FUNCTION

EXERCISE # 1

Question based on Domain

Q.1 Domain of $y = \log_{10}\left(\dfrac{5x - x^2}{4}\right)$:

(A) (0, 5) (B) [1, 4]
(C) $(-\infty, 0) \cup (5, \infty)$ (D) $(-\infty, 1) \cup (4, \infty)$

Sol. **[A]**

$\dfrac{5x - x^2}{4} > 0 \Rightarrow x^2 - 5x < 0 \Rightarrow x(x - 5) < 0$

+   −   +
0   5

Hence $x \in (0, 5)$

Q.2 The domain of definition of

$f(x) = \dfrac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2 + 2x + 8}}$ is:

(A) (1, 4) (B) (–2, 4)
(C) (2, 4) (D) $[2, \infty)$

Sol. **[D]**

$x^2 + 2x + 8 \geq 0$ here $D = 4 - 8(4) < 0$

$\therefore x^2 + 2x + 8 \geq 0 \quad \forall x \in R$

$-\log_{0.3}(x-1) \geq 0 \Rightarrow \log_{0.3}(x-1) \leq 0$

$\Rightarrow (x - 1) \geq 1$

$\Rightarrow x \geq 2$

$\therefore R \cap [2, \infty) = [2, \infty)$

Q.3 The function

$f(x) = \cot^{-1}\sqrt{(x+3)x} + \cos^{-1}\sqrt{x^2 + 3x + 1}$ is defined on the set S, where S is equal to:

(A) {0, 3} (B) (0, 3)
(C) {0, –3} (D) [–3, 0]

Sol. **[C]**

$x(x + 3) \geq 0$ …..(i)

$\Rightarrow$ +   −   +
–3   0

$\Rightarrow x \in (-\infty, -3] \cup [0, \infty)$ …(ii)

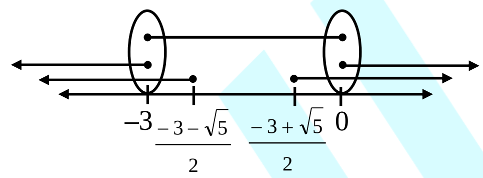
$-1 \leq \sqrt{x^2 + 3x + 1} \leq 1$

$\Rightarrow x^2 + 3x + 1 \leq 1$

$x^2 + 3x + 1 \geq 0$ …(iii)

(i) $\Rightarrow x \in [-3, 0]$

(ii) $\Rightarrow x \in \left(-\infty, \dfrac{-3-\sqrt{5}}{2}\right] \cup \left[\dfrac{-3-\sqrt{5}}{2}, \infty\right)$



$x \in \{-3, 0\}$

Q.4 The domain of $\sqrt{\sec^{-1}\left(\dfrac{2-|x|}{4}\right)}$ is

(A) R (B) R – (–1, 1)
(C) R – (–3, 3) (D) R – (– 6, 6)

Sol. **[D]**

$\sec^{-1}\left(\dfrac{2-|x|}{4}\right) \geq 0$ and domain of $\sec^{-1}x$

is $x \in R - (-1, 1)$

$\dfrac{2-|x|}{4} \geq 1$ or $\dfrac{2-|x|}{4} \leq -1$

$2 - |x| \geq 4$ | $2 - |x| \leq -4$

$-|x| \geq 2$ | $-|x| \leq -6$

$|x| \leq -2$ | $|x| \geq 6$

Not possible $\Rightarrow x \in (-\infty, -6] \cup [6, \infty)$

$\therefore x \in R - (-6, 6)$

Q.5 The domain of the function

$f(x) = {}^{24-x}C_{3x-1} + {}^{40-6x}C_{8x-10}$ is -

(A) {2, 3} (B) {1, 2, 3}
(C) {1, 2, 3, 4} (D) None of these

Sol. **[A]**

${}^{24-x}C_{3x-1} + {}^{40-6x}C_{8x-10} = f(x)$

Case I :

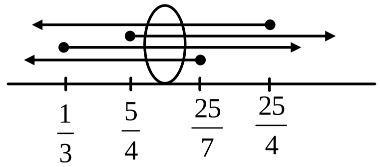
$24 - x \geq 3x - 1$ & $3x - 1 \geq 0$

$\Rightarrow x \leq \dfrac{25}{4}$ & $x \geq \dfrac{1}{3}$

Case II :

$40 - 6x \geq 8x - 10$ & $8x - 10 \geq 0$

$x \leq \dfrac{25}{7}$ & $x \geq \dfrac{5}{4}$



$x \in \left[\frac{5}{4}, \frac{25}{7}\right] \Rightarrow x \in [1.25, 3.57]$

but $x \in I \;\Rightarrow\; x \in \{2, 3\}$

Question based on

Range

Q.6 The range of the function $y = \dfrac{1}{2-\sin 3x}$ is :

(A) $\left(\frac{1}{3}, 1\right)$ (B) $\left[\frac{1}{3}, 1\right)$

(C) $\left[\frac{1}{3}, 1\right]$ (D) None of these

Sol. **[C]**

$y = \dfrac{1}{2-\sin 3x}$

$-1 \le \sin\theta \le 1$

$-1 \le \sin 3x \le 1$

$1 \ge -\sin 3x \ge -1$

$3 \ge 2 - \sin 3x \ge 1$

$\dfrac{1}{3} \le \dfrac{1}{2-\sin 3x} \le 1$

$\therefore$ range $\in \left[\frac{1}{3}, 1\right]$

Q.7 The value of the function

$f(x) = \dfrac{x^2-3x+2}{x^2+x-6}$ lies in the interval -

(A) $(-\infty, \infty) - \left\{\frac{1}{5}, 1\right\}$ (B) $(-\infty, \infty)$

(C) $(-\infty, \infty) - \{1\}$ (D) None of these

Sol. **[A]**

$$f(x) = \frac{x^2-3x+2}{x^2+x-6} = \frac{x^2-2x-x+2}{x^2+3x-2x-6}$$

$$= \frac{x(x-2)-1(x-2)}{x(x+3)-2(x+3)} = \frac{(x-1)\,(x-2)}{(x+3)(x-2)} \;\forall\; x \ne 2$$

$$y = \frac{x-1}{x+3} \quad \forall\; x \ne 2$$

$xy + 3y = x - 1 \Rightarrow \dfrac{3y+1}{1-y} = x$

$\Rightarrow y \in R - \{1\}$

at $x = 2$, $y = \dfrac{1}{5}$

range $y \in R - \left\{\frac{1}{5}, 1\right\}$

Q.8 Find the range of the following function,

$y = \log_{\sqrt{7}}(\sqrt{2}(\sin x - \cos x) + 5)$

(A) R (B) Z

(C) $[\log_7 4, \log_7 5]$ (D) $[2\log_7 3, 2]$

Sol. **[D]**

$y = \log_{\sqrt{7}}(\sqrt{2}(\sin x - \cos x) + 5)$

$-\sqrt{2+2} \le \sqrt{2}\,(\sin x - \cos x) \le \sqrt{2+2}$

$-2 + 5 \le \sqrt{2}\,(\sin x - \cos x) + 5 \le 2 + 5$

$\log_{\sqrt{7}} 3 \le \log_{\sqrt{7}}[\sqrt{2}(\sin x - \cos x) + 5] \le \log_{\sqrt{7}} 7$

$2\log_7 3 \le y \le 2$

$\therefore$ range $\in [2\log_7 3, 2]$

Q.9 Which of the following function (s) has the range $[-1, 1]$

(A) $f(x) = \cos(2\sin x)$

(B) $g(x) = \cos\left(1 - \dfrac{1}{1+x^2}\right)$

(C) $h(x) = \sin(\log_2 x)$

(D) $k(x) = \tan(e^x)$

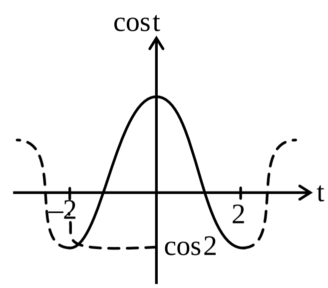
Sol. **[C,D]**

(A) $f(x) = \cos(2\sin x)$

$-1 \le \sin x \le 1$

$-2 \le 2\sin x \le 2$

$-2 \le t \le 2$



range $\in (\cos 2, 1]$

(B) $f(x) = \cos\left(1 - \dfrac{1}{1+x^2}\right)$

$-1 \le 1 - \dfrac{1}{1+x^2} \le 1$

$-2 \le \dfrac{-1}{1+x^2} \le 0$

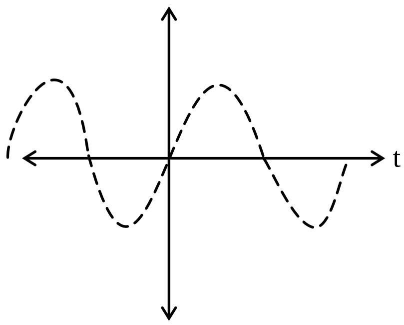
$2 \ge \dfrac{1}{1+x^2} \ge 0$

$\dfrac{1}{2} \le 1 + x^2 \le \infty$

(C) $h(x) = \sin(\log_2 x)$

$-1 \le \log_2 x \le 1$

$-1 \le t \le 1$



(D) $f(x) = \cos(e^x)$

$-1 \le \cos e^x \le 1$

Question based on — Kinds of functions

Q.10 Let $f : R \to R$ be a function defined by

$$f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1} \text{ is :}$$

(A) one-one and into (B) one-one and onto

(C) many-one and onto (D) many-one and into

Sol. **[D]**

$$f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1} = \frac{1 + \frac{2}{x} + \frac{5}{x^2}}{1 + \frac{1}{x} + \frac{1}{x^2}}$$

$f(\infty) = 1$ & $f(-\infty) = 1$

$\therefore$ many one function

if $f(x) \to \infty \Rightarrow x^2 + x + 1 \longrightarrow 0$ which

is false as $x^2 + x + 1 \ne 0 \ \forall \ x \in R$

$\therefore$ into function.

