SOLVED EXAMPLES

Ex. 1 $\sim[(p \wedge q] \rightarrow (\sim p \vee q)]$ is

(A) tautology **(B)** contradiction **(C)** neither A nor B **(D)** either A or B

Sol.

p	q	~q	p ∧ q	~p ∨ q	(p ∧ q) → (~p ∨ q)	~[(p ∧ q) → (~p ∨ q)]
T	T	F	T	T	T	F
T	F	F	F	F	T	F
F	T	T	F	T	F	T
F	F	T	F	T	F	T

Ex. 2 $\sim(p \Rightarrow q)$ is equivalent to

(A) $p \wedge q$ **(B)** $\sim pVq$ **(C)** $\sim p \wedge \sim q$ **(D)** $p \wedge \sim q$

Sol.

p	q	~q	p ⇒ q	~(p ⇒ q)	p∧~q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	F	F
F	F	T	F	F	F

Ex. 3 Choose the correct answer. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to which of the following ?

(A) $p \rightarrow (p \rightarrow q)$ **(B)** $p \rightarrow (p \vee q)$ **(C)** $p \rightarrow (p \wedge q)$ **(D)** $p \leftrightarrow (p \rightarrow q)$

Sol. $p \rightarrow (q \rightarrow p) \equiv \sim p \vee (q \rightarrow p) \equiv \sim p \vee (\sim q \vee p)$

$\equiv \sim q \vee (p \vee \sim p) \equiv \sim q \vee T = T$

$\Rightarrow$ $p \rightarrow (q \rightarrow p)$ is tautology.

Also $p \rightarrow (p \vee q) \equiv \sim p \vee (p \vee q) \equiv (p \vee p) \vee q \equiv T \vee q \equiv T$

$\Rightarrow$ $p \rightarrow (p \vee q)$ is also tautology.

$\Rightarrow$ $p \rightarrow (q \rightarrow p) \equiv p \rightarrow (p \vee q)$

Ex.4 If t and c denote a tautology and a contradiction respectively, then for any statement p, $p \wedge t$ is equal to

(A) c **(B)** t **(C)** p **(D)** None of these

Sol.

p	t	c	p ∧ t
T	T	F	T
F	T	F	F

Ex. 5 If p, q, r are simple propositions with truth T, F, T then the truth value of $[(\sim pVq) \wedge \sim r] \Rightarrow p$ is

(A) false **(B)** true **(C)** true, if r is false **(D)** true, if q is true

Sol. **(B)**

~pVq means FVF = F, ~ r means F,

$(\sim pVq) \wedge \sim r$ means F

So, $[(\sim pVq) \wedge \sim r] \Rightarrow p$ means T i.e,. True

Ex. 6 If p, q, r are simple propositions, then $(p \wedge q) \wedge (q \wedge r)$ is true, then

(A) p, q, r are all false **(C)** p, q are true and r is false

(B) p, q, r are all true **(D)** p is true and q, r are false

Sol. **(B)**

$(p \wedge q) \wedge (q \wedge r)$ is true means $p \wedge q, q \wedge r$ are both true, it implies that p, q, r are all true.

Ex. 7 If the following statements are true: r, $p \to q$ and $q \to r$, then :

(A) r is true **(B)** p is true **(C)** q is true **(D)** p is false

Sol. $\sim r$ is true $\to$ r is false. Therefore **(A)** is not the answer.

Also, $q \to r$ is true and r is false $\to$ q is false. Therefore **(C)** is not the answer.

Similarly, since q is false and $p \to q$ is true, p must be false $\to$ **(D)** is the answer, not **(B)**.

Ex. 8 Let $f : R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then which of the following statements is true ?

(A) $f(x) \geq 1$ for all $x \in R$

(B) f(x) is not differentiable at $x = 1$

(C) f(x) is differentiable everywhere

(D) f(x) is not differentiable at $x = 0$

Sol. **(C)**

$f(x) = \min\{x + 1, |x| + 1\}$

$f(x) = x + 1 \ \forall \ \in R$

Ex. 9 Let S be a non empty subset of R. Consider the following statement : P : There is a rational number $x \in S$ such that $x > 0$. Then, which of the following statements is the negation of statement P :

(A) $x \in$ and $x \leq 0 \Rightarrow$ x is not rational

(B) Every rational number $x \in$ satisfies $x \leq 0$

(C) There is no rational number $x \in$ such that $x \leq 0$

(D) There is a rational number $x \in$ such that $x \leq 0$

Sol. The given statement is P : There is a rational number $x \in S$ such that $x > 0$

The negation would be : There is no rational number $x \in S$ such that $x > 0$ which is equivalent to all rational number $x \in S$ satisfy $x \leq 0$.

