CONTINUITY AND DIFFERENTIABILITY

DIFFERENTIABILITY

EXERCISE

Q.1 If $f(x) = \begin{cases} [2x] + x\,, & x < 1 \\ \{x\} + 1\,, & x \geq 1 \end{cases}$, then comment on the continuity and differentiable at $x = 1$,

where [.] is greatest integer function and {.} is fractional part function.

Q.2 If $f(x) = \begin{cases} x\tan^{-1}1/x, & x \neq 0 \\ 0\,, & x = 0 \end{cases}$, then comment on the derivability of f(x) at $x = 0$.

Q.3 If possible find the equation of tangent to the following curves at the given points.

(i) $y = x^3 + 3x^2 + 28x + 1$ at $x = 0$.

(ii) $y = (x - 8)^{2/3}$ at $x = 8$.

Q.4 If $f(x) = \begin{cases} \left(\dfrac{e^{[x]} + |x| - 1}{[x] + \{2x\}}\right) & x \neq 0 \\ 1/2 & x = 0 \end{cases}$, comment on the continuity at $x = 0$ and

differentiability at $x = 0$, where [.] is greatest integer function and {.} is fractional part function.

Q.5 If $f(x) = [x] + [1 - x]$, $-1 \leq x \leq 3$, then draw its graph and comment on the continuity and differentiability of f(x), where [.] is greatest integer function.

Q.6 If $f(x) = \begin{cases} |1 - 4x^2| & , \quad 0 \leq x < 1 \\ [x^2 - 2x] & , \quad 1 \leq x \leq 2 \end{cases}$, then draw the graph of f(x) and comment on the

differentiability and continuity of f(x), where [.] is greatest integer function.

Q.7 $f\left(\dfrac{x}{y}\right) = f(x) - f(y)\ \forall\ x, y \in R^+$ and $f'(1) = 1$, then show that $f(x) = \lambda nx$.

Q.8 If f(x) and g(x) are differentiable, then prove that $f(x) \pm g(x)$ will be differentiable.

Q.9 If $f'(2) = 4$, then find the value of $\lim_{h\to 0} \frac{f(2+h) - f(2+\sinh)}{h.\ \sinh.\tanh}$.

Q.10 If f(x) is a polynomial function satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ $\forall\ x \in R - \{0\}$ and $f(3) = -8$, then find f(4)

Q.11 If $f(x + y) = f(x) \cdot f(y)$ for all real x, y and $f(0) \neq 0$, then prove that the function, $g(x) = \frac{f(x)}{1+f^2(x)}$ is an even function.

ANSWER KEY

1. Discontinuous and non-differentiable at $x = 1$

2. non-differentiable at $x = 0$

3. (i) $y = 28x + 1$

(ii) $x = 8$

4. discontinuous hence non-differentiable at $x = 0$

5. f(x) is discontinuous at $x = -1, 0, 1, 2, 3$ hence non-differentiable.

6. f(x) is discontinuous at $x = 1, 2$ & non differentiable at $x = \frac{1}{2}$, 1, 2.

9. 2/3

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