Q.11 The function $f : [2, \infty) \to Y$ defined by $f(x) = x^2 - 4x + 5$ is both one–one & onto if:

(A) $Y = R$ (B) $Y = [1, \infty)$

(C) $Y = [4, \infty)$ (D) $Y = [5, \infty)$

Sol. **[B]**

$f(x) = x^2 - 4x + 5 \to$ its one-one & onto

$f'(x) = 2x - 4$ if $f'(x) = 0 \Rightarrow 2x = 4 \Rightarrow x = 2$

(i) $f(2) = 1$

(ii) $f(\infty) = 1 - \frac{4}{x} + \frac{5}{x^2} = 1$

$\therefore f(x) \in [1, \infty)$

Q.12 Let $f : R \to R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is :

(A) one– one & onto (B) one –one & into

(C) many one & onto (D) many one & into

Sol. **[A]**

$f(x) = x^3 + x^2 + 3x + \sin x$

$f(x) \to$ cubic polynomial $\to$ onto function

$f'(x) = 3x^2 + 2x + 3 + \cos x > 0$ always as coeff of x^2 is +ve

$\therefore$ No sign change $\to$ NOT many one

Q.13 Which of the following function from $A = \{x : -1 \le x \le 1\}$ to itself are bijections-

(A) $f(x) = x/2$ (B) $g(x) = \sin(\pi x/2)$

(C) $h(x) = |x|$ (D) $k(x) = x^2$

Sol. **[B]**

We have to check for every options as :

For A : $f(x) = x/2 = y$; $-1 \le x \le 1$

$x = -1,\ y = -1/2 \Rightarrow x \ne y$

$x = 1,\ y = 1/2 \Rightarrow x \ne y$

$\therefore$ option (A) is wrong

For B : $y = \sin\left(\frac{\pi x}{2}\right)$; $-1 \le x \le 1$

For $x = -1$, $y = \sin\left(\frac{-\pi}{2}\right) = -1 \Rightarrow x = y$

For $x = 1$, $y = \sin\left(\frac{\pi}{2}\right) = 1 \Rightarrow x = y$

Hence option (B) is correct

For C $y = |x| \Rightarrow y = \pm x$

For $x = -1$, $y = \mu\ 1$

For $x = 1$, $y = \pm 1$

$\therefore$ Option (C) is wrong

For D: $y = x^2$

For $x = -1$, $y = 1 \Rightarrow x \ne y$

For $x = 1$, $y = 1 \Rightarrow x = y$

Both must be satisfied simultaneously.

$\therefore$ Option (D) is wrong.

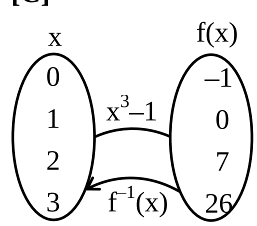
Question based on — Inverse function

Q.14 If $f(x) = x^3 - 1$ and domain of $f = \{0, 1, 2, 3\}$, then domain of f^{-1} is -

(A) $\{0, 1, 2, 3\}$ (B) $\{1, 0, -7, -26\}$

(C) $\{-1, 0, 7, 26\}$ (D) $\{0, -1, -2, -3\}$

Sol. **[C]**



$\therefore$ Domain of $f^{-1}(x)$

= range of $f(x)$

$= \{-1, 0, 7, 26\}$

Q.15 The inverse of the function $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ is

(A) $\frac{1}{2}\log\frac{1+x}{1-x}$ (B) $\frac{1}{2}\log\frac{2+x}{2-x}$

(C) $\frac{1}{2}\log\frac{1-x}{1+x}$ (D) $2\log(1+x)$

Sol. **[C]**

$y = \dfrac{e^{2x}-1}{e^{2x}+1}$; Let $e^{2x} = t$

$y = \dfrac{t-1}{t+1} \Rightarrow ty + y = t - 1$

$\Rightarrow \dfrac{y+1}{1-y} = t$

$\Rightarrow e^{2x} = \dfrac{y+1}{1-y} \Rightarrow 2x = \log\left(\dfrac{y+1}{1-y}\right)$

$\Rightarrow f^{-1}(x) = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right)$

Question based on **Composite function**

Q.16 The function f(x) is defined in [0, 1] then the domain of definition of the function $f[\lambda n(1-x^2)]$ is given by :

(A) $x \in \{0\}$

(B) $x \in [-\sqrt{1+e} - 1] \cup [1 + \sqrt{1+e}\,]$

(C) $x \in (-\infty, \infty)$

(D) None of these

Sol. **[A]**

f(x) where $x \in [0, 1]$

$f(\lambda n(1-x^2))$ where $0 \le \lambda n(1-x^2) \le 1$

$1 \le 1 - x^2 \le e$

$0 \le -x^2 \le e - 1$

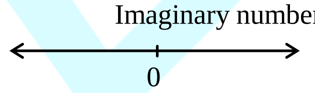
$0 \ge x^2 \ge 1 - e$

(i) $x = 0$

(ii) $x \in (-\infty, -\sqrt{1-e}\,] \cup [\sqrt{1-e}, \infty)$

$\downarrow$ $\downarrow$

Imaginary numbers as $1 < e$



$\Rightarrow x \in \{0\}$

Question base don **Periodic function**

Q.17 If $f : R \to R$ is a function satisfying the property $f(x+1) + f(x+3) = 2 \ \forall\ x \in R$ then the period (may not be fundamental period) of f (x) is

(A) 3 (B) 4 (C) 7 (D) 6

Sol. **[B]**

$x \longrightarrow x + 2$

(ii) $f(x+3) + f(x+5) = 2$

(i) $f(x+1) + f(x+3) = 2$

(ii) – (i) $\Rightarrow f(x+1) = f(x+5)$

$f(x+1) = f((x+1)+4)$

Hence T = 4

Q.18 The fundamental period of the function:
$f(x) = x + a - [x+b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots + \sin(2n-1)\pi x + \cos 2n\pi x$ for every $a, b \in R$ is:
(where [.] denotes the greatest integer function)

(A) 2 (B) 4

(C) 1 (D) 0

Sol. **[A]**

$f(x) = x + b - [x+b] + \sin \pi x + \cos 2\pi x + \ldots + \cos 2\pi n x + a - b$

$= \{x+b\} + \sin \pi x + \cos 2\pi x + \ldots + a - b$

$\downarrow$

T 1, 2, ………

L.C.M. = 2

Thus period = 2

Q.19 Let $f(x) = \sin\sqrt{[a]}\,x$ (where [ ] denotes the greatest integers function). If f is periodic with fundamental period π, then a belongs to -

(A) [2, 3) (B) {4, 5}

(C) [4, 5] (D) [4, 5)

Sol. **[C]**

$f(x) = \sin\sqrt{[a]}.x$

$\sin\sqrt{[a]}.x$ is Periodic with Period $\dfrac{2\pi}{\sqrt{[a]}}$

$\therefore \dfrac{2\pi}{\sqrt{[a]}} = \pi \Rightarrow \sqrt{[a]} = 2 \Rightarrow [a] = 4 \Rightarrow a = 5$

$a \in [4, 5]$

$\therefore$ option (C) is correct Answer.

Question based on **Even and odd function**

Q.20 Which of the following is an even function?

(A) $x\dfrac{a^x - 1}{a^x + 1}$ (B) tan x

(C) $\dfrac{a^x - a^{-x}}{2}$ (D) $\dfrac{a^x + 1}{a^x - 1}$

Sol. **[A]**

(A) $f(-x) = (-x)\dfrac{a^{-x} - 1}{a^{+x} + 1} + (-x)\dfrac{1 - a^x}{1 + a^x} = f(x)$

(B) $f(-x) = -f(x)$
(C) $f(-x) = -f(x)$
(D) $f(-x) = \frac{1+a^x}{1-a^x} = -f(x)$
Hence (A) is an even function

Q.21 Which of the following function is an odd function

(A) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$
(B) $f(x) = x\left(\frac{a^x+1}{a^x-1}\right)$
(C) $f(x) = \log\left(\frac{1-x}{1+x^2}\right)$
(D) $f(x) = k$ (constant)

Sol. **[A]**

(A) $f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2} = -f(x)$
(B) $f(-x) = (-x)\left(\frac{1+a^x}{1-a^x}\right) = f(x)$
(C) $f(-x) = \log\left(\frac{1+x}{1+x^2}\right)$
(D) $f(x) = k$ (even)

Question based on **Miscellaneous**

Q.22 The set of points for which $f(x) = \cos(\sin x) > 0$ contains -

(A) $(-\infty, 0]$ (B) $[-1, 1]$
(C) $(-\infty, \infty)$ (D) All are correct

Sol. **[D]**

Since, $-1 \le \sin x \le 1$
hence for
$\sin x \in [-1, 1]$
$\Rightarrow -1 \le x \le 1$
$\therefore$ Option (B) is correct Answer

Q.23 If $[x]$ stands for the greatest integer function, then the value of

$$\left[\frac{1}{2}+\frac{1}{1000}\right]+\left[\frac{1}{2}+\frac{2}{1000}\right]+\ldots\ldots\ldots+\left[\frac{1}{2}+\frac{999}{1000}\right]$$

(A) 498 (B) 499
(C) 500 (D) 501

Sol. **[C]**

Greatest integer function defined as $[x] \le x$ for all $x \in R$.
then

$$\left[\frac{1}{2}+\frac{1}{1000}\right]+\left[\frac{1}{2}+\frac{2}{1000}\right]+\ldots\ldots\ldots\ldots$$
$$+\left[\frac{1}{2}+\frac{999}{1000}\right] \text{ (999 term)}$$

i.e. $\left[\frac{1}{2}+\frac{1}{1000}\right]+\left[\frac{1}{2}+\frac{2}{1000}\right]+\ldots\ldots\ldots\ldots$
$+\left[\frac{1}{2}+\frac{499}{1000}\right]+\left[\frac{1}{2}+\frac{500}{1000}\right]+\ldots\ldots\ldots\ldots$
$+\left[\frac{1}{2}+\frac{999}{1000}\right]$

First 499 terms, each will be zero and remaining 500 terms will be as follows

$$\left[\frac{1}{2}+\frac{500}{1000}\right]+\left[\frac{1}{2}+\frac{501}{1000}\right]+\ldots\ldots+\left[\frac{1}{2}+\frac{999}{1000}\right]$$

$= 1 + 1 + 1$ .... $+ 1$(500 terms) $= 500$
$\therefore$ Option (C) is correct Answer.