Ex. 10 The statement "If x is divisible by 8, then it is divisible by 6" is false if x equals

(A) 6 **(B)** 14 **(C)** 32 **(D)** 48

Sol. **(C)**

A sentence in "If....then..." form is called an implication. The only time when an implication is false is when the "If" part of the sentence is true and the "then" part of the sentence is false. The number 32 makes the first part of the statement true and the second part of the statement false.

Ex. 11 What is true about the statement "If two angles are right angles, the angles have equal measure" and its converse "If two angles have equal measure then the two angles are right angles" ?

(A) The statement is true but its converse is false.

(B) The statement is false but its converse is true.

(C) Both the statement and its converse are false.

(D) Both the statement and its converse are true.

Sol. **(A)**

All right angles have 90 degrees, therefore they are all equal in measure. However, two angles can be equal in measure (each being 30 degrees), but they are not necessarily right angles.

Ex. 12 Mary says "The number I am thinking is divisible by 2 or it is divisible by 3". This statement is false if the number Mary is thinking of is

(A) 6 **(B)** 8 **(C)** 11 **(D)** 15

Sol. **(C)**

This sentence contains the word "or" (disjunction). A disjunction is only false if BOTH conditions are false. The only number that is not divisible by 2 and is not divisible by 3 is 11.

Ex. 13 "If Tom buys a red skateboard, then Amanda buys green in-line skates." Which statement below is logically equivalent ?

(A) If Amanda does not buy green in-line skates, then Tom does not buy a red skateboard.

(B) If Tom does not buy a red skateboard, then Amanda does not buy green in-line skates.

(C) If Amanda buys green in-line skates, then Tom buys a red skateboard.

(D) If Tom buys a red skateboard, then Amanda does not buy green in-line skates.

Sol. **(A)**

Ex. 14 Which statement represents the inverse of the statement "If it is snowing, then Skeeter wears a sweater." ?

(A) If Skeeter wears a sweater, then it is snowing.

(B) If Skeeter does not wear a sweater, then it is not snowing.

(C) If it is not snowing, then Skeeter does not wear a sweater.

(D) If it is not snowing, then Skeeter wears a sweater.

Sol. **(C)**

Remember : to form the INVERSE of a statement, insert "NOT" into the "If" and the "then" sections of the sentence.

Ex. 15 Which of the following is the converse of the statement: "If $x > 4$, then $x + 2 > 5$" ?

(A) If $x + 2 < 5$, then $x < 4$.

(B) If x is not greater than 4, then $x + 2$ is not greater than 5.

(C) If $x + 2 > 5$, then $x > 4$,

(D) If $x + 2$ is not greater than 5, then x is not greater than 4.

Sol. **(C)**

Remember: to form the CONVERSE of a statement, the "If" and the "then" sections of the statement switch places.

Ex. 16 Which statement is logically equivalent to "If Yoda cannot use a lightsaber, then he cannot help Luke win the battle."

(A) Yoda cannot use a lightsaber and he will help Luke win the battle.

(B) If Yoda can help Luke win the battle, then he can use a lightsaber.

(C) Yoda can use a lightsaber if and only if he can help Luke win the battle.

(D) Yoda cannot use a lightsaber and will not help Luke win the battle.

Sol. **(B)**

A statement is logically equivalent to its contrapositive. To form the contrapositive, switch the "If" and "then" sections of the statement AND insert "NOTs" into each section. Notice in this situation, that inserting "NOTs" turns the thoughts positive (you are negating negative thoughts).

Ex. 17 If Shelly does not like John, what is the truth value of the statement "Shelly likes Mike and she likes John."?

