CONTINUITY AND DIFFERENTIABILITY

INTRODUCTION, CONTINUITY

EXERCISE

Q.1 If possible find value of λ for which f(x) is continuous at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} \dfrac{1-\sin x}{1+\cos 2x}, & x < \dfrac{\pi}{2} \\ \lambda \quad , & x = \dfrac{\pi}{2} \\ \dfrac{\sqrt{2x-\pi}}{\sqrt{4+\sqrt{2x-\pi}}-2} \quad , & x > \dfrac{\pi}{2} \end{cases}$$

Q.2 Find the values of a and b such that the function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & ; \quad 0 \le x < \dfrac{\pi}{4} \\ 2x\cot x + b & ; \quad \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \\ a\cos 2x - b\sin x & ; \quad \dfrac{\pi}{2} < x \le \pi \end{cases}$$ is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

Q.3 If $f(x) = \begin{cases} (1+ax)^{1/x} & ; \quad x < 0 \\ b & ; \quad x = 0 \\ \dfrac{(x+c)^{1/3}-1}{x} & ; \quad x > 0 \end{cases}$, then find the values of a, b, c, for which f(x) is continuous at $x = 0$

Q.4 If $f(x) = \begin{cases} [\ell n\ x]\ .\ \operatorname{sgn}\left(\left\{x - \dfrac{1}{2}\right\}\right) ; & 1 < x \le 3 \\ \{x^2\} \quad ; & 3 < x \le 3.5 \end{cases}$.

Find the points where the continuity of f(x), should be checked, where [.] is greatest integer function and {.} fractional part function.

ANSWER KEY

1. discontinuous

2. $a = \frac{\pi}{6}$, $b = \frac{-\pi}{12}$

3. $a = -\lambda n\ 3$, $b = \frac{1}{3}$, $c = 1$

4. $\{ 1, \frac{3}{2}, \frac{5}{2}, e, 3, \sqrt{10}, \sqrt{11}, \sqrt{12}, 3.5 \}$