Q.24 Let the function $f(x) = 3x^2 - 4x + 8\log(1 + |x|)$ be defined on the interval $[0, 1]$. The even extension of $f(x)$ to the interval $[-1, 0]$ is -

(A) $3x^2 + 4x + 8\log(1 + |x|)$
(B) $3x^2 - 4x + 8\log(1 + |x|)$
(C) $3x^2 + 4x - 8\log(1 + |x|)$
(D) $3x^2 - 4x - 8\log(1 + |x|)$

Sol. **[A]**

$f(x) = 3x^2 - 4x + 8\log(1 + |x|)$; $x \in [0, 1]$
$f(-x) = 3x^2 + 4x + 8\log(1 + |x|)$; $x \in [-1, 0]$
$\therefore$ Option (A) is correct Answer.

Q.25 Let $f : N \to N$ where $f(x) = x + (-1)^{x-1}$ then f is-

(A) Inverse of itself (B) even function
(C) periodic (D) identity

Sol. **[A]**

$f(x) = x + (-1)^{x-1}$

$$f(x) = \begin{cases} x+1 \ ; & x \in \text{odd natural number} \\ x-1 \ ; & x \in \text{even natural number} \end{cases}$$

$y = f(x)$

$$x = \begin{cases} y-1 \ ; & y \in \text{odd natural number} \\ y+1 \ ; & y \in \text{even natural number} \end{cases}$$

$$f^{-1}(x) = \begin{cases} x-1 \ ; & x \in \text{odd natural number} \\ x+1 \ ; & x \in \text{even natural number} \end{cases}$$

$\therefore$ $f(x)$ is inverse to itself

True or False type Questions

Q.26 The range of the function $f(x) = \cos(\cos^{-1}\{x\})$ is $[0, 1]$.
(where $\{\cdot\}$ denotes the fractional part function)

Sol. **[False]**

$f(x) = \cos(\cos^{-1}(x))$
$\{x\}$ = Fractional Part $= x - [x]$
$y = \cos(\cos^{-1}\{x\})$
$\Rightarrow \cos^{-1}\{x\} = \cos^{-1} y$
$\Rightarrow \{x\} = y$
But $0 \le \{x\} < 1$

$\Rightarrow\ 0 \le y < 1 \Rightarrow\ y \in [0, 1)$

Range $\in [0, 1)$

$\therefore$ Answer is False

Q.27 The function $f(x) = \left|\cos^5\left(\dfrac{x}{2}\right)\right|$ is periodic with period π.

Sol. **[False]**

$f(x) = \cos(\cos^{-1}(x))$

$\{x\}$ = Fractional Part = $x - [x]$

$y = \cos(\cos^{-1}\{x\})$

$\Rightarrow \cos^{-1}\{x\} = \cos^{-1}y$

$\Rightarrow\ \{x\} = y$

But $0 \le \{x\} < 1$

$\Rightarrow\ 0 \le y < 1 \Rightarrow\ y \in [0, 1)$

Range $\in [0, 1)$

$\therefore$ Answer is False

Q.28 Domain of the function $f(x) = (1 - 3x)^{1/3} + 3\cos^{-1}\left(\dfrac{2x-1}{3}\right) + 3^{3\tan^{-1}x}$ is $\left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$.

Sol. **[False]**

$f(x) = (1 - 3x)^{1/3} + 3\cos^{-1}\left(\dfrac{2x-1}{3}\right) + 3^{3\tan^{-1}x}$

$(1 - 3x)^{1/3}$ to be defined for $x \in R$.

$\cos^{-1}\left(\dfrac{2x-1}{3}\right)$ to be defined for $-1 \le \dfrac{2x-1}{3} \le 1$

$\Rightarrow\ -3 \le 2x - 1 \le 3$

$\Rightarrow\ -2 \le 2x \le 4$

$\Rightarrow\ -1 \le x \le 2$

$\tan^{-1}x$ to be defined for $x \in R$

Hence, domain $\in [-1, 2]$

$\therefore$ Answer is False

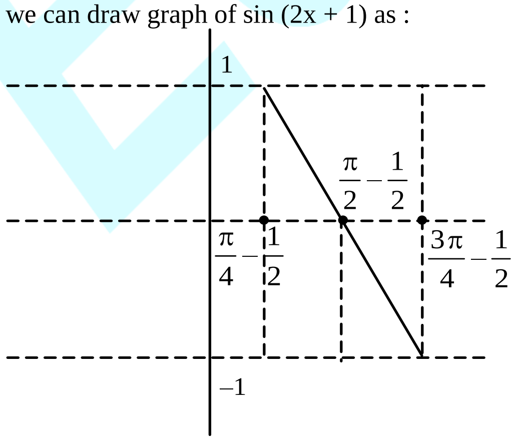
Q.29 If $f : \left[\dfrac{\pi}{4} - \dfrac{1}{2}, \dfrac{3\pi}{4} - \dfrac{1}{2}\right] \to [-1, 1]$ is defined by $f(x) = \sin(2x + 1)$, then f is one-one and onto.

Sol. **[True]**

$f : \left[\dfrac{\pi}{4} - \dfrac{1}{2}, \dfrac{3\pi}{4} - \dfrac{1}{2}\right] \to [-1, 1]$

$f(x) = \sin(2x + 1)$

we can draw graph of sin (2x + 1) as :



Which shows f(x) is one-one

$y = \sin(2x + 1) \Rightarrow\ 2x + 1 = \sin^{-1}y$

$$\Rightarrow x = \frac{\sin^{-1}y - 1}{2}$$

$$f(x) = f\left[\frac{\sin^{-1}y - 1}{2}\right] = \sin\left[2 \times \frac{\sin^{-1}y - 1}{2} + 1\right]$$

$$= \sin[\sin^{-1}y - 1 + 1]$$

$\Rightarrow\ f(x) = y$

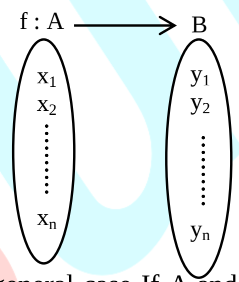
$\therefore$ f(x) is onto mapping

$\therefore$ Option is true.

Fill in the blanks questions

Q.30 The number of bijective functions from set A to itself when A contains 106 elements is……..

Sol.



We take general case If A and B are finite sets and $f : A \to B$ is a bijection. Then A and B have the same number of elements. If A has n elements, then number of bijection $A \to B$

Here, x_1 can take (n) images

x_2 can take (n – 1) images

x_3 can take (n – 2) images

....

....

x_n can take 1 image

Thus, total number of mapping are,

n(n–1) (n –2) .... (2) (1)

Therefore, number of bijection from

$A \to B = n!$

But our case is for n = 106

$\therefore$ Number of bijective functions from set A to itself = 106!

EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 The function f(x) = $\sqrt{\log_{10}\cos(2\pi x)}$ exists -

(A) for any rational x

(B) only when x is a positive integer

(C) only when x is fractional

(D) for any integer value of x including zero

Sol. **[D]**

If f(x) = $\sqrt{\log_{10}\cos 2\pi x}$ exists, then

$\log_{10}\cos 2\pi x \ge 0$ and $\cos 2\pi x > 0$

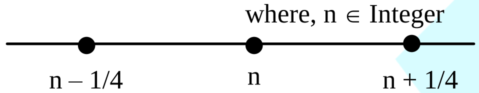
or $\cos 2\pi x \ge 1$ and $\cos 2\pi x > \cos \pi/2$

or $\cos 2\pi x \ge \cos 0°$ and $\cos 2\pi x > \cos\pi/2$

or $2\pi x \ge 2n\pi$ and $2\pi x > 2n\pi \pm \pi/2$

or $x \ge n$ and $x > n \pm \pi/4$

where, n ∈ Integer



If we take x > n + 1/4, then x ≥ n will be excluded which is not possible

If we take x > n – 1/4, then x ≥ n will be included which is possible

Therefore,

Option (D) is correct answer.