(A) True **(B)** False

(C) Shelly likes Peter. **(D)** none of the above

Sol. **(B)**

The statement in question contains the word "AND". A statement with "AND" is only true when BOTH conditions are true. The section "she likes John" is false, so the entire statement is false.

Ex. 18 Which is logically equivalent to "If today is Sunday, Matt cannot play hockey." ?

(A) Today is Sunday and Matt can play hockey.

(B) If Matt plays hockey, then today is not Sunday.

(C) Today is Sunday and Matt cannot play hockey.

(D) Today is not Sunday if and only if Matt plays hockey.

Sol. **(B)**

Statements are logically equivalent to their contrapositives.

Ex. 19 The sentence "_____ if and only if x + x = 3x" is TRUE. Which of the following could be used to fill in the blank ?

(A) $x + x = 2x$ **(B)** $2x - x = 2x$

(C) both 1 and 2 could be used **(D)** neither 1 nor 2 could be used

Sol. **(B)**

"if and only if" is TRUE when both conditions are the SAME truth value. Since "$x + x = 3x$" is false, the first condition must also be false. $2x - x = 2x$ is false.

Ex. 20 The inverse of the converse of a conditional statement is the _______________.

(A) converse **(B)** inverse

(C) contrapositive **(D)** none of the above

Sol. **(C)**

The inverse will insert NOT into the statement.

The converse will switch the IF and THEN.

The contrapositive does both. The contrapositive is the inverse of the converse of the conditional statement.

Ex. 21 "If Deb and Sam go to the mall, then it is snowing." Which statement below is logically equivalent ?

(A) If Deb and Sam do not go to the mall, then it is not snowing.

(B) If Deb and Sam do not go to the mall, them it is snowing.

(C) If it is snowing, then Deb and Sam go to the mall.

(D) If it is not snowing, then Deb and Sam do not go to the mall.

Sol. **(D)**

A statement is logically equivalent to its contrapositive.

Ex. 22 S_1 : $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

S_2 : $\sim(p \leftrightarrow \sim q)$ is a tautology.

(A) S_1 is true, S_2 is true, S_2 is a correct explanation for S_1

(B) S_1 is true, S_2 is true, S_2 is not a correct explanation for S_1

(C) S_1 is true, S_2 is false **(D)** S_1 is false, S_2 is true

Sol. **(C)**

p	q	$p \leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	T	T	F
F	F	T	F	F	T

Ex. 23 The proposition $p \Rightarrow \sim(p \wedge \sim q)$ is

(A) contradiction **(C)** either A or B **(B)** tautology **(D)** None of these

Sol. **(D)**

p	p	~q	$(p \wedge \sim q)$	$\sim(p \wedge \sim q)$	$p \Rightarrow \sim(p \wedge \sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

It is clear from the table that given proposition is neither tautology nor contradiction.

Ex. 24 The proposition $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is a ;

(A) contradiction **(B)** tautology

(C) neither a contradiction nor a tautology **(D)** both a contradiction nor a tautology

Sol. Let us construct the truth table for this proposition :

P	Q	$P \vee Q$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(P \vee Q) \vee (\sim P \wedge \sim Q)$
T	T	T	F	F	F	T
F	T	T	T	F	F	T
T	F	T	F	T	F	T
F	F	F	T	T	T	T

Since the proposition is True for every assignment of truth values to its components, it is a tautology.

Ex. 25 Which of the following are tautologies and which are contradictions ?

(1) $(p \wedge q) \rightarrow (p \vee q)$ **(2)** $(p \wedge (\sim p)) \wedge ((\sim q) \wedge p)$ **(3)** $q \vee (p \vee (\sim q))$

(A) All are contradictions. **(B)** All are tautologies.

(C) Only Statement **(2)** is a contradiction, **(1)** and **(3)** are tautologies.

(D) Only Statement **(1)** is a tautology, **(2)** and **(3)** are contradictions

Sol. Let us construct the truth tables for the statements :

(1) $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \rightarrow p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

p	q	~p	~q	$p \vee (\sim q)$	$(\sim q) \vee p$	$(p \wedge (\sim p)) \wedge ((\sim q) \wedge p)$
T	T	F	F	F	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Therefore, this is a contradiction.

(3) $q \vee (p \vee (\sim q))$

p	q	~q	$p \vee (\sim q)$	$q \vee (p \vee (\sim q))$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Therefore, this is a tautology.

