SOLVED EXAMPLES

Ex. 1 Evaluate : $\lim\limits_{x\to 1}\dfrac{4-\sqrt{15x+1}}{2-\sqrt{3x+1}}$

Sol. $\lim\limits_{x\to 1}\dfrac{4-\sqrt{15x+1}}{2-\sqrt{3x+1}}=\lim\limits_{x\to 1}\dfrac{(4-\sqrt{15x+1})(2+\sqrt{3x+1})(4+\sqrt{15x+1})}{(2-\sqrt{3x+1})(4+\sqrt{15x+1})(2+\sqrt{3x+1})}$

$$=\lim_{x\to 1}\frac{(15-15x)}{(3-3x)}\times\frac{2+\sqrt{3x+1}}{4+\sqrt{15x+1}}=\frac{5}{2}$$

Ex. 2 If $\lim\limits_{x\to\infty}\left(\dfrac{x^3+1}{x^2+1}-(ax+b)\right)=2$, then find value of a and b.

Sol. $\lim\limits_{x\to\infty}\left(\dfrac{x^3+1}{x^2+1}-(ax+b)\right)=2$ $\quad\Rightarrow\quad$ $\lim\limits_{x\to\infty}\dfrac{x^3(1-a)-bx^2-ax+(1-b)}{x^2+1}=2$

$\Rightarrow\quad \lim\limits_{x\to\infty}\dfrac{x(1-a)-b-\dfrac{a}{x}+\dfrac{(1-b)}{x^2}}{1+\dfrac{1}{x^2}}=2 \quad\Rightarrow\quad 1-a=0,\ -b=2 \quad\Rightarrow\quad a=1,\ b=-2$

Ex. 3 Evaluate : $\lim\limits_{x\to 1}\dfrac{x^{P+1}-(P+1)x+P}{(x-1)^2}$

Sol. $\lim\limits_{x\to 1}\dfrac{x^{P+1}-(P+1)x+P}{(x-1)^2}$ $\quad\left(\dfrac{0}{0}\text{ form}\right)$

$$=\lim_{x\to 1}\frac{x^{P+1}-Px-x+P}{(x-1)^2}=\lim_{x\to 1}\frac{x(x^P-1)-P(x-1)}{(x-1)^2}$$

Dividing numerator and denominator by (x –1), we get

$$=\lim_{x\to 1}\frac{\dfrac{x(x^P-1)}{x-1}-P}{(x-1)}=\lim_{x\to 1}\frac{(x+x^2+x^3+\ldots.+x^P)-P}{(x-1)}$$

$$=\lim_{x\to 1}\frac{(x+x^2+x^3+\ldots.+x^P)-(1+1+1+\ldots\ldots\text{upto P times})}{(x-1)}$$

$$=\lim_{x\to 1}\left\{\frac{(x-1)}{(x-1)}+\frac{(x^2-1)}{(x-1)}+\frac{(x^3-1)}{(x-1)}+\ldots\ldots+\frac{(x^P-1)}{(x-1)}\right\}$$

$$=1+2(1)^{2-1}+3(1)^{3-1}+\ldots\ldots+P(1)^{P-1}=1+2+3+\ldots\ldots+P=\frac{P(P+1)}{2}$$

Ex. 4 Evaluate : $\lim\limits_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]$

Sol. We have

$$\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{x-3}{x(x-1)}\right]=-\frac{1}{2}$$

Ex. 5 Evaluate $\lim\limits_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$.

Sol. $$\lim_{x\to\infty}\frac{x}{2}\left(2\sin\frac{\pi}{4x}\cos\frac{\pi}{4x}\right)=\lim_{x\to\infty}\frac{x}{2}\sin\left(\frac{\pi}{2x}\right)$$

$$=\lim_{x\to\infty}\frac{\sin\left(\frac{\pi}{2x}\right)}{\frac{\pi}{2x}}\cdot\frac{\pi}{4}=\frac{\pi}{4}\qquad \lim_{y\to 0}\frac{\sin y}{y}=\frac{\pi}{4},\quad \text{where } y=\frac{\pi}{2x}$$

Ex. 6 Evaluate $\lim\limits_{x\to 1}(\log_3 3x)^{\log_x 3}$

Sol. $$\lim_{x\to 1}(\log_3 3x)^{\log_x 3}=\lim_{x\to 1}(\log_3 3+\log_3 x)^{\log_x 3}$$

$$=\lim_{x\to 1}(1+\log_3 x)^{1/\log_3 x}=e$$

✈ $\log_b a=\frac{1}{\log_a b}$

Ex. 7 Evaluate : $\lim\limits_{x\to 0}\frac{x^3\cot x}{1-\cos x}$

Sol. $$\lim_{x\to 0}\frac{x^3\cos x}{\sin x(1-\cos x)}=\lim_{x\to 0}\frac{x^3\cos x(1+\cos x)}{\sin x.\sin^2 x}=\lim_{x\to 0}\frac{x^3}{\sin^3 x}.\cos x(1+\cos x)=2$$

Ex. 8 Evaluate $\lim\limits_{x\to 1}\frac{(7+x)^{\frac{1}{3}}-2}{x-1}$

Sol. Put $x=1+h$

$$\lim_{h\to 0}\frac{(8+h)^{\frac{1}{3}}-2}{h}$$

$$=\lim_{h\to 0}\frac{2.\left(1+\frac{h}{8}\right)^{\frac{1}{3}}-2}{h}=\lim_{h\to 0}\frac{2\left\{1+\frac{1}{3}.\frac{h}{8}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{h}{8}\right)^2}{1.2}+\ldots\ldots-1\right\}}{h}=\lim_{h\to 0}2\times\frac{1}{24}=\frac{1}{12}$$

Ex. 9 Evalute $\lim\limits_{x\to\infty}\frac{1}{1+n^2}+\frac{1}{2+n^2}+...+\frac{n}{n+n^2}$.

Sol. $P_n = \frac{1}{1+n^2}+\frac{2}{2+n^2}+...+\frac{n}{n+n^2}$

Now, $P_n < \frac{1}{1+n^2}+\frac{2}{1+n^2}+...+\frac{n}{1+n^2}$

$= \frac{1}{1+n^2}(1+2+3+....+n)$

$= \frac{n(n+1)}{2(1+n^2)}$

Also, $P_n > \frac{1}{n+n^2}+\frac{2}{n+n^2}+\frac{3}{n+n^3}+...+\frac{n}{n+n^2}$

$= \frac{n(n+1)}{2(n+n^2)}$

Thus, $\frac{n(n+1)}{2(n+n^2)} < P_n < \frac{n(n+1)}{2(1+n^2)}$

or $\lim\limits_{n\to\infty}\frac{n(n+1)}{2(n+n^2)} < \lim\limits_{n\to\infty}P_n < \lim\limits_{n\to\infty}\frac{n(n+1)}{2(1+n^2)}$

or $\lim\limits_{n\to\infty}\frac{1\left(1+\frac{1}{n}\right)}{2\left(\frac{1}{n}+1\right)} < \lim\limits_{n\to\infty}P_n < \lim\limits_{n\to\infty}\frac{1\left(1+\frac{1}{n}\right)}{2\left(\frac{1}{n^2}+1\right)}$

or $\frac{1}{2} < \lim\limits_{n\to\infty}P_n < \frac{1}{2}$

or $\lim\limits_{n\to\infty}P_n = \frac{1}{2}$

Ex. 10 Evalute $\lim\limits_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$

Sol. $\lim\limits_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x} = \lim\limits_{x\to 0}\left(1+\frac{a^x+b^x+c^x-3}{3}\right)^{1/x}$

$= \lim\limits_{x\to 0}\left[\left(1+\frac{(a^x-1)}{3}+\frac{(b^x-1)}{3}+\frac{(c^x-1)}{3}\right)^{\frac{3}{(a^x-1)+(b^x-1)+(c^x-1)}}\right]^{\frac{a^x-1+b^x-1+c^x-1}{3x}}$

$= e^{1/3}\lim\limits_{x\to 0}\left[\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right] = e^{1/3\ (\log a+\log b+\log c)} = e^{\log (abc)1/3} = (abc)^{1/3}$

Ex. 11 Evaluate $\lim_{x \to a} \left(2 - \frac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$.

