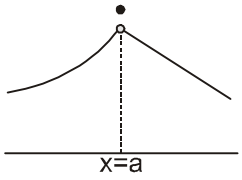
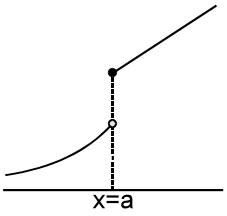
APPLICATIONS OF DERIVATIVES

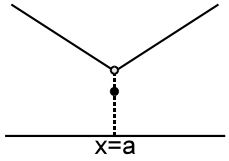
MAXIMA & MINIMA

EXERCISE

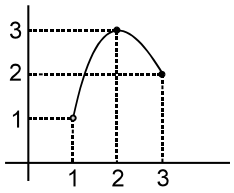
Q.1 In each of following graphs identify if $x = a$ is point of local maxima, minima or neither

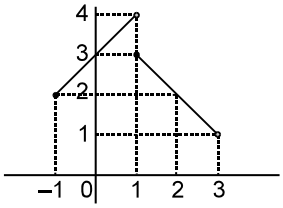
(i) 

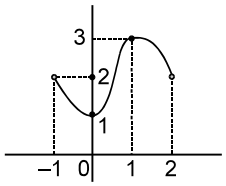
(ii) 

(iii) 

Q.2 Examine the graph of following functions in each case identify the points of global maximum/minimum and local maximum / minimum.

(i) 

(ii) 

(iii) 

Q.3 Find the points of local maxima or minima of following functions

(i) $f(x) = (x - 1)^3 (x + 2)^2$

(ii) $f(x) = x^3 + x^2 + x + 1$.

Q.4 Let $f(x) = 2x^3 + 12x + x + 6$

(i) Find the possible points of Maxima/Minima of $f(x)$ for $x \in R$.

(ii) Find the number of critical points of $f(x)$ for $x \in [0, 2]$.

(iii) Discuss absolute (global) maxima/minima value of $f(x)$ for $x \in [0,2]$

(iv) Prove that for $x \in (1, 3)$, the function does not has a Global maximum.

Q.5 Let $f(x) = \frac{x}{2} + \frac{2}{x}$. Find local maximum and local minimum value of f(x). Can you explain this discrepancy of locally minimum value being greater than locally maximum value.

Q.6 If $f(x) = \begin{cases} (x+\lambda)^2 & x < 0 \\ \cos x & x \geq 0 \end{cases}$, find possible values of λ such that f(x) has local maxima at $x = 0$.

Q.7 Let $f(x) = \sin x\,(1 + \cos x)$; $x \in (0, 2\pi)$. Find the number of critical points of f(x). Also identify which of these critical points are points of Maxima/Minima.

Q.8 Find the two positive numbers x and y whose sum is 35 and the product $x^2\,y^5$ maximum.

Q.9 A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the slops to form a box. What should be the side of the square to be cut off such that volume of the box is maximum possible.

Q.10 Prove that a right circular cylinder of given surface area and maximum volume is such that the height is equal to the diameter of the base.

Q.11 A normal is drawn to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Find the maximum distance of this normal from the centre.

Q.12 A line is drawn passing through point P(1, 2) to cut positive coordinate axes at A and B. Find minimum area of ΔPAB.

Q.13 Two towns A and B are situated on the same side of a straight road at distances a and b respectively perpendiculars drawn from A and B meet the road at point C and D respectively. The distance between C and D is c. A hospital is to be built at a point P on the road such that the distance APB is minimum. Find position of P.

ANSWER KEY

1. (i) Maxima

(ii) Neither maxima nor minima

(iii) Minima

2. (i) Local maxima at $x = 2$, Local minima at $x = 3$, Global maximum at $x = 2$. No global minimum

(ii) Local minima at $x = -1$, No point of Global minimum, no point of local or Global maxima

(iii) Local & Global maximum at $x = 1$, Local & Global minimum at $x = 0$.

3. (i) Maxima at $x = -2$, Minima at $x = -\frac{4}{5}$

(ii) No point of local maxima or minima.

4. (i) $x = 1,.2$ (ii) one

(iii) $f(0) = 6$ is global maximum, $f(1) = 11$ is global maximum

5. Local maxima at $x = -2$, $f(-2) = -2$;Local minima at $x = 2$, $f(2) = 2$

6. $\lambda \in [-1, 1)$

7. Three $x = \frac{\pi}{3}$ is point of maxima.

$x = \pi$ is not a point of extrema.

$x = \frac{5\pi}{3}$ is point of minima

8. $x = 25, y = 10$.

9. 3 cm

11. 1 unit

12. 4 units

13. P is at distance of $\frac{ac}{a+b}$ from C.