Therefore the correct answer is **(C)** Only Statement **(2)** is a contradiction, **(1)** and **(3)** are tautologies.

Exercise # 1

[Single Correct Choice Type Questions]

1. If the compound statement $p \to (\sim p \vee q)$ is false then the truth value of p and q are respectively-

(A) T, T **(B)** T, F **(C)** F, T **(D)** F, F

2. Consider the statement p : ''New Delhi is a city''. Which of the following is not negation of p ?

(A) New Delhi is not a city **(B)** It is false that New Delhi is a city

(C) It is not the case that New Delhi is not a city **(D)** None of these

3. If p is any statement, t is a tautology and c is a contradiction then which of the following is not correct-

(A) $p \wedge (\sim c) \equiv p$ **(B)** $p \vee (\sim t) \equiv p$ **(C)** $t \vee c \equiv p \vee t$ **(D)** $(p \wedge t) \vee (p \vee c) \equiv (t \wedge c)$

4. Which of the following is not a component statement of the statement '100 is divisible by 5, 10 and 11' ?

(A) 100 is divisible by 5 **(B)** 100 is divisible by 10

(C) 100 is not divisible by 11 **(D)** 100 is divisible by 11

5. The inverse of the statement $(p \wedge \sim q) \to r$ is-

(A) $\sim(p \vee \sim q) \to \sim r$ **(B)** $(\sim p \wedge q) \to \sim r$

(C) $(\sim p \vee q) \to \sim r$ **(D)** None of these

6. Which of the following is logically equivalent to $\sim (p \leftrightarrow q)$

(A) $(\sim p) \leftrightarrow q$ **(B)** $(\sim p) \leftrightarrow (\sim q)$ **(C)** $p \to (\sim q)$ **(D)** $p \to q$

7. Suppose that x and y are positive real numbers.

Let $p : x < y \Rightarrow x^2 < y^2$ and $q : x^2 < y^2 \Rightarrow x < y$. Then

(A) $p \Rightarrow q$ and $q \not\Rightarrow p$ **(B)** $p \not\Rightarrow q$ and $q \Rightarrow p$ **(C)** $p \Leftrightarrow q$ **(D)** $p \Rightarrow q$ and $\sim q \Rightarrow p$

8. The statement $\sim(p \to q) \leftrightarrow (\sim p \vee \sim q)$ is-

(A) a tautology **(B)** a contradiction

(C) neither a tautology nor a contradiction **(D)** None of these

9. Which of the following statements is using an 'inclusive Or" ?

(A) A number is either rational or irrational **(B)** All integers are positive or negative

(C) The office is closed if it is a holiday or a Sunday **(D)** Sum of two integers is odd or even

10. If statement $p \to (q \vee r)$ is true then the truth values of statements p, q, r respectively-

(A) T, F, T **(B)** F, T, F **(C)** F, F, F **(D)** All of these

11. If $x = 5$ and $y = -2$ then $x - 2y = 9$. The contrapositive of this statement is

(A) If $x - 2y = 9$ then $x = 5$ and $y = -2$ **(B)** If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$

(C) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$ **(D)** If $x - 2y \neq 9$ then either $x \neq 5$ or $y = -2$

12. If S*(p, q) is the dual of the compound statement S(p, q) then S*(~p, ~q) is equivalent to-

(A) S(~p, ~q) **(B)** ~S(p, q) **(C)** ~S*(p, q) **(D)** None of these

13. Which of the following is a logical statement ?

(A) Open the door **(B)** What an intelligent student !

(C) Are you going to Delhi **(D)** All prime numbers are odd numbers

14. $(\sim p \vee \sim q)$ is logically equivalent to-

(A) $p \wedge q$ **(B)** $\sim p \rightarrow q$ **(C)** $p \rightarrow \sim q$ **(D)** $\sim p \rightarrow \sim q$

15. If p is true and q is false, then which of the following statement is not true?

(A) $p \vee q$ **(B)** $p \Rightarrow q$ **(C)** $p \wedge (\sim q)$ **(D)** $q \Rightarrow p$

16. The contrapositive of statement "Something is cold implies that it has low temperature" is

(A) If something does not have low temperature, then it is not cold.