Sol. $\lim_{x \to a} \left(2 - \frac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$ put $x = a + h$

$$= \lim_{h \to 0} \left(1 + \frac{h}{(a+h)}\right)^{\tan\left(\frac{\pi}{2} + \frac{\pi h}{2a}\right)} = \lim_{h \to 0} \left(1 + \frac{h}{a+h}\right)^{-\cot\left(\frac{\pi h}{2a}\right)} = e^{\lim_{h \to 0} -\cot\frac{\pi h}{2a} \cdot \left(1 + \frac{h}{a+h} - 1\right)}$$

$$= e^{\lim_{h \to 0} -\left(\frac{\frac{\pi h}{2a}}{\tan\frac{\pi h}{2a}}\right) \cdot \frac{\frac{2a}{\pi}}{a+h}} \qquad = e^{-\frac{2}{\pi}}$$

Ex. 12 Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$

Sol. Put $x = \frac{\pi}{4} + h$

✈ $x \to \frac{\pi}{4}$

$\Rightarrow$ $h \to 0$

$$\lim_{h \to 0} \frac{1 - \tan\left(\frac{\pi}{4} + h\right)}{1 - \sqrt{2}\sin\left(\frac{\pi}{4} + h\right)} = \lim_{h \to 0} \frac{1 - \frac{1 + \tan h}{1 - \tan h}}{1 - \sin h - \cos h} = \lim_{h \to 0} \frac{\frac{-2\tan h}{1 - \tan h}}{2\sin^2\frac{h}{2} - 2\sin\frac{h}{2}\cos\frac{h}{2}}$$

$$= \lim_{h \to 0} \frac{-2\tan h}{2\sin\frac{h}{2}\left[\sin\frac{h}{2} - \cos\frac{h}{2}\right]} \frac{1}{(1 - \tanh)} = \lim_{h \to 0} \frac{-2\frac{\tanh}{h}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}}\left[\sin\frac{h}{2} - \cos\frac{h}{2}\right]} \frac{1}{(1 - \tanh)} = \frac{-2}{-1} = 2.$$

Ex. 13 If $\lim_{x \to 0} \frac{\cos 4x + a\cos 2x + b}{x^4}$ is finite, find a and b using expansion formula.

Sol. $\lim_{x \to 0} \frac{\cos 4x + a\cos 2x + b}{x^4}$ = Finite

Using expansion formula for cos 4x and cos 2x, we get

$$\lim_{x \to 0} \frac{\left(1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!}\right) + a\left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!}\right) + b}{x^4} = \text{finite}$$

or $$\lim_{x \to 0} \frac{(1 + a + b) + (-8 - 2a)x^2 + \left(\frac{32}{3} + \frac{2}{3}a\right)x^4 + \ldots}{x^4}$$

or $1 + a + b = 0$ ....**(i)**

$-8 - 2a = 0$ ....**(ii)**

Solving **(i)** and **(ii)** for a and b, we get

$a = -4$ and $b = 3$

Also, $L = \frac{32}{3} + \frac{2}{3}a = \frac{32-8}{3} = 8$

Ex. 14 Evaluate $\lim_{n\to\infty} \frac{n^p \sin^2(n!)}{n+1}$.

Sol. $\lim_{n\to\infty} \frac{n^p \sin^2(n!)}{n+1} = \lim_{n\to\infty} \frac{\sin^2(n!)}{n^{1-p}\left(1+\frac{1}{n}\right)}$

$$= \frac{\text{some number between 0 and 1}}{\infty} = 0$$

Ex. 15 Evalute $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x-\sin x}\right)}$

Sol. Since $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{\sin x}{x-\sin x} = \lim_{x\to 0}\frac{1}{\left(\frac{x}{\sin x}-1\right)}$

$$= \frac{1}{1-1} = \infty$$

we have, $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x-\sin x}\right)} = e^{\lim_{x\to 0}\left(\frac{\sin x}{x}-1\right)\left(\frac{\sin x}{x-\sin x}\right)}$

$$= e^{\lim_{x\to 0} -\frac{\sin x}{x}} = e^{-1} = \frac{1}{e}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. $\lim\limits_{x \to \infty} x\left(\tan^{-1}\frac{x+1}{x+2} - \cot^{-1}\frac{x+2}{x}\right)$ is

(A) -1 **(B)** $\frac{1}{2}$ **(C)** $-\frac{1}{2}$ **(D)** non existent

2. $\text{Limit}_{x \to 0}\left(\frac{3}{1+\sqrt{4+x}}\right)^{\text{cosec}\, x}$ has the value equal to :

(A) $e^{-1/12}$ **(B)** $e^{-1/6}$ **(C)** $e^{-1/4}$ **(D)** $e^{-1/3}$

3. $\text{Limit}_{x\to\infty}\frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^{x}\right\}}$ is equal to

(A) 1 **(B)** 0 **(C)** $\pi/2$ **(D)** non existent

4. $\lim\limits_{n\to\infty}\left[\frac{1}{1.3}+\frac{1}{3.5}+.......+\text{ to n terms}\right]$ is equal to -

(A) 1/4 **(B)** 1/2 **(C)** 1 **(D)** 2

5. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4-1}}$ is equal to -

(A) 1 **(B)** -1 **(C)** 0 **(D)** none of these

6. If $\lim\limits_{x\to a}[f(x)+g(x)]=2$ and $\lim\limits_{x\to a}[f(x)-g(x)]=1$, then $\lim\limits_{x\to a} f(x)\, g(x)$ -

(A) need not exist **(B)** exist and is $\frac{3}{4}$ **(C)** exists and is $-\frac{3}{4}$ **(D)** exists and is $\frac{4}{3}$

7. Let α and β be the distinct roots of $ax^2+bx+c=0$ then $\lim\limits_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to -

(A) $\frac{1}{2}(\alpha-\beta)^2$ **(B)** $\frac{-a^2}{2}(\alpha-\beta)^2$ **(C)** 0 **(D)** $\frac{a^2}{2}(\alpha-\beta)^2$

8. If $\lim\limits_{x\to 0}(x^{-3}\sin 3x + ax^{-2}+b)$ exists and is equal to zero then :

(A) $a=-3$ & $b=9/2$ **(B)** $a=3$ & $b=9/2$ **(C)** $a=-3$ & $b=-9/2$ **(D)** $a=3$ & $b=-9/2$

9. If α, β are the roots of the quadratic equation $ax^2+bx+c=0$ then $\lim\limits_{x\to\alpha}\frac{1-\cos\left(ax^2+bx+c\right)}{(x-\alpha)^2}$ equals

(A) 0 **(B)** $\frac{1}{2}(\alpha-\beta)^2$ **(C)** $\frac{a^2}{2}(\alpha-\beta)^2$ **(D)** $-\frac{a^2}{2}(\alpha-\beta)^2$