Q.2 The domain of the function $\sec^{-1}[x^2 - x + 1]$, is given by–

where [·] is greatest integer function -

(A) [0, 1] (B) $(-\infty, 0] \cup [1, \infty)$

(C) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ (D) None of these

Sol. **[B]**

$\sec^{-1}[x^2 - x + 1]$ to be defined, if

$\sec^{-1}[x^2 - x + 1] \le -1$ or $\sec^{-1}[x^2 - x + 1] \ge 1$

From the definition of greatest integer,

$[x] \le x$

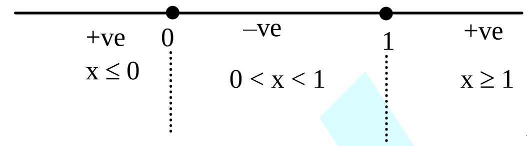
$\sec^{-1}[x^2 - x + 1] \ge 1$ possible

Then $\sec^{-1}[x^2 - x + 1] \ge 1$

or $[x^2 - x + 1] \ge 0$

or $(x^2 - x) \ge 0$

or $x(x - 1) \ge 0$



i.e

. $x \le 0$ or $x \ge 1$

Domain ∈ $(-\infty, 0] \cup [1, \infty)$

Q.3 The domain of definition of the function

f(x) = $\frac{\cot^{-1}x}{\sqrt{\{x^2 - [x^2]\}}}$, where [x] denotes the greatest integer less than or equal to x is -

(A) R

(B) $R - \{\pm\sqrt{n} : n \in I^+ \cup \{0\}\}$

(C) R – {0}

(D) R – {n : n ∈ I}

Sol. **[B]**

f(x) = $\frac{\cot^{-1}x}{\sqrt{\{x^2 - [x^2]\}}}$ to be defined,

$\cot^{-1}x$, $x \in R$

Also, $x^2 - [x^2] > 0$ and $x^2 - [x^2] \ne 0$

or, $x^2 > [x^2]$ and $x^2 \ne [x^2]$

from definition of greatest integer, we know

$[x] \le x \Rightarrow x^2 \ge [x]^2$

But $[x^2] > [x]^2$

⇒ $x^2 > [x]^2$ and $x^2 \ne [x]^2$

⇒ $x < -[x]$ or $x > [x]$

and $x \ne \pm [x]$

⇒ $x > 0$ and $x \ne 0$

Only possibility is option (B).

Q.4 The domain of the definition of

f(x) = log{(log x)² – 5 log x + 6} is equal to-

(A) $(0, 10^2)$ (B) $(10^3, \infty)$

(C) $(10^2, 10^3)$ (D) $(0, 10^2) \cup (10^3, \infty)$

Sol. **[D]**

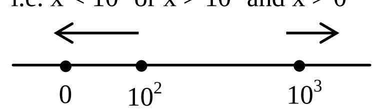
f(x) to be defined if

$\{(\log_{10}x)^2 - 5\log_{10}x + 6\} > 0$ and $x > 0$

i.e. $(\log_{10}x - 2)(\log_{10}x - 3) > 0$ and $x > 0$

i.e. $\log_{10}x < 2$ or $\log_{10}x > 3$ and $x > 0$

i.e. $x < 10^2$ or $x > 10^3$ and $x > 0$



Domain $\in (0, 10^2) \cup (10^3, \infty)$

Q.5 If $A = \left\{x : \frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ and $f(x) = \cos x - x(1 + x)$ then f(A) is equal to-

(A) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(B) $\left[-\frac{\pi}{3}, -\frac{\pi}{6}\right]$

(C) $\left[\frac{1}{2} - \frac{\pi}{3}\left(1+\frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$

(D) $\left[\frac{1}{2} + \frac{\pi}{3}\left(1-\frac{\pi}{3}\right), \frac{\sqrt{3}}{2} + \frac{\pi}{6}\left(1-\frac{\pi}{6}\right)\right]$

Sol. **[C]**

We have to find value of f(A).

Simply we can put A = x at x = π/6

and A = x at π/3 in f(x)

A = x at x = π/6

$f(A) = f(\pi/6) = \cos\pi/6 - \pi/6(1+\pi/6)$

$= \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)$

A = x at x = π/3

$f(A) = f(\pi/3) = \cos\pi/3 - \pi/3(1+ \pi/3)$

$= \frac{1}{2} - \frac{\pi}{3}(1+ \pi/3)$

Hence f(A) in $\pi/6 \le x \le \pi/3$ is

$$\left[\frac{1}{2} - \frac{\pi}{3}(1+\pi/3), \frac{\sqrt{3}}{2} - \frac{\pi}{6}(1+\pi/6)\right]$$

∴ Option (C) is correct Answer.

Q.6 If A be the set of all triangles and B that of positive real numbers, then the mapping $f : A \to B$ given by $f(\Delta)$ = area of Δ, (Δ ∈ A) is

(A) one-one into mapping

(B) one-one onto mapping

(C) many-one into mapping

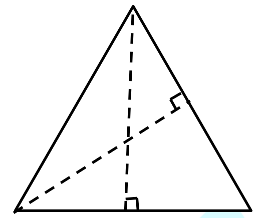
(D) many-one onto mapping

Sol. **[D]**

$f : A \to B$

$f(\Delta)$ = area of Δ, (Δ ∈ A)

Area of Δ will be same for different sides combination



∴ f(x) = f(y) but x ≠ y

∴ It is many one mapping

It will also be onto mapping

∴ Given function is many-one onto mapping

Q.7 Let $f : R \to A = \left\{y \mid 0 \le y < \frac{\pi}{2}\right\}$ be a function such that $f(x) = \tan^{-1}(x^2 + x + k)$, where k is a constant. The value of k for which f is an onto function, is -

(A) 1 (B) 0

(C) 1/4 (D) None of these

Sol. **[C]**

$f : R \to A = \{y ;\ 0 \le y < \pi/2\}$

$f(x) = \tan^{-1}(x^2 + x + k) = y$

$\tan y = x^2 + x + k$

for y = 0, $x^2 + x + k = 0$

$x = \frac{-1 \pm \sqrt{1-4k}}{2\times 1}$

For onto function, $1 - 4k = 0$

$k = 1/4$

∴ Option (C) is correct answer.

Q.8 Which of the following functions are not injective mapping-

(A) $f(x) = |x + 1|$, $x \in [-1, \infty)$

(B) $g(x) = x + \frac{1}{x}$; $x \in (0, \infty)$

(C) $h(x) = x^2 + 4x - 5$; $x \in (0, \infty)$

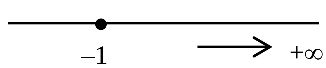
(D) $k(x) = e^{-x}$; $x \in [0, \infty)$

Sol. **[B]**

We have to check for every option

For A : $f(x) = |x + 1|$; $x \in [-1, \infty)$

$|x + 1| = x + 1$



For one-one

$f(x) = f(y) \Rightarrow x + 1 = y + 1$

$\Rightarrow x = y$ for $x \in [-1, \infty)$

option (A) is Injective

For B : $g(x) = x + \frac{1}{x}$; $x \in (0, \infty)$

If $g(x) = g(y) \Rightarrow x + \frac{1}{x} = y + \frac{1}{y}$

$\Rightarrow \qquad (x-y) + \frac{1}{x} - \frac{1}{y} = 0$

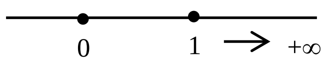
$\Rightarrow \qquad (x-y) + \frac{y-x}{xy} = 0$

$\Rightarrow \qquad (x-y)\frac{[xy-1]}{xy} = 0$

$\Rightarrow \qquad x = y$ and $xy - 1 \neq 0$

$\Rightarrow \qquad x^2 - 1 \neq 0 \;\Rightarrow\; x \neq \pm 1$

But $x \in (0, \infty)$



Hence option (B) must-not be Injective

For C : $h(x) = x^2 + 4x - 5$; $x \in (0, \infty)$

If $h(x) = h(y) \Rightarrow x^2 + 4x - 5 = y^2 + 4y - 5$

$\Rightarrow (x-y)(x+y) + (x-y).4 = 0$

$\Rightarrow (x-y)(x+y+4) = 0$

$\Rightarrow x = y$ and $x + y + 4 \neq 0$

$\Rightarrow x \neq -2$

But $x \in (0, \infty)$

$\therefore$ Option (C) must be Injective

For D : $k(x) = e^{-x}$; $x \in [0, \infty)$

If $k(x) = k(y) \Rightarrow e^{-x} = e^{-y}$

$\Rightarrow -x = -y$

$\Rightarrow x = y \quad$ for $x \in [0, \infty)$

$\therefore$ Option (D) is Injective

$\therefore$ Correct option is (B)

Q.9 Let f be an injective map. with domain {x, y, z} and range {1, 2, 3}, such that exactly one of the following statements is correct and the remaining are false : $f(x) = 1$, $f(y) \neq 1$, $f(z) \neq 2$

The value of $f^{-1}(1)$ is -

(A) x (B) y

(C) z (D) None of these

Sol. **[B]**

From given statement, following combinations may be generated as –

$f(x) = 1, f(y) = 1, f(z) = 2$ one-one not exist

$f(y) = 2, f(z) = 2, f(x) = 3$ one-one not exist

$f(y) = 3, f(z) = 2, f(x) = 3$ one-one not exist

$f(z) = 1, f(x) = 2, f(y) = 1$ one-one not exist

$f(z) = 3, f(y) = 1, f(x) = 2$ one-one mapping

Hence, $f^{-1}(1) = y$

$\therefore$ Option (B) is correct Answer.

