HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

6.

p	q	$\sim p$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$\sim p \leftrightarrow q$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	F

7. Both statements p and q are true

so $p \Rightarrow q$ and $q \Rightarrow p$

$\therefore$ $p \Leftrightarrow q$

8. When p and q both are true then

$\sim(p \to q)$ and $(\sim p \vee \sim q)$ both are false

i.e. $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is true

when p and q both are false then

$\sim(p \to q)$ is false and $(\sim p \vee \sim q)$ is true

i.e. $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is false

Hence $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is neither tautology nor contradiction.

10. $\because$ $p \to (q \vee r)$ is false

$\Rightarrow$ p is true and $(q \vee r)$ is false

$\Rightarrow$ p is true, q and r both are false

i.e. $p \to (q \vee r)$ is false when truth values of p, q, r are T, F, F resp. otherwise it is true.

11. Let p, q, r be the three statements such that

$p : x = 5$, $q : y = -2$ and $r : x - 2y = 9$

Here given statement is $(p \wedge q) \to r$ and its contrapositive is $\sim r \to \sim(p \wedge q)$

i.e. $\sim r \to (\sim p \vee \sim q)$ i.e. if $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$

12. Let $S(p, q) \equiv (p \vee \sim q) \wedge \sim p$

$\Rightarrow$ $S(\sim p, \sim q) \equiv (\sim p \vee q) \wedge p$

Now $S^*(\sim p, \sim q) \equiv (\sim p \wedge q) \vee p$

and $\sim S(p, q) \equiv \sim[(p \vee \sim q) \wedge \sim p] \equiv \sim(p \vee \sim q) \vee p$

$\equiv (\sim p \wedge q) \vee p$

Hence $S^*(\sim p, \sim q) \equiv \sim S(p, q)$

17. $\because$ $(p \wedge q) \vee (q \wedge r)$ is false

$\Rightarrow$ $(p \wedge q)$ and $(q \wedge r)$ both are false

$\Rightarrow$ p and r both are false or q is false.

otherwise $(p \wedge q) \vee (q \wedge r)$ is true

18. The negation of ''Everyone in Germany speaks German'' is - there is at least one person in Germany who does not speak German.

20. $\sim(p \to (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$

$(\because \sim(p \to q) \equiv p \wedge \sim q)$

$\equiv p \wedge (\sim q \vee \sim r)$

21. $\because$ $\sim q \to \sim p \equiv \sim(\sim q) \vee \sim p$ ($\because p \to q \equiv \sim p \vee q$)

$\equiv q \vee \sim p$

$\equiv \sim p \vee q$ (by commutative law)

$\equiv p \to q$ ($\because p \to q = \sim p \vee q$)

Hence $p \to q \equiv \sim q \to \sim p$

26. Let p, q, r three statement defined as

p : a number N is divisible by 15

q : number N is divisible by 5

r : number N is divisible by 3

Here given statement is $p \to (q \vee r)$

Here negative of above statement is

$\sim(p \to (q \vee r)) \equiv p \wedge (\sim(q \vee r)$

$\equiv p \wedge (\sim q \wedge \sim r)$

i.e. A number is divisible by 15 and it is not divisible by 5 and 3.

31. $(\sim T \vee F) \wedge \sim T \Rightarrow T$

$\therefore$ $(F \vee F) \wedge F \Rightarrow T$

$\therefore$ $F \wedge F \Rightarrow T$

$\therefore$ $F \Rightarrow T$

35. $\because$ $(\sim p \vee \sim q) \vee (p \vee \sim q)$

$\equiv$ $(\sim p \vee \sim q) \vee (\sim q \vee p)$ (by commutative law)

$\equiv$ $\sim p \vee [\sim q \vee (\sim q \vee p)]$ (by Associative law)

$\equiv$ $\sim p \vee [(\sim q \vee \sim q) \vee p]$ (by Associative law)

$\equiv$ $\sim p \vee (\sim q \vee p)$ ($\because p \vee p = p$)

$\equiv$ $\sim p \vee (p \vee \sim q)$ (by commutative law)

$\equiv$ $(\sim p \vee p) \vee \sim q$ (by Associative law)

$\equiv$ $t \vee \sim q \equiv t$ t is a tautology

Hence $(\sim p \vee \sim q) \vee (p \vee \sim q)$ is a tautology.

37. ∵ $(p \wedge q) \rightarrow p$ is false

$\Rightarrow$ $(p \wedge q)$ is true and p is false

which is not possible

so $(p \wedge q) \rightarrow p$ is always true i.e. it is a tautology.