10. Let $l = \lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x$ then $\{l\}$ where $\{x\}$ denotes the fractional part function is

(A) $8 - e^2$ **(B)** $7 - e^2$ **(C)** $e^2 - 6$ **(D)** $e^2 - 7$

11. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ is equal to -

(A) $2/\pi$ **(B)** $\pi/2$ **(C)** π **(D)** none of these

12. The value of $\lim_{x\to 0}\frac{(\tan(\{x\}-1))\sin\{x\}}{\{x\}(\{x\}-1)}$ where $\{x\}$ denotes the fractional part function -

(A) is 1 **(B)** is tan 1 **(C)** is sin 1 **(D)** is not existent

13. $\lim_{x\to\infty}\left(\sin\sqrt{x+1} - \sin\sqrt{x}\right)$ is equal to -

(A) 1 **(B)** –1 **(C)** 0 **(D)** none of these

14. $\lim_{x\to\pi/2}\frac{[1-\tan x/2][1-\sin x]}{[1+\tan x/2][\pi-2x]^3}$ is equal to -

(A) 0 **(B)** 1/32 **(C)** ∞ **(D)** 1/8

15. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of *a* and *b* are -

(A) $a \in R, b = 2$ **(B)** $a = 1, b \in R$ **(C)** $a \in R, b \in R$ **(D)** $a = 1$ and $b = 2$

16. $\lim_{x\to 0}\frac{\sqrt[3]{1+3x}-1-x}{(1+x)^{101}-1-101x}$ has the value equal to -

(A) $-\frac{3}{5050}$ **(B)** $-\frac{1}{5050}$ **(C)** $\frac{1}{5051}$ **(D)** $\frac{1}{4950}$

17. $\lim_{x\to 0}\frac{e^{x^3} - \tan x + \sin x - 1}{x^n}$ exists and is non-zero, then the value of n is -

(A) 1 **(B)** 3 **(C)** 2 **(D)** 0

18. $\lim_{x\to 0}\frac{\left(\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\ldots\ldots\infty}}}\right)-1}{x^2}$ equals

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2

19. $\lim_{x\to\infty}\left(\sqrt[3]{(x+a)(x+b)(x+c)} - x\right) =$

(A) $\sqrt{abc}$ **(B)** $\frac{a+b+c}{3}$ **(C)** abc **(D)** $(abc)^{1/3}$

20. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2+n+r}$ equals

(A) 0 **(B)** 1/3 **(C)** 1/2 **(D)** 1

21. $\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\left[\sqrt{n^2+n+1}\right]\right)$ $(n \in I)$ where [ ] denotes the greatest integer function is

(A) 0 (B) 1/2 (C) 2/3 (D) 1/4

22. $\lim_{x\to 0}\frac{\sin^{-1}x - \tan^{-1}x}{x^3}$ is equal to

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) none of these

23. If α and β be the roots of equation $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha}\left(1+ax^2+bx+c\right)^{\frac{1}{x-\alpha}}$ is equal to

(A) $a(\alpha-\beta)$ (B) ●n $|a(\alpha-\beta)|$ (C) $e^{a(\alpha-\beta)}$ (D) $e^{a|\alpha-\beta|}$

24. $\lim_{x\to 0}\left[\left(1-e^x\right)\frac{\sin x}{|x|}\right]$, where [·] represents greatest integer function, is equal to

(A) – 1 (B) 1 (C) 0 (D) does not exist

25. $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+............+\frac{1}{\sqrt{n^2+2n}}\right)$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2

26. $\lim_{y\to 0}\left(\lim_{x\to\infty}\frac{\exp\left(x\ln\left(1+\frac{ay}{x}\right)\right)-\exp\left(x\ln\left(1+\frac{by}{x}\right)\right)}{y}\right)$ is equal to

(A) a + b (B) a – b (C) b – a (D) – (a + b)

27. Let $f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right), & x\neq 0 \\ 0, & x=0\end{cases}$, then $\lim_{x\to\infty} f(x)$ is equal to

(A) 0 (B) $-\frac{1}{2}$ (C) 1 (D) none of these.

28. If $\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}\,(bx-\sin x)}=1$, then constants 'a' and 'b' are (where a > 0)

(A) b = 1, a = 36 (B) a = 1, b = 6 (C) a = 1, b = 36 (D) b = 1, a = 6

29. $\lim_{x\to\infty}\sec^{-1}\left(\frac{x}{x+1}\right)$ is equal to

(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) does not exist

30. The value of $\lim_{x\to 0}(\cos ax)^{\operatorname{cosec}^2 bx}$ is -

(A) $e^{\left(\frac{8b^2}{a^2}\right)}$ (B) $e^{\left(-\frac{8a^2}{b^2}\right)}$ (C) $e^{\left(-\frac{a^2}{2b^2}\right)}$ (D) $e^{\left(-\frac{b^2}{2a^2}\right)}$

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Let $f(x) = \lim\limits_{n\to\infty} \dfrac{2x^{2n}\sin\frac{1}{x} + x}{1 + x^{2n}}$ then which of the following alternative(s) is/are correct?

(A) $\lim\limits_{x\to\infty} x f(x) = 2$ **(B)** $\lim\limits_{x\to 1} f(x)$ does not exist.

(C) $\lim\limits_{x\to 0} f(x)$ does not exist. **(D)** $\lim\limits_{x\to -\infty} f(x)$ is equal to zero.

2. If $\bullet = \lim\limits_{x\to\infty}\left(\dfrac{x+1}{x-1}\right)^x$, then $\{\bullet\}$ and $[\bullet]$ (where $\{.\}$ & $[.]$ denotes the fractional part function & greatest integer function respectively), is/are -

(A) 7 **(B)** $7 - e^2$ **(C)** -7 **(D)** $e^2 - 7$

3. If $\lim\limits_{x\to 0} \dfrac{\sin 2x + a\sin x}{x^3} = p$ (finite), then

(A) $a = -2$ **(B)** $a = -1$ **(C)** $p = -2$ **(D)** $p = -1$

4. For $\lim\limits_{x\to 0} \dfrac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$

(A) R.H.L exists

(B) L.H.L does not exists

(C) limit does not exists as R.H.L is 1 and L.H.L is –1

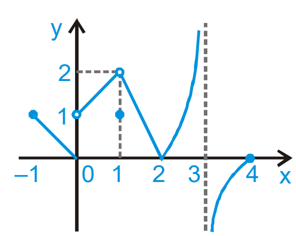
(D) limit does not exists as R.H.L and L.H.L both are non-existent.