(B) If something does not have low temperature then it is cold

(C) Something is not cold implies that it has low temperature

(D) If something have low temperature, then it is not cold.

17. For any three simple statement p, q, r the statement $(p \wedge q) \vee (q \wedge r)$ is true when-

(A) p and r true and q is false **(B)** p and r false and q is true

(C) p, q, r all are false **(C)** q and r true and p is false

18. Consider the statement p : ‘’Everyone in Germany speaks German” which of the following is not negation of p”

(A) not everyone in Germany speaks German

(B) no one in Germany speaks German

(C) there are persons in Germany who do not speak German

(D) There is atleast one person in Germany who does not speak German.

19. If p is any statement, t and c are a tautology and a contradiction respectively then which of the following is not correct-

(A) $p \wedge t \equiv p$ **(B)** $p \wedge c \equiv c$ **(C)** $p \vee t \equiv c$ **(D)** $p \vee c \equiv p$

20. Negation of the statement $p \rightarrow (q \wedge r)$ is

(A) $\sim p \rightarrow \sim(q \wedge r)$ **(B)** $\sim p \vee (q \wedge r)$ **(C)** $(q \wedge r) \rightarrow p$ **(D)** $p \wedge (\sim q \vee \sim r)$

21. Which of the following is correct-

(A) $(\sim p \vee \sim q) \equiv (p \wedge q)$ **(B)** $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$

(C) $\sim(p \rightarrow \sim q) \equiv (p \wedge \sim q)$ **(B)** $\sim(p \leftrightarrow q) \equiv (p \rightarrow q) \vee (q \rightarrow p)$

22. Which one statement gives the same meaning of statement
"The Banana trees will bloom if it stays warm for a month."

(A) It stays warm for a month and the banana trees will bloom

(B) If it stays warm for a month, then the Banana trees will bloom

(C) It stays warm for a month or the banana trees will bloom

(D) None of these

23. The negation of the statement $q \vee (p \wedge \sim r)$ is equivalent to-

(A) $\sim q \wedge (p \rightarrow r)$ **(B)** $\sim q \wedge \sim(p \rightarrow r)$ **(C)** $\sim q \wedge (\sim p \wedge r)$ **(D)** None of these

24. Which of the following statement are not logically equivalent-

(A) $\sim(p \vee \sim q)$ and $(\sim p \wedge q)$ **(B)** $\sim(p \to q)$ and $(p \wedge \sim q)$

(C) $(p \to q)$ and $(\sim q \to \sim p)$ **(D)** $(p \to q)$ and $(\sim p \wedge q)$

25. The negation of the statement "If a quadrilateral is a square then it is a rhombus".

(A) If a quadrilateral is not a square then it is a rhombus

(B) If a quadrilateral is a square then it is not a rhombus

(C) a quadrilateral is a square and it is not a rhombus

(D) a quadrilateral is not a square and it is a rhombus

26. The negative of the statement "If a number is divisible by 15 then it is divisible by 5 or 3"

(A) If a number is divisible by 15 then it is not divisible by 5 and 3

(B) A number is divisible by 15 and it is not divisible by 5 or 3

(C) A number is divisible by 15 or it is not divisible by 5 and 3

(D) A number is divisible by 15 and it is not divisible by 5 and 3

27. The statement 'x is an even number implies that x is divisible by 4' means the same as

(A) x is divisible by 4 is necessary condition for x to be an even number

(B) x is an even number is a necessary condition for x to divisible by 4

(C) x is divisible by 4 is a sufficient condition for x to be an even number

(D) x is divisible by 4 implies that x is not always an even number

28. The negation of the statement "The sand heats up quickly in the sun and does not cool down fast at night"is

(A) The sand does not heat up quickly in the sun and it does not cool down fast at night

(B) Either the sand does not heat up quickly in the sun or it cools down fast at night

(C) The sand heats up quickly in the sun and it cools down fast at night

(D) The sand heats up quickly in the sun or it cools down fast at night

29. The negation of the statement "$2 + 3 = 5$ and $8 < 10$" is-

(A) $2 + 3 \neq 5$ and $8 \not< 10$ **(B)** $2 + 3 \neq 5$ or $8 > 10$

(C) $2 + 3 \neq 5$ or $8 \geq 10$ **(D)** None of these

30. The statement "If $2^2 = 5$ then I get first class" is logically equivalent to-

(A) $2^2 = 5$ and I don't get first class **(B)** $2^2 = 5$ or I do not get first class

(C) $2^2 \neq 5$ or I get first class **(D)** None of these

31. If p, q, r are simple propositions, with truth values T, F, T respectively then the truth value of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is

(A) true **(B)** false **(C)** true if r is false **(D)** true if q is true

32. Which of the following statement convey different meaning from the others?

(A) p is a necessary condition for q **(B)** p only if q

(C) p implies q **(D)** $\sim$ q implies $\sim$ p

33. Negation of the statement $(p \wedge r) \rightarrow (r \vee q)$ is-

(A) $\sim(p \wedge r) \rightarrow \sim(r \vee q)$ **(B)** $(\sim p \vee \sim r) \vee (r \vee q)$ **(C)** $(p \wedge r) \wedge (r \wedge q)$ **(D)** $(p \wedge r) \wedge (\sim r \wedge \sim q)$

34. The converse of the statement 'p implies q' is

(A) ~ q implies ~ p **(B)** q implies p **(C)** p only if q **(D)** ~ p implies q

35. Which of the following statement is a tautology-

(A) $(\sim p \vee \sim q) \vee (p \vee \sim q)$ **(B)** $(\sim p \vee \sim q) \wedge (p \vee \sim q)$

(C) $\sim p \wedge (\sim p \vee \sim q)$ **(D)** $\sim q \wedge (\sim p \vee \sim q)$

36. The negation of the statement '$\sqrt{2}$ is not a complex number' is

(A) $\sqrt{2}$ is a rational number **(B)** $\sqrt{2}$ is an irrational number

(C) $\sqrt{2}$ is a real number **(D)** $\sqrt{2}$ is a complex number

37. Statement $(p \wedge q) \rightarrow p$ is-

(A) a tautology **(B)** a contradiction

(C) neither (1) nor (2) **(D)** None of these

38. For the compound statement

'All prime numbers are either even or odd'. Which of the following is true?

(A) Both component statements are false

(B) Exactly one of the component statements is true

(C) At least one of the component statements is true

(D) Both the component statements are true

39. Which of the following is wrong-

(A) $p \vee \sim p$ is a tautology **(B)** $\sim(\sim p) \leftrightarrow p$ is a tautology

(C) $p \wedge \sim p$ is a contradiction **(D)** $((p \wedge p) \rightarrow q) \rightarrow p$ is a tautology

40. Which of the following is a statement-

(A) I am Lion **(B)** Logic is an interesting subject

(C) A triangle is a circle and 10 is a prime number **(D)** None of these

Exercise # 2 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. Match the following columns :

	Column - I		Column - II
(A)	Negation of $(\sim p \to q)$ is	**(p)**	$[\sim(p \vee q)] \wedge [p \vee (\sim p)]$
(B)	Negation of $(p \leftrightarrow q)$ is	**(q)**	$(p \wedge \sim q) \vee (\sim p \wedge q)$
(C)	Negation of $(p \vee q)$ is	**(r)**	$(\sim p \vee q) \wedge (p \vee \sim q)$
(D)	$p \leftrightarrow q$ is equivalent to	**(s)**	$\sim p \wedge \sim q$

Part # II [Comprehension Type Questions]

Comprehension # 1

$p \to q$ is called a conditional statement. It is read as "If p then q". It is false only when p is true and q is false. Following points may be noted in context of $p \to q$:

(i) p is a sufficient condition for q
(ii) q is a necessary condition for p
(iii) "If p then q" also means "p only if q"
(iv) $p \to q$ is equivalent to $\sim q \to \sim p$

1. Consider statement "If you are born in India then you are a citizen of India". Which of the following is logical equivalent to the given statement ?
(A) If you are not born in India then you are not a citizen of India
(B) If you are a citizen of India then you are not born in India
(C) You are born in India only if you are a citizen of India
(D) Taking birth in India is not sufficient condition to be a citizen of India

2. Consider statement "If there are clouds in the sky then it will rain". Which of the following give same meaning ?
(A) Having clouds in the sky is sufficient to have rain
(B) It is not necessary to have rain if there are clouds in the sky.
(C) If it is raining then there are clouds in the sky.
(D) All of these

3. If p : Mumbai is in Japan and q : Delhi is in South Africa then
(A) $p \to q$ is true **(B)** $p \to q$ is false **(C)** $p \to \sim q$ is false **(D)** $\sim q \to \sim p$ is false

Exercise # 3 [Subjective Type Questions]

1. Write down the negation of :

(i) It is raining and it is cool.

(ii) $3 + 6 > 8$ and $2 + 3 < 6$

(iii) The roof is red and the wall is white

(iv) I study or I fail.

(v) Sun rises or moon sets.

(vi) If she works then she will earn money

(vii) If it snows, then they do not drive the car.

(viii) If you feel thirsty, then it is not hot outside.

(ix) ABC is an equilateral triangle if and only if it is equiangular

(x) He swims if and only if the water is warm

2. Find the truth values of the following compound statements :

(i) $\sim(p \wedge \sim q)$

(ii) $\sim(\sim p \vee \sim q)$

(iii) $(p \wedge q) \vee \sim(p \vee q)$

(iv) $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$

(v) $(p \wedge \sim q) \vee r$

(vi) $\sim p \vee (q \wedge \sim r)$

(vii) $(\sim p \wedge \sim q) \vee \sim r$

(viii) $\sim((p \wedge q) \vee \sim r$

(ix) $(p \wedge q) \rightarrow \sim p$

(x) $(p \wedge q) \rightarrow (p \vee q)$

(xi) $(p \rightarrow q) \rightarrow r$

(xii) $(p \vee q) \leftrightarrow r$

3. Show that the following argument is valid :
"If he works hard, he will be successful. He was not successful. Therefore he did not work hard".

4. Find the negation of the compound proposition p v (~ p v q).

5. Are the following statements equivalent :
"If the traders do not reduce the price then the government will take action against them"
"It is not true that the trader do not reduce the prices and government does not take action against them".

6. Check whether the following statement is true or false by proving its contrapositive.
"If x, y are integers such that xy is odd, then both x and y are odd integers"

7. Show that $((\sim q) \wedge p) \vee (p \vee \sim p)$ is a tautology

8. Write the converse, contradiction and contrapositive of the statement given below :
If ΔABC is right angled at B, then $AB^2 + BC^2 = AC^2$

9. By using laws of algebra prove that

(i) $\sim(\sim p \wedge q) \equiv p \vee \sim q$

(ii) $p \wedge (p \vee q) \equiv p \vee (p \wedge q)$

(iii) $(p \wedge q) \vee \sim p \equiv \sim p \vee q$

(iv) $p \wedge (\sim p \vee q) \equiv p \wedge q$

10. Given below are two statements :
p : 25 is a multiple of 5.
q : 25 is a multiple of 8.
Write the compound statement connecting these two statements with "OR" and check its validity.

11. Show that the following statement is true by the method of contrapositive.

p : If x is an integer and x^2 is even, then x is also even.

12. Prove that statements $(p \wedge q) \vee [\sim p \vee (p \wedge \sim q)]$ and $(\sim p \wedge q) \vee t$ where t is tautology are logically equivalent

13. Test the validity of the following statement :

"If it rains tomorrow, I shall carry my umbrella if its cloth is mended. It will rain tomorrow and the cloth will not be mended. Therefore, I shall not carry my umbrella".

14. Examine the validity of the argument $(S_1, S_2 ; S)$, where

S_1 : If Hema is not in team A, then Rita is in team B.

S_2 : If Rita is not in team B, then Mamta is in team A.

S : Mamta is in team A or Rita is not in team B.

15. If p, q, r are three statements, then test the validity of the argument $(S_1, S_2, S_3;S)$, where $S_1 : (p \wedge \sim q) \rightarrow r$; S_2 ; $p \vee q$; S_3 ; $q \rightarrow p$ and S ; r

.

