1}(x/a) - \sqrt{a^2 - x^2} + c$

(B) $\cos^{-1}(x/a) - \sqrt{a^2 - x^2} + c$

(C) $a\sin^{-1}(x/a) - \sqrt{a^2 - x^2} + c$

(D) $a\cos^{-1}(x/a) - \sqrt{a^2 - x^2} + c$

Sol.[C] I = $\int \sqrt{\frac{a+x}{a-x}}$ dx

Put x = a sin 2θ ⇒ dx = 2a cos 2θ dθ

I = $\int \left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$. 2a cos 2θ dθ

= $\int 2a(1 + \sin 2\theta)\,d\theta$

= 2aθ – a cos 2θ + c

= $a\sin^{-1}\frac{x}{a} - \sqrt{a^2 - x^2} + c$

Questions based on Integration by parts

Q.5 $\int (\log x)^2 dx$ equals -

(A) $(x \log x)^2 - 2x \log x + 2x + c$

(B) $x(\log x)^2 - 2x \log x + 2x + c$

(C) $x(\log x)^2 + 2x \log x + 2x + c$

(D) None of these

Sol. [B] I = $\int 1.(\log x)^2$ dx

Take 1 is II[nd] function then

I = $x(\log x)^2 - \int x.\frac{2\log x}{x}$ dx

= $x(\log x)^2 - 2\int \log x\,dx$

Again by parts

I = $x(\log x)^2 - 2[x\log x - \int x.\frac{1}{x}\,dx]$

= $x(\log x)^2 - 2x\log x + 2x + c$

Q.6 $\int e^x\left(\frac{\sin x + \cos x}{\cos^2 x}\right)$dx is equal to -

(A) $e^x \cos x + c$ (B) $e^x \sec x + c$

(C) $e^x \sin x + c$ (D) None of these

Sol. [B] $I = \int e^x (\sec x \tan x + \sec x)dx$

$\because \frac{d}{dx} \sec x = \sec x \tan x$

$I = e^x \sec x + c$

Q.7 $\int \left(\log (\log x) + \frac{1}{(\log x)^2} \right) dx =$

(A) $x \log \log x + \frac{x}{\log x} + c$

(B) $x \log \log x + \frac{2x}{\log x} + c$

(C) $x \log \log x - \frac{x}{\log x} + c$

(D) None of these

Sol.[C] $I = \int \frac{x}{x} \left(\log(\log x). + \frac{1}{(\log x)^2} \right) dx$

Let $\log x = t \Rightarrow \frac{1}{x} dx = dt$

and $x = e^t$

$I = \int e^t \left(\log t + \frac{1}{t^2} \right) dt$

$= \int e^t \left(\log t + \frac{1}{t} \right) dt - \int e^t \left(\frac{1}{t} - \frac{1}{t^2} \right) dt$

$= e^t \log t - e^t . \frac{1}{t} + c$

$= x \log (\log x) - \frac{x}{\log x} + c$

Q.8 $\int e^x \frac{(1-x)^2}{(1+x^2)^2} dx =$

(A) $\frac{e^x}{x+1} + c$ (B) $\frac{e^x}{x^2+1} + c$

(C) $\frac{e^x}{x^2-1} + c$ (D) None of these

Sol.[B] $\int e^x \left(\frac{1}{1+x^2} + \frac{-2x}{(1+x^2)^2} \right) dx$

$= \frac{e^x}{1+x^2} + c$

$\because \frac{d}{dx} \left(\frac{1}{1+x^2} \right) = -\frac{2x}{(1+x^2)^2}$

Questions based on Integration of rational function

Q.9 $\int \frac{3x+1}{2x^2-2x+3} dx$ equals -

(A) $\frac{1}{4} \log(2x^2-2x+3) - \frac{\sqrt{5}}{2} \tan^{-1} \left(\frac{2x-1}{\sqrt{5}} \right) + C$

(B) $\frac{3}{4} \log(2x^2-2x+3) + \frac{\sqrt{5}}{2} \tan^{-1} \left(\frac{2x-1}{\sqrt{5}} \right) + C$

(C) $\frac{3}{4} \log(2x^2-2x+3) + \frac{\sqrt{5}}{2} \tan^{-1} \left(\frac{4x-2}{5} \right) + C$

(D) None of these

Sol.[B] $I = \int \frac{3x+1}{2x^2-2x+3} dx$

$= \frac{3}{4} \int \frac{4x-2}{2x^2-2x+3} dx + \frac{5}{2} \int \frac{1}{2x^2-2x+3} dx$

$= \frac{3}{4} \log (2x^2 - 2x + 3) + \frac{5}{4} \int \frac{1}{\left(x - \frac{1}{2} \right)^2 + \frac{5}{4}} dx$

$= \frac{3}{4} \log (2x^2 - 2x + 3) + \frac{\sqrt{5}}{2} \tan^{-1} \left(\frac{2x-1}{\sqrt{5}} \right) + c$

Q.10 $\int \frac{x^3 - x - 2}{(1-x^2)} dx =$

(A) $\log \left(\frac{x+1}{x-1} \right) - \frac{x^2}{2} + c$

(B) $\log \left(\frac{x-1}{x+1} \right) + \frac{x^2}{2} + c$

(C) $\log \left(\frac{x+1}{x-1} \right) + \frac{x^2}{2} + c$

(D) $\log \left(\frac{x-1}{x+1} \right) - \frac{x^2}{2} + c$

Sol.[D] $I = \int \frac{x(x^2-1)-2}{(1-x^2)} dx = \int \left(\frac{2}{x^2-1} - x \right) dx$

$= \log \frac{x-1}{x+1} - \frac{x^2}{2} + c$

Q.11 The value of $\int \frac{x^2-1}{x^4+1} dx$ equals -

(A) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)+C$

(B) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+C$

(C) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+C$

(D) None of these

Sol.[A] $I = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}dx$

Let $x+\frac{1}{x}=t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int\frac{dt}{t^2-2} = \frac{1}{2\sqrt{2}}\log\frac{t-\sqrt{2}}{t+\sqrt{2}}+c$

$= \frac{1}{2\sqrt{2}}\log\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)+c$

Questions based on Integration of irrational function

Q.12 $\int\frac{dx}{\sqrt{5x-6-x^2}}$ equals-

(A) $\sin^{-1}(2x+5)+c$

(B) $\cos^{-1}(2x+5)+c$

(C) $\sin^{-1}(2x-5)+c$

(D) None of these

Sol.[C] $I = \int\frac{dx}{\sqrt{\frac{1}{4}-\left(x-\frac{5}{2}\right)^2}} = \sin^{-1}(2x-5)+c$

Q.13 $\int\frac{2x+3}{\sqrt{x^2+1}}dx$ is equal to -

(A) $2\sqrt{x^2+1}+3\lambda n\,|x+\sqrt{x^2+1}|+c$

(B) $\sqrt{x^2+1}+3\lambda n\,|x+\sqrt{x^2+1}|+c$

(C) $2\sqrt{x^2+1}+3\lambda n\,|x+\sqrt{x^2-1}|+c$

(D) None of these

Sol.[C] $I = \int\frac{2x}{\sqrt{x^2+1}}dx + \int\frac{3}{\sqrt{x^2+1}}dx$

$= 2\sqrt{x^2+1}+3\sinh^{-1}x+c$

Questions based on Integration of Trigonometric function

Q.14 $\int\frac{dx}{3+\sin^2 x}$ equals -

(A) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}\right)+c$

(B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right)+c$

(C) $\frac{1}{2}\tan^{-1}(2\tan x)+c$

(D) None of these

Sol.[A] $I = \int\frac{\sec^2 x}{4\tan^2 x+3}dx$

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$I = \frac{1}{4}\int\frac{dt}{t^2+\frac{3}{4}} = \frac{1}{4}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t}{\sqrt{3}}\right)+c$

$= \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{3}}\right)+c$

Q.15 The value of $\int\frac{\sin x}{\sin x-\cos x}dx$ equals-

(A) $\frac{1}{2}x+\frac{1}{2}\log(\sin x-\cos x)+C$

(B) $\frac{1}{2}x-\frac{1}{2}\log(\sin x-\cos x)+C$

(C) $x+\log(\sin x+\cos x)+C$

(D) None of these

Sol.[A] $I = \frac{1}{2}\int\frac{2\sin x}{\sin x-\cos x}dx$

$= \frac{1}{2}\int\frac{(\sin x-\cos x)+(\sin x+\cos x)}{\sin x-\cos x}dx$

$= \frac{1}{2}\int\left[1+\frac{\sin x+\cos x}{\sin x-\cos x}\right]dx$

$= \frac{1}{2}x+\frac{1}{2}\log(\sin x-\cos x)+c$

Q.16 $\int\frac{dx}{a\sin x+b\cos x}$ equals -

(A) $\frac{1}{\sqrt{a^2+b^2}}\log\left[\tan\frac{1}{2}\left(x+\tan^{-1}\frac{b}{a}\right)\right]+C$

(B) $\frac{1}{\sqrt{a^2+b^2}} \log \left[\tan\left(x+\tan^{-1}\frac{b}{a}\right)\right] + C$

(C) $\frac{1}{\sqrt{a^2+b^2}} \log \left[\tan\frac{1}{2}\left(x-\tan^{-1}\frac{b}{a}\right)\right] + C$

(D) None of these

Sol.[A] $I = \frac{1}{\sqrt{a^2+b^2}} \int \frac{1}{\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x} dx$

$= \frac{1}{\sqrt{a^2+b^2}} \int \text{cosec}\,(x+\alpha)\,dx \left[\frac{b}{\sqrt{a^2+b^2}} = \sin\alpha\right]$

$= \frac{1}{\sqrt{a^2+b^2}} \log [\tan \frac{1}{2} (x+\alpha)] + c$

$= \frac{1}{\sqrt{a^2+b^2}} \log [\tan \frac{1}{2} (x+\tan^{-1}\frac{b}{a})] + c$

Questions based on **Some special integrals**

Q.17 Evaluate $\int \frac{1}{(x-3)\sqrt{x+1}} dx$ -

(A) $-\frac{1}{2}\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + C$

(B) $\frac{1}{2}\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + C$

(C) $\frac{1}{3}\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| - C$

(D) None of these

Sol.[B] $I = \int \frac{1}{(x-3)\sqrt{x+1}} dx$

Let $x + 1 = t^2 \Rightarrow dx = 2t\,dt$

$I = 2\int \frac{1}{t^2-4} dt = 2 \,.\, \frac{1}{4} \log\left|\frac{t-2}{t+2}\right| + c$

$= \frac{1}{2} \log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c$

Q.18 Evaluate $\int \frac{1}{(x^2-4)\sqrt{x+1}} dx$ -

(A) $\frac{1}{4\sqrt{3}} \lambda n\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}(\sqrt{x+1}) + c$

(B) $-\frac{1}{4}\tan^{-1}(\sqrt{x+1}) + \frac{1}{4}\lambda n\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| + c$

(C) $-\frac{1}{4\sqrt{3}}\tan^{-1}(\sqrt{x+1}) + \frac{1}{4}\lambda n\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + c$

(D) None of these

Sol.[D] $I = \int \frac{1}{(x^2-4)\sqrt{x+1}} dx$

Let $x + 1 = t^2 \Rightarrow dx = 2t\,dt$

$I = \int \frac{2}{((t^2-1)^2-4)} dt = 2\int \frac{dt}{(t^2-3)(t^2+1)}$

$= \frac{2}{4}\int\left(\frac{1}{t^2-3} - \frac{1}{t^2+1}\right) dt$

$= \frac{1}{2}\left[\frac{1}{2\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| - \tan^{-1} t\right]$

$= \frac{1}{4\sqrt{3}}\log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}\sqrt{x+1} + c$

Q.19 Evaluate $\int \frac{1}{(x-1)\sqrt{x^2+4}} dx$ -

(A) $-\frac{1}{\sqrt{5}}\log\left|\frac{1}{x-1} + \frac{1}{5} + \sqrt{\frac{x^2+4}{5(x-1)^2}}\right| + C$

(B) $\frac{1}{\sqrt{5}}\log\left|\frac{1}{x-1} + \frac{1}{5} + \sqrt{\frac{x^2+4}{5(x-1)^2}}\right| + C$

(C) $\frac{1}{\sqrt{5}}\log\left|\frac{1}{x+1} - \frac{1}{5} - \sqrt{\frac{x^2+4}{5(x-1)^2}}\right| + C$

(D) $\frac{1}{\sqrt{5}}\log\left|\frac{1}{x+1} + \frac{1}{5} - \sqrt{\frac{x^2+4}{5(x-1)^2}}\right| - C$

Sol.[A] $I = \int \frac{1}{(x-1)\sqrt{x^2+4}} dx$

Put $x - 1 = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$I = \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}+1\right)^2+4}} = -\int \frac{dt}{\sqrt{5t^2+2t+1}}$

$= -\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{\left(t+\frac{1}{5}\right)^2+\frac{4}{25}}}$

$= -\frac{1}{\sqrt{5}} \log\left|\left(t+\frac{1}{5}\right) + \sqrt{\left(t+\frac{1}{5}\right)^2 + \frac{4}{25}}\right| + c$

$= -\frac{1}{\sqrt{5}} \log\left|\frac{1}{x-1} + \frac{1}{5} + \sqrt{\frac{1}{(x-1)^2} + \frac{2}{5(x-1)} + \frac{1}{5}}\right| + c$

$$= -\frac{1}{\sqrt{5}}\log\left|\frac{1}{x-1}+\frac{1}{5}+\sqrt{\frac{x^2+4}{5(x-1)^2}}\right| + c$$

Q.20 Evaluate $\int \frac{\sqrt{1+x^2}}{1-x^2}dx$ -

(A) $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| + \log|x+\sqrt{1+x^2}| + C$

(B) $\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| + \log|x+\sqrt{1+x^2}| - C$

(C) $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| - \log|x-\sqrt{1+x^2}| - C$

(D) $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| - \log|x+\sqrt{1+x^2}| + C$

Sol.[D] $I = \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2}}dx$

$$= 2\int \frac{1}{(1-x^2)\sqrt{1+x^2}}dx - \int \frac{1}{\sqrt{1+x^2}}dx$$

$$= 2I_1 - \log\left|x+\sqrt{1+x^2}\right| + c$$

$$I_1 = \int \frac{1}{(1-x^2)\sqrt{1+x^2}}dx$$

put $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt$

$$= -\int \frac{t\,dt}{(t^2-1)\sqrt{t^2+1}}$$

put $t^2 + 1 = u^2 \Rightarrow 2t\,dt = 2u\,du$

$$I_1 = -\int \frac{u\,du}{(u^2-2)u} = -\int \frac{du}{u^2-2}$$

$$= -\frac{1}{2\sqrt{2}}\log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right|$$

$$= -\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right|$$

$$I = -\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right|$$

$$-\log\left|x+\sqrt{1+x^2}\right| + c$$

Questions based on Euler's substitutions

Q.21 $\int \frac{dx}{x+\sqrt{x^2+2x+2}} =$

(A) $\log_e|x+1+\sqrt{x^2+2x+2}| + \frac{2}{x+2+\sqrt{x^2+2x+2}} + C$

(B) $\log_e|x+1+\sqrt{x^2+2x+2}| - \frac{2}{x+2+\sqrt{x^2+2x+2}} + C$

(C) $\log_e|x+1-\sqrt{x^2+2x+2}| + \frac{2}{x+2+\sqrt{x^2+2x+2}} + C$

(D) None of these

Sol. [A]