5. Let $f(x) = \dfrac{|x + \pi|}{\sin x}$, then

(A) $f(-\pi^+) = -1$ **(B)** $f(-\pi^-) = 1$

(C) $\lim\limits_{x\to -\pi} f(x)$ does not exist **(D)** $\lim\limits_{x\to \pi} f(x)$ does not exist

6. The graph of the function $y = f(x)$ is shown in the adjacent figure, then correct statement is -



(A) $\lim\limits_{x\to 0^+} f(x) = 1$ **(B)** $\lim\limits_{x\to 1} f(x) = 2$

(C) $\lim\limits_{x\to 3} f(x)$ does not exist. **(D)** $\lim\limits_{x\to 4} f(x) = 0$

7. If $\lim\limits_{x\to 0} \left(1 + ax + bx^2\right)^{\frac{2}{x}} = e^3$, then possible values of a and b is/are :

(A) $a = 3, b = 0$ **(B)** $a = \frac{3}{2}, b = \frac{1}{2}$ **(C)** $a = \frac{3}{2}, b = \frac{3}{2}$ **(D)** $a = \frac{3}{2}, b = 0$

8. Which of the following limits is/are unity ?

(A) $\lim_{x\to\pi}\frac{\sin(\pi-x)}{(\pi-x)}$ **(B)** $\lim_{x\to0}\frac{\tan(\sin x)}{\tan x}$ **(C)** $\lim_{x\to\infty}\frac{x^2+x}{x^2-x}$ **(D)** $\lim_{x\to1}\frac{\tan(\pi x)}{\pi(x-1)}$

9. Assume that $\lim_{\theta\to-1} f(\theta)$ exists and $\frac{\theta^2+\theta-2}{\theta+3} \le \frac{f(\theta)}{\theta^2} \le \frac{\theta^2+2\theta-1}{\theta+3}$ holds for certain interval containing the point $\theta = -1$ then $\lim_{\theta\to-1} f(\theta)$

(A) is equal to $f(-1)$ **(B)** is equal to 1 **(C)** is non existent **(D)** is equal to – 1

10. Which of the following limits vanishes ?

(A) $\lim_{x\to\infty} x^{\frac{1}{4}}\sin\frac{1}{\sqrt{x}}$

(B) $\lim_{x\to\pi/2}(1-\sin x).\tan x$

(C) $\lim_{x\to\infty}\frac{2x^2+3}{x^2+x-5}.\text{sgn}(x)$

(D) $\lim_{x\to3^+}\frac{[x]^2-9}{x^2-9}$

where [ ] denotes greatest integer function

11. $\lim_{x\to c}$ f(x) does not exist when -

(A) $f(x)=[[x]]-[2x-1], c=3$

(B) $f(x)=[x]-x,\ c=1$

(C) $f(x)=\{x\}^2-\{-x\}^2, c=0$

(D) $f(x)=\frac{\tan(\text{sgn } x)}{\text{sgn } x}, c=0$

where [x] denotes step up function & {x} fractional part function.

12. For $a > 0$, let $\bullet = \lim_{x\to\frac{\pi}{2}}\frac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}$ and $m = \lim_{x\to-\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2-ax}\right)$ then -

(A) '●' is always greater than 'm' for all values of $a > 0$

(B) '●' is always greater than 'm' only when $a \ge 1$

(C) '●' is always greater than 'm' for all values of 'a' satisfying $a > e^{-a}$

(D) $e^{\bullet} + m = 0$

13. Which of the following limits is/are unity ?

(A) $\lim_{t\to0}\frac{\sin(\tan t)}{\sin t}$

(B) $\lim_{x\to\pi/2}\frac{\sin(\cos x)}{\cos x}$

(C) $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

(D) $\lim_{x\to0}\frac{\sqrt{x^2}}{x}$

14. If $f(x)=\begin{cases}\dfrac{\sin[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{cases}$ where [x] denotes the greatest integer less than or equal to x, then

(A) $\lim\limits_{x\to 0^-} f(x)=\sin 1$ **(B)** $\lim\limits_{x\to 0^+} f(x)=0$

(C) limit does not exist at x = 0 **(D)** limit exist at x = 0

15. Let $f(x)=\dfrac{x.2^x - x}{1-\cos x}$ & $g(x)=2^x \sin\left(\dfrac{\ln 2}{2^x}\right)$ then -

(A) $\lim\limits_{x\to 0} f(x) =$ ●n 2 **(B)** $\lim\limits_{x\to\infty} g(x) =$ ●n 4 **(C)** $\lim\limits_{x\to 0} f(x) =$ ●n 4 **(D)** $\lim\limits_{x\to\infty} g(x) =$ ●n 2

16. Which of the following limits vanishes ?

(A) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{\tan x}\right)$ **(B)** $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+1}{2x^2-1}\right)^{\frac{x^3}{1-x}}$

(C) $\lim\limits_{x\to\frac{\pi}{4}^+}\left[\tan\left(x+\dfrac{\pi}{8}\right)\right]^{\tan 2x}$ **(D)** $\lim\limits_{x\to 1}\dfrac{x^4-2x^2+1}{x^3-1}$

17. If $\lim\limits_{x\to 3}\dfrac{x^3+Cx^2+5x+12}{x^2-7x+12}=$ ● (finite real number) then -

(A) ● = 4 **(B)** C = –6 **(C)** C = 4 **(D)** $C \in R$

18. Consider the function $f(x)=\left(\dfrac{ax+1}{bx+2}\right)^x$ where a, b > 0 then $\lim\limits_{x\to\infty} f(x)$ -

(A) exists for all values of a and b **(B)** is zero for a < b

(C) is non existent for a > b **(D)** is $e^{-\left(\frac{1}{a}\right)}$ or $e^{-\left(\frac{1}{b}\right)}$ if a = b

19. Identify the true statement(s) -

(A) $\lim\limits_{n\to\infty}\left[\sum\limits_{r=1}^{n}\dfrac{1}{2^r}\right]=1$, where [.] denotes the greatest integer function.

(B) If f(x) = (x – 1) {x}, then limit of f(x) does not exist at all integers.

(C) $\lim\limits_{x\to 0^+}\left[\dfrac{\tan x}{x}\right]=1$, where [.] denotes the greatest integer function.

(D) $\left[\lim\limits_{x\to 0^+}\dfrac{\tan x}{x}\right]=1$, where [.] denotes the greatest integer function.

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement - I** If $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ exists finitely, then $\lim_{x\to 0} f(x).g(x)$ exists finitely.

Statement - II If $\lim_{x\to 0} f(x).g(x)$ exists finitely then $\lim_{x\to 0} f(x).g(x) = \lim_{x\to 0} f(x).\lim_{x\to 0} g(x)$

2. **Statement - I** $\lim_{x\to\infty} \frac{x^n + nx^{n-1} + 1}{[x^n]} = 0,\ n \in I$ (where [.] represents greatest integer function).

Statement - II $x - 1 < [x] \le x$, (where [.] represents greatest integer function).

3. **Statement - I** $\lim_{x\to 0} \frac{\sqrt{\frac{1-\cos 2x}{2}}}{x}$ does not exist.

Statement - II $\lim_{x\to\frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln \sin x} = 2$

4. **Statement - I** $\lim_{x\to\infty} \frac{2x^4 + 3x^3 + 7x}{3x^4 + 2x^2 + 3x} = \frac{2}{3}$.

Statement - II If P(x) and Q(x) are two polynomials with rational coefficients, then

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \frac{\text{coefficient of highest power of } x \text{ in } P(x)}{\text{coefficient of highest power of } x \text{ in } Q(x)}$$

5. **Statement - I** $\lim_{x\to 0} \left\{1^{1/\sin^2 x} + 2^{1/\sin^2 x} + 3^{1/\sin^2 x} + \ldots\ldots + n^{1/\sin^2 x}\right\}^{\sin^2 x} = n$

Statement- II For $0 < a < 1$, $\lim_{x\to\infty} a^x = 0$

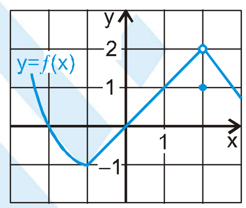
Exercise # 3 Part # I [Matrix Match Type Questions]

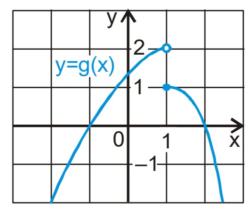
Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. Let $\phi(x) = \dfrac{a_0x^m + a_1x^{m+1} + ... + a_kx^{m+k}}{b_0x^n + b_1x^{n+1} + ... + b_1x^{n+1}}$, where $a_0 \neq 0$, $b_0 \neq 0$ and $m, n \in N$, then $\lim\limits_{x\to 0} \phi(x)$ is equal to

