APPLICATIONS OF DERIVATIVES

INCREASING & DECREASING FUNCTIONS

EXERCISE

Q.1 Find the intervals of monotonicity of the following functions.

(i) $f(x) = -x^3 + 6x^2 - 9x - 2$

(ii) $f(x) = x + \frac{1}{x+1}$

(iii) $f(x) = x \cdot e^{x-x^2}$

(iv) $f(x) = x - \cos x$

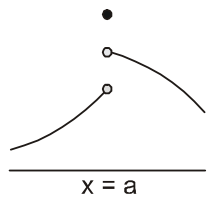
Q.2 Let $f(x) = x - \tan^{-1}x$. Prove that $f(x)$ is monotonically increasing for $x \in R$.

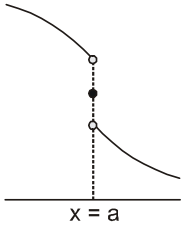
Q.3 If $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ monotonically increases for $\forall\, x \in R$, then find range of values of a.

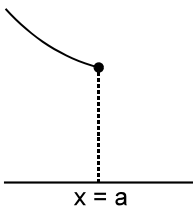
Q.4 Let $f(x) = e^{2x} - ae^x + 1$. Prove that $f(x)$ cannot be monotonically decreasing for $\forall\, x \in R$ for any value of 'a'.

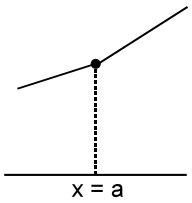
Q.5 The values of 'a' for which function $f(x) = (a + 2)x^3 - ax^2 + 9ax - 1$ monotonically decreasing for $\forall\, x \in R$.

Q.6 For each of the following graph comment on monotonicity of $f(x)$ at $x = a$.

(i)


(ii)


(iii)


(iv)


Q.7 Let $f(x) = x^3 - 3x^2 + 3x + 4$, comment on the monotonic behaviour of f(x) at
(i) $x = 0$ (ii) $x = 1$.

Q.8 Draw the graph of function $f(x) = \begin{cases} x & 0 \le x \le 1 \\ [x] & 1 \le x \le 2 \end{cases}$. Graphically comment on the monotonic behavior of f(x) at $x = 0, 1, 2$. Is f(x) M.I. for $x \in [0, 2]$?

ANSWER KEY

1. (i) I in $[1, 3]$; D in $(-\infty, 1] \cup (3, \infty)$

(ii) I in $(-\infty, -2] \cup [0, \infty)$; D in $[-2, -1) \cup (-1, 0]$

(iii) I in $\left[-\frac{1}{2}, 1\right]$; D in $\left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$

(iv) I for $x \in R$

3. $a \ge 0$

5. $-\infty < a \le -3$

6. (i) neither M.I. nor M.D. (ii) M.D.

(iii) M.D (iv) M.I.

7. M.I. both at $x = 0$ and $x = 1$.

8. M.I. at $x = 0, 2$; neither M.I. nor M.D. at $x = 1$. No, f(x) is not M.I. for $x \in [0, 2]$.