Q.10 Let $f : R \to R$ and $g : R \to R$ be two one-one onto functions such that they are mirror image of each other about the line $y = 0$, then $h(x) = f(x) + g(x)$ is-

(A) one-one and onto

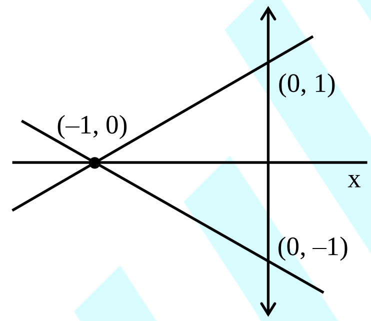
(B) one-one but not onto

(C) not one-one but onto

(D) Neither one-one nor onto

Sol. **[D]**

We have to choose two functions which are Bijective and mirror image about y = 0 i.e. about x-axis.



Let : $f : R \to R$ such that $-x + y = 1$

i.e. $y = x + 1$

$f(x) = y = x + 1$ is one-one as well as onto.

Also $g : R \to R$ such that $-x - y = 1$

i.e. $y = -x - 1$

$g(x) = y = -x - 1$ is one-one as well as onto.

$f(x) = x + 1$ and $g(x) = -x - 1$ are image about line $y = 0$

From given condition, $h(x) = f(x) + g(x)$

$h(x) = x + 1 - x - 1 = 0$

$h(x) = 0$

which shows that h(x) is neither one-one nor onto

$\therefore$ Option (D) is correct Answer.

Q.11 Which of the following functions is inverse of itself -

(A) $f(x) = \frac{1-x}{1+x}$ (B) $g(x) = 5^{\log x}$

(C) $h(x) = 2^{x(x-1)}$ (D) None of these

Sol. **[A]**

We have to check for every options as :

For A : $y = \frac{1-x}{1+x} \Rightarrow 1 - x = y + yx$

$\Rightarrow 1 - y = x(1+y)$

$\Rightarrow x = \frac{1-y}{1+y}$

which shows functions is inverse of itself.

$\therefore$ Option (A) is correct Answer.

For B : $y = 5^{\log x} \Rightarrow \log y = \log x \log 5$

$\Rightarrow \log x = \log_5 y$

$\Rightarrow x = e^{\log_5 y}$

which is not inverse of itself.

Similarly for C; $y = 2^{x(x-1)}$

$\log y = x(x-1)\log 2$

$\log_2 y = x(x-1)$

which certainly not show inverse of itself

$\therefore$ Only option (A) is correct Answer.

Q.12 Period of $f(x) = e^{\cos\{x\}} + \sin \pi [x]$ is (where, [.] and { } denote the greatest integer function and fractional part of function respectively).

(A) 1 (B) 2

(C) π (D) 2π

Sol. **[A]**

$f(x) = e^{\cos\{x\}} + \sin\pi [x]$

period of $\{x\} = x - [x]$ is 1

Also Period of $\sin\pi [x]$ is $\frac{\pi}{|\pi|} = 1$

L.C.M of (1, 1) is 1

$\therefore$ Option (A) is correct Answer.

Q.13 If $f(x) = \cos(ax) + \sin(bx)$ is periodic, then which of the followings is false -

(A) a and b both are rational

(B) non-periodic if a is rational but b is irrational

(C) non-periodic if a is irrational but b is rational

(D) none of these

Sol. **[D]**

$f(x) = \cos(ax) + \sin bx$

$\cos ax$ is periodic with period $= 2\pi/a$

$\sin bx$ is periodic with period $= 2\pi/b$

$\frac{1}{2}$ L.C.M of $\left[\frac{2\pi}{a}, \frac{2\pi}{b}\right]$

Option (A), (B), (C) are correct

$\therefore$ Option (D) is correct Answer.

Q.14 The function

$f(x) = 2(x - [x]) + \sin^2\pi (x - [x])$ is -

(Where [.] denotes greatest integer function)

(A) Non periodic

(B) periodic with period 1

(C) periodic with period 2

(D) None of these

Sol. **[B]**

$f(x) = 2(x - [x] + \sin^2\pi(x - [x])$

$x - [x]$ is periodic with period 1

$\sin^2\pi (x-[x])$ is periodic with period 1

$f(x)$ is periodic with period 1

$\therefore$ Option (B) is correct Answer.

Q.15 If f: [–20, 20] $\to$ R is defined by

$f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$, is an even function,

then the set of values of a is-

(A) $(-\infty, 100)$ (B) $(400, \infty)$

(C) $(-400, 400)$ (D) None of these

Sol. **[B]**

$f : [-20, 20] \to R$

$f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$

since $f(-x) = f(x)$

$\left[\frac{x^2}{a}\right](-\sin x) + \cos x = \left[\frac{x^2}{a}\right] \sin x + \cos x$

or $\left[\frac{x^2}{a}\right] \sin x = 0$

$\left[\frac{x^2}{a}\right] \sin x = 0$

$\sin x = 0$ But $\left[\frac{x^2}{a}\right] \neq 0$ or $\left[\frac{x^2}{a}\right] = 0$

If $\left[\frac{x^2}{a}\right] = 0$. It means $\frac{x^2}{a}$ must be +ve fractional number less than unity

i.e. $a \to$ value greater than x

If $\left[\frac{x^2}{a}\right] \neq 0$. It means, value of $\frac{x^2}{a}$ is 1

$\therefore a \in (400, \infty)$

$\therefore$ Option (B) is correct Answer.

Q.16 Let f be a function satisfying $f(x + y) = f(x).f(y)$ for all $x, y \in R$. If $f(1) = 3$ then $\sum_{r=1}^{n} f(r)$ is equal to -

(A) $\frac{3}{2}(3^n - 1)$ (B) $\frac{3}{2} n(n + 1)$

(C) $3^{n+1} - 3$ (D) None of these

Sol. **[A]**

Given $f(x+y) = f(x).f(y)$ for all $x, y \in R$ and $f(1) = 3$

then $\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + \ldots\ldots\ldots + f(n)$

For x = y = 1; f(2) = f(1). f(1) = 3.3 = 9
For x = 1, y = 2; f(1+2)= f(3) = f(1).f(2) = 3.9 = 27
For x = 2, y = 2; f(2 + 2) = f(4) = f(2).f(2) = 9.9 = 81

Then $\sum_{r=1}^{n} f(r)$ = f(1) + f(2) + f(3) + f(4) + ... n terms

= 3 + 9 + 27 + 81 + ... n terms
= 3[1 + 3 + 9 + 27 + ... n terms]
= 3[A G.P. with common ratio 3 and first term 1]

$= 3.\dfrac{1(3^n-1)}{3-1} = \dfrac{3}{2}(3^n-1)$

∴ Option (A) is correct Answer.

Q.17 If f(θ) =

$$\frac{(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1).....(2\cos 2^{n-1}\theta-1)}{2\cos 2^n\theta+1}$$

for n ∈ N and θ ≠ $2m\pi \pm \dfrac{2\pi}{3}$, m ∈ I,

then f(π/4) =

(A) $1-\sqrt{2}$ (B) $\sqrt{2}-1$

(C) $\sqrt{2}+1$ (D) None of these

Sol. **[B]**

Given

f(θ) =

$$\frac{(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1).....(2\cos 2^{n-1}\theta-1)}{(2\cos 2^n\theta+1)}$$

For n ∈ N and θ ≠ 2mπ ± 2π/3, m ∈ I.

we have to find f(π/4) = ?

f(θ)=

$$\frac{(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1).....(2\cos 2^{n-1}\theta-1)}{1+2\cos 2\theta+2\cos 4\theta+2\cos 8\theta+2\cos 16\theta+........+...n\text{ terms}}$$

$$=\frac{(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1).....(2\cos 2^{n-1}\theta-1)}{1+2[\cos 2\theta+\cos 4\theta+\cos 6\theta+\cos 8\theta+........]}$$

f(π/4) =

$$\frac{(2\cos\pi/4-1)(2\cos\pi/2-1)(2\cos\pi-1)(2\cos 2\pi-1)(2\cos 4\pi-1)(2\cos 8\pi-1).....}{1+2\cos\pi/2+2\cos\pi+2\cos 2\pi+2\cos 4\pi+........}$$

$$=\frac{(\sqrt{2}-1)(-1)(-3)(1)(1)(1)........}{1+0+2(-1)+2+2+..........}$$

$$f(\pi/4)=\frac{(\sqrt{2}-1)3}{1+0-2+2+2}=\frac{3(\sqrt{2}-1)}{1+2(n-3)}$$

$$=\frac{3(\sqrt{2}-1)}{2n-5}$$

f(π/4) = 3($\sqrt{2}$ – 1) / (2n –5), n ∈ Natural number

Q.18 If f(x) = $[x^2]-[x]^2$ where [·] denotes the greatest integer function and x ∈ [0, 2], the set of values of f(x) is -

(A) {–1, 0} (B) {–1, 0, 1}

(C) {0} (D) {0, 1, 2}

Sol. **[D]**

f(x) = $[x^2]-[x]^2$; x ∈ [0, 2]

We can explain this example with general case as.