	Column – I		Column – II
(A)	$m > n$	(p)	∞
(B)	$m = n$	(q)	$-\infty$
(C)	$m < n$ and $n - m$ is even, $\dfrac{a_0}{b_0} > 0$	(r)	$\dfrac{a_0}{b_0}$
(D)	$m < n$ and $n - m$ is even, $\dfrac{a_0}{b_0} < 0$	(s)	0

2. The graphs of f and g are given. Use them to evaluate each limit.





	Column-I		Column-II
(A)	$\lim\limits_{x\to 1} f(g(x))$	(p)	1
(B)	$\lim\limits_{x\to 2} \sqrt{3f(x) - 2}$	(q)	does not exist
(C)	$\lim\limits_{x\to 0} \dfrac{f(x)}{g(x)} + f(x)g(x)$	(r)	0
(D)	$\lim\limits_{x\to 1^+} \dfrac{3f(x) - g(x)}{f(x) + g(x)}$	(s)	2

3. $\lim\limits_{x\to 0} f(x)$, where f (x) is as in column - I, is

	Column – I		Column – II
(A)	$f(x) = \dfrac{\tan[e^2]x^2 - \tan[-e^2]x^2}{\sin^2 x}$ where [.] represents greatest integer function.	(p)	$\dfrac{\sqrt{2}}{8}$
(B)	$f(x) = \left[\left(\min(t^2 + 4t + 6)\right)\dfrac{\sin x}{x}\right]$ where [.] represents greatest integer function.	(q)	15
(C)	$f(x) = \dfrac{\sqrt[3]{1 + x^2} - \sqrt[4]{1 - 2x}}{x + x^2}$	(r)	1
(D)	$f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$	(s)	$\dfrac{1}{2}$

4. **Column-I** | **Column-II**

(A) If $L = \lim_{x \to \pi/2^+} \frac{\cos(\tan^{-1}(\tan x))}{x - \pi/2}$ then $\cos(2\pi L)$ is **(p)** 1

(B) Number of solutions of the equation $\text{cosec}\theta = k$ in $[0, \pi]$ where $k = \lim_{n\to\infty} \prod_{r=2}^{n} \frac{r^3 - 1}{r^3 + 1}$ **(q)** –1

(C) If c satisfies the equation $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = 4$ then $-\frac{e^c}{2}$ is **(r)** 0

(D) If $\lim_{x\to-\infty} \frac{(3x^4 + 2x^2)\sin(1/x) + |x|^3 + 5}{|x|^3 + |x|^2 + |x| + 1} = k$, then $\frac{k}{2}$ is **(s)** 2

5. **Column I** | **Column II**

(A) $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$ equals **(p)** e^2

(B) $\lim_{x\to\infty}\left(\sin\frac{1}{x} + \cos\frac{1}{x}\right)^x$ **(q)** $e^{-1/2}$

(C) $\lim_{x\to 0}(\cos x)^{\cot^2 x}$ **(r)** e

(D) $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}$ **(s)** e^{-1}

6. **Column-I** | **Column-II**

(A) $\lim_{x\to 0} \frac{1-\cos 2x}{e^{x^2} - e^x + x}$ equals **(p)** 1

(B) If the value of $\lim_{x\to 0^+}\left(\frac{(3/x)+1}{(3/x)-1}\right)^{1/x}$ can be expressed in the form of $e^{p/q}$, where *p* and *q* are relative prime then $(p + q)$ is equal to **(q)** 2

(C) $\lim_{x\to 0} \frac{\tan^3 x - \tan x^3}{x^5}$ equals **(r)** 4

(D) $\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ **(s)** 5

Part # II [Comprehension Type Questions]

Comprehension # 1

Consider $f(x) = \dfrac{\sin x + ae^{x} + be^{-x} + c\ln(1+x)}{x^3}$, where a, b, c are real numbers.

1. If $\lim\limits_{x\to 0^+} f(x)$ is finite, then the value of a + b + c is

(A) 0 (B) 1 (C) 2 (D) – 2

2. If $\lim\limits_{x\to 0^+} f(x) = \bullet$ (finite), then the value of $\bullet$ is

(A) – 2 (B) $-\frac{1}{2}$ (C) – 1 (D) $-\frac{1}{3}$

3. Using the values of a, b, c as found in Q.No. 1 or Q. No. 2 above, the value of $\lambda\text{im}_{x\to 0^+}$ x f(x) is

(A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 2

Comprehension # 2

Consider two functions $f(x) = \lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$ and $g(x) = -x^{4b}$, where $b = \lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1} - \sqrt{x^2+1}\right)$.

1. f (x) is -

(A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$

2. g(x) is -

(A) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$

3. Number of solutions of f (x) + g(x) = 0 is -

(A) 2 (B) 4 (C) 0 (D) 1

Comprehension # 3

Consider two functions $f(x) = \lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$ and $g(x) = -x^{4b}$, where $b = \lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1} - \sqrt{x^2+1}\right)$, then

1. f(x) is

(A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$

2. g(x) is

(A) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$

3. Number of solutions of equation f(x) + g(x) = 0 is

(A) 2 (B) 4 (C) 0 (D) 1

Exercise # 4 [Subjective Type Questions]

1. Evaluate $\lim_{x\to\frac{3\pi}{4}} \dfrac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$

2. Evaluate $\lim_{x\to\infty}\left(\dfrac{a_1^{1/x}+a_2^{1/x}+a_3^{1/x}+........+a_n^{1/x}}{n}\right)^{nx}$ where $a_1, a_2, a_3, ... a_n > 0$

3. Evaluate $\lim_{x\to\infty} x^2 \sin\left(\ln\sqrt{\cos\dfrac{\pi}{x}}\right)$

4. Evaluate $\lim_{x\to 1}\dfrac{(\ln(1+x)-\ln 2)\left(3.4^{x-1}-3x\right)}{\left[(7+x)^{\frac{1}{3}}-(1+3)^{\frac{1}{2}}\right].\sin(x-1)}$

5. Using Sandwich theorem, evaluate $\lim_{n\to\infty}\left(\dfrac{1}{\sqrt{n^2}}+\dfrac{1}{\sqrt{n^2+1}}+\dfrac{1}{\sqrt{n^2+2}}+........+\dfrac{1}{\sqrt{n^2+2n}}\right)$

6. Find the values of a, b & c so that $\lim_{x\to 0}\dfrac{ae^x-b\cos x+ce^{-x}}{x.\sin x}=2$.

7. Evaluate $\lim_{x\to\frac{\pi}{4}}\dfrac{\sqrt{1-\sqrt{\sin 2x}}}{\pi-4x}$

8. Evaluate $\lim_{n\to\infty}\dfrac{[1.x]+[2.x]+[3.x]+...........+[n.x]}{n^2}$, where [.] denotes the greatest integer function.

9. Using Sandwich theorem, evaluate

$$\lim_{n\to\infty}\frac{1}{1+n^2}+\frac{2}{2+n^2}+...........+\frac{n}{n+n^2}$$

10. Evaluate $\lim_{x\to 0}\dfrac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$

11. Verify the following limits

(i) $\lim_{x\to 0}\left(\dfrac{(1+x)^{\frac{1}{x}}}{e}\right)^{\frac{1}{x}} = e^{-\frac{1}{2}}$ **(ii)** $\lim_{x\to 0}\left[\sin^2\left(\dfrac{\pi}{2-ax}\right)\right]^{\sec^2\left(\frac{\pi}{2-bx}\right)} = e^{-\frac{a^2}{b^2}}$

12. Evaluate $\lim_{x\to 1}\dfrac{\left[\sum_{k=1}^{100}x^k\right]-100}{x-1}$

13. Let $f(x)=\dfrac{\sin^{-1}(1-\{x\}).\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\ (1-\{x\})}$, then find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$.