Q.22 $\int \frac{dx}{\sqrt{1-x^2}-1} =$

(A) $\frac{1+\sqrt{1-x^2}}{x} - 2\tan^{-1}\sqrt{\frac{1-x}{1+x}} + C$

(B) $\frac{1+\sqrt{1-x^2}}{x} - 2\tan^{-1}\sqrt{\frac{1+x}{1-x}} + C$

(C) $\frac{1+\sqrt{1-x^2}}{x} + 2\tan^{-1}\sqrt{\frac{1+x}{1-x}} + C$

(D) $\frac{1+\sqrt{1-x^2}}{x} + 2\tan^{-1}\sqrt{\frac{1-x}{1+x}} + C$

Sol. [C]

Q.23 $\int \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{\sqrt{1+x^2}}dx =$

(A) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{30} + C$

(B) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{15} + C$

(C) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{30x} + C$

(D) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{15x} + C$

Sol. [B]

Questions based on Miscellaneous Forms

Q.24 If x = f″ (t) cos t + f′(t) sin t, y = – f″(t) sin t + f′(t) cos t, then $\int\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]^{1/2}$ dt is equal to -

(A) f ′(t) + f ″ (t) + c (B) f ″(t) + f ″ (t) + c
(C) f(t) + f ″ (t) + c (D) f ′ (t) – f ″ (t) + c

Sol.[C] x = f " (t) cost + f '(t) sin t

y = – f " (t) sin t + f '(t) cos t

$\frac{dx}{dt}$ = f '''(t) cost– f "(t) sint +f "(t) sint +f "(t) cost

= f '''(t) cost– f '(t) cos t = (f ''' (t) + f '(t)) cos t

Similarly $\frac{dy}{dt}$ = – (f '''(t) + f '(t)) sin t

$$\int\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]^{1/2} dt = \int[f'(t)+f'''(t)]\ dt$$

= f(t) + f "(t) + c

Q.25 $\int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$ is equal to -

(A) $e^{x/2}$ sin x/2 + c (B) $e^{x/2}$ cos x/2 + c
(C) $\sqrt{2}\ e^{x/2}$ sin x/2 + c (D) $\sqrt{2}\ e^{x/2}$ cos x/2 + c

Sol.[C] $\int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

Put $\frac{x}{2}$ = t ⇒ dx = 2dt

$\Rightarrow 2\int e^t \sin\left(\frac{\pi}{4}+t\right)\ dt$

$\Rightarrow \sqrt{2}\int e^t$ (sint + cost) dt

$\Rightarrow \sqrt{2}\ e^t$ sin t + c $\Rightarrow \sqrt{2}\ e^{x/2}\sin\frac{x}{2}$ + c

Q.26 $\int\frac{dx}{\sin(x-a)\cos(x-b)}$ is equal to -

(A) cos (a – b) log $\frac{\sin(x-a)}{\cos(x-b)}$ + c

(B) sec (a – b) log $\frac{\sin(x-a)}{\cos(x-b)}$ + c

(C) sin (a – b) log $\frac{\cos(x-a)}{\sin(x-b)}$ + c

(D) cosec (a – b) log $\frac{\cos(x-a)}{\sin(x-b)}$ + c

Sol.[B] $\frac{1}{\cos(a-b)}\int\frac{\cos[(x-b)-(x-a)]}{\sin(x-a)\cos(x-b)}dx$

= sec (a – b) $\int$ [cot (x–a)+tan(x–b)]dx

= sec(a–b) [logsin(x–a) – logcos(x–b)] + c

= sec(a–b) log $\frac{\sin(x-a)}{\cos(x-b)}$ + c

Q.27 If $\int\frac{1}{\sqrt{(x-\alpha)(\beta-x)}}$ dx (α < x < β) is equal to

(i) $2\sin^{-1}\sqrt{(x-\alpha)/(\beta-\alpha)}$

(ii) $2\sin^{-1}\sqrt{(\beta-x)/(\beta-\alpha)}$

(iii) $-2\cos^{-1}\sqrt{(\beta-x)/(\beta-\alpha)}$

(iv) $\sin^{-1}\frac{\left[x-\frac{1}{2}(\alpha+\beta)\right]}{\left[\frac{1}{2}(\beta-\alpha)\right]}$

Then which of the above are correct -

(A) (i) only (B) (i) and (ii) only
(C) (ii) & (iii) only (D) All

Sol.[D]

Q.28 If $\int f(x)\,dx$ = f(x), then $\int[f(x)]^2$ dx is equal to

(A) $\frac{1}{2}[f(x)]^2$ (B) $[f(x)]^3$

(C) $\frac{1}{3}[f(x)]^3$ (D) $[f(x)]^2$

Sol.[A] Consider a function f(x) = e^x

So that $\int e^x$ dx = e^x

$\Rightarrow \int[f(x)]^2$ dx = $\int e^{2x}$ dx

= $\frac{e^{2x}}{2} = \frac{1}{2}[f(x)]^2$

Q.29 $\int u\frac{d^2v}{dx^2}dx - \int v\frac{d^2u}{dx^2}dx$ is equal to -

(A) uv + c (B) $u\frac{dv}{dx} - v\frac{du}{dx}$ + c

(C) $2\frac{du}{dx}+\frac{dv}{dx}$ + c (D) $u\frac{dv}{dx} + v\frac{du}{dx}$ + c

Sol.[B] $\int u\frac{d^2v}{dx^2}dx - \int v\frac{d^2u}{dx^2}dx$

$= u\frac{dv}{dx} - \int\frac{du}{dx}\frac{dv}{dx}dx - v\frac{du}{dx} + \int\frac{dv}{dx}\frac{du}{dx}dx$

$= u\,.\frac{dv}{dx} - v\frac{du}{dx}$ + c

Q.30 $\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}dx$ is equal to -

(A) $\cos x - \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(B) $\cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

(C) $\cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(D) $\cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

Sol.[B] $\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}\,dx$

$= \int 2\sin x(\cos 2x + \cos x)\,dx$

$= \int (2\sin x\cos 2x + 2\sin x\cos x)\,dx$

$= \int (\sin 3x - \sin x + \sin 2x)\,dx$

$= -\frac{\cos 3x}{3} + \cos x - \frac{1}{2}\cos 2x + C$

Q.31 $\int e^{\tan\theta}(\sec\theta - \sin\theta)\,d\theta$ is equal to -

(A) $-e^{\tan\theta}\sin\theta + c$ (B) $e^{\tan\theta}\sin\theta + c$

(C) $e^{\tan\theta}\sec\theta + c$ (D) $e^{\tan\theta}\cos\theta + c$

Sol.[D] $\int e^{\tan\theta}(\sec\theta - \sin\theta)\,d\theta$

$= \int e^{\tan\theta}\sec\theta\,d\theta - \int \underset{I}{e^{\tan\theta}}\,\underset{II}{\sin\theta}\,d\theta$

$= \int e^{\tan\theta}\sec\theta\,d\theta + e^{\tan\theta}\cos\theta -$

$\int e^{\tan\theta}\sec^2\theta\cos\theta\,d\theta + C$

$= e^{\tan\theta}\cos\theta + C$

Q.32 $\int \sin x.\cos x.\cos 2x.\cos 4x.\cos 8x.\cos 16x\,dx$ is equal to -

(A) $\frac{\sin 16x}{1024} + c$ (B) $-\frac{\cos 32x}{1024} + c$

(C) $\frac{\cos 32x}{1096} + c$ (D) $-\frac{\cos 32x}{1096} + c$

Sol.[B] $\int \sin x\cos x\cos 2x\cos 4x\cos 8x\cos 16x\,dx$

$= \frac{1}{2}\int \sin 2x\cos 2x\cos 4x\cos 8x\cos 16x\,dx$

$= \frac{1}{4}\int \sin 4x\cos 4x\cos 8x\cos 16x\,dx$

$= \frac{1}{8}\int \sin 8x\cos 8x\cos 16x\,dx$

$= \frac{1}{16}\int \sin 16x\cos 16x\,dx = \frac{1}{32}\int \sin 32x\,dx$

$= \frac{-\cos 32x}{1024} + C$

Q.33 $\int\left\{\ln(1+\sin x) + x\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}dx$ is equal to -

(A) x ln (1 + sin x) + C

(B) ln (1 + sin x) + C

(C) – x ln (1 + sin x) + C

(D) ln (1 – sin x) + C

Sol. [A] $\int \lambda n(1+\sin x)\,dx + \int x\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)dx$

$= x\,\lambda n\,(1 + \sin x) - \int x.\frac{\cos x}{1+\sin x}\,dx$

$+ \int x\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)dx + C$

$= x\,\lambda n\,(1 + \sin x) + C$

True or false type questions

Q.34 If $\int \frac{x\tan^{-1}x}{\sqrt{1+x^2}}dx = \sqrt{1+x^2}\,(\tan^{-1}x) +$ $K\log\left(x + \sqrt{x^2+1}\right) + c$ then K must be 1.

Sol.[False] $\int \frac{x}{\sqrt{1+x^2}}.\tan^{-1}x\,dx$

$= \sqrt{1+x^2}\,\tan^{-1}x - \int \frac{1}{\sqrt{1+x^2}}dx$

$= \sqrt{1+x^2}\,\tan^{-1}x - \log\,(x + \sqrt{1+x^2}\,) + c$

$\Rightarrow K = -1$

Q.35 If $\int \frac{dx}{\sqrt{2-3x-x^2}} = fog(x) + C$, then $g(x) = \frac{2x+3}{\sqrt{17}}$ and $f(x) = \cos^{-1}x$.

Sol.[False] $\int \frac{dx}{\sqrt{\frac{17}{4} - \left(x + \frac{3}{2}\right)^2}}$

$= \sin^{-1}\frac{\left(x+\frac{3}{2}\right)}{\frac{\sqrt{17}}{2}} = \sin^{-1}\frac{2x+3}{\sqrt{17}}$

$\Rightarrow f(x) = \sin^{-1}x,\ g(x) = \frac{2x+3}{\sqrt{17}}$

Fill in the blanks type questions

Q.36 If a function f is such that $f''(x) = \sec^4 x + 4$, $f'(0) = 0$ and $f(0) = 0$, then the function is..................

Sol. $f'(x) = \int f''(x)\,dx = \int (\sec^4 x + 4)\,dx$

$= \int [\sec^2 x(1 + \tan^2 x) + 4]\,dx$

$= \tan x + \frac{\tan^3 x}{3} + 4x + c$

$\because f'(0) = 0$ weget $c = 0$

Thus $f(x) = \int f'(x)\,dx$

$= \int \left(\tan x + \frac{1}{3}\tan^3 x + 4x\right) dx$

$= \log|\sec x| + \frac{1}{3}\int \tan x\ (\sec^2 x - 1)dx + 2x^2 + k$

$= \log|\sec x| + \frac{1}{3}\ \frac{\tan^2 x}{2} - \frac{1}{3}\log|\sec x| + 2x^2 + k$

Hence from $f(0) = 0$ we get $k = 0$

$f(x) = \frac{2}{3}\log|\sec x| + \frac{1}{6}\tan^2 x + 2x^2$

Q.37 If $\int \frac{dx}{\sqrt{x^2+2x+1}} = A\log|x+1| + c$ for $x < -1$, then A =.............

Sol. $\int \frac{dx}{\sqrt{x^2+2x+1}} = \int \frac{dx}{|x+1|}$

$= \begin{cases} \log|x+1| + c & \text{if } x > -1 \\ -\log|x+1| + c & \text{if } x < -1 \end{cases}$

$\Rightarrow A = -1$

Q.38 If $\int \frac{\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx = fog(x) + \text{const}$, then f(x) = ..., g(x) = ...

Sol. Putting $\log(x+\sqrt{1+x^2}) = t$ we have

$\frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)dx = dt$

$\Rightarrow \frac{1}{\sqrt{1+x^2}}\ dx = dt$

So $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx = \int t\,dt = \frac{1}{2}t^2 + c$

$= \frac{1}{2}[\log(x+\sqrt{1+x^2})]^2$

Thus $f(x) = \frac{x^2}{2}$ and

$g(x) = \log(x+\sqrt{1+x^2})$

Q.39 $\int \frac{d(x^2+1)}{\sqrt{(x^2+2)}}$ is equal to …………………

Sol. $I = \int \frac{d(x^2+1)}{\sqrt{x^2+2}}$

Let $x^2 + 1 = t \Rightarrow d(x^2+1) = dt$

$I = \int \frac{dt}{\sqrt{t+1}} = 2\sqrt{t+1} + c = 2\sqrt{x^2+2} + c$

EXERCISE # 2

Part-A **(Only single correct answer type questions)**

Q.1 $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$ equals -

(A) $-\frac{1}{2}\cos 4x + C$ (B) $-\frac{1}{2}\cos 4x + C$

(C) $-\frac{1}{8}\cos 4x + C$ (D) None of these

Sol.[C] $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$

$= \int \frac{2\cos^2 2x . \cos x \sin x}{\cos^2 x - \sin^2 x} dx$

$= \int \cos 2x . \sin 2x \, dx$

$= \frac{1}{2} \int \sin 4x dx = -\frac{1}{8}\cos 4x + C$

Q.2 $\int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ equals-

(A) $\log (x^2 + \sin 2x + 2x) + c$

(B) $-\log (x^2 + \sin 2x + 2x) + c$

(C) $\frac{1}{2}\log (x^2 + \sin 2x + 2x) + c$

(D) None of these

Sol.[C] $\int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$

Let $x^2 + \sin 2x + 2x = t$

$(x + \cos 2x + 1)\, dx = \frac{1}{2} dt$

$\Rightarrow \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t$

$= \frac{1}{2}\log (x^2 + \sin 2x + 2x) + c$

Q.3 $\int 5^{5^{5^x}} . 5^{5^x} . 5^x \, dx$ is equal to -

(A) $\frac{5^{5^x}}{(\log 5)^3} + c$ (B) $5^{5^{5^x}} (\log 5)^3 + c$

(C) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$ (D) None of these

Sol.[C] $\int 5^{5^{5^x}} . 5^{5^x} . 5^x \, dx$

Let $5^{5^{5^x}} = t$

$5^{5^{5^x}} (\log 5).\ 5^{5^x} (\log 5).5^x(\log 5)\, dx = dt$

$\Rightarrow 5^{5^{5^x}} . 5^{5^x} . 5^x dx = \frac{dt}{(\log 5)^3}$

$\Rightarrow \int \frac{dt}{(\log 5)^3} = \frac{t}{(\log 5)^3} + c$

$= \frac{5^{5^{5^x}}}{(\log 5)^3} + c$

Q.4 $\int \frac{x}{x^4 + x^2 + 1} dx$ equals -

(A) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

(B) $\frac{1}{3} \tan^{-1}\left(\frac{2x^2+1}{3}\right) + c$

(C) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

(D) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{3}\right) + c$

Sol.[A] $\int \frac{x}{x^4 + x^2 + 1} dx$

Let $x^2 = t \Rightarrow xdx = \frac{1}{2} dt$

$= \frac{1}{2}\int \frac{dt}{t^2 + t + 1} = \frac{1}{2}\int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{2} . \frac{2}{\sqrt{3}} \tan^{-1}\frac{(t + 1/2)}{\sqrt{3}/2} + c$

$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x^2+1}{\sqrt{3}} + c$

Q.5 $\int \frac{dx}{\left[(x-1)^3(x+2)^5\right]^{1/4}}$ is equal to -

(A) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (B) $\frac{1}{3}\left(\frac{x+2}{x-2}\right)^{1/4} + c$