Let f(x) = $[x^2]-[x]^2$; x ∈ [0, n], n ∈ N

For n – 1 ≤ x ≤ n

[x] = n – 1 and x = n ⇒ $x^2 = n^2$

⇒ $[x^2] = n^2-1$

∴ f(x) = $[x^2]-[x]^2$

$= n^2-1-(n-1)^2$

$= n^2-1-n^2-1+2n$

= 2n – 2

i.e. $f(x) = \underbrace{0, 2, 4, 6, ... 2n-2}_{\text{Total elements }(2n-1)}$

Similarly, f(x) = 2 (For our Problem)

i.e., f(x) will have 3 elements

But x ∈ [0, 2]

Q.19 Domain of $y=\sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\dfrac{2x-1}{3+x}\right)}$

(A) (–4, – 3) ∪ (4, ∞) (B) (–∞, –3) ∪ (4, ∞)

(C) (–∞, –4) ∪ (3, ∞) (D) (–4, – 3) ∪ (3, 4)

Sol. **[A]**

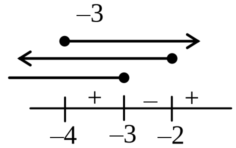
$$y=\sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)}$$

(i) $0<\dfrac{x+4}{2}<1 \Rightarrow \log_2\dfrac{2x-1}{3+x}\geq 1$

–4 < x < –2 $\dfrac{2x-1}{3+x}\geq 2$

$\dfrac{2x-1}{3+x}-2\geq 0 \Rightarrow \dfrac{-7}{3+x}\geq 0$

+ | – (at –3) x ∈ (–∞,–3)



∴ x ∈ (–4, –3)

(ii) $\dfrac{x+4}{2}>1 \Rightarrow 0<\log_2\left(\dfrac{2x-1}{x+3}\right)\leq 1$

x > –2 $\Rightarrow 1<\dfrac{2x-1}{3+x}\leq 2$

$$\frac{2x-1}{3+x}-2\le 0 \ \& \ \frac{2x-1}{3+x}-1>0$$
$$\Downarrow$$

$\frac{-7}{3+x}\le 0$ $\quad$ $x\in(-\infty,-3)\cup(4,\infty)$ …(iii)

$\frac{7}{3+x}\ge 0 \longrightarrow x\in(-3,\infty)\to$ (iv)

(iv) $\cap$ iii) $\Rightarrow x\in(4,\infty)$

$\therefore$ Final (i $\cup$ ii)

$\Rightarrow(-4,-3)\cup(4,\infty)$

Part-B **One or more than one correct answer type questions**

Q.20 If $f(x)=\sqrt{x^2-|x|}$, $g(x)=\frac{1}{\sqrt{9-x^2}}$ then D_{f+g} contains –

(A) $(-3,-1)$ $\quad$ (B) $[1, 3)$

(C) $[-3, 3]$ $\quad$ (D) $\{0\}\cup[1, 3)$

Sol. **[A, B, D]**

$f(x)=\sqrt{x^2-|x|}$, $g(x)=\frac{1}{\sqrt{9-x^2}}$

f(x) to be defined if

$x^2-|x|\ge 0\Rightarrow x^2\ge|x|$

$\Rightarrow x^2-x\ge 0$

$\Rightarrow x(x-1)\ge 0$

+ve $\quad$ 0 $\quad$ –ve $\quad$ 1 $\quad$ +ve

i.e. $x\le 0$ or $x\ge 1$

g(x) to be defined if $9-x^2>0$ and $9-x^2\ne 0$

$\Rightarrow x^2<9$ and $x^2\ne 9$

$\Rightarrow -3<x<3$ and $x\ne\pm 3$

–3 $\quad$ 0 $\quad$ 1 $\quad$ 3

$-3<x\le 0$ $\quad$ $1\le x<3$

Required Domain $\in(-3, 0]\cup[1, 3)$

$\therefore$ Option (A), (B) and (D) are correct answers.

Q.21 If $f(x)=\frac{3x-1}{3x^3+2x^2-x}$ and $S=\{x|\ f(x)>0\}$ then S contains –

(A) $(-\infty,-2)$ $\quad$ (B) $\left(\frac{1}{3},5\right)$

(C) $(-\infty,-1)$ $\quad$ (D) $(0,\infty)-\left\{\frac{1}{3}\right\}$

Sol. **[A,B,C,D]**

$f(x)=\frac{3x-1}{3x^3+2x^2-x}$ and $S=\{x|f(x)>0\}$

$f(x)=\frac{3x-1}{3x^3+2x^2-x}>0$

i.e. $\frac{3x-1}{x(3x^2+2x-1)}>0$

i.e. $\frac{3x-1}{x(3x-1)(x+1)}>0$

Either $(3x-1)<0$ and $x(3x-1)(x+1)<0$

or $(3x-1)>0$ and $x(3x-1)(x+1)>0$

Case I : When $(3x-1)<0$ and $x(3x-1)(x+1)<0$

–1 $\quad$ 0 $\quad$ 1/3

or $x<1/3$ and $x(3x-1)(x+1)<0$

For $0<x<1/3$, $f(x)>0$

For $-1<x<0$, $f(x)<0$

For $x<-1$, $f(x)>0$

$x\in(-\infty,-1)\cup(0, 1/3)$

Case II : When $(3x-1)>0$ and $x(3x-1)(x+1)>0$

i.e. $x>1/3$ and $x(3x-1)(x+1)>0$

$x\in\left(\frac{1}{3},\infty\right)$

Required Domain $\in(-\infty,-1)\cup(0,\frac{1}{3})\cup(\frac{1}{3},\infty)$

$\therefore$ Options (A), (B), (C), (D) are correct answers

Q.22 If D is the domain of the function $f(x)=\sqrt{1-2x}+3\sin^{-1}\left(\frac{3x-1}{2}\right)$ then D contains-

(A) $\left[-\frac{1}{3},\frac{1}{2}\right]$ $\quad$ (B) $\left[-\frac{1}{3},0\right]$

(C) $\left[-\frac{1}{3},1\right]$ $\quad$ (D) $\left[\frac{1}{2},1\right]$

Sol. **[A,B]**

$f(x)=\sqrt{1-2x}+3\sin^{-1}\left(\frac{3x-1}{2}\right)$

f(x) to be defined if

$1-2x\ge 0$ and $-1\le\left(\frac{3x-1}{2}\right)\le 1$

or $x\le 1/2$ and $-2\le 3x-1\le 2$

or $x\le 1/2$ and $-1\le 3x\le 3$

or $x\le 1/2$ and $-1/3\le x\le 1$

–1/3 $\quad$ 1/2 $\quad$ 1

$x\in\left[\frac{-1}{3},\frac{1}{2}\right]$

$\therefore$ Option (A) and (B) are correct answers.

Q.23 Let A = R – {2} and B = R – {1}.

Let $f : A \to B$ be defined by $f(x) = \dfrac{x-3}{x-2}$

then–

(A) f is one-one (B) f is onto
(C) f is bijective (D) None of these

Sol. **[A,B,C]**

Given A = R – {2}
B = R – {1}

$f : A \to B$; $f(x) = \dfrac{x-3}{x-2}$

For one-one : f(x) = f(y)

$$\Rightarrow \frac{x-3}{x-2} = \frac{y-3}{y-2}$$

$\Rightarrow (x-3)(y-2) = (y-3)(x-2)$

$\Rightarrow xy - 3y - 2x + 6 = xy - 3x - 2y + 6$

$\Rightarrow x = y$

i.e. f(x) is one-one mapping

For onto : $y = \dfrac{x-3}{x-2} \Rightarrow x - 3 = yx - 2y$

$\Rightarrow 2y - 3 = x(y-1)$

$\Rightarrow x = \dfrac{2y-3}{y-1}$

$$f(x) = \frac{x-3}{x-2} = \frac{\frac{2y-3}{y-1}-3}{\frac{2y-3}{y-1}-2} = \frac{2y-3-3y+3}{2y-3-2y+2}$$

f(x) = y

i.e, f(x) is onto mapping

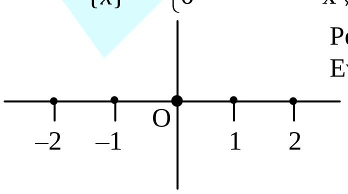
$\therefore$ Option (A), (B) and (C) are correct Answer.

Q.24 If $F(x) = \dfrac{\sin \pi[x]}{\{x\}}$, then F(x) is:

(A) Periodic with fundamental period 1
(B) even
(C) range is singleton
(D) identical to $\text{sgn}\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where {x} denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function.

Sol. **[A,B,C,D]**

$$f(x) = \frac{0}{\{x\}} = \begin{cases} \text{not defined} & x \in I \\ 0 & x \notin I \end{cases}$$



Period =1
Even function

$$g(x) = \text{sgn}\left(\underbrace{\underbrace{\text{signum}\frac{\{x\}}{\sqrt{\{x\}}}}_{1}}_{1}\right) - 1$$

= 0 $\to$ same domain & range

Q.25 Let f: [–1, 1] $\to$ [0, 2] be a linear function which is onto then f(x) is/are.

(A) 1 – x (B) 1 + x
(C) x –1 (D) x +2

Sol. **[A, B]**

$f : [-1, 1] \longrightarrow [0, 2]$

$f(x) = ax+b \longrightarrow$ onto $\therefore$ range = codomain

checking options

Q.26 Function $f(x) = \sin x + \tan x + \text{sgn}(x^2 - 6x + 10)$ is

(A) periodic with period 2π
(B) periodic with period π
(C) non-periodic
(D) periodic with period 4π

Sol. **[A]**

$$f(x) = \underset{\downarrow}{\sin x} + \underset{\downarrow}{\tan x} + \underset{\downarrow}{\text{sgn}(x^2 - 6x + 10)}$$

L.C.M.: T = 2π 2π, 2π

Q.27 In the following functions defined from [–1, 1] to [–1, 1] the functions which are not bijective are:

(A) $\sin(\sin^{-1}x)$ (B) $\dfrac{2}{\pi}\sin^{-1}(\sin x)$
(C) $(\text{sgn } x)\, \ell n\, e^x$ (D) x^3 sgn x

Sol. **[B,C,D]**

$f : [-1, 1] \longrightarrow [-1, 1]$

Checking equations

Part-C Assertion-Reason type Questions

The following questions 28 to 31 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true

Q.28 **Assertion** : The domain of the function $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is [–1, 1]

Reason : $\sin^{-1}x$, $\cos^{-1}x$ are defined for

$|x| \le 1$ and $\tan^{-1} x$ is defined for all x.