(where {.} denotes the fractional part function)

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $f(1) = 1, f'(1) = 2$, then $\lim_{x\to1} \frac{\sqrt{f(x)}-1}{\sqrt{x}-1} =$ **[AIEEE - 2002]**

(1) 2 (2) 1 (3) 3 (4) 4

2. The values of $\lim_{x\to0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$ is- **[AIEEE - 2002]**

(1) 10/3 (2) 3/10 (3) 6/5 (4) 5/6

3. $\lim_{x\to\infty} \left(\frac{x^2+5x+3}{x^2+x+3}\right)^x =$ **[AIEEE - 2002]**

(1) e^4 (2) e^2 (3) e^3 (4) e

4. $\lim_{x\to\infty} \frac{\log x^n - [x]}{[x]}$, $n \in N$, (where [x] denotes greatest integer less than or equal to x)- **[AIEEE - 2002]**

(1) Has value -1 (2) Has values 0 (3) Has value 1 (4) Does not exist

5. If $\lim_{x\to0} \frac{\log(3+x)-\log(3-x)}{x} = k$, the value of k is- **[AIEEE - 2003]**

(1) $-\frac{2}{3}$ (2) 0 (3) $-\frac{1}{3}$ (4) $\frac{2}{3}$

6. Let f(a) = g(a) = k and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x\to a} \frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$ then the value of k is- **[AIEEE - 2003]**

(1) 0 (2) 4 (3) 2 (4) 1

7. $\lim_{x\to\pi/2} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is- **[AIEEE - 2003]**

(1) ∞ (2) $\frac{1}{8}$ (3) 0 (4) $\frac{1}{32}$

8. If $\lim_{x\to\infty} \left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are- **[AIEEE - 2004]**

(1) $a \in R, b \in R$ (2) $a = 1, b \in R$ (3) $a \in R, b = 2$ (4) $a = 1$ and $b = 2$

9. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to- **[AIEEE - 2005]**

(1) $\frac{a^2}{2}(\alpha-\beta)^2$ (2) 0 (3) $\frac{-a^2}{2}(\alpha-\beta)^2$ (4) $\frac{1}{2}(\alpha-\beta)^2$

10. Let f : R → R be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$ **[AIEEE-2010]**

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

11. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[AIEEE-2011]**

(1) equals $-\sqrt{2}$ (2) equals $\frac{1}{\sqrt{2}}$ (3) does not exist (4) equals $\sqrt{2}$

12. Let f : R →[0, ∞) be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$. Then $\lim_{x\to 0} f(x)$ equal - **[AIEEE-2011]**

(1) 3 (2) 0 (3) 1 (4) 2

13. If function f(x) is differentiable at x = a then $\lim_{x\to a}\frac{x^2 f(a)-a^2 f(x)}{x-a}$ **[AIEEE-2011]**

(1) $2a\,f(a) + a^2 f'(a)$ (2) $-a^2 f'(a)$ (3) $af(a) - a^2 f'(a)$ (4) $2af(a) - a^2 f'(a)$

14. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to : **[JEE(Main)-2013]**

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

15. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to : **[JEE(Main)-2014]**

(1) $\frac{\pi}{2}$ (2) 1 (3) $-\pi$ (4) π

16. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **[JEE(Main)-2015]**

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 3

17. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then f(2) is equal to : **[JEE(Main)-2015]**

(1) 0 (2) 4 (3) – 8 (4) – 4

18. Let $p=\lim_{x\to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to : **[JEE(Main)-2016]**

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

19. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to : **[JEE(Main)-2016]**

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) $3\log 3-2$ (4) $\frac{18}{e^4}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. $\lim\limits_{x\to 0}\dfrac{\sin(\pi\cos^2 x)}{x^2}$ equals **[JEE 2001]**

(A) $-\pi$ **(B)** π **(C)** $\dfrac{\pi}{2}$ **(D)** 1

2. Evaluate $\lim\limits_{x\to 0}\dfrac{a^{\tan x}-a^{\sin x}}{\tan x-\sin x}$, $a>0$. **[JEE 2001]**

3. The integer n for which $\lim\limits_{x\to 0}\dfrac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number is :-

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **[JEE 2002]**

4. If $\lim\limits_{x\to 0}\dfrac{\sin(nx)[(a-n)nx-\tan x]}{x^2}=0$ $(n>0)$ then the value of 'a' is equal to :- **[JEE 2003]**

(A) $\dfrac{1}{n}$ **(B)** n^2+1 **(C)** $\dfrac{n^2+1}{n}$ **(D)** None

5. Find the value of $\lim\limits_{n\to\infty}\left[\dfrac{2}{\pi}(n+1)\cos^{-1}\left(\dfrac{1}{n}\right)-n\right]$. **[JEE 2004]**

6. Let $L=\lim\limits_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\dfrac{x^2}{4}}{x^4}$, $a>0$. If L is finite, then :- **[JEE 2009]**

(A) $a=2$ **(B)** $a=1$ **(C)** $L=\dfrac{1}{64}$ **(D)** $L=\dfrac{1}{32}$

7. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right)=4$, then - **[JEE 2012]**

(A) $a=1, b=4$ **(B)** $a=1, b=-4$ **(C)** $a=2, b=-3$ **(D)** $a=2, b=3$

8. Let α(A) and β(A) be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$.

Then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{a\to 0^+}\beta(a)$ are **[JEE 2012]**

(A) $-\dfrac{5}{2}$ and 1 **(B)** $-\dfrac{1}{2}$ and -1 **(C)** $-\dfrac{7}{2}$ and 2 **(D)** $-\dfrac{9}{2}$ and 3

9. The largest value of the nonnegative integer a for which **[JEE Ad. 2014]**

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

10. Let $\alpha, \beta \in R$ be such that $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{ax-\sin x}=1$. Then $6(\alpha+\beta)$ is equals **[JEE Ad. 2016]**

11. Let $f(x)=\lim\limits_{n\to\infty}\left(\dfrac{n^n(x+n)(x+\frac{n}{2})...(x+\frac{n}{n})}{n!(x^2+n^2)(x^2+\frac{n^2}{4})...(x^2+\frac{n^2}{n^2})}\right)^{\frac{1}{n}}$, for all $x>0$. Then

(A) $f\left(\dfrac{1}{2}\right)\geq f(1)$ **(B)** $f\left(\dfrac{1}{3}\right)\leq f\left(\dfrac{2}{3}\right)$ **(C)** $f'(2)\leq 0$ **(D)** $\dfrac{f'(3)}{f(3)}\geq\dfrac{f'(2)}{f(2)}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $\lim\limits_{x\to 0} \dfrac{e^{-nx} + e^{nx} - 2\cos\dfrac{nx}{2} - kx^2}{(\sin x - \tan x)}$ exists and finite, then possible values of 'n' and 'k' is :

(A) $k = 3,\ n = 2$ **(B)** $k = 3,\ n = -2$ **(C)** $k = 5,\ n = 2$ **(D)** $k = -5,\ n = 2$