39. ∵ $[(p \wedge p) \rightarrow q] \rightarrow p \equiv (p \rightarrow q) \rightarrow p$ (∵ $p \wedge p \equiv p$)

when p is false and q is true (or false) then

$(p \rightarrow q)$ is true i.e. $(p \rightarrow q) \rightarrow p$ is false

Hence $[(p \wedge p) \rightarrow q] \rightarrow p$ is not a tautology.

EXERCISE - 2

Part # II : Comprehension

1. If p then q means p only if q

2. If p then q $\Rightarrow$ p is sufficient for q

3. p is false, q is false so $p \rightarrow q$ is true.

EXERCISE - 3

Subjective Type

2. (i)

p	q	(i)	(ii)
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

(ii)

p	q	(i)	(ii)
T	T	T	F
T	F	F	F
F	T	F	F
F	F	T	T

(iii)

p	q	r	(i)	(ii)	(iii)	(iv)
T	T	T	T	F	F	F
T	T	F	F	T	T	F
T	F	T	T	F	F	T
F	T	T	T	T	F	T
T	F	F	T	F	T	F
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	F	T	T	F

(iv)

p	q	(i)	(ii)
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

(v)

p	q	r	$(p \rightarrow q) \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	F	F	T
F	T	F	F
F	F	T	T
F	F	F	F

(vi)

p	q	r	$(p \vee q) \leftrightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	F	F	F
F	T	F	F
F	F	T	F
F	F	F	T

4. Fallacy

5. Let p be the statement "Traders do not reduce the prices" and q be the statement "Government takes action against them"

The first statement in symbolic form is $p \rightarrow q$ and the second statement is $\sim(p \wedge \sim q)$.

In order to check the equivalence of the above statements let us prepare the following truth table.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \rightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Clearly, $\sim q \rightarrow \sim p$ and $\sim(p \wedge \sim q)$ have same truth values for all the values of p and q. Hence, the two statements are equivalent.

Aliter : We have, $\sim(p \wedge \sim q) \equiv (\sim p \vee q) \equiv (p \rightarrow q)$

Hence the two statements are equivalent.

6. True

7.

p	q	~p	~q	~q ∧ p	p∨~p	(~q ∧ p)∨(p∨~p)
T	T	F	F	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

8. Given Statement is

(ΔABC is right angled at B) $\Rightarrow$ ($AB^2 + BC^2 = AC^2$)

(A) Its converse is :

(In ΔABC, $AB^2 + BC^2 = AC^2$) $\Rightarrow$ ΔABC is right angled at B.

(B) Its contradiction is :

(In ΔABC, $AB^2 + BC^2 = AC^2$) $\Rightarrow$ ΔABC is not right angled at B.

(C) Its contrapositive is :

(In ΔABC, $AB^2 + BC^2 \neq AC^2$) $\Rightarrow$ ΔABC is not right angled at B.

10. The compound statement is :

"25 is a multiple of 5 or 8"

Let us assume that the statement q is false i.e. 25 is not a multiple of 8. Clearly, p is true.

Thus, if we assume that q is false, then p is true.

Hence, the compound statement is true i.e. valid.

11. Let q and r be the statements given by

q : If x is an integer and x^2 is even

r : x is an even integer

then p : "If q, then r"

If possible, let r be false then r is false

$\Rightarrow$ x is not an even integer

$\Rightarrow$ x is an odd integer

$\Rightarrow$ x = (2n + 1) for some integer n

$\Rightarrow$ $x^2 = 4n^2 + 4n + 1$

$\Rightarrow$ $x^2 = 4n(n + 1) + 1$

$\Rightarrow$ x^2 is an odd integer

$\Rightarrow$ q is false

[$\therefore$ 4n(n + 1) is even]

Thus, r is false $\Rightarrow$ q is false

Hence, p : "If q, then r" is a true statement.

12. $(p \wedge q) \vee [\sim p \vee (p \wedge \sim q)]$

$= (p \wedge q) \vee [(\sim p \vee p) \wedge (\sim p \vee \sim q)] = (p \wedge q) \vee [t \wedge (\sim p \vee \sim q)] = (p \wedge q) \vee (\sim p \vee \sim q)$

$= (p \wedge q) \vee [\sim (p \wedge q)] = t$

also $(\sim p \wedge q) \vee t = t$

13. p : it rains tomorrow

q : I shall carry my umbrella

r : cloth is mended

P : $p \to (r \to q)$

Q : $p \wedge \sim r$

S : ~q

P : T, Q : T $\not\Rightarrow$ S : T

$\therefore$ S not valid

p	q	r	P	Q	S
T	T	T	T	F	F
T	T	F	T	T	F
T	F	T	F	F	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	T

14. Consider the following statements :

p : Hema is not in team A

q : Rita is not in team B.

r : Mamta is in team A.

We have, $S_1 : p \to \sim q$, $S_2 : q \to r$, $S : r \vee q$

In order to test the validity of the given argument, let us first prepare the truth table for $S_1 \wedge S_2 \to S$ as given below.

Truth table for $S_1 \wedge S_2 \to S$

p	q	r	~q	S_1:p→~q	S_2:q→r	$S_1 \wedge S_2$	S:q∨r	$S_1 \wedge S_2 \to S$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	F	F
F	T	T	F	T	T	T	T	T
F	T	F	F	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	F	F

We observe that the last column of the truth table contains F also. Thus $S_1 \wedge S_2 \to S$ is not a tautology hence argument is invalid.

15. The truth table showing the truth values of hypotheses and conclusion is as given below :

p	q	r	~q	p∧~q	Hypotheses S_1 (p∧~q)→r	S_2 p∨q	S_3 q→p	Conclusion S r	
T	T	T	F	F	**T**	**T**	**T**	**T**	← Critical Rows
T	T	F	F	F	**T**	**T**	**T**	**F**	←
T	F	T	T	T	**T**	**T**	**T**	**T**	←
T	F	F	T	T	F	T	T	F	
F	T	T	F	F	T	T	F	T	
F	T	F	F	F	T	T	F	F	
F	F	T	T	F	T	F	T	T	
F	F	F	T	F	T	F	T	F	

We observe that there are three critical rows, namely I, II, & III such that the conclusion is not true in the IInd row. Hence the given argument is invalid.

EXERCISE - 4

Part # I : AIEEE/JEE-MAIN

1. ✈ $p \to (q \to p)$ is false

⇒ p is true and $(q \to p)$ is false. which is not possible.

So $p \to (q \to p)$ is always true i.e. it is a tautology.

again $p \to (p \vee q)$ is false

p is true and $(p \vee q)$ is false. Which is not possible.

So $p \to (p \vee q)$ is always true i.e. it is a tautology.

Hence $p \to (q \to p) \equiv p \to (p \vee q)$

2. Given r : $\sim p \leftrightarrow q$

statement-I $r \equiv q \vee p$

statement-II $r \equiv (p \leftrightarrow \sim q)$

p	q	~ p	~ q	(~ p ↔ q)	q ∨ p	(p ↔ ~ q)
T	T	F	F	F	T	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	F	F

Hence Statement-I is false and Statement-II is true.

3. statement-I : $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

statement-II : $\sim(p \leftrightarrow \sim q)$ is a tautology.

p	q	~ q	(p ↔ q)	(p ↔ ~ q)	~ (p ↔ ~ q)
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

Hence statement-1 is true, statement-2 is false.

4. Given $S \subseteq R$ and

p = There is a rational number $x \in S$ such that $x > 0$

then ~p : Any rational number $x \in S$ such that $x \ngtr 0$

i.e. ~p : Every rational number $x \in S$ satisfy $x \le 0$

5. Given Statement :

$(p \wedge \sim r) \Leftrightarrow q$

Negations of $p \Leftrightarrow q$ are

$\sim (p \Leftrightarrow q), \sim (q \Leftrightarrow p)$,

$\sim p \Leftrightarrow q$ and $\sim q \Leftrightarrow p$

Hence negations of given statement

are $\sim(q \Leftrightarrow (p \wedge \sim r))$ and $\sim (p \wedge \sim r) \Leftrightarrow q$

6. $[p \wedge (p \rightarrow q)] \rightarrow q$

$[p \wedge (\sim p \vee q)] \rightarrow q$

$[(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q$

$[c \vee (p \wedge q)] \rightarrow q$

$\Rightarrow (p \wedge q) \rightarrow q$ $\quad \begin{cases} p \wedge \sim p \equiv c \equiv \text{contradiction} \\ c \vee p \equiv p \end{cases}$

$\Rightarrow \sim(p \wedge q) \vee q$

$\Rightarrow (\sim p \vee \sim q) \vee q$

$\Rightarrow \sim p \vee (q \vee \sim q)$

$\Rightarrow \sim p \vee (t) \equiv$ tautology

9. $\sim S \vee (\sim r \wedge S)$

S	r	~r	~ r ∧ S	~ S	~ S ∨ (~ r ∧ S)	~ (~ S ∨ (~ r ∧ S))
T	T	F	F	F	F	T
T	F	T	T	F	T	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

10. $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= [(p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)$

$= [(p \vee q) \wedge t] \vee (\sim p \wedge q)$

$= (p \vee q) \vee (\sim p \wedge q)$

$= [(p \vee q \vee \sim p) \wedge (p \vee q \vee q)]$

$= (t \vee q) \wedge (p \vee q) = t \wedge (p \vee q) = p \vee q$