Exercise # 4 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The statement $p \to (q \to p)$ is equivalent **[AIEEE-2008]**

(1) $p \to (p \to q)$ **(2)** $p \to (p \vee q)$

(3) $p \to (p \wedge q)$ **(4)** $p \to (p \leftrightarrow q)$

2. Let p be the statement "x is an irrational number", q be the statement "y is a trascendental number", and r be the statement "x is a rational number iff y is a transcendental number". **[AIEEE-2008]**

Statement –I : r is equivalent to either q or p.

Statement –II : r is equivalent to $(p \leftrightarrow \sim q)$

(1) Statement –I is false, Statement –II is true

(2) Statement–I is true, Statement–II is false

(3) Statement–I is true, Statement–II is true; Statement–II is a correct explanation for Statement–I

(4) Statement–I is true, Statement–II is true; Statement–II is not a correct explanation for Statement–I

3. **Statement–I :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement–II : $\sim(p \leftrightarrow \sim q)$ is a tautology. **[AIEEE-2009]**

(1) Statement–I is true, Statement–II is false.

(2) Statement–I is false, Statement–II is true.

(3) Statement–I is true, Statement–II is true ; Statement–II is a correct explanation for Statement–I.

(4) Statement–I is true, Statement–II is true ; Statement–II is not a correct explanation for statement–I.

4. Let S be a non-empty subset of R.

Consider the following statement :

p : There is a rational number $x \in S$ such that $x > 0$

which of the following statements is the negation of the statement p ? **[AIEEE-2010]**

(1) There is a rational number $x \in S$ such that $x \le 0$ **(2)** There is no rational number $x \in S$ such that $x \le 0$

(3) Every rational number $x \in S$ satisfies $x \le 0$ **(4)** $x \in S$ and $x \le 0 \Rightarrow$ x is not rational

5. Consider the following statements

p : Suman is brilliant

q : Suman is rich

r : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :- **[AIEEE-2011]**

(1) $\sim q \leftrightarrow \sim p \wedge r$ **(2)** $\sim (p \wedge \sim r) \leftrightarrow q$

(3) $\sim p \wedge (q \leftrightarrow \sim r)$ **(4)** $\sim (q \leftrightarrow (p \wedge \sim r))$

6. The only statement among the followings that is a tautology is : **[AIEEE-2011]**

(1) $q \to [p \wedge (p \to q)]$ **(2)** $p \wedge (p \vee q)$

(3) $p \vee (p \wedge q)$ **(4)** $[p \wedge (p \to q)] \to q$

7. The negation of the statement

"If I become a teacher, then I will open a school", is **[AIEEE-2012]**

(1) I will not become a teacher or I will open a school.

(2) I will become a teacher and I will not open a school.

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school.

8. The statement $\sim (p \leftrightarrow \sim q)$ is : **[JEE Main 2014]**

(1) equivalent to $p \leftrightarrow q$ **(2)** equivalent to $\sim p \leftrightarrow q$

(3) a tautology **(4)** a fallacy

9. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[JEE Main 2015]**

(1) $s \vee (r \vee \sim s)$ **(2)** $s \wedge r$ **(3)** $s \wedge \sim r$ **(4)** $s \wedge (r \wedge \sim s)$

10. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim q \wedge p)$ is equivalent to : **[JEE Main 2016]**

(1) $p \wedge q$ **(2)** $p \vee q$ **(3)** $p \vee \sim q$ **(4)** $\sim p \wedge q$

ANSWER KEY

EXERCISE - 1

1. B **2.** C **3.** D **4.** C **5.** C **6.** A **7.** C **8.** C **9.** C **10.** D **11.** C **12.** B **13.** D
14. C **15.** B **16.** A **17.** D **18.** B **19.** C **20.** D **21.** B **22.** B **23.** A **24.** D **25.** C **26.** D
27. A **28.** B **29.** C **30.** C **31.** A **32.** A **33.** D **34.** B **35.** A **36.** D **37.** A **38.** A **39.** D
40. C

EXERCISE - 2 : PART - I

1. $A \to p,s$ $B \to q$ $C \to p,s$ $D \to r$

PART - II

Comprehension # 1 : **1.** C **2.** A **3.** A

EXERCISE - 4

1. 2 **2.** 1 **3.** 1 **4.** 3 **5.** 2,4 **6.** 4 **7.** 2 **8.** 1 **9.** 2 **10.** 2