(C) $\frac{1}{3}\left(\frac{x-1}{x-2}\right)^{1/4} + c$ (D) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + c$

Sol.[A] $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$= \int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}$

Let $\frac{x-1}{x+2} = t \Rightarrow \frac{1}{(x+2)^2}dx = \frac{1}{3}dt$

$= \frac{1}{3}\int t^{-3/4}\,dt = \frac{1}{3}.4\,t^{1/4} + c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$

Q.6 $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\,dx$ equals-

(A) $\frac{2}{3}\sin^{-1}(\cos^{3/2} x) + c$ (B) $\frac{3}{2}\sin^{-1}(\cos^{3/2} x) + c$

(C) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + c$ (D) None of these

Sol.[C] $\int \frac{\sqrt{\cos x}.\sin x}{\sqrt{1-(\cos^{3/2}x)^2}}\,dx$

Let $\cos^{3/2} x = t \Rightarrow -\frac{3}{2}\sqrt{\cos x}\,.\sin x\,dx = dt$

$\Rightarrow \sqrt{\cos x}\,.\sin x\,dx = -\frac{2}{3}dt$

$\Rightarrow -\frac{2}{3}\int \frac{1}{\sqrt{1-t^2}}dt$

$= \frac{2}{3}\cos^{-1}t + c$

$= \frac{2}{3}\cos^{-1}(\cos^{3/2}x) + c$

Q.7 $\int \frac{x}{x^4-1}\,dx$ equals -

(A) $\frac{1}{2}\log\left(\frac{x^2-1}{x^2+1}\right)$+c (B) $\frac{1}{2}\log\left(\frac{x^2+1}{x^2-1}\right)$+c

(C) $\frac{1}{4}\log\left(\frac{x^2+1}{x^2-1}\right)$+c (D) $\frac{1}{4}\log\left(\frac{x^2-1}{x^2+1}\right)$ + c

Sol.[D] $\int \frac{x}{x^4-1}\,dx$

let $x^2 = t \Rightarrow xdx = \frac{1}{2}dt = \frac{1}{2}\int\frac{1}{t^2-1}dt$

$= \frac{1}{2}\,.\,\frac{1}{2}\log\frac{t-1}{t+1} + c = \frac{1}{4}\log\frac{x^2-1}{x^2+1} + c$

Q.8 If $f(x) = \int \frac{dx}{\sin^{1/2}x\cos^{7/2}x}$, then $f\left(\frac{\pi}{4}\right) - f(0) =$

(A) 2.2 (B) 2.3 (C) 2.4 (D) 2.5

Sol.[C] $\mathbf{f(x)} = \int \frac{dx}{\tan^{1/2}x\cos^4 x}$

$= \int \frac{\sec^2 x(1+\tan^2 x)}{\tan^{1/2}x}\,dx$

Let tanx = t $\Rightarrow \sec^2x\,dx = dt$

$= \int \left(t^{-1/2} + t^{3/2}\right)dt$

$= 2.\,t^{1/2} + \frac{2}{5}t^{5/2} + c$

$= 2\,(\tan^{1/2}x) + \frac{2}{5}(\tan x)^{5/2} + c$

$f\left(\frac{\pi}{4}\right) - f(0) = 2 + \frac{2}{5} + 0 = \frac{12}{5} = 2.4$

Q.9 The value of $\int \frac{dt}{t^2+2xt+1}$ $(x^2 > 1)$ is -

(A) $\frac{1}{\sqrt{1-x^2}}\tan^{-1}\left(\frac{t+x}{\sqrt{1+x^2}}\right) + c$

(B) $\frac{1}{2\sqrt{x^2-1}}\log\left(\frac{t+x-\sqrt{x^2-1}}{t+x+\sqrt{x^2-1}}\right) + c$

(C) $\frac{1}{2}\log(t^2 + 2xt + 1) + c$

(D) None of these

Sol.[B] $\int \frac{dt}{t^2 + 2xt + x^2 - x^2 + 1}$

$= \int \frac{dt}{(t+x)^2 - (x^2 - 1)} \quad \therefore x^2 > 1$

$= \frac{1}{2\sqrt{x^2-1}} \log \left(\frac{t + x - \sqrt{x^2-1}}{t + x + \sqrt{x^2-1}} \right) + c$

Q.10 $\int \frac{dx}{(1+e^x)(1-e^{-x})}$ equals -

(A) $\log\left(\frac{e^x - 1}{e^x + 1}\right) + c$ (B) $\log\left(\frac{e^x + 1}{e^x - 1}\right) + c$

(C) $\frac{1}{2} \log\left(\frac{e^x + 1}{e^x - 1}\right) + c$ (D) $\frac{1}{2} \log\left(\frac{e^x - 1}{e^x + 1}\right) + c$

Sol.[D] $\int \frac{e^x}{(e^x+1)(e^x-1)} dx$

Let $e^x = t \Rightarrow e^x\, dx = dt \Rightarrow \int \frac{dt}{t^2 - 1}$

$= \frac{1}{2} \log \frac{t-1}{t+1} + c = \frac{1}{2} \log\left(\frac{e^x - 1}{e^x + 1}\right) + c$

Q.11 $\int \frac{dx}{\sqrt{2e^x - 1}}$ equals -

(A) $\sec^{-1}\sqrt{2e^x} + c$ (B) $\sec^{-1}\left(\sqrt{2e^x}\right) + c$

(C) $2\sec^{-1}\left(\sqrt{2e^x}\right) + c$ (D) $2\sec^{-1}\sqrt{2e^x} + c$

Sol.[D] $\int \frac{e^{x/2}}{e^{x/2}\sqrt{2e^x - 1}} dx$

Let $e^{x/2} = t \Rightarrow e^{x/2}\, dx = 2dt$

$\Rightarrow 2\int \frac{dt}{t\sqrt{2t^2 - 1}} = \frac{2}{\sqrt{2}} \int \frac{dt}{t\sqrt{t^2 - 1/2}}$

$= \frac{2}{\sqrt{2}} . \sqrt{2} \sec^{-1} \sqrt{2}\, t + c = 2 \sec^{-1} \sqrt{2e^x} + c$

Q.12 $\int \frac{e^x (x+1)}{(x+2)^2} dx$ is equal to -

(A) $\frac{e^x}{(x+2)} + c$ (B) $\frac{2e^x}{(x+2)} + c$

(C) $\frac{e^x}{(x+2)^2} + c$ (D) $\frac{2e^x}{(x+2)^2} + c$

Sol.[A] $\int \frac{e^x (x+2-1)}{(x+2)^2} dx$

$= \int e^x \left(\frac{1}{(x+2)} - \frac{1}{(x+2)^2} \right) dx = \frac{e^x}{x+2} + c$

Q.13 If $f(x) = \lim_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}$, $0 < x < 1$, $n \in N$ then $\int (\sin^{-1} x)\, f(x)\, dx$ is equal to -

(A) $-\left[x\sin^{-1} x + \sqrt{1-x^2}\right] + c$

(B) $x \sin^{-1}x + \sqrt{1-x^2} + c$

(C) $\frac{x^2}{2} + c$

(D) $\frac{1}{2}(\sin^{-1}x)^2 + c$

Sol.[A] $\Theta \lim_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}$, $0 < x < 1$

$f(x) = \lim_{n\to\infty} \frac{x^{2n} - 1}{x^{2n} + 1} = -1$

so $\int (\sin^{-1} x) f(x) dx$

$= -\int \sin^{-1}x\, dx$

$= -\left[x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx \right]$

$= -[x\sin^{-1}x + \sqrt{1-x^2}] + c$

Q.14 Consider the following :

If $\int \frac{ax^2 - b}{x\sqrt{c^2x^2 - (ax^2+b)^2}} dx =$

(i) $\sin^{-1} \frac{ax + bx}{c} + k$ (ii) $\sin^{-1} \frac{ax + b/x}{c} + k$

(iii) $\sin^{-1} \frac{ax^2 + b}{cx} + k$ (iv) $\sin^{-1} \frac{ax^2 + b}{c} + k$

then which of the above are correct -

(A) (ii) & (iii) only (B) (i) & (iii) only

(C) (ii) & (iv) only (D) (iii) & (iv) only

Sol.[A] $\int \frac{ax^2 - b}{x\sqrt{c^2x^2 - (ax^2+b)^2}}\, dx$

$= \int \frac{a - b/x^2}{\sqrt{c^2 - (ax + b/x)^2}}\, dx$

Put $ax + \frac{b}{x} = t \Rightarrow \left(a - \frac{b}{x^2}\right) dx = dt$

$= \int \frac{dt}{\sqrt{c^2 - t^2}}$

$= \sin^{-1}\frac{t}{c} + k$

$= \sin^{-1}\frac{ax + \frac{b}{x}}{c} + k$

$= \sin^{-1}\frac{ax^2 + b}{cx} + k$

Q.15 If $f\left(\frac{3x-4}{3x+4}\right) = x + 2$, then $\int f(x)\,dx$ is equal to

(A) $e^{x+2}\,\lambda n\left|\frac{3x-4}{3x+4}\right| + c$

(B) $-\frac{8}{3}\lambda n\,|(1-x)| + \frac{2}{3}x + c$

(C) $\frac{8}{3}\lambda n\,|(x-1)| + \frac{x}{3} + c$

(D) None of these

Sol.[B] Put $\frac{3x-4}{3x+4} = t \Rightarrow \frac{6x}{-8} = \frac{t+1}{t-1}$

$\Rightarrow x = \frac{4}{3}\left(\frac{t+1}{1-t}\right)$

so $f(t) = \frac{4}{3}\left(\frac{t+1}{1-t}\right) + 2$

Again put t = x and solving weget

$f(x) = \frac{10-2x}{3(1-x)}$

so $\int f(x)\,dx = \int \frac{10-2x}{3(1-x)}\,dx$

$= \frac{8}{3}\int \frac{1}{1-x}\,dx + \frac{2}{3}\int dx$

$= -\frac{8}{3}\lambda n\,|1-x| + \frac{2}{3}x + c$

Q.16 Let $f : \left[0, \frac{\pi}{2}\right] \to R$ be such that f(0) = 3 and $f'(x) = \frac{1}{1+\cos x}$ .if $a < f\left(\frac{\pi}{2}\right) < b$, then a and b can be -

(A) $\frac{\pi}{2}, \pi$ (B) 3 , 4

(C) $3 + \frac{\pi}{4}, 3 + \frac{\pi}{2}$ (D) $3 + \frac{\pi}{2}, 3 + \frac{3\pi}{4}$

Sol.[C] $f'(x) = \frac{1}{1+\cos x} = \frac{1}{2}\sec^2\frac{x}{2}$

$f(x) = \int \frac{1}{2}\sec^2\frac{x}{2}\,dx = \tan\frac{x}{2} + c$

given that f(0) = 3

f(0) = tan 0 + c = 3

$\Rightarrow c = 3$

$f(x) = \tan\frac{x}{2} + 3$

$f\left(\frac{\pi}{2}\right) = 1 + 3 = 4$

Now $\pi = 3.14$

$\therefore 3 + \frac{\pi}{4} = 3 + .78 = 3.78$

and $3 + \frac{\pi}{2} = 3 + 1.57 = 4.57$

so $a < f\left(\frac{\pi}{2}\right) < b$

$\Rightarrow 3 + \frac{\pi}{4} < f\left(\frac{\pi}{2}\right) < 3 + \frac{\pi}{2}$

$a = 3 + \frac{\pi}{4}, b = 3 + \frac{\pi}{2}$

Q.17 $\int \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$ -

(A) $\frac{e^x}{\sqrt{1-x^2}}$ + c (B) $e^x\sqrt{1-x^2}$ + c

(C) $\frac{e^x(2-x^2)}{\sqrt{1-x^2}}$ + c (D) $\frac{e^x(1+x)}{\sqrt{1-x^2}}$ + c

Sol.[D] $\int \frac{e^x\{1+(1-x^2)\}}{(1-x)\sqrt{1-x^2}}\,dx$

$= \int e^x\left[\frac{1}{(1-x)\sqrt{1-x^2}} + \frac{1+x}{\sqrt{1-x^2}}\right]dx$

$= \int e^x\left[\frac{1}{(1-x)^{3/2}\sqrt{1+x}} + \sqrt{\frac{1+x}{1-x}}\right]dx$

$= \int e^x\sqrt{\frac{1+x}{1-x}}\,dx + \int e^x\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{(1-x)^2}\,dx$

$= e^x\sqrt{\frac{1+x}{1-x}} - \int e^x\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{(1-x)^2}\,dx +$

$\int e^x\sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{(1-x)^2}\,dx = e^x\sqrt{\frac{1+x}{1-x}} = e^x\frac{(1+x)}{\sqrt{1-x^2}}$

Q.18 If $\int \frac{x^4+1}{x^6+1}\,dx = \tan^{-1}f(x) - \frac{2}{3}\tan^{-1}g(x) + c$, then -

(A) $f(x) = x + \frac{1}{x}$, $g(x) = x^3$

(B) $f(x) = x^2 + \frac{1}{x^2}$, $g(x) = x^{-3}$

(C) $f(x) = x^2 - \frac{1}{x^2}$, $g(x) = x^{-3}$

(D) $f(x) = x - \frac{1}{x}$, $g(x) = x^3$

Sol.[D] $\int \frac{x^4+1+2x^2-2x^2}{x^6+1}\,dx$

$= \int \frac{(x^2+1)^2}{x^6+1}\,dx - \int \frac{2x^2}{x^6+1}$

$= \int \frac{(x^2+1)^2}{(x^2+1)(x^4-x^2+1)}dx - \frac{2}{3}\int \frac{3x^2}{(x^3)^2+1}dx$

$= \int \frac{x^2+1}{x^4-x^2+1}\,dx - \frac{2}{3}\int \frac{dt}{t^2+1}$ $x^3 = t$

$= \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}\,dx - \frac{2}{3}\tan^{-1}t$

$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+1}\,dx - \frac{2}{3}\tan^{-1}x^3$

$= \int \frac{1}{p^2+1}\,dp - \frac{2}{3}\tan^{-1}x^3$ $\because x - \frac{1}{x} = p$

$= \tan^{-1}p - \frac{2}{3}\tan^{-1}x^3 + c$

$= \tan^{-1}\left(x-\frac{1}{x}\right) - \frac{2}{3}\tan^{-1}x^3 + c$

$f(x) = x - \frac{1}{x}$, $g(x) = x^3$

Q.19 $\int \frac{10x^9+10^x\log 10}{x^{10}+10^x}\,dx$ is equal to -

(A) $10^x - x^{10} + c$

(B) $10^x + x^{10} + c$

(C) $(10^x - x^{10})^{-1} + c$

(D) $\log(10^x + x^{10}) + c$

Sol.[D] $\int \frac{10x^9+10^x\log 10}{x^{10}+10^x}\,dx$

put $x^{10} + 10^x = t$

$(10x^9+10^x\log 10)\,dx = dt$

$\int \frac{1}{t}\,dt = \log t + c = \log(x^{10} + 10^x) + c$

Q.20 If $\int f(x)\,\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log(f(x)) + c$, then f(x) is equal to -