2. $\lim\limits_{x\to 1} \left(\sec\dfrac{\pi}{2^x}\right)(\ln x)$ is equal to

(A) $-\dfrac{\pi}{2\ln 2}$ **(B)** $\dfrac{\pi}{2}$ **(C)** $\dfrac{\pi}{2\ln 2}$ **(D)** $\dfrac{2}{\pi\ln 2}$

3. $\lim\limits_{x\to 0} |x(x-1)|^{[\cos 2x]}$, where [.] denotes greatest integer function, is equal to

(A) 1 **(B)** 0 **(C)** e **(D)** Does not exist

4. The value of $\lim\limits_{x\to 1}\left(\dfrac{p}{1-x^p} - \dfrac{q}{1-x^q}\right)$; $p, q \in N$ equals

(A) $\dfrac{p+q}{2}$ **(B)** $\dfrac{pq}{2}$ **(C)** $\dfrac{p-q}{2}$ **(D)** $\sqrt{\dfrac{p}{q}}$

5. The value of $\lim\limits_{x\to\pi} \dfrac{1}{(x-\pi)}\left(\sqrt{\dfrac{4\cos^2 x}{2+\cos x}} - 2\right)$ is

(A) 0 **(B)** 2 **(C)** -2 **(D)** does not exist

6. The value of $\lim\limits_{x\to \frac{1}{\sqrt{2}}} \dfrac{\cos^{-1}(2x\sqrt{1-x^2})}{x - \dfrac{1}{\sqrt{2}}}$ equals

(A) $2\sqrt{2}$ **(B)** $-2\sqrt{2}$ **(C)** 0 **(D)** does not exist

7. $\lim\limits_{x\to 0} \dfrac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^4(e^{2x^4} - 1 - 2x^4)}$ is equal to

(A) 0 **(B)** $-\dfrac{1}{6}$ **(C)** $\dfrac{1}{6}$ **(D)** does not exist

8. $\lim\limits_{n\to\infty} \dfrac{1^2\, n + 2^2\,(n-1) + 3^2\,(n-2) + \ldots.. + n^2.1}{1^3 + 2^3 + 3^3 + \ldots\ldots + n^3}$ is equal to:

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{2}{3}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{1}{6}$

9. $\lim_{x\to 0^+} (x)^{\frac{1}{\ln \sin x}}$ is equal to

(A) 1 **(B)** 0 **(C)** e **(D)** does not exist

10. S_1 **:** If $\lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ exists, then it is not necessary that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ will exist

S_2 **:** $\lim_{x\to 0^+} \sqrt{x}\,(\log x)^2 = 0$

S_3 **:** $\lim_{x\to 0} \left\{\frac{\sin x}{x}\right\} = 1,$ (where {.} denotes fractional part function)

S_4 **:** $\lim_{x\to\infty} \frac{\ln \cos^2 x}{x^2} = \frac{1}{2}$

(A) T T T F **(B)** F T T F **(C)** T T F F **(D)** F F T F

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $f(x) = \begin{cases} x + \frac{1}{2} & , \quad x < 0 \\ 2x + \frac{1}{3} & , \quad x \ge 0 \end{cases}$, identify the correct statement(s)

([ ] denotes greatest integer function)

(A) $\lim_{x\to 0} [f(x)] = 0$

(B) $\lim_{x\to 0} f(x)$ does not exist

(C) $\left[\lim_{x\to 0} f(x)\right]$ exists

(D) $\lim_{x\to 0} \frac{[f(x)]}{x}$ does not exist

12. Which of the following limits tend to unity ?

(A) $\lim_{t\to 0} \frac{\sin(\tan t)}{\sin t}$ **(B)** $\lim_{t\to\pi/2} \frac{\sin(\cos x)}{\cos x}$ **(C)** $\lim_{x\to 0} \frac{\sqrt{x^2}}{x}$ **(D)** $\lim_{x\to\pi/2} \left(\frac{1-\cos x}{x^2}\right)$

13. Which of the following is/are true

(A) If $\lim_{x\to a} \{f(x) + g(x)\}$ exists, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist

(B) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} \{f(x) + g(x)\}$ exists

(C) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} f(x)\ g(x)$ exists

(D) If $\lim_{x\to a} \{f(x)\,g(x)\}$ exists, then both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist

14. $f(x) = \left(\frac{x}{2+x}\right)^{2x}$, then

(A) $\lim_{x\to\infty} f(x) = -4$ **(B)** $\lim_{x\to\infty} f(x) = 2$ **(C)** $\lim_{x\to\infty} f(x) = e^{-4}$ **(D)** $\lim_{x\to 1} f(x) = \frac{1}{9}$

15. It x is a real number in [0, 1]. then the value of $\lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\,\pi x)]$ is -

(A) 1 if $x \notin Q$ **(B)** 2 if $x \notin Q$ **(C)** 1 if $x \in Q$ **(D)** 2 if $x \in Q$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** $\lim_{x\to 0}\sin^{-1}\{x\}$ does not exist

Statement-II : $\{x\}$ is discontinuous at $x = 0$ (where $\{.\}$ denotes fractional part function).

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** $\lim_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+\frac{3}{x^2}+\ldots\ldots\ldots\ldots+\frac{x}{x^2}\right) = \lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}\frac{2}{x^2}+\ldots\ldots\ldots\ldots+\lim_{x\to\infty}\frac{x}{x^2} = 0$

Statement-II : $\lim_{x\to a}(f_1(x)+f_2(x)+\ldots\ldots\ldots f_n(x)) = \lim_{x\to a} f_1(x)+\lim_{x\to a} f_2(x)+\ldots\ldots\ldots+\lim_{x\to a} f_n(x), n \in N.$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** If $a_1, a_2, a_3, \ldots\ldots. a_n > 0$, then $\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+a_3^{1/x}+\ldots..+a_n^{1/x}}{n}\right\}^{nx} = \prod_{i=1}^{n} a_n$

Statement-II : If $\lim_{x\to a} f(x)\to 1$, $\lim_{x\to a} g(x)\to\infty$, then $\lim_{x\to a}\{f(x)\}^{g(x)} = e^{\lim_{x\to a}(f(x)-1)g(x)}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement-I :** If $f(x)=\frac{2}{\pi}\cot^{-1}\left(\frac{3x^2+1}{(x-1)(x-2)}\right)$, then $\lim_{x\to 1^-} f(x)=0$ and $\lim_{x\to 2^-} f(x)=2$

Statement-II : $\lim_{x\to\infty}\cot^{-1}x=0$ and $\lim_{x\to-\infty}\cot^{-1}x=\pi$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** $\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\dots\dots\dots+\frac{x^2}{x^3}\right)=\frac{1}{3}$

Statement-II : $\lim_{x\to a}(f_1(x)+f_2(x)+\dots\dots+f_n(x))=\lim_{x\to a}f_1(x)+\lim_{x\to a}f_2(x)+\dots\dots\dots+\lim_{x\to a}f_n(x)$ where $n\in N$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I	Column – II
(A) Let $f: R\to R$ be a differentiable function and $f(1)=1, f'(1)=3$. Then the value of $\lim_{x\to1}\int_1^{x^2}\frac{(f(t)-t)}{(x-1)^2}dt$ is	**(p)** 0
(B) $\lim_{n\to\infty}\left(\frac{1+\sqrt[n]{4}}{2}\right)^n$ is equal to	**(q)** – 1
(C) If $f(x)=\lim_{n\to\infty}\frac{2x}{\pi}.\tan^{-1}(nx), x>0$, then $\lim_{x\to0^+}[f(x)-1]$ is {where [.] represents greatest integer function}	**(r)** 2
(D) $\lim_{n\to\infty}\left[\sum_{r=1}^{n}\frac{1}{2^r}\right]=$ (where [.] denotes the greatest integer function)	**(s)** 1
	(t) 4

22.