Sol. **[A]**

Assertion : The domain of $\sin^{-1}x$ is $[-1, 1]$

Domain of $\cos^{-1}x$ is $[-1, 1]$

Domain of $\tan^{-1}x$, $x \in R$

Hence, domain of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is $[-1, 1]$

Assertion true.

Reason : $\sin^{-1}x$, $\cos^{-1}x$ are defined for $-1 \le x \le 1$ and $\tan^{-1}x$ is defined for all $x \in R$

Reason is true and correct explanation of Assertion

$\therefore$ Option (A) is correct answer.

Q.29 **Assertion** : Function $f(x) = \sin x + \{x\}$ is periodic with period 2π.

Reason : $\sin x$ and $\{x\}$ are both periodic functions with period 2π and 1 respectively.

Sol. **[D]**

Assertion : $f(x) = \sin x + \{x\}$

sinx is periodic with period 2π

$\{x\}$ is periodic with period 1

LCM of $(2\pi, 1)$ not possible

$\therefore$ f(x) is non periodic

$\therefore$ Assertion is false.

Reason : Reason is true.

$\therefore$ Option (D) is correct Answer.

Q.30 **Assertion:** If f(x) & g(x) both are one-one, then f (g (x)) is also one-one.

Reason : If, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ then f(x) is one-one

Sol. **[A]**

Q.31 **Assertion:** Let f: $[0, 3] \to [1, 13]$ is defined by $f(x) = x^2 + x + 1$ then inverse is

$$f^{-1}(x) = \frac{-1+\sqrt{4x-3}}{2}$$

Reason: Many-one function is not invertible

Sol. **[B]**

Part-D Column Matching type questions

Match the entry in Column 1 with the entry in Column 2.

Q.32

Column 1	**Column 2**
(A) Range of $\sqrt{[\sin 2x]-[\cos 2x]}$	(P) {1, 2, 3}
(B) Domain of $\sqrt{{}^{x^2+4x}C_{2x^2+3}}$	(Q) {1}
(C) Range of $\sqrt{\log(\cos(\sin x))}$	(R) {0, 1}
(D) Range of $[\lvert\sin x\rvert + \lvert\cos x\rvert]$ [where [ ] denotes G.I.F]	(S) {0}

Sol. **$A \to R$, $B \to P$, $C \to S$, $D \to Q$**

$y = \sqrt{[\sin 2x]-[\cos 2x]}$

$[\sin 2x] - [\cos 2x] = y^2$

$-1 \le \sin 2x \le 1$ and $-1 \le \cos 2x \le 1$

$0 \le [\sin 2x] \le 1$ and $0 \le [\cos 2x] \le 1$

$0 \le [\sin 2x] - [\cos 2x] \le 1$

$0 \le y^2 \le 1 \Rightarrow$ ..(i) $\to$ (C)

$-1 \le y \le 1$

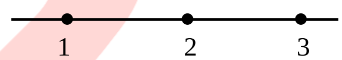
$f(x) = {}^{x^2+4x}C_{2x^2+3}$

$x^2 + 4x \ge 2x^2 + 3$ and $2x^2 + 3 > 0$

$-x^2 + 4x - 3 \ge 0$

or $x^2 - 4x + 3 \le 0$

or $(x - 1)(x - 3) \le 0$



$1 \le x \le 3$

$y = \sqrt{\log(\cos(\sin x))} \Rightarrow \log(\cos(\sin x)) = y^2$

$\cos(\sin x) = e^{y^2}$

$\sin x = \cos^{-1} e^{y^2}$

$x = \sin^{-1}(\cos^{-1}(e^{y^2}))$

If x to be defined, $-1 \le \cos^{-1} e^{y^2} \le 1$

$\Rightarrow \cos(-1) \le e^{y^2} \le \cos 1$

$\Rightarrow \cos 1 \le e^{y^2} \le \cos 1$

That means only possibility is

$e^{y^2} = \cos 1$

$y^2 = \log_e \cos 1 = -ve$

y is imginary

If we take y = 0

Then x = 0

Hence Range has only element {0}

$y = [|\sin x| + |\cos x|]$

Since, $|\sin x| \le 1$ and $|\cos x| \le 1$

$|\sin x| + |\cos x| \le 1$

$y = 1$

Q.33 **Column 1** **Column 2**

Column 1	Column 2
(A) Period of $\frac{1}{2}\left\{\frac{\lvert\sin x\rvert}{\cos x}+\frac{\lvert\cos x\rvert}{\sin x}\right\}$	(P) 2
(B) Range of $\cos^{-1}\sqrt{\log_{[x]}\frac{\lvert x\rvert}{x}}$	(Q) 2π
(C) Total number of solution $x^2 - 4 - [x] = 0$	(R) 1
(D) Period of $e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$	(S) $\frac{\pi}{2}$

[where [ ] denotes G.I.F]

Sol. **A → Q, B → S, C → P, D → R**

$y = \cos^{-1}\sqrt{\log_{[x]}\frac{|x|}{x}}$

$\sqrt{\log_{[x]}\frac{|x|}{x}} = \cos y$

$\log_{[x]}\frac{|x|}{x} = \cos^2 y$

$\frac{|x|}{x} = \begin{cases} 1; & x \ge 0 \\ -1; & x < 0 \end{cases}$

$\log_{[x]}^{1} = \cos^2 y = 0$

$y = \pi/2$

$\therefore$ (B) is matched with (S)

Period of $e^{\cos^4 \pi x} + x - [x] + \cos^2 \pi x$

Period of $\cos^4 \pi x$ is 1

Period of $x - [x]$ is 1

Period of $\cos^2 \pi x$ is 1

L.C.M of (1, 1, 1) is 1

$\therefore$ (D) is matched (R)

Total number of solution will be (2)

Because $[x] \le x$

$x^2 - x - 4 = 0$

$\therefore$ (C) is matched with (P)

Obviously (A) will be matched with (Q) Hence, required combinations as :

Q.34 **Column 1** **Column 2**

Column 1	Column 2
(A) Domain of $f(x) = \sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$ is	(P) [0, 1]
(B) Solution set of equation $2\cos^2 x/2 . \sin^2 x = x^2 + 1/x^2$ is	(Q) $[0, \infty)$
(C) If $A = \{(x, y);\ y = 1/x, x \in R_0\}$ & $B = \{(x, y); y = -x, x \in R\}$ then $A \cap B$ is	(R) $[1, \infty)$
(D) The functions $f(x) = \sqrt{x}\ \sqrt{x-1}$ & $\phi(x) = \sqrt{x^2 - x}$ are identical in	(S) ϕ

Sol. **A → S, B – S, C → S, D – R**

(A) $f(x) = \sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$

f(x) to be defined if

$2^x - 3^x \ge 0$ and $x \le 0$

or $2^x \ge 3^x$ and $x \le 0$

or $x(\log 2 - \log 3) \ge 0$ and $x \le 0$

$\Rightarrow x \le 0 \Rightarrow x \in (-\infty, 0]$

Also $\log_{1/2} x > 0$ and $x > 0$

i.e., $x > 1$ and $x > 0$

$x \in (1, \infty)$

$\therefore$ Domain would be empty set.

(B) $2\cos^2\frac{x}{2}.\sin^2 x = x^2 + \frac{1}{x^2}$

From the Property of A.M ≥ G.M.

$$\frac{x^2 + \frac{1}{x^2}}{2} \ge \sqrt{x^2 . \frac{1}{x^2}}$$

$\Rightarrow x^2 + \frac{1}{x^2} \ge 2$

$\Rightarrow 2\cos^2\frac{x}{2}.\sin^2 x \ge 2$

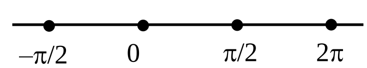
$\Rightarrow \cos^2\frac{x}{2}.\sin^2 x \ge 1$

since $|\sin x| \le 1 \Rightarrow \sin^2 x \le 1$

$\Rightarrow x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -1 \le \cos\frac{x}{2} \le 1$

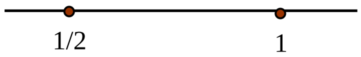
$\Rightarrow 0 \le \frac{x}{2} \le \pi \Rightarrow x \in [0, 2\pi]$