(A) $\frac{1}{a^2\sin^2x + b^2\cos^2x}$

(B) $\frac{1}{a^2\sin^2x - b^2\cos^2x}$

(C) $\frac{1}{a^2\cos^2x + b^2\sin^2x}$

(D) $\frac{1}{a^2\cos^2x - b^2\sin^2x}$

Sol.[A] Differentiate both side w.r.to x we have

$f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \frac{f'(x)}{f(x)}$

$\Rightarrow 2b^2 \sin x \cos x - 2a^2 \sin x \cos x = \frac{f'(x)}{[f(x)]^2}$

$\Rightarrow 2b^2 \int \sin x \cos x\, dx - 2a^2 \int \sin x \cos x\, dx$

$= \int \frac{f'(x)}{[f(x)]^2} dx \quad \Rightarrow \quad -2b^2 \frac{\cos^2 x}{2} - 2a^2 \frac{\sin^2 x}{2} = -\frac{1}{f(x)}$

$\Rightarrow f(x) = \frac{1}{a^2 . \sin^2 x + b^2 \cos^2 x}$

Q.21 $\int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - 2\log x]}{x^4}$ dx is equal to -

(A) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right] + c$

(B) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right] + c$

(C) $\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right] + c$

(D) None of these

Sol.[B] $\int \frac{\sqrt{x^2+1}\log\left(\frac{x^2+1}{x^2}\right)}{x^4} dx$

$= \int \frac{\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)}{x^3} dx$

let $1 + \frac{1}{x^2} = t^2 \Rightarrow -\frac{1}{x^3} dx = t\,dt$

$= -\int t^2 \log t^2\, dt = -2\int t^2 \log t\, dt$

$= -2\left[\frac{t^3}{3}\log t - \int \frac{t^3}{3}.\frac{1}{t} dt\right] = -2\left[\frac{t^3}{3}\log t - \frac{t^3}{9}\right] + c$

$= -\frac{2}{3} t^3 \left(\log t - \frac{1}{3}\right) + c = -\frac{1}{3} t^3 \left(\log t^2 - \frac{2}{3}\right) + c$

$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right] + c$

Q.22 $\int \frac{\sin^3 x dx}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)} =$

(A) $\tan^{-1}(\sec x + \cos x) + c$

(B) $\log \tan^{-1}(\sec x + \cos x) + c$

(C) $\frac{1}{(\sec x + \cos x)^2} + c$

(D) None of these

Sol.[B] $\int \frac{\sin^3 x dx}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)}$

Let $\tan^{-1}(\sec x + \cos x) = t$

$\Rightarrow \frac{\sin^3 x}{(\cos^4 x + 3\cos^2 x + 1)} dx = dt$

$= \int \frac{1}{t} dt = \log t + c = \log \tan^{-1}(\sec x + \cos x) + c$

Q.23 $\int \frac{\sqrt{\cot x} - \sqrt{\tan x}}{\sqrt{2}(\cos x + \sin x)}$ dx equals -

(A) $\sec^{-1}(\sin x + \cos x) + C$

(B) $\sec^{-1}(\sin x - \cos x) + C$

(C) $\lambda n\,|(\sin x + \cos x) + \sqrt{\sin 2x}\,| + C$

(D) $\lambda n\,|(\sin x - \cos x) + \sqrt{\sin 2x}\,| + C$

Sol.[A]

Q.24 $\int e^{x^4}(x + x^3 + 2x^5)e^{x^2}$ dx is equal to -

(A) $\frac{1}{2}xe^{x^2}.e^{x^4} + C$ (B) $\frac{1}{2}x^2.e^{x^4} + C$

(C) $\frac{1}{2}e^{x^2}.e^{x^4} + C$ (D) $\frac{1}{2}x^2.e^{x^2}.e^{x^4} + C$

Sol. [D]

Q.25 $\int \frac{1-x^7}{x(1+x^7)}$dx equals -

(A) $\lambda n\,|x| + \frac{2}{7}\,\lambda n\,|1 + x^7| + c$

(B) $\lambda n\,|x| - \frac{2}{4}\,\lambda n\,|1 - x^7| + c$

(C) $\lambda n\,|x| - \frac{2}{7}\,\lambda n\,|1 + x^7| + c$

(D) $\lambda n\,|\,x\,| + \frac{2}{4}\ \lambda n\,|\,1 - x^7\,| + c$

Sol. [C]

Q.26 $\int \frac{x\,dx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$ is equal to -

(A) $\frac{1}{2}\ln(1+\sqrt{1+x^2})+c$

(B) $2\sqrt{1+\sqrt{1+x^2}}+c$

(C) $2(1+\sqrt{1+x^2})+c$

(D) None of these

Sol. [B]

Q.27 $\int e^x \frac{(1+n.x^{n-1}-x^{2n})}{(1-x^n)\sqrt{1-x^{2n}}}\,dx$ is equal to -

(A) $e^x\sqrt{\frac{1-x^n}{1+x^n}}+C$ (B) $e^x\sqrt{\frac{1+x^n}{1-x^n}}+C$

(C) $-e^x\sqrt{\frac{1-x^n}{1+x^n}}+C$ (D) $-e^x\sqrt{\frac{1+x^n}{1-x^n}}+C$

Sol. [B]

Part-B	**(One or more then one correct answer type questions)**

Q.28 $\int \frac{\lambda n\left(\frac{x-1}{x+1}\right)}{x^2-1}\,dx$ equal -

(A) $\frac{1}{2}\lambda n^2\frac{x-1}{x+1}+c$ (B) $\frac{1}{4}\lambda n^2\frac{x-1}{x+1}+c$

(C) $\frac{1}{2}\lambda n^2\frac{x+1}{x-1}+c$ (D) $\frac{1}{4}\lambda n^2\frac{x+1}{x-1}+c$

Sol.[B, D]

$I = \int \frac{\lambda n\left(\frac{x-1}{x+1}\right)}{x^2-1}\,dx$

Let $\lambda n\left(\frac{x-1}{x+1}\right) = t \Rightarrow \left(\frac{x+1}{x-1}\right).\frac{x+1-x+1}{(x+1)^2}\,dx = dt$

$\Rightarrow \frac{2}{x^2-1}\,dx = dt$

$I = \frac{1}{2}\int t\,dt = \frac{1}{4}t^2 + c$

$= \frac{1}{4}\ \lambda n^2\ \frac{x-1}{x+1} + c = \frac{1}{4}\ \lambda n^2\ \frac{x+1}{x-1} + c$

Q.29 $\int \frac{\lambda n(\tan x)}{\sin x\ \cos x}\,dx$ equal -

(A) $\frac{1}{2}\lambda n^2(\cot x)+c$

(B) $\frac{1}{2}\lambda n^2(\sec x)+c$

(C) $\frac{1}{2}\lambda n^2(\sin x\ \sec x)+c$

(D) $\frac{1}{2}\lambda n^2(\cos x\ \mathrm{cosec}\ x)+c$

Sol.[A, C, D]

$I = \int \frac{\lambda n(\tan x)}{\sin x\ \cos x}\,dx$

Let $\lambda n\ (\tan x) = t \Rightarrow \frac{1}{\sin x\ \cos x}\,dx = dt$

$I = \int t\,dt = \frac{t^2}{2} + c$

$= \frac{1}{2}\ \lambda n^2\ (\tan x) + c = \frac{1}{2}\ \lambda n^2\ (\cot x) + c$

$= \frac{1}{2}\ \lambda n^2(\sin x \sec x) + c$

$= \frac{1}{2}\ \lambda n^2(\cos x\ \mathrm{cosec}\ x) + c$

Q.30 If $\int \mathrm{cosec}\ 2x\,dx = f(g(x)) + C$, then -

(A) range $g(x) = (-\infty, \infty)$

(B) dom $f(x) = (-\infty, \infty) - \{0\}$

(C) $g'(x) = \sec^2 x$

(D) $f'(x) = 1/x$ for all $x \in (0, \infty)$

Sol.[A,B,C] $\int \mathrm{cosec}\ 2x\,dx$

$= \frac{1}{2}\int \frac{2\,\mathrm{cosec}2x(\mathrm{cosec}2x-\cot 2x)}{(\mathrm{cosec}2x-\cot 2x)}\,dx$

$= \frac{1}{2}\log|\mathrm{cosec}\ 2x - \cot 2x| + c$

$= \frac{1}{2}\log\left|\frac{1-\cos 2x}{\sin 2x}\right| + c = \frac{1}{2}\log|\tan x| + c$

Thus $f(x) = \frac{1}{2}\log|x|$ and $g(x) = \tan x$

Now,

Range $g(x) = (-\infty, \infty)$

dom $f(x) = (-\infty, \infty) - \{0\}$

$g'(x) = \sec^2 x$ and

$f'(x) = \frac{1}{2x}$ for all $x \in (0,\infty)$

So option A,B,C are correct.

Q.31 If $\int \frac{\sin x + \sin^3 x}{\cos 2x}\,dx$

$= A\cos x + B\log|f(x)| + C$, then -

(A) $A = 1/4,\ B = -1/\sqrt{2},\ f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(B) $A = 1/2,\ B = -3/4\sqrt{2},\ f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(C) $A = -1/2,\ B = 3/\sqrt{2},\ f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

(D) $A = 1/2,\ B = 3/4\sqrt{2},\ f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

Sol.[B,D] $\int \frac{\sin x\,(2-\cos^2 x)}{2\cos^2 x - 1}\,dx$

Let $\cos x = t \Rightarrow -\sin x\,dx = dt$

$\int \frac{t^2-2}{2t^2-1}\,dt = \frac{1}{2}\int \frac{2t^2-4}{2t^2-1}\,dt$

$= \frac{1}{2}\int dt - \frac{3}{2}\int \frac{dt}{2t^2-1}$

$= \frac{1}{2}t - \frac{3}{2\sqrt{2}}\cdot\frac{1}{2}\log\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| + c$

$= \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}}\log\left|\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}\right| + c$

Hence $A = \frac{1}{2}$, $B = -\frac{3}{4\sqrt{2}}$

and $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

and if $B = \frac{3}{4\sqrt{2}}$ then $f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

So option B, D are correct.

Q.32 If $\int \frac{3x+4}{x^3-2x-4}\,dx = \log|x-2| + K\log f(x) + c$,

then -

(A) $K = -1/2$ (B) $f(x) = x^2 + 2x + 2$

(C) $f(x) = |x^2 + 2x + 2|$ (D) $K = 1/4$

Sol.[A,B,C] $\int \frac{3x+4}{x^3-2x-4}\,dx$

Since $x^3 - 2x - 4 = (x-2)(x^2+2x+2)$

so $(3x+4) = A(x^2+2x+2) + (Bx+C)(x-2)$

Comparing the coefficient we get

$A = 1, B = -1, C = -1$

thus, we have

$\int \frac{3x+4}{x^3-2x-4}\,dx = \int \frac{1}{x-2}\,dx - \int \frac{x+1}{x^2+2x+2}\,dx$

$= \log|x-2| - \frac{1}{2}\log|x^2+2x+2| + c$

Hence $k = -\frac{1}{2}$, $f(x) = |x^2 + 2x \ \ + 2|$

$\Rightarrow f(x) = x^2 + 2x + 2\ \Theta\ x^2 + 2x + 2 > 0$

Hence option A, B, C are correct.

Q.33 If $\int \tan^4 x\,dx = a\tan^3 x + b\tan x + \phi(x)$ then

(A) $a = \frac{1}{3}$ (B) $b = 1$

(C) $\phi(x) = x + c$ (D) $b = -1$

Sol.[A,C,D] $\int \tan^4 x\,dx$

$= \int \tan^2 x\,(\sec^2 x - 1)\,dx$

$= \int \tan^2 x\,d(\tan x) - \int (\sec^2 x - 1)\,dx$

$= \frac{1}{3}\tan^3 x - \tan x + x + c$

Hence $a = \frac{1}{3}$, $b = -1$, $\phi(x) = x + c$

$\Rightarrow$ Option A , C , D are correct.

Q.34 Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}\,dx = \frac{2}{3}\,gof(x) + C$ then -

(A) $f(x) = \sqrt{x}$ (B) $f(x) = x^{3/2}$

(C) $f(x) = x^{2/3}$ (D) $g(x) = \sin^{-1} x$

Sol.[B,D] $\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx$

Put $x^{3/2} = t \Rightarrow x^{1/2}\,dx = \frac{2}{3}\,dt$

$= \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}} = \frac{2}{3}\sin^{-1} t + c$

$= \frac{2}{3}\sin^{-1} x^{3/2} + c$

comparing we get

$f(x) = x^{3/2}$ and $g(x) = \sin^{-1}x$

Hence option B, D are correct.

Q.35 If $\int x\log(1+x^2)dx = \phi(x)\log(1+x^2) + \Psi(x) + c$ then -

(A) $\phi(x) = \frac{1+x^2}{2}$ (B) $\Psi(x) = \frac{1+x^2}{2}$

(C) $\Psi(x) = -\frac{1+x^2}{2}$ (D) $\phi(x) = -\frac{1+x^2}{2}$

Sol.[A,C] $\int x\log(1+x^2)dx$

$= \frac{x^2}{2}\log(1+x^2) - \int \frac{x^2}{2}\cdot\frac{2x}{1+x^2}dx$

$= \frac{x^2}{2}\log(1+x^2) - \int x dx + \int \frac{x}{1+x^2}dx$

$= \frac{x^2}{2}\log(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\log(1+x^2) + c'$

$= \frac{1+x^2}{2}\log(1+x^2) - \frac{x^2+1}{2} + c$

$\phi(x) = \frac{1+x^2}{2}$, $\Psi(x) = -\frac{1+x^2}{2}$

Hence option A,C are correct.

Q.36 Let $f'(x) = 3x^2\sin\frac{1}{x} - x\cos\frac{1}{x}$, if $x \neq 0$; $f(0) = 0$ and $f(1/\pi) = 0$ then -

(A) f(x) is continuous at x = 0

(B) f(x) is non derivable at x = 0

(C) f '(x) is continuous at x = 0

(D) f '(x) is non derivable at x = 0

Sol.[A,C,D]

$f(x) = \int f'(x)\,dx$

$= \int \underset{II}{3x^2}\,\underset{I}{\sin}\frac{1}{x}\,dx - \int x\cos\frac{1}{x}\,dx$

$= \sin\frac{1}{x}\cdot x^3 - \int \left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\cdot x^3\,dx$

$- \int x\cos\frac{1}{x}\,dx + C$

$= x^3\sin\frac{1}{x} + \int x\cos\frac{1}{x}\,dx$

$- \int x\cos\frac{1}{x}\,dx + C$

$= x^3\sin\frac{1}{x} + C$

as $f\left(\frac{1}{\pi}\right) = 0$ $\therefore$ C = 0

$\therefore$ $f(x) = x^3 + \sin\frac{1}{x}$

Q.37 $\int \frac{x^2+\cos^2 x}{1+x^2}\,\mathrm{cosec}^2 x\,dx$ is equal to -

(A) $\cot x - \cot^{-1} x + c$

(B) $c - \cot x + \cot^{-1} x$

(C) $-\tan^{-1} x - \frac{\mathrm{cosec}\,x}{\sec x} + c$

(D) $-e^{\lambda n \tan^{-1}x} - \cot x + c$

Sol.[B,C,D]

$\int \frac{x^2+1-1+\cos^2 x}{x^2+1}\cdot \mathrm{cosec}^2 x\,dx$

$= \int \mathrm{cosec}^2 x\,dx - \int \frac{1}{1+x^2}dx$

$= -\cot x - \tan^{-1} x + C$

Part-C Column matching type questions

Q.38 **Column-I** **Column-II**

(A) If $F(x) = \int \frac{x+\sin x}{1+\cos x}dx$ and $F(0) = 0$, then $F(\pi/2) =$ (P) $\frac{\pi}{2}$

(B) Let $F(x) = \int e^{\sin^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)dx$ (Q) $\frac{\pi}{3}$

and F(0) = 1, if

F(1/2) = $\frac{k\sqrt{3}e^{\pi/6}}{\pi}$, then k =

(C) Let F(x) = $\int \frac{dx}{(x^2+1)(x^2+9)}$ (R) $\frac{\pi}{4}$

and F(0) = 0, if

F($\sqrt{3}$) = $\frac{5}{36}$ k, then k =

(D) Let F(x) = $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ (S) 2π

and F(0) = 0, if F(π/4) = $\frac{k}{\pi}$

then k =

Sol. A → P, B → P, C → R, D → S

(A) F(x) = $\int \frac{x}{2}\sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$

$= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx + c$

$= x\tan\frac{x}{2} + c$

∵ F(0) = 0 ⇒ c = 0

F(x) = $x\tan\frac{x}{2} \Rightarrow F\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

(B) F(x) = $\int e^{\sin^{-1}x}\left(\frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}}\right) dx$

Let $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$

F(x) = $\int e^t(\cos t - \sin t)\, dt$

$= e^t\cos t + c = e^{\sin^{-1}x}\cos(\sin^{-1}x) + c$

∵ F(0) = 1 ⇒ c = 0

F(x) = $e^{\sin^{-1}x}\cos(\sin^{-1}x)$

$F\left(\frac{1}{2}\right) = e^{\pi/6}.\frac{\sqrt{3}}{2} = \frac{k\sqrt{3}e^{\pi/6}}{\pi}$ given

$\Rightarrow k = \frac{\pi}{2}$

(C) F(x) = $\frac{1}{8}\int\left(\frac{1}{x^2+1} - \frac{1}{x^2+9}\right) dx$

$= \frac{1}{8}\left[\tan^{-1}x - \frac{1}{3}\tan^{-1}\frac{x}{3}\right] + c$

∵ F(0) = 0 ⇒ c = 0

F(x) = $\frac{1}{8}\tan^{-1}x - \frac{1}{24}\tan^{-1}\frac{x}{3}$

$F(\sqrt{3}) = \frac{\pi}{24} - \frac{1}{24}.\frac{\pi}{6} = \frac{\pi}{24}\left(1-\frac{1}{6}\right)$

$= \frac{5\pi}{36\times4} = \frac{5}{36}$ k given

$\Rightarrow k = \frac{\pi}{4}$

(D) F(x) = $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Let $\tan x = t^2 \Rightarrow \sec^2 x\, dx = 2t\, dt$

F(x) = $\int \frac{2t\,dt}{t} = 2t + c = 2\sqrt{\tan x} + c$

∵ F(0) = 0 ⇒ c = 0

F(x) = $2\sqrt{\tan x}$

$F\left(\frac{\pi}{4}\right) = 2 = \frac{2k}{\pi}$ given

⇒ k = π

Q.39 For solving each integral in column-I match suitable substitutions in column-II.

Column-I	Column-II
(A) I = $\int \frac{\sin x \cos x\, dx}{\sin^2 x + 3\cos^2 x}$	(P) sin x = t
(B) I = $\int \frac{\sin x \cos x\, dx}{\sin^2 x + 2\cos^2 x + \cos x}$	(Q) cos x = t
(C) I = $\int \frac{dx}{3 + \sin x \cos x}$	(R) tan x = t
(D) I = $\int \frac{(\cos^2 x - \sin^2 x)dx}{1 - \sin^2 x \cos^2 x}$	(S) sin x cos x = t

Sol. A → P,Q,R; B → Q, C → R, D → R,S

(A) $I = \int \frac{\sin x \cos x}{\sin^2 x + 3\cos^2 x} dx$

we can put sin x = t or cos x = t

(B) $I = \int \frac{\sin x \cos x}{\sin^2 x + 2\cos^2 x + \cos x} dx$

we can put cos x = t

(C) $I = \int \frac{dx}{3 + \sin x \cos x} = \int \frac{\sec^2 x\, dx}{3(1+\tan^2 x) + \tan x}$

we can put tan x = t

(D) $I = \int \frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x \cos^2 x} dx$

we can put sin x cos x = t

$\Rightarrow I = \int \frac{(1-\tan^2 x)\sec^2 x}{(1+\tan^2 x)^2 - \tan^2 x} dx$

we can put tan x = t

Q.40 Column-I and column-II contains four entries each. Entry of column-I are to be matched with one or more than one entries of column-II.

Column-I	Column-II
(A) $\int \left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) dx$	(P) $\ln\left|\cos\left(x - \frac{\pi}{4}\right)\right| + C$
(B) $\int \frac{\cos 2x}{1 + \sin 2x} dx$	(Q) $(\cos^{-1}\sqrt{x})^2 - \pi\cos^{-1}\sqrt{x} + C$
(C) $\int \frac{1 - \sin 2x}{\cos 2x} dx$	(R) $(\sin^{-1}\sqrt{x})^2 + C$
(D) $\int \frac{\sin^{-1}\sqrt{x}}{\sqrt{x - x^2}} dx$	(S) ln | cos x + sin x | + C

Sol. **A → P,S; B → P,S; C → P,S; D → Q,R**

(A) $I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

Let cos x + sin x = t

(–sinx + cos x) dx = dt

$I = \int \frac{dt}{t} = \lambda n\, |t| + C = \lambda n\, |\sin x + \cos x| + C$

$= \lambda n\, \left|\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)\right| + C$

$= \lambda n\, \left|\cos\left(x - \frac{\pi}{4}\right)\right| + C$

(B) $I = \int \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} dx = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$

$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

(C) $\int \frac{1 - \sin 2x}{\cos 2x} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$

(D) $I = \int \frac{\sin^{-1}\sqrt{x}}{\sqrt{x - x^2}} dx$

Let $\sin^{-1}\sqrt{x} = t$

$\frac{1}{2\sqrt{x}\sqrt{1-x}} dx = 2dt$

$I = 2\int t\, dt = t^2 + C = (\sin^{-1}\sqrt{x})^2 + C$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Integrate $\int (x^2+a^2)\ (x^4 + a^2 x^2 + a^4)^{-1}\,dx$

Sol. $\int \frac{(x^2+a^2)}{x^4+a^2x^2+a^4}dx$

$= \int \frac{\frac{a^2}{x^2}+1}{\frac{a^4}{x^2}+x^2-2a^2+3a^2}dx$

$= \int \frac{\frac{a^2}{x^2}+1}{\left(\frac{a^2}{x}-x\right)^2+3a^2}dx$

Let $\frac{a^2}{x} - x = t \Rightarrow \left(\frac{a^2}{x^2}+1\right) dx = -\,dt$

$= -\int \frac{dt}{t^2+(\sqrt{3}a)^2}$

$= \frac{1}{\sqrt{3}a}\cot^{-1}\frac{t}{\sqrt{3}a} + c$

$= \frac{1}{\sqrt{3}a}\cot^{-1}\frac{a^2-x^2}{\sqrt{3}ax} + c$

$= \frac{1}{\sqrt{3}a}\tan^{-1}\frac{\sqrt{3}ax}{a^2-x^2} + c$

Q.2 Integrate $\int\sqrt{\frac{1-\sqrt[3]{x}}{1+\sqrt[3]{x}}}\,dx$

Sol. Let $x = t^3 \Rightarrow dx = 3t^2dt$

$I = \int 3t^2\sqrt{\frac{1-t}{1+t}}\,dt = \int\frac{3t^2(1-t)}{\sqrt{1-t^2}}dt$

Again let $t = \sin\theta \Rightarrow dt = \cos\theta\, d\theta$

Put this value and solving weget

$\frac{3}{2}\int\theta d\theta - \frac{3}{2}\int\cos 2\theta d\theta - 3\int\sin\theta\ (1-\cos^2\theta)d\theta$

Solving weget

$= \frac{3}{2}\theta + \frac{1}{2}\cos\theta\,(2\sin^2\theta + 4 - 3\sin\theta) + c$

$\Rightarrow \frac{3}{2}\sin^{-1}t - \frac{1}{2}\sqrt{1-t^2}\ (2t^2 - 3t + 4) + c$

Where $t = x^{1/3}$

Q.3 Evaluate : $\int\frac{x^5+2x^3+4x+4}{x^4+2x^3+2x^2}dx.$

Sol. $I = \int\left[x-2+\frac{4x^3+4x^2+4x+4}{x^2(x^2+2x+2)}\right]dx$

$= \int(x-2)\,dx + \int\frac{2x^2(2x+2)+2(x^2+2x+2)-2x^2}{x^2(x^2+2x+2)}dx$

$= \int(x-2)dx + \int\frac{2(2x+2)}{(x^2+2x+2)}dx$

$+ \int\frac{2}{x^2}dx - \int\frac{2}{1+(1+x)^2}dx$

$= \frac{x^2}{2} - 2x + 2\lambda n(x^2+2x+2) - \frac{2}{x} - 2\tan^{-1}(1+x)+c$

Q.4 Evaluate : $\int\frac{2e^{5x}+e^{4x}-4e^{3x}+4e^{2x}+2e^x}{(e^{2x}+4)(e^{2x}-1)^2}dx$

Sol. Let $e^x = t \Rightarrow e^x\,dx = dt$

$I = \int\frac{2t^4+t^3-4t^2+4t+2}{(t^2+4)(t^2-1)^2}dt$

$= \int\frac{2(t^4-2t^2+1)+t(t^2+4)}{(t^2+4)(t^2-1)^2}dt$

$= \int\frac{2}{t^2+4}dt + \int\frac{t}{(t^2-1)^2}dt$

$= \tan^{-1}\frac{t}{2} + \left(-\frac{1}{2}\right)\frac{1}{(t^2-1)} + c$

$= \tan^{-1}\frac{e^x}{2} - \frac{1}{2}\cdot\frac{1}{(e^{2x}-1)} + c$

Q.5 Find the integral $\int\frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx$

Sol. Let $I = \int\frac{\sin^{-1}\sqrt{x}}{\sqrt{1-x}}$

Let $\sin^{-1}\sqrt{x} = t$

$\Rightarrow \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \frac{1}{\sqrt{1-x}}\,dx = 2\sin t\,dt$

$I = 2\int t\sin t\,dt = -2t\cos t + 2\int\cos t$

$= -2t\cos t + 2\sin t + c = 2\left(\sqrt{x} - 2\sqrt{1-x}\sin^{-1}\sqrt{x}\right) + c$

Q.6 Applying integration by parts, derive the following reductions formulas :

$I_n = \int (\lambda n\, x)^n\, dx = x(\lambda n\, x)^n - nI_{n-1}$

Sol. $I_n = \int (\lambda n\, x)^n\, dx$

$= x(\lambda n\, x)^n - \int x.\frac{n(\lambda n x)^{n-1}}{x}\, dx$

$= x(\lambda n x)^n - n\int (\lambda n x)^{n-1} dx$

$= x(\lambda n\, x)^n - n\, I_{n-1}$

L.H.S hence proved.

Q.7 If $u = \int e^{ax}\cos bx\, dx$, $v = \int e^{ax}\sin bx\, dx$, prove that $\tan^{-1}\frac{v}{u} + \tan^{-1}\frac{b}{a} = bx$, and that $(a^2+b^2)(u^2+v^2) = e^{2ax}$

Sol. $u = \int e^{ax}\cos bx\, dx$

we put $a = r\cos\alpha$, $b = r\sin\alpha$

$\Rightarrow r^2 = a^2 + b^2$ and $\tan\alpha = \frac{b}{a}$

$u = \int e^{ax}\cos bx\, dx$

$= \frac{e^{ax}}{r}(r\cos\alpha\cos bx + r\sin\alpha\sin bx)$

$= \frac{e^{ax}}{r^2}\cos(bx-\alpha)$ ....(i)

Similarly $v = \frac{e^{ax}}{r}\sin(bx-\alpha)$ ....(ii)

Squaring and adding weget

$u^2 + v^2 = \frac{e^{2ax}}{r^2} = \frac{e^{2ax}}{a^2+b^2}$

$\Rightarrow (a^2+b^2)(u^2+v^2) = e^{2ax}$

Divide (ii) & (i) we have

$\frac{v}{u} = \tan(bx-\alpha) \Rightarrow \tan^{-1}\frac{v}{u} + \alpha = bx$

$\Rightarrow \tan^{-1}\frac{v}{u} + \tan^{-1}\frac{b}{a} = bx$

Hence proved.

Q.8 Evaluate : $\int \lambda n(\sqrt{1-x} + \sqrt{1+x})\,dx$

Sol. $I = \int 1.\lambda n(\sqrt{1-x}+\sqrt{1+x})\, dx$

$= x\,\lambda n\,(\sqrt{1-x}+\sqrt{1+x}) -$

$\int x\frac{1}{\sqrt{1-x}+\sqrt{1+x}}.\left(-\frac{1}{2\sqrt{1-x}} + \frac{1}{2\sqrt{1+x}}\right)dx$

$= x\,\lambda n\,(\sqrt{1-x}+\sqrt{1+x})$

$- \int \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}.\frac{x}{2\sqrt{1-x^2}}dx$

Let $I_1 = -\int \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}.\frac{x}{2\sqrt{1-x^2}}dx$

Put $x = \cos2\theta$ $dx = -2\sin\theta d\theta$

and solving weget

$= -\frac{x}{2} + \frac{\sin^{-1}x}{2} + c$

So, $I = x\lambda n(\sqrt{1-x}+\sqrt{1+x}) - \frac{x}{2} + \frac{\sin^{-1}x}{2} + c$

Q.9 Evaluate : $\int \frac{1+x^{-2/3}}{1+x}dx$

Sol. $I = \int \frac{1}{1+x}dx + \int \frac{dx}{x^{2/3}(1+x)}$

$= \lambda n(1+x) + \int \frac{dx}{x^{2/3}(1+x)}$

Let $I_1 = \int \frac{dx}{x^{2/3}(1+x)}$

Put $x = t^3 \Rightarrow dx = 3t^2$

$I_1 = \int \frac{3dt}{1+t^3}$

using partial fraction weget

$I_1 = \int\left(\frac{1}{1+t} - \frac{t-2}{1-t+t^2}\right)dt$

$= \lambda n(1+t) - \frac{1}{2}\int \frac{2t-1}{1-t+t^2}dt$

$+ \frac{3}{2}\int \frac{1}{(t-1/2)^2 + (\sqrt{3}/2)^2}dt$

$= \lambda n(1+t) - \frac{1}{2}\lambda n(1-t+t^2)\, dt$

$+ \frac{3}{2}.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right) + c$

$I = \lambda n\,(1 + x)+\lambda n(1 + x^{1/3})-\frac{1}{2}\lambda n\,(1-x^{1/3} + x^{2/3})$

$+\sqrt{3}\tan^{-1}\left(\frac{2x^{1/3}-1}{\sqrt{3}}\right) + c$

Q.10 Evaluate : $\int \frac{2-x^2}{(1+x)\sqrt{1-x^2}}\,dx$

Sol. $I = \int \frac{1-x^2+1}{(1+x)\sqrt{1-x^2}}\,dx$

$= \int \sqrt{\frac{1-x}{1+x}}\,dx + \int \frac{1}{(1+x)\sqrt{1-x^2}}\,dx$

$\Rightarrow I = I_1 + I_2$; $I_1 = \int \frac{1-x}{\sqrt{1-x^2}}dx$

$= \sin^{-1}x + \sqrt{1-x^2} + c_1$

for I_2 Let $1 + x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}\,dt$

$I_2 = \int \frac{-1}{\sqrt{2t-1}}\,dt = -\sqrt{2t-1} + c_2 = -\sqrt{\frac{1-x}{1+x}} + c_2$

$I = \sin^{-1}x + \sqrt{1-x^2} + \frac{-(1-x)}{\sqrt{1-x^2}} + c$

$= \sin^{-1}x + \sqrt{1-x^2} + \frac{x-1}{\sqrt{1-x^2}} + c$

Q.11 Evaluate : $\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx.$

Sol. $I = \int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx$

$= \int \frac{1}{x^{13}\sqrt{1+\frac{1}{x^4}}}\,dx$

Put $1+\frac{1}{x^4} = t^2 \Rightarrow -\frac{4}{x^5}dx = 2t\,dt$

$\Rightarrow \frac{1}{x^5}\,dx = -\frac{t}{2}\,dt$

$= \frac{1}{2}\int \frac{-t\,dt}{x^8\ t.} = -\frac{1}{2}\int (t^2-1)^2\,dt$

$= -\frac{1}{2}\int (t^4-2t^2+1)\,dt$

$= -\frac{1}{2}\left[\frac{t^5}{5} - \frac{2t^3}{3} + t\right] + c$

Where $t^2 = 1 + \frac{1}{x^4}$.

Q.12 Evaluate : $\int \sqrt{x+\sqrt{x^2+2}}\,dx$

Sol. $I = \int \sqrt{x+\sqrt{x^2+2}}\,dx$

Put $x + \sqrt{x^2+2} = t^2$ ......(i)

$\Rightarrow \frac{x+\sqrt{x^2+2}}{\sqrt{x^2+2}}\,dx = 2t\,dt$

$\Rightarrow \frac{1}{\sqrt{x^2+2}}\,dx = \frac{2}{t}\,dt$

But $\frac{x^2-(x^2+2)}{x-\sqrt{x^2+2}} = t^2$

$\Rightarrow \frac{-2}{t^2} = x - \sqrt{x^2+2}$ ......(ii)

Subtracting (i) & (ii) weget

$\sqrt{x^2+2} = \frac{1}{2}\left(t^2+\frac{2}{t^2}\right)$

So, $dx = \frac{1}{t}\left(t^2+\frac{2}{t^2}\right)dt$

$I = \int t.\,\frac{1}{t}\left(t^2+\frac{2}{t^2}\right)dt$

$= \int\left(t^2+\frac{2}{t^2}\right)dt = \frac{1}{3}t^3 - \frac{2}{t} + c$

$= \frac{1}{3}\,(x+\sqrt{x^2+2})^{3/2} - 2\left(x+\sqrt{x^2+2}\right)^{-1/2} + c$

Q.13 Evaluate : $\int \frac{dx}{(x+\sqrt{x^2-1})^2}$

Sol. $I = \int \frac{dx}{(x+\sqrt{x^2-1})^2}$

$= \int (x-\sqrt{x^2-1})^2\,dx$

$= \int (x^2 - 2x\sqrt{x^2-1} + x^2 - 1)\,dx$

$= \int (2x^2 - 1 - 2x\sqrt{x^2-1}) + c$

$= \frac{2}{3}x^3 - x - \frac{2}{3}(x^2-1)^{3/2} + c$

Q.14 Integrate $\int \frac{5\cos x + 6}{2\cos x + \sin x + 3}dx$.

Sol. $I = \int \frac{5\cos x + 6}{2\cos x + \sin x + 3}dx$

$\Rightarrow$ 5 cos x + 6 = A(2cosx + sinx + 3)+
B (–2sinx + cosx) + C
This gives A =2, B =1, C = 0

So, $I = \int 2dx + \int \frac{-2\sin x + \cos x}{2\cos x + \sin x + 3}dx$

= 2x + λn (2 cosx + sinx + 3) + c

Q.15 Integrate $\int \frac{dx}{\sin^2 x + \sin 2x}$

Sol. $I = \int \frac{\sec^2 x}{\tan^2 x + 2\tan x}dx$

Put tan x = t $\Rightarrow \sec^2 x\, dx = dt$

$I = \int \frac{dt}{(t+1)^2 - 1}$

$= \frac{1}{2}\log\left|\frac{t}{t+2}\right| + c = \frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right| + c$

Q.16 Integrate $\int \frac{dx}{\sin x(2+\cos x)}$.

Sol. $I = \int \frac{\sin x}{(1-\cos^2 x)(2+\cos x)}dx$

Put cosx = t $\Rightarrow$ –sinx dx = dt

$I = \int \frac{-dt}{(1-t)(1+t)(2+t)}$

using partial fraction weget

$I = \frac{-1}{6}\int \frac{1}{1-t}dt - \frac{1}{2}\int \frac{dt}{1+t} + \frac{1}{3}\int \frac{1}{2+t}dt$

$= \frac{1}{6}$ λn|1–cosx|$-\frac{1}{2}$ λn|1+cosx|+$\frac{1}{3}$ λn|2+cosx| + c

Q.17 Evaluate : $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}dx$

Sol. $I = \int \frac{\cos x - \sin x}{\sqrt{9 - (1 + \sin 2x)}}dx$

$= \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}}dx$

Put sin x + cosx = t $\Rightarrow$ (cosx– sinx) dx = dt

$I = \int \frac{dt}{\sqrt{3^2 - t^2}}$

$= \sin^{-1}\frac{t}{3} + c = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$

Q.18 Evaluate : $\int \frac{x^2 dx}{(x\sin x + \cos x)^2}$

Sol. $I = \int \frac{x^2 dx}{(x\sin x + \cos x)^2}$

$= \int \underset{I}{\frac{x}{\cos x}} . \underset{II}{\frac{(x\cos x)}{(x\sin x + \cos x)^2}}dx$

$I = \frac{x}{\cos x}\left\{-\frac{1}{x\sin x + \cos x}\right\}$

$- \int \frac{\cos x + \sin x}{\cos^2 x} \times \frac{-1}{x\sin x + \cos x}dx$

$= \frac{-x}{\cos x(x\sin x + \cos x)} + \int \sec^2 x\, dx$

$= \frac{-x}{\cos(x\sin x + \cos x)}$ + tan x + C

Q.19 Integrate

$\int \frac{\sin\theta - \cos\theta}{(\sin\theta + \cos\theta)\sqrt{\sin\theta\cos\theta + \sin^2\theta\cos^2\theta}}d\theta$

Sol. $I = \int \frac{\sin^2\theta - \cos^2\theta}{(\sin\theta + \cos\theta)^2\sqrt{\sin\theta\cos\theta(1 + \sin\theta\cos\theta)}}d\theta$

$= -\int \frac{2\cos 2\theta}{(1 + \sin 2\theta)\sqrt{\sin 2\theta(2 + \sin 2\theta)}}d\theta$

Put sin2θ = t

2 cos2θ dθ = dt

$I = -\int \frac{dt}{(1+t)\sqrt{t^2 + 2t}}$

$= -\int \frac{dt}{(1+t)\sqrt{(t+1)^2 - 1}}$

$= \text{cosec}^{-1}(1 + t) + c$

$= \text{cosec}^{-1}(1+\sin 2\theta) + c$

$= \sin^{-1}\left(\frac{1}{1+\sin 2\theta}\right) + c$

Q.20 Evaluate : $\int \frac{\cot x + \text{cosec } x - 1}{\cot x - \text{cosec } x + 1} dx$

Sol. **I =** $\int \frac{\cos x - \sin x + 1}{\cos x + \sin x - 1} dx$

$= \int \frac{2\cos^2 x/2 - 2\sin x/2 \cos x/2}{2\sin x/2 \cos x/2 - 2\sin^2 x/2} dx$

$= \int \frac{\cos x/2}{\sin x/2} dx$

$= 2\ \lambda n\ |\sin x/2| + c$

$= \lambda n\ |\sin^2 x/2| + c$

Q.21 Evaluate : $\int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2} dx$

Sol. **I =** $\int \{1 + 2\tan^2 x + 2\tan x \sec x\}^{1/2} dx$

$= \int \{\sec^2 x + \tan^2 x + 2\tan x \sec x\}^{1/2} dx$

$= \int (\sec x + \tan x) dx$

$= \lambda n\ (\sec x + \tan x) + \lambda n\ (\sec x) + c$

$= \lambda n\ \sec x. (\sec x + \tan x) + c$

Q.22 Evaluate : $\int \frac{dx}{(x^3 + 3x^2 + 3x + 1)\sqrt{x^2 + 2x - 3}}$

Sol. $\frac{\sqrt{x^2 + 2x - 3}}{8(x+1)^2} + \frac{1}{16}.\cos^{-1}\left(\frac{2}{x+1}\right) + c$

Q.23 Evaluate : $\int e^x \left(\frac{x^3 - x + 2}{(x^2+1)^2}\right) dx$

Sol. $e^x \left(\frac{x+1}{x^2+1}\right) + c$

Q.24 Evaluate : $\int \frac{(\cos 2x - 3)}{\cos^4 x\sqrt{4 - \cot^2 x}} dx$

Sol. $c - \frac{1}{3} \tan x. (2 + \tan^2 x). \sqrt{4 - \cot^2 x}$

Part-B Passage based objective questions

Passage - 1 (Q.25 to 27)

Integrals of class of functions following a definite pattern can be found by the method of reduction and recursion. If the integral of member of the class having positive integral power can be found then the other members of the class can be found by successive application of the recursion formula. While deriving a recursion formula. We integrate by parts and try to bring back the original integral.

For example, $I_n = \int (\sin x)^n dx$ can be expressed

as $I_n = -\frac{(\sin x)^{n-1}\cos x}{n} + \frac{n-1}{n} I_{n-2}$,

Which is the required reduction formula and is true for all $n \geq 2$.

On the basis of above passage, answer the following questions :

Q.25 If $I_n = \int \tan^n x\, dx$, then the value of I_3 must be equal to -

(A) $\sec^2 x + \log \cos x + c$

(B) $\sec^2 x - \log \cos x + c$

(C) $\frac{1}{2} \sec^2 x - \log \cos x + c$

(D) $\frac{1}{2} \sec^2 x + \log \cos x + c$

Sol.[D] $I_3 = \int \tan^3 x\, dx = \int \tan x(\sec^2 x - 1) dx$

$= \frac{\tan^2 x}{2} + \log|\cos x| + c'$

$= \frac{\sec^2 x}{2} + \log|\cos x| + c$

Q.26 If $I_n = \int \sec^n x dx$, then the relation between I_5 and I_3 will be -

(A) $3I_5 = -\sec^3 x \tan x - 2I_3$

(B) $3I_5 = \sec^3 x \tan x - 3I_3$

(C) $4I_5 = \sec^3 x \tan x + 3I_3$

(D) $4I_5 = \sec^3 x \tan x - 3I_3$

Sol.[C] $I_n = \int \sec^n x dx = \int \sec^{n-2} x \sec^2 x\, dx$

Integrating by parts we have

$I_n = \sec^{n-2} x \tan x - \int (n-2) \sec^{n-2} x \tan^2 x\, dx$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^n x dx + (n-2)$

$$\int \sec^{n-2} x dx$$

$I_n = \sec^{n-2}x \tan x - (n-2) I_n + (n-2) I_{n-2}$

Put n = 5 weget

$I_5 = \sec^3 x \tan x - 3I_5 + 3I_3$

$4I_5 = \sec^3 x \tan x + 3I_3$

Q.27 If $I_n = \int x \sin^n x \, dx$, then the value of $I_3\Big|_{x=\frac{\pi}{2}}$ must be equal to -

(A) $\frac{2}{3}$ (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) 1

Sol.[C] $I_3 = \int x \sin^3 x \, dx = \int \frac{x(3\sin x - \sin 3x)}{4} dx$

$= \frac{3}{4} \int x \sin x dx - \frac{1}{4} \int x \sin 3x \, dx$

$= -\frac{3}{4} x \cos x + \frac{3}{4} \int \cos x \, dx + \frac{1}{4} \frac{x \cos 3x}{3} - \frac{1}{4} \int \frac{\cos 3x}{3} dx$

$= -\frac{3}{4} x \cos x + \frac{3}{4} \sin x + \frac{1}{12} x \cos 3x - \frac{1}{36} \sin 3x$

$I_3\Big|_{x=\frac{\pi}{2}} = 0 + \frac{3}{4} + 0 + \frac{1}{36} = \frac{28}{26} = \frac{7}{6}$

Passage - 2 (Q.28 to 30)

It is known that $\sqrt{\tan x} + \sqrt{\cot x} =$

$$\begin{cases} \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } 0 < x < \frac{\pi}{2} \\ \frac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \frac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \pi < x < \frac{3\pi}{2} \end{cases}$$

$\frac{d}{dx}(\sqrt{\tan x} - \sqrt{\cot x})$

$= \frac{1}{2}(\sqrt{\tan x} + \sqrt{\cot x})(\tan x + \cot x)$

$\forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$ and

$\frac{d}{dx}(\sqrt{\tan x} + \sqrt{\cot x})$

$= \frac{1}{2}(\sqrt{\tan x} - \sqrt{\cot x})(\tan x + \cot x)$

$\forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

Q.28 Value of integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$, where $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$ is -

(A) $\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(B) $\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(C) $-\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(D) $-\sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

Sol. **[A]**

Q.29 Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$, where $x \in \left(0, \frac{\pi}{2}\right)$, is -

(A) $\sqrt{2} \sin^{-1}(\cos x - \sin x) + C$

(B) $\sqrt{2} \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2} \sin^{-1}(\sin x + \cos x) + C$

(D) $-\sqrt{2} \sin^{-1}(\sin x + \cos x) + C$

Sol. **[B]**

Q.30 Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$, where $x \in \left(\pi, \frac{3\pi}{2}\right)$, is -

(A) $\sqrt{2} \sin^{-1}(\cos x - \sin x) + C$

(B) $\sqrt{2} \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2} \sin^{-1}(\sin x + \cos x) + C$

(D) $-\sqrt{2} \sin^{-1}(\sin x + \cos x) + C$

Sol. **[A]**

EXERCISE # 4

Old IIT-JEE Subjective type questions

Q.1 $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$ **[IIT- 2001]**

Sol. $I = \int \sin^{-1}\left(\frac{2x+2}{\sqrt{(2x+2)^2+9}}\right)dx$

Let 2x + 2 = 3 tanθ

$\Rightarrow dx = \frac{3}{2}\sec^2\theta\, d\theta$

$I = \frac{3}{2}\int \sin^{-1}\left(\frac{3\tan\theta}{3\sec\theta}\right)\sec^2\theta\, d\theta$

$= \frac{3}{2}\int \theta \sec^2\theta\, d\theta$

$= \frac{3}{2}\left|\theta\tan\theta - \int \tan\theta d\theta\right|$

$= \frac{3}{2}[\theta\tan\theta - \lambda n \sec\theta] + c$

$= \frac{3}{2}\left[\tan^{-1}\left(\frac{2x+2}{3}\right).\frac{2x+2}{3} - \lambda n\sqrt{1+\left(\frac{2x+2}{3}\right)^2}\right] + c$

$= (x+1)\tan^{-1}\frac{2x+2}{3} - \frac{3}{2}\lambda n\sqrt{4x^2+8x+13} + c$

Q.2 For any natural m, evaluate

$\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m}dx,\ x > 0.$

[IIT- 2002]

Sol. $I = \int \frac{(x^{3m}+x^{2m}+x^m)}{x}(2x^{3m}+3x^{2m}+6x^m)^{1/m}dx$

Let $2x^{3m} + 3x^{2m} + 6x^m = t$

$\Rightarrow \frac{6m(x^{3m}+x^{2m}+x^m)}{x}dx = dt$

$I = \frac{1}{6m}\int t^{1/m}dt$

$= \frac{1}{6m}\frac{t^{1/m+1}}{\frac{1}{m}+1} + c$

$= \frac{1}{6(m+1)}(2x^{3m}+3x^{2m}+6x^m)^{\frac{m+1}{m}} + c$

Old IIT-JEE Objective type questions

Q.3 $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to -

[IIT- 2006]

(A) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$ (B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$

(C) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$ (D) $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$

Sol.[A] $I = \int \frac{x^2-1}{x^5\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}dx$

$= \frac{1}{4}\int \frac{\frac{4}{x^3}-\frac{4}{x^5}}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}dx$

Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = t^2$

$\Rightarrow \left(\frac{4}{x^3}-\frac{4}{x^5}\right)dx = 2t\,dt$

$I = \frac{1}{4}\int 2dt = \frac{1}{2}t + c$

$= \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}} + c$

$= \frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$

Q.4 Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \ge 2$ and

$g(x) = \underbrace{(fofo....of)}_{f \text{ occurs n times}}(x)$. Then $\int x^{n-2}g(x)dx$

equals - **[IIT-2007]**

(A) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$

(B) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + K$

(C) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

(D) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + K$

Sol.[A] $f(x) = \frac{x}{(1+x^n)^{1/n}} \quad n \geq 2$

Then $f(f(x)) = \frac{\frac{x}{(1+x^n)^{1/n}}}{\left(1+\frac{x^n}{1+x^n}\right)^{1/n}} = \frac{x}{(1+2x^n)^{1/n}}$

and $\underbrace{(fofo....of)}_{n\text{ times}}$ weget

$g(x) = \frac{x}{(1+nx^n)^{1/n}}$

$I = \int x^{n-2}.\frac{x}{(1+nx^n)^{1/n}}dx$

$= \int x^{n-1}(1 + nx^n)^{-1/n}\,dx$

Put $1 + nx^n = t \Rightarrow x^{n-1}\,dx = \frac{1}{n^2}\,dt$

$I = \frac{1}{n^2}\int t^{-1/n}\,dt$

$= \frac{1}{n^2}.\frac{n}{n-1}t^{1-1/n} + k$

$= \frac{1}{n(n-1)}(1 + nx)^{1-1/n} + k$

Q.5 Let

$I = \int\frac{e^x}{e^{4x}+e^{2x}+1}dx,\ J = \int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$

Then for an arbitrary constant C, the value of J – I equals- **[IIT-2008]**

(A) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

(B) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

(D) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

Sol. [C] $J = \int\frac{e^{3x}}{1+e^{2x}+e^{4x}}\,dx$

$J - I = \int\frac{e^{3x}-e^{x}}{e^{4x}+e^{2x}+1}\,dx$

Let $e^x = t$

$= \int\frac{t^2-1}{t^4+t^2+1}\,dt = \int\frac{1-\frac{1}{t^2}}{t^2+1+\frac{1}{t^2}}\,dt$

$= \int\frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-1}\,dt = \frac{1}{2}\ \ln\left(\frac{t+\frac{1}{t}-1}{t+\frac{1}{t}+1}\right)+C$

$= \frac{1}{2}\ \ln\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

Q.6 Let F(x) be an indefinite integral of $\sin^2x$.

STATEMENT-1 : The function F(x) satisfies $F(x+\pi) = F(x)$ for all real x.

Because

STATEMENT-2: $\sin^2(x+\pi) = \sin^2x$ for all real x

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol. **[D]**

Q.7 The integral $\int\frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K) **[IIT-2012]**

(A) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

Sol. [C]

$$I = \int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$$

sex x + tan x = t

(sec x tan x + $\sec^2$ x) dx = dt

$$\sec x\, dx = \frac{dt}{t}$$

$$\text{also, } \sec x - \tan x = \frac{1}{t}$$

$$\sec x = \frac{1}{2}\left(t+\frac{1}{t}\right)$$

$$\text{So, } I = \frac{1}{2}\int \frac{\left(t+\frac{1}{t}\right)dt}{t^{11/2}}$$

$$= \frac{1}{2}\int\left(\frac{1}{t^{9/2}}+\frac{1}{t^{13/2}}\right)dt$$

$$= \frac{1}{2}\left[-\frac{2}{7}\frac{1}{t^{7/2}}-\frac{2}{11}\frac{1}{t^{11/2}}\right]+K$$

$$= -\frac{1}{t^{11/2}}\left[\frac{t^2}{7}+\frac{1}{11}\right]+K$$

$$= -\frac{1}{(\sec x+\tan x)^{11/2}}\left[\frac{1}{7}(\sec x+\tan x)^2+\frac{1}{11}\right]$$

+ K

EXERCISE # 5

Old IIT-JEE questions

Q.1 Find the value of $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ **[IIT-1985]**

Sol. $I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

putting $x = \cos^2\theta$, $dx = -2\sin\theta\cos\theta\, d\theta$, we get

$I = \int \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\,.(-2\sin\theta\cos\theta)\, d\theta$

$= -\int 2\tan\frac{\theta}{2}\,.\,\sin\theta\cos\theta\, d\theta$

$= -4\int \sin^2\frac{\theta}{2}\,.\,\cos\theta\, d\theta$

$= -2\int (1-\cos\theta)\cos\theta d\theta$

$= -2\int (\cos\theta - \cos^2\theta)\, d\theta$

$= -2\int \cos\theta d\theta + \int (1+\cos 2\theta)\, d\theta$

$= -2\sin\theta + \theta + \frac{\sin 2\theta}{2} + c$

$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x(1-x)}\; c$

Q.2 Evaluate :

$\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$ **[IIT-1986]**

Sol. $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$

$= \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\pi/2}\,.dx$

$= \frac{2}{\pi}\int \left\{2\sin^{-1}\sqrt{x} - \frac{\pi}{2}\right\} dx$

$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - x + c$ ....(i)

Now, $\int \sin^{-1}\sqrt{x}\,dx$

$= \int \theta\,.\,\sin 2\theta\, d\theta$, where $x = \sin^2\theta$

$= -\frac{\theta\cos 2\theta}{2} + \int \frac{1}{2}\cos 2\theta\, d\theta$

$= -\frac{\theta}{2}\cos 2\theta + \frac{1}{4}\sin 2\theta$

$= -\frac{1}{2}\theta(1 - 2\sin^2\theta) + \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta}$

$= -\frac{1}{2}\sin^{-1}\sqrt{x}\,(1-2x) + \frac{1}{2}\sqrt{x}\sqrt{1-x}$ ....(ii)

From (i) and (ii), we get

$I = \frac{4}{\pi}\left\{-\frac{1}{2}(1-2x)\sin^{-1}\sqrt{x} + \frac{1}{2}\sqrt{x-x^2}\right\} - x + c$

$= \frac{2}{\pi}\{\sqrt{x-x^2} - (1-2x)\sin^{-1}\sqrt{x}\} - x + c$

Q.3 Evaluate :

$\int \frac{(\cos 2x)^{1/2}}{\sin x}\,dx$ **[IIT-1987]**

Sol. $I = \frac{\sqrt{\cos 2x}}{\sin x}\,dx$

$= \int \frac{\sqrt{\cos^2 x - \sin^2 x}}{\sin x}\,dx = \int \sqrt{\cot^2 x - 1}\;dx$

putting, $\cot x = \sec\theta$

$\Rightarrow -\mathrm{cosec}^2 x\, dx = \sec\theta\tan\theta\, d\theta$.

We get,

$I = \int \sqrt{\sec^2\theta - 1}\,.\,\frac{\sec\theta.\tan\theta}{-(1+\sec^2\theta)}\,d\theta$.

$= -\int \frac{\sec\theta.\tan^2\theta}{1+\sec^2\theta}\,d\theta = -\int \frac{\sin^2\theta}{\cos\theta + \cos^3\theta}\,d\theta$

$= -\int \frac{1-\cos^2\theta}{\cos\theta(1+\cos^2\theta)}\,d\theta$

$= -\int \frac{(1+\cos^2\theta) - 2\cos^2\theta}{\cos\theta(1+\cos^2\theta)}\,d\theta$

$= -\int \sec\theta\, d\theta + 2\int \frac{\cos\theta}{1+\cos^2\theta}\,d\theta$

$= -\log|\sec\theta + \tan\theta| + 2\int \frac{\cos\theta}{2-\sin^2\theta}\,d\theta$

$= -\log|\sec\theta + \tan\theta| + \int \frac{dt}{2-t^2}$, where $\sin\theta = t$.

$= -\log|\sec\theta + \tan\theta| + 2.\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin\theta}{\sqrt{2}-\sin\theta}\right| + c$

$= -\log|\cot x + \sqrt{\cot^2 x - 1}| +$

$$\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+\sqrt{1-\tan^2 x}}{\sqrt{2}-\sqrt{1-\tan^2 x}}\right| + c$$

Q.4 Evaluate : $\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$ **[IIT-1988]**

Sol. $I = \int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$

$= \int\frac{\tan x+1}{\sqrt{\tan x}}dx$, put $\tan x = t^2$

$\Rightarrow (\sec^2 x)\,dx = 2t\,dt$

or $dx = \frac{2t}{1+t^4}dt. = \int\frac{t^2+1}{\sqrt{t^2}}\,\frac{2t}{t^4+1}dt$

$= 2\int\frac{t^2+1}{t^4+1}dt$

$= 2\int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-2+2}dt$

$= 2\int\frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{2}\right)^2}dt$

$= 2\int\frac{du}{u^2+(\sqrt{2})^2}$, where $u = t-\frac{1}{t}$.

$\Rightarrow I = \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$

$= \sqrt{2}\tan^{-1}\left(\frac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right)+c$

Q.5 $\int\left(\frac{1}{\sqrt[3]{x}+\sqrt[4]{x}}+\frac{\log(1+\sqrt[6]{x})}{\sqrt[3]{x}+\sqrt{x}}\right)dx$ **[IIT-1990]**

Sol. $\int\left(\frac{1}{\sqrt[3]{x}+\sqrt[6]{x}}+\frac{\ln(1+\sqrt[4]{x})}{\sqrt[3]{x}+\sqrt{x}}\right)dx$

Let $I = I_1 + I_2$ where

$I_1 = \int\left(\frac{1}{\sqrt[3]{x}+\sqrt[6]{x}}\right)dx$, $I_2 = \frac{\ln(1+\sqrt[4]{x})}{\sqrt[3]{x}+\sqrt{x}}\,dx$

$I_1 = \int\left(\frac{1}{\sqrt[3]{x}+\sqrt[6]{x}}\right)dx$

Now let $x = t^{12}$

$\Rightarrow dx = 12t^{11}\,dt$

$= 12\int\frac{t^{11}}{t^4+t^3}dt = 12\int\frac{t^8dt}{t+1}$

$= 12\int(t^7-t^6+t^5-t^4+t^3-t^2+t-1)\,dt$

$+ 12\int\frac{dt}{t+1}$

$= 12.\left(\frac{t^8}{8}-\frac{t^7}{7}+\frac{t^6}{6}-\frac{t^5}{5}+\frac{t^4}{4}-\frac{t^3}{3}+\frac{t^2}{2}-t\right)$

$+ 12\ln(t+1)$

and for $I_2 = \int\left\{\frac{\ln(1+\sqrt[6]{x})}{\sqrt[3]{x}+\sqrt{x}}\right\}dx$

put $x = u^6$

$\Rightarrow dx = 6u^5\,du$

$= \int\frac{\ln(1+u)}{u^2+u^3}6u^5\,du = \int\frac{\ln(1+u)}{u^2(u+1)}.6u^5du$

$= 6\int\frac{u^3}{(u+1)}\ln(1+u)\,du$

$= 6\int\left(\frac{u^3-1+1}{u+1}\right)\ln(1+u)\,du$

$= 6\int\left(u^2-u+1-\frac{1}{u+1}\right)\ln(1+u)\,du$

$= 6\int(u^2-u+1)\ln(1+u)du - 6\int\frac{\ln(1+u)}{(u+1)}du$

Integrating the first intergral by parts, we get

$I_2 = 6\left(\frac{u^3}{3}-\frac{u^2}{2}+4\right)\ln(1+u)$

$-\int\frac{2u^3-3u^2+6u}{u+1}du - 6.\frac{1}{2}[\ln(1+u)]^2$

$= (2\mu^3-3\mu^2+6\mu)\ln(1+\mu) -$

$\int\left(2\mu^2-5\mu+\frac{11-11}{\mu+1}\right)du - 3[\ln(1+\mu)]^2$

$= (2\mu^3-3\mu^2+6\mu)\ln(1+\mu) -$

$\left(\frac{2\mu^3}{3}-\frac{5}{2}\mu^2+11\mu-11\ln(\mu+1)\right) - 3(\ln(1+\mu))^2$

Therefore,

$I = \frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2x^{1/2} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4}$

$-6x^{1/6} - 12x^{1/12} + 12\ln(x^{1/12}+1)$

$+ (2x^{1/2} - 3x^{1/3} + 6x^{1/6})\ln(1+x^{1/6})$

$$-\left(\frac{2}{3}x^{1/2}-\frac{5}{2}x^{1/3}11x^{1/6}-11\ln(1+x^{1/6})\right)$$

$$-3[\ln(1+x^{1/6})]^2+c$$

$$=\frac{3}{2}x^{2/3}-\frac{12}{7}x^{7/12}+\frac{4}{3}x^{1/2}-\frac{12}{5}x^{5/12}$$

$$+\frac{1}{2}x^{1/3}-4x^{1/4}-7x^{1/6}-12x^{1/12}$$

$$+(2x^{1/2}-3x^{1/3}+6x^{1/6}+11)\ln(1+x^{1/6})$$

$$+12\ln(1+x^{12})-3[\ln(1+x^{1/6})]^2+c$$

Q.6 $\int \frac{4e^{x}+6e^{-x}}{9e^{x}-4e^{-x}}dx = Ax + B\log(9e^{2x}-4)+c$, then

A= ………., B = ………., C = ……………..

[IIT 1990]

Sol. $A=-\frac{3}{2}$, $B=\frac{35}{36}$, C = any real value

Q.7 Find the Indefinite integral

$\int \cos 2\theta \ \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta$ **[IIT-1994]**

Sol. $I=\int \cos 2\theta \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$ dx (given)

We integrate it by parts taking

$\ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$ as first function

$$=\frac{\sin 2\theta}{2}\ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$-\frac{1}{2}\int \frac{d}{d\theta}\left(\ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\right).\ \sin 2\theta\ d\theta \ .....(1)$$

But ,

$$\frac{d}{d\theta}\left[\ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\right]$$

$$=\frac{d}{d\theta}[\ln(\cos\theta+\sin\theta)-\ln(\cos\theta-\sin\theta)]$$

$$=\frac{1}{\cos\theta+\sin\theta}.(-\sin\theta+\cos\theta)-\frac{(-\sin\theta-\cos\theta)}{\cos\theta-\sin\theta}$$

$$(\cos\theta-\sin\theta)(\cos\theta-\sin\theta)$$

$$=\frac{-(\cos\theta+\sin\theta)(-\sin\theta-\cos\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$$

$$=\frac{2(\cos^2\theta+\sin\theta)}{\cos 2\theta}$$

$$=\frac{2}{\cos 2\theta}$$

Therefore putting this value in (1), we get

$$I=\frac{1}{2}\sin^2\theta\ \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$-\frac{1}{2}\int \sin 2\theta \frac{2}{\cos 2\theta}d\theta$$

$$=\frac{1}{2}\sin 2\theta\ \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)+1/2\ \ln(\cos 2\theta)+c$$

Q.8 The value of the integral $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$ is - **[IIT-1995]**

(A) $\sin x - 6\tan^{-1}(\sin x) + c$

(B) $\sin x - 2(\sin x)^{-1} + c$

(C) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$

(D) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

Sol.[C] $I=\int \frac{(\cos^2 x+\cos^4 x)\cos x}{\sin^2 x+\sin^4 x}dx$

Put sinx = t $\Rightarrow$ cosx dx = dt

$$I=\int \frac{(1-t^2)+(1-t^2)^2}{t^2+t^4}dt$$

$$=\int \frac{t^4-3t^2+2}{t^4+t^2}dt$$

$$=\int\left[1+\frac{-4t^2+2}{t^4+t^2}\right]dt$$

using partial fraction weget

$$I=\int dt+\int \frac{2}{t^2}dt-6\int \frac{1}{1+t^2}dt$$

$$=t-\frac{2}{t}-6\tan^{-1}t+c$$

$$=\sin x-2(\sin x)^{-1}-6\tan^{-1}(\sin x)+c$$

Q.9 Evaluate : $\int \frac{x+1}{x(1+xe^x)^2}dx$ **[IIT-1996]**

Sol. $I=\frac{e^x(1+x)}{xe^x(1+xe^x)^2}dx$

Put $1+xe^x=t \Rightarrow e^x(1+x)\,dx = dt$

$$I=\frac{dt}{(t-1)t^2}$$

$= \int \left(\frac{1}{t-1} - \frac{t+1}{t^2} \right) dt = \int \left(\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2} \right) dt$

$= \lambda n\,(1-t) - \lambda n\, t + \frac{1}{t} + c = \lambda n \left(\frac{1-t}{t} \right) + \frac{1}{t} + c$

$= \lambda n \frac{xe^x}{1+xe^x} + \frac{1}{1+xe^x} + c$

Q.10 Evaluate : $\int \frac{\cos 2x \,.\, \sin 4x}{\cos^4 x \,.\, (1+\cos^2 2x)} dx$

[REE-1996]

Sol. $I = 4\int \frac{2\cos^2 2x \sin 2x dx}{(1+\cos 2x)^2 (1+\cos^2 2x)}$

Let $\cos 2x = t \Rightarrow 2 \sin 2x\, dx = -\, dt$

$I = -4 \int \frac{t^2}{(1+t)^2 (1+t^2)} dt$

Using partial fraction weget

$I = -4 \int \left[\frac{1}{2(1+t)^2} - \frac{1}{2(1+t)} + \frac{t}{2(1+t^2)} \right] dt$

$= -2 \left[-\frac{1}{1+t} - \lambda n(1+t) + \frac{1}{2} \lambda n(1+t^2) \right] + c$

$= \frac{2}{2\cos^2 x} + 2\, \lambda n\,(1+\cos 2x) - \lambda n\,(1+\cos^2 2x) + c$

$= \sec^2 x + 2\, \lambda n\,(1 + \cos 2x) - \lambda n\,(1+\cos^2 2x) + c$

Q.11 Evaluate : $\int \left(\frac{1-\sqrt{x}}{1+\sqrt{x}} \right)^{1/2} \frac{dx}{x}$ **[IIT-1997]**

Sol. $I = \int \left(\frac{1-\sqrt{x}}{1+\sqrt{x}} \right)^{1/2} \frac{dx}{x}$

Put $x = t^2 \Rightarrow dx = 2t\, dt$

$I = \int \sqrt{\frac{1-t}{1+t}} \,.\, \frac{2t}{t^2} dt$

$= 2\int \frac{1-t}{t\sqrt{1-t^2}} dt$

$= 2\int \frac{1}{t\sqrt{1-t^2}} dt - 2\int \frac{1}{\sqrt{1-t^2}}$

$I = I_1 - I_2$

for I_1 Put $t = \frac{1}{z} \Rightarrow dt = -\frac{1}{z^2} dz$

$I_1 = -2\int \frac{1}{\sqrt{z^2-1}} dz.$

$= -2\, \lambda n\, [z + \sqrt{z^2-1}\,] = -2\lambda n \left(\frac{1+\sqrt{1-x}}{\sqrt{x}} \right) + c_1$

$I_2 = -2 \sin^{-1} t + c = -2 \sin^{-1} \sqrt{x} + c_2$

$I = -2\lambda n \left(\frac{1+\sqrt{1-x}}{\sqrt{x}} \right) - 2\sin^{-1} \sqrt{x} + c$

Q.12 Evaluate : $\int \frac{x^3+3x+2}{(x^2+1)^2 (x+1)} dx$ **[IIT-1999]**

Sol. $I = \int \frac{x(x^2+1) + 2(x+1)}{(x^2+1)^2 (x+1)} dx$

$= \int \frac{x}{(x^2+1)(x+1)} dx + \int \frac{2}{(x^2+1)^2} dx$

$I = I_1 + I_2$

for I_1 using partial fraction weget

$I_1 = -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx$

$= -\frac{1}{2} \lambda n\, |x+1| + \frac{1}{4} \lambda n |x^2+1| + \frac{1}{2} \tan^{-1} x + c_1$

and for $I_2 = \int \frac{2}{(x^2+1)^2}$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$I_2 = 2\int \frac{\sec^2\theta d\theta}{(\tan^2\theta+1)^2} = 2 \int \cos^2\theta\, d\theta$

$= \int (1+\cos 2\theta)\, d\theta = \theta + \frac{\sin 2\theta}{2} + c_2$

$= \theta + \frac{\tan\theta}{1+\tan^2\theta} + c_2$

$= \tan^{-1} x + \frac{x}{1+x^2} + c_2$

$I = -\frac{1}{2} \lambda n|x+1| + \frac{1}{4} \lambda n\, |x^2+1| + \frac{3}{2} \tan^{-1} x + \frac{x}{x^2+1} + c$

Q.13 $\int \frac{1}{\sqrt{1+\sin x}} dx = ?$ **[REE-2000]**

(A) $\sqrt{2}$ log tan(x/4 + π/8)

(B) $\sqrt{2}$ log cot(x/2 + π/8)

(C) $\sqrt{2}$ log cos(x/2 + π/8)

(D) None of these

Sol. [A]

Q.14 Evaluate : $\int \tan^{-1} x.\ln(1+x^2)dx$

Sol. $x \tan^{-1} x. \ln (1 + x^2) + (\tan^{-1} x)^2 - 2x \tan^{-1} x + \ln (1 + x^2) - \left(\ln \sqrt{1+x^2}\right)^2 + c$

Q.15 Let $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 5 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ x^2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ 2\alpha x + \beta x^2 \\ 5x + \gamma x^2 + 3 \end{bmatrix}$,

∀ x ∈ R and f(x) is a differentiable functions satisfying, $f(xy) = f(x) + x^2 (y^2 - 1) + x(y - 1)$; ∀ x, y ∈ R and f(1) = 3.

Evaluate $\int \frac{\alpha x^2 + \beta x + \gamma}{f(x)} dx$

Sol. $3x - \ln\left(\sqrt{x^2 + x + 1}\right) + \sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

Q.16 Evaluate : $\int \frac{e^{\cos x}(x\sin^3 x + \cos x)}{\sin^2 x} dx$

Sol. $C - e^{\cos x}(x + \text{cosec } x)$

Q.17 Match the column :

Column-I **Column-II**

(A) $\int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}} dx$

(P) $\ln\left(\frac{(x^2+1) + \sqrt{x^4+1}}{x}\right) + C$

(B) $\int \frac{x^2 - 1}{x\sqrt{1+x^4}} dx$

(Q) $C - \frac{1}{\sqrt{2}}\ln\left(\frac{\sqrt{x^4+1} - \sqrt{2}x}{(x^2-1)}\right)$

(C) $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$

(R) $C - \tan^{-1}\left(\sqrt{\sqrt{1+\frac{1}{x^4}} - 1}\right)$

(D) $\int \frac{1}{(1+x^4)\sqrt{\sqrt{1+x^4} - x^2}} dx$

(S) $\frac{\sqrt{x^4 + x^2 + 1}}{x} + C$

Sol. **A → S ; B → P ; C → Q ; D → R**

ANSWER KEY

EXERCISE # 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	C	C	B	B	C	B	B	D	A	C	A	A	A
Qus.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	A	A	D	A	C	B	C	C	B	D	A	B	B
Qus.	31	32	33												
Ans.	D	B	A												

34. False

35. False

36. $\frac{2}{3}\log|\sec x| + \frac{1}{6}\tan^2 x + 2x^2$

37. -1

38. $\frac{x^2}{2}$, $\log(x+\sqrt{1+x^2})$

39. $2\sqrt{x^2+2} + c$

EXERCISE # 2

PART-A

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	C	A	A	C	D	C	B	D	D	A	A	A	B
Qus.	16	17	18	19	20	21	22	23	24	25	26	27			
Ans.	C	D	D	D	A	B	B	A	D	C	B	B			

PART-B

Qus.	28	29	30	31	32	33	34	35	36	37
Ans.	B,D	A,C,D	A,B,C	B, D	A,B,C	A,C,D	B,D	A,C	A,C,D	B,C,D

PART-C

38. A $\rightarrow$ P; B $\rightarrow$ P; C $\rightarrow$ R; D $\rightarrow$ S

39. A $\to$ P,Q, R; B $\to$ Q; C $\to$ R; D $\to$ R, S **40.** A $\to$ P,S ; B $\to$ P,S ; C $\to$ P,S ; D $\to$ Q,R

EXERCISE # 3

PART-A

1. $\frac{1}{a\sqrt{3}}\tan^{-1}\frac{x^2-a^2}{ax\sqrt{3}} + c$

2. $\frac{3}{2}\sin^{-1} t + \frac{1}{2}\sqrt{1-t^2}\,(2t^2 - 3t + 4) + C$; where $t = x^{1/3}$

3. $-\frac{x^2}{2} - 2x + 2\lambda n\,(x^2 + 2x + 2) - \frac{2}{x} - 2\tan^{-1}(x + 1) + c$

4. $\tan^{-1}\frac{e^x}{2} - \frac{1}{2}(e^{2x} - 1)^{-1} + c$

5. $2\left(\sqrt{x} - \sqrt{1-x}\,\sin^{-1}\sqrt{x}\right)$

8. $x\lambda n\left(\sqrt{1-x}+\sqrt{1+x}\right) - \frac{x}{2} + \frac{\sin^{-1}x}{2} + c$

9. $\lambda n\,(1 + x) + \lambda n\,(1 + x^{1/3}) - \frac{1}{2}\lambda n\,(x^{2/3} + 1 - x^{1/3}) + \sqrt{3}\tan^{-1}\frac{2x^{1/3}-1}{\sqrt{3}} + c$

10. $-\sin^{-1} x + \sqrt{1-x^2} + \frac{x-1}{\sqrt{1-x^2}} + c$

11. $\frac{-1}{2}\left(\frac{t^5}{5} - \frac{2}{3}t^3 + t\right) + c\ \forall\ t = 1 + x^{-4}$

12. $\frac{1}{3}\left(x + \sqrt{x^2+2}\right)^{3/2} - 2\left(x + \sqrt{x^2+2}\right)^{-1/2} + c$

13. $\frac{2}{3}x^3 - x - \frac{2}{3}(x^2 - 1)^{3/2} + c$

14. $2x + \log(2\cos x + \sin x + 3) + C$

15. $\frac{1}{2}\log\left|\frac{\tan x}{\tan x + 2}\right| + C$

16. $\frac{1}{6}\log|1 - \cos x| - \frac{1}{2}\log|1 + \cos x| + \frac{1}{3}\log|2 + \cos x| + C$

17. $\sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$

18. $\frac{\sin x - x\cos x}{x\sin x + \cos x} + c$

19. $\sin^{-1}\left(\frac{1}{1+\sin 2\theta}\right) + c$

20. $\lambda n|\,2\sin^2 x/2| + c$

21. $\lambda n\sec x\,.\,(\sec x + \tan x) + c$

22. $\frac{\sqrt{x^2+2x-3}}{8(x+1)^2} + \frac{1}{16}.\cos^{-1}\left(\frac{2}{x+1}\right) + c$

23. $e^x\left(\frac{x+1}{x^2+1}\right) + c$

24. $c - \frac{1}{3}\tan x.\,(2 + \tan^2 x).\sqrt{4-\cot^2 x}$

PART-B

Qus.	25	26	27	28	29	30
Ans.	D	C	C	A	B	A

EXERCISE # 4

1. $(x+1)\tan^{-1}\frac{2x+2}{3} - \frac{3}{2}\log_e\sqrt{4x^2+8x+13} + C$

2. $\frac{1}{6(m+1)}\left[2x^{3m}+3x^{2m}+6x^m\right]^{\frac{m+1}{m}} + C$

Qus.	**3**	**4**	**5**	**6**	**7**
Ans.	A	A	C	D	C

EXERCISE # 5

1. $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x(1-x)}\ ) + C$

2. $\frac{2}{\pi}\{\sqrt{x-x^2} - (1-2x)\sin^{-1}\sqrt{x}\} - x + C$

3. $-\log|\cot x + \sqrt{\cot^2 x - 1}| + \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+\sqrt{1-\tan^2 x}}{\sqrt{2}-\sqrt{1-\tan^2 x}}\right| + C$

4. $\sqrt{2}\tan^{-1}\left(\frac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right) + C$

5. $\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + \frac{4}{3}x^{1/2} - \frac{12}{5}x^{5/12} + \frac{1}{2}x^{1/3} - 4x^{1/4} - 7x^{1/6} - 12x^{1/12} +$

$(2x^{1/2} - 3x^{1/3} + 6x^{1/6} + 11)\log(1+x^{1/6}) + 12\log(1+x^{1/12}) - 3[\log(1+x^{1/6})]^2 + C$

6. $A = -\frac{3}{2}$, $B = \frac{35}{36}$, C = any real value

7. $\frac{1}{2}\sin 2\theta \ \lambda n \left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)+\frac{1}{2}\lambda n(\cos 2\theta) + C$

8. **[C]**

9. $\frac{1}{1+xe^x} + \log\frac{xe^x}{1+xe^x} + C$

10. $\sec^2 x + 2\log(1 + \cos 2x) - \log(1 + \cos^2 2x) + C$

11. $-2\log\frac{1+\sqrt{1-x}}{\sqrt{x}} - 2\sin^{-1}\sqrt{x} + C$

12. $\frac{1}{4}\lambda n(x^2+1) + \frac{5}{4}\tan^{-1} x - \lambda n\frac{1}{2}(x+1) + \frac{x}{(1+x^2)} + C$

13. **[A]**

14. $x\tan^{-1} x. \ln(1+x^2) + (\tan^{-1} x)^2 - 2x\tan^{-1} x + \ln(1+x^2) - \left(\ln\sqrt{1+x^2}\right)^2 + c$

15. $3x - \ln\left(\sqrt{x^2+x+1}\right) + \sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

16. $C - e^{\cos x}(x + \operatorname{cosec} x)$

17. $A \to S$; $B \to P$; $C \to Q$; $D \to R$