Column – I		Column – II	
(A)	Let $f : R \to R$ be such that $f(A) = 1$, $f'(A) = 2$ and $\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{1/x} = e^k$, then $k =$	**(p)**	0
(B)	$\lim_{x\to\frac{\pi}{2}^+}\frac{\cos(\tan^{-1}(\tan x))}{x-\frac{\pi}{2}} =$	**(q)**	1
(C)	$\lim_{x\to\pi}\frac{\sin(\cos x+1)}{\cos\left(\frac{x}{2}\right)} =$	**(r)**	4
(D)	$\lim_{x\to 0}\frac{xe^{\sin x}-e^x\sin^{-1}(\sin x)}{\sin^2 x - x\sin x} =$	**(s)**	3
		(t)	does not exist

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

If $\lim_{x\to 0^+} f(x) =$ finite where $f(x) = \frac{\sin x + ae^x + be^{-x} + c\ln(1+x)}{x^3}$ and a, b, c are real numbers.

1. The value of a is

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** 1

2. The value of b is

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$ **(C)** 0 **(D)** 1

3. The value of c is

(A) –1/2 **(B)** 1/2 **(C)** 0 **(D)** 2

24. Read the following comprehension carefully and answer the questions.

Let $f(x) = \lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x) = \lim_{n\to\infty}\left(1 - x + x\sqrt[n]{e}\right)^n$. Now, consider the function $y = h(x)$, where $h(x) = \tan^{-1}(g^{-1}f^{-1}(x))$.

1. $\lim_{x\to 0^+}\frac{\ln(f(x))}{\ln(g(x))}$ is equal to

(A) $\frac{1}{2}$ **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** 1

2. Domain of the function $y = h(x)$ is

(A) $(0, \infty)$ (B) R (C) $(0, 1)$ (D) $[0, 1]$

3. Range of the function $y = h(x)$ is

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) R (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

25. **Read the following comprehension carefully and answer the questions.**

Let $f(x) = \max\{a, b, c\}$, where

$$a = \lim_{n\to\infty} \lim_{\alpha\to 1^+} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$b = \lim_{n\to\infty} \lim_{\alpha\to 1^-} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$c = \lim_{n\to\infty} \frac{\pi}{4n}\left[1 + \cos\frac{\pi}{2n} + \cos\frac{2\pi}{2n} + \ldots\ldots\ldots + \cos\frac{(n-1)\pi}{2n}\right]$$. Then

1. The value of a is

(A) $2|\sin x|$ (B) $|\cos x|$ (C) $|\sin x|$ (D) $\frac{1}{2}$

2. The value of $b + c - \frac{1}{2}$ is

(A) $|\cos x|$ (B) $2|\cos x| - 1$ (C) $|\sin x| + 1$ (D) $|\sin x| + |\cos x|$

3. Range of f(x) is

(A) $[0, 1]$ (B) $\left[\frac{1}{2}, 1\right]$ (C) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (D) $\left[\frac{1}{2}, 2\right]$

SECTION - VI : INTEGER TYPE

26. If $\lim_{x\to\infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d}\right) = 4$, then value of $(a + c)$.

27. If $\lim_{x\to 0} (x^{-3} \sin 3x + ax^{-2} + b)$ exists and is equal to zero then value of $(a + 2b)$.

28. Let $f(x) = \frac{\tan x}{x}$ and $\lim_{x\to 0} ([f(x)] + x^2)^{\frac{1}{\{f(x)\}}} = e^{\lambda}$, then find λ

(where [.] and {.} denotes greatest integer and fractional part function respectively)

29. The value of $\lim_{x\to\infty} \frac{\cot^{-1}\left(x^{-a} \log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}$ $(a > 1)$.

30. Let $P = \frac{\left(1^4 + \frac{1}{4}\right)\left(3^4 + \frac{1}{4}\right)\left(5^4 + \frac{1}{4}\right)\ldots\ldots\ldots\left((2n-1)^4 + \frac{1}{4}\right)}{\left(2^4 + \frac{1}{4}\right)\left(4^4 + \frac{1}{4}\right)\left(6^4 + \frac{1}{4}\right)\ldots\ldots\left((2n)^4 + \frac{1}{4}\right)}$ and $\lim_{n\to\infty} (n^{\alpha} P)$ exists, then find α

ANSWER KEY

EXERCISE - 1

1. B **2.** A **3.** A **4.** B **5.** A **6.** B **7.** D **8.** A **9.** C **10.** D **11.** A **12.** D **13.** C
14. B **15.** B **16.** B **17.** B **18.** B **19.** B **20.** C **21.** B **22.** B **23.** C **24.** A **25.** D **26.** B
27. C **28.** A **29.** D **30.** C

EXERCISE - 2 : PART # I

1. ABD **2.** AD **3.** AD **4.** AB **5.** ABCD **6.** ABCD **7.** BCD **8.** ABCD **9.** AD
10. ABD **11.** BC **12.** CD **13.** ABC **14.** ABC **15.** CD **16.** ABCD **17.** AB **18.** BCD
19. CD

PART - II

1. C **2.** D **3.** B **4.** C **5.** A

EXERCISE - 3 : PART # I

1. **A** $\to$ s **B** $\to$ r **C** $\to$ p **D** $\to$ q **2.** **A** $\to$ q **B** $\to$ s **C** $\to$ r **D** $\to$ p
3. **A** $\to$ q **B** $\to$ r **C** $\to$ s **D** $\to$ p **4.** **A** $\to$ p **B** $\to$ r **C** $\to$ q **D** $\to$ q
5. **A** $\to$ s **B** $\to$ r **C** $\to$ q **D** $\to$ p **6.** **A** $\to$ r **B** $\to$ s **C** $\to$ p **D** $\to$ q

PART - II

Comprehension # 1 : **1.** A **2.** D **3.** A **Comprehension # 2 :** **1.** B **2.** A **3.** A
Comprehension # 3 : **1.** B **2.** A **3.** A

EXERCISE - 5 : PART # I

1. 1 **2.** 1 **3.** 1 **4.** 1 **5.** 4 **6.** 2 **7.** 4 **8.** 2 **9.** 1 **10.** 1 **11.** 3 **12.** 1 **13.** 4
14. 4 **15.** 4 **16.** 1 **17.** 1 **18.** 2 **19.** 1

PART - II

1. B **2.** ●na **3.** C **4.** C **5.** $1-\frac{2}{\pi}$ **6.** A, C **7.** B **8.** B **9.** 0 **10.** 7 **11.** B,C

MOCK TEST

1. C **2.** D **3.** A **4.** C **5.** A **6.** B **7.** C **8.** A **9.** C **10.** A **11.** A, B **12.** A, B
13. B,C **14.** **C** **15.** A,D **16.** B **17.** D **18.** A **19.** A **20.** C
21. **A** $\rightarrow$ t **B** $\rightarrow$ **r** **C** $\rightarrow$ **q** **D** $\rightarrow$ **p** **22.** **A** $\rightarrow$ **r** **B** $\rightarrow$ **q** **C** $\rightarrow$ **p** **D** $\rightarrow$ **q** **23.** **1.** A **2.** B **3.** C **24.** **1.** B
2. C **3.** D **25.** **1.** C **2.** **A** **3.** C **26.** 7 **27.** 6 **28.** 3 **29.** 1 **30.** 2

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