Common interval is $0 \le x \le \pi/2$

$0 \le x \le \pi/2 \Rightarrow \sin^2 x \le 1$

$0 \le x/2 \le \pi/4 \Rightarrow \frac{1}{2} \le \cos^2\frac{x}{2} \le 1$



$\cos^2 x/2.\ \sin^2 x \geq 1$ Not Possible

$\therefore$ Solution set is empty

(C) If $A = \{\{x, y\}\ ;\ y = 1/x,\ x \in R_0\}$ and

$B = \{(x, y);\ y = -x,\ x \in R\}$

$A \cap B$ is $x \in R - \{0\}$

(D) $f(x) = \sqrt{x}\ \ \sqrt{x-1}$

$\phi(x) = \sqrt{x^2 - x}$

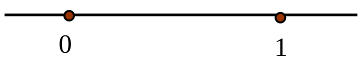
If f(x) and ϕ(x) are identical then

Domain of f = Domain of ϕ

and Range of f = Range ϕ

f(x) to be defined if $x \geq 0$ and $(x - 1) \geq 0$

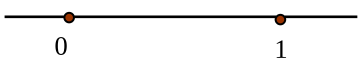
i.e. $x \geq 0$ and $x \geq 1$



$x \in [1, \infty)$

$\phi(x)$ to be defined if $x^2 - x \geq 0$

$\Rightarrow\ x(x - 1) \geq 0$



$\Rightarrow x \leq 0$ or $x \geq 1$

$\Rightarrow$ Domain of $\phi \in [1, \infty)$

Range : $y^2 = x(x - 1) \Rightarrow x^2 - x - y^2 = 0$

$$\Rightarrow\ x = \frac{1 \pm \sqrt{1 + 4y^2}}{2}$$

$\Rightarrow$ Range $\in R$

Q .35 **Column 1** **Column 2**

	Column 1	Column 2
(A)	The period of the function $y = \sin(2\pi t + \pi/3) + 2\sin(3\pi t + \pi/4) + 3\sin 5\pi t$	(P) 1/2
(B)	$y = \{\sin(\pi x)\}$ is a many one function for $x \in (0, a)$ where {} denotes fractional part of x and a may be	(Q) 8
(C)	The Fundamental period of the function $y = \frac{1}{2}\left(\frac{\lvert\sin(\pi/4)x\rvert}{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{\lvert\cos(\pi/4)x\rvert}\right)$	(R) 2
(D)	If $f : [0, 2] \to [0, 2]$ is bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non - zero real numbers, then f(2) is equal to	(S) 0

Sol. **A $\to$ R, B – Q,R, C $\to$ Q, D – S**

(A) $\sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin(3\pi t + \pi/4) + 3\sin 5\pi t.$

$$T_1 = \frac{2\pi}{2\pi},\ T_2 = \frac{2\pi}{3\pi},\ T_3 = \frac{2\pi}{3\pi}$$

$$\text{L.C.M.} = \frac{2}{1} = 2$$

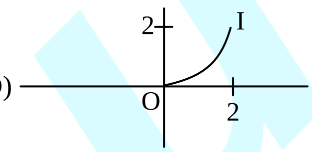
(B) $y = \{\sin(\pi x)\}$, $x \in (0, a)$

(C) $y = \frac{1}{2}\left(\frac{\lvert\sin(\pi/4)x\rvert}{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{\lvert\cos(\pi/4)x\rvert}\right)$

$$T_1 = \frac{\pi}{4} \times 4,\ T_2 = \frac{2\pi}{\pi}, 4$$

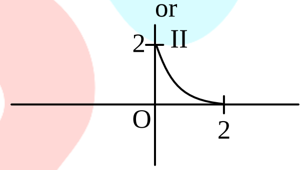
L.C.M. = 8

(D)



here f(0) = 0

or



f (0) = 2

but, $f(x) = ax^2 + bx + c \Rightarrow f(0) = c$

but $c \neq 0 \Rightarrow$ graph II

$\therefore\ f(0) = 2 \Rightarrow f(2) = 0$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Find the domains of definitions of the following functions:
(Read the symbols [*] and {*} as greatest integers and fractional part functions respectively)

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7\log_5\log_3\log_2 (2x^3 + 5x^2 - 14x)$

(iii) $f(x) = \lambda n \left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

(iv) $f(x) = \sqrt{\dfrac{1-5^x}{7^{-x} - 7}}$

(v) $y = \log_{10} \sin (x - 3) + \sqrt{16 - x^2}$

(vi) $f(x) = \log_{100x}\left(\dfrac{2\log_{10} x + 1}{-x}\right)$

(vii) $f(x) = \dfrac{1}{\sqrt{4x^2 - 1}} + \lambda n\, x\, (x^2 - 1)$

(viii) $f(x) = \sqrt{\log_{1/2} \dfrac{x}{x^2 - 1}}$

(ix) $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9 - x^2}}$

(x) $f(x) = \sqrt{\log_x (\cos 2\pi x)}$

(xi) $f(x) = \dfrac{\sqrt{\cos x - (1/2)}}{\sqrt{6 + 35x - 6x^2}}$

(xii) $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$

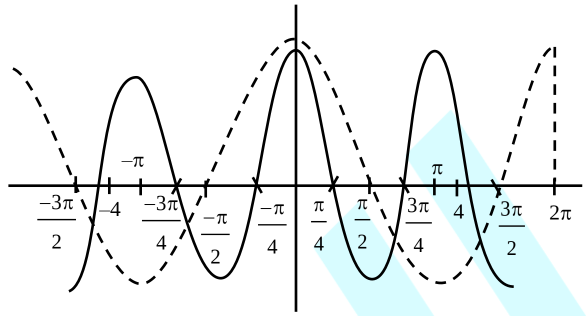
(xiii) $f(x) = \dfrac{[x]}{2x - [x]}$

(xiv) $f(x) = \log_x \sin x$

(xv) $f(x) = \log_{\left[x + \frac{1}{x}\right]} |x^2 - x - 6| + {}^{16-x}C_{2x-1} + {}^{20-3x}P_{2x-5}$

Sol. (i) $\sqrt{\cos 2x} + \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow -4 \leq x \leq 4$



$\cos 2x \geq 0$

$2x \in \left[2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{2}\right]$

$x \in \left[n\pi - \dfrac{\pi}{4}, n\pi + \dfrac{\pi}{4}\right]$

$\cos 2x \geq 0 \quad \& \quad -4 \leq x \leq 4$

$\Rightarrow x \in \left[-4, \dfrac{-3\pi}{4}\right] \cup \left[\dfrac{-\pi}{4}, \dfrac{\pi}{4}\right] \cup \left[\dfrac{3\pi}{4}, 4\right]$

(ii) $\log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 0$

$\log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$\log_2 (2x^3 + 5x^2 - 14x) > 3$

$(2x^3 + 5x^2 - 14x) > 8$

$(x - 2)(2x^2 + 9x + 4) > 0$

$(x - 2)(2x^2 + 8x + x + 4) > 0$

$(x - 2)(2x + 1)(x + 4) > 0$

$\Rightarrow x \in [2, \infty) \cup \left(-4, \dfrac{-1}{2}\right)$

at $x = 2$

$2x^3 + 5x^2 - 14x - 8 = 0$

$$\Rightarrow \begin{array}{r|l} & 2x^2 + 9x + 4 \\ x - 2 & 2x^3 + 5x^2 - 14x - 8 \\ & 2x^3 - 4x^2 \\ & - \quad + \\ \hline & 9x^2 - 14x - 8 \\ & 9x^2 - 18x \\ \hline & 4x - 8 \\ & 4x - 8 \\ \hline & 0 \end{array}$$

(iii) $\sqrt{x^2-5x-24}-x-2>0$

$\downarrow$

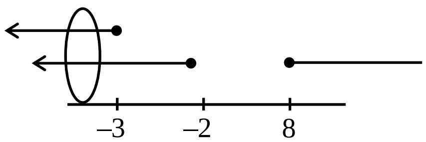
$x^2-5x-24 \geq 0 \Rightarrow (x-8)(x+3) \geq 0$

$\Rightarrow x \in (-\infty, -3] \cup [8, \infty) \rightarrow$ (i)

II $x \in (-\infty, -3]$

if $\sqrt{x^2-5x-24} = 0 \Rightarrow -x-2>0$

$\Rightarrow x < -2$



$\Rightarrow x \in (-\infty, -3]$

(iv) $f(x)=\sqrt{\dfrac{7^x(1-5^x)}{1-7^{x+1}}}$

$(7^{x+1} \neq 1$; $x+1 \neq 0$; $x \neq -1)$

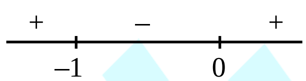
always +ve $\forall$ $x \in R$

$\dfrac{7^x(1-5^x)}{1-7^{x+1}} \geq 0 \Rightarrow \dfrac{1-5^x}{1-7^{x+1}} \geq 0$

(i) $1-5^x = 0 \Rightarrow 5^x = 1 \Rightarrow x = 0$

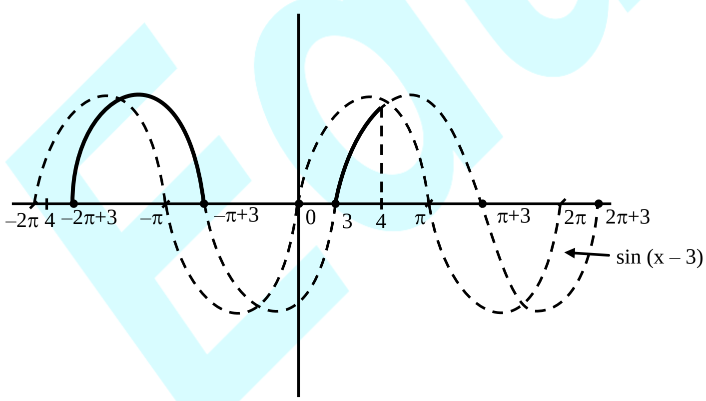
(ii) $1-7^{x+1} = 0 \Rightarrow 7^{x+1} = 1 \Rightarrow x+1 = 0$

$x = -1$



$\Rightarrow$ $x \in (-\infty, -1) \cup [0, \infty)$

(v) $16-x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow -4 \leq x \leq 4$



$\therefore$ $x \in (-2\pi+3, -\pi+3) \cup (3, 4]$

$(x-3) \in (2n\pi, (2n+1)\pi)$

$x \in (3+2n\pi, 3+(2n+1)\pi)$

(vi) $100x > 0$ & $100x \neq 1 \Rightarrow x > