RELATIONS AND FUNCTIONS

BINARY OPERATIONS

EXERCISE

Q.1. Determine whether the following operation define a binary operation on the given set or not:

(i) '*' on N defined by $a * b = a^b$ for all $a, b \in N$.

(ii) 'O' on Z defined by $a\ O\ b = a^b$ for all $a, b \in Z$.

(iii) '*' on N defined by $a * b = a + b - 2$ for all $a, b \in N$

(iv) '$\times_6$' on $S = \{1, 2, 3, 4, 5\}$ defined by $a \times_6 b$ = Remainder when $a\ b$ is divided by 6.

(v) '$+_6$' on $S = \{0, 1, 2, 3, 4, 5\}$ defined by $a +_6 b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b\geq 6 \end{cases}$

(vi) '$\odot$' on N defined by $a \odot b = a^b + b^a$ for all $a, b \in N$

(vii) '*' on Q defined by $a * b = (a - 1)/(b + 1)$ for all $a, b \in Q$

Q.2. Determine whether or not the definition of * given below gives a binary operation. In the event that * is not a binary operation give justification of this.

(i) On Z^+, defined * by $a * b = a - b$

(ii) On Z^+, define * by $a*b = ab$

(iii) On R, define * by $a*b = ab^2$

(iv) On Z^+ define * by $a * b = |a - b|$

(v) On Z^+ define * by $a * b = a$

(vi) On R, define * by $a * b = a + 4b^2$

Here, Z^+ denotes the set of all non-negative integers.

Q.3. Let * be a binary operation on the set I of integers, defined by $a * b = 2a + b - 3$. Find the value of 3 * 4.

Q.4. Is * defined on the set {1, 2, 3, 4, 5} by a * b = LCM of a and b a binary operation? Justify your answer.

Q.5. Let '*' be a binary operation on N defined by a * b = l.c.m. (a, b) for all a, b ∈ N

(i) Find 2 * 4, 3 * 5, 1 * 6.

(ii) Check the commutativity and associativity of '*' on N.

Q.6. Determine which of the following binary operation is associative and which is commutative:

(i) * on N defined by a * b = 1 for all a, b ∈ N

(ii) * on Q defined by $a * b = (a + b)/2$ for all a, b ∈ Q

Q.7 Let A be any set containing more than one element. Let '*' be a binary operation on A defined by a * b = b for all a, b ∈ A Is '*' commutative or associative on A?

Q.8. Check the commutativity and associativity of each of the following binary operations:

(i) '*' on Z defined by $a * b = a + b + a\,b$ for all a, b ∈ Z

(ii) '*' on N defined by $a * b = 2^{ab}$ for all a, b ∈ N

(iii) '*' on Q defined by a * b = a – b for all a, b ∈ Q

(iv) '⊙' on Q defined by $a \odot b = a^2 + b^2$ for all a, b ∈ Q

(v) 'o' on Q defined by $a \circ b = (ab/2)$ for all a, b ∈ Q

(vi) '*' on Q defined by $a * b = ab^2$ for all a, b ∈ Q

(vii) '*' on Q defined by $a * b = a + a\,b$ for all a, b ∈ Q

(viii) '*' on R defined by a * b = a + b -7 for all a, b ∈ R

(ix) '*' on Q defined by $a * b = (a - b)^2$ for all a, b ∈ Q

(x) '*' on Q defined by $a * b = a\,b + 1$ for all $a, b \in Q$

(xi) '*' on N defined by $a * b = a^b$ for all $a, b \in N$

(xii) '*' on Z defined by $a * b = a - b$ for all $a, b \in Z$

(xiii) '*' on Q defined by $a * b = (ab/4)$ for all $a, b \in Q$

(xiv) '*' on Z defined by $a * b = a + b - ab$ for all $a, b \in Z$

(xv) '*' on Q defined by $a * b = \gcd(a, b)$ for all $a, b \in Q$

Q.9. If the binary operation o is defined by $a0b = a + b - ab$ on the set $Q - \{-1\}$ of all rational numbers other than 1, show that o is commutative on $Q - [1]$.

Q.10. Show that the binary operation * on Z defined by $a * b = 3a + 7b$ is not commutative?

Q.11. On the set Z of integers a binary operation * is defined by $a\ 8\ b = ab + 1$ for all $a, b \in Z$. Prove that * is not associative on Z.

ANSWER KEY

1. (i) Thus, * is a binary operation on N.

   (ii) Thus, * is not a binary operation on Z

   (iii) Thus, there exist $a = 1$ and $b = 1$ such that $a * b \notin N$

   (iv) Thus, $\times_6$ is not a binary operation on S.

   (v) Thus, $\odot$ is a binary operation on N.

   (vii)So, * is not a binary operation in Q.

2. (i) Thus, * is not a binary operation on Z^+.

   (ii) Thus, * is a binary operation on R.

(iii) Thus, * is a binary operation on R.

(iv) Thus, * is a binary operation on Z^+.

(v) Thus, * is a binary operation on Z^+.

(vi) Thus, * is a binary operation on R.

3. 7

4. Thus, * is not a binary operation on {1, 2, 3, 4, 5}.

5. (i) $2 * 4 = \text{l.c.m. } (2, 4) = 4$

$3 * 5 = \text{l.c.m. } (3, 5) = 15$

$1 * 6 = \text{l.c.m. } (1, 6) = 6$

(ii) Thus, * is associative on N.

6. (i) Thus, * is associative on N.

(ii) Thus, * is not associative on N

7. Thus, * is associative on A

8. (i) Thus, * is associative on Z.

(ii) Thus, * is not associative on N

(iii) Thus, * is not associative on Q

(iv) Thus, $\odot$ is not associative on Q.

(v) Thus, o is associative on Q.

(vi) Thus, * is not associative on Q.

(vii) Thus, * is not associative on Q

(viii) Thus, * is associative on R.

(ix) Thus, * is not associative on Q.

(x) Thus, * is not associative on Q.

(xi) Thus, * is not associative on N

(xii) Thus, * is not associative on Z

(xiii) Thus, * is associative on Q

(xiv) Thus, * is associative on N

9. Thus, o is commutative on Q – {-1}

10. Thus, * is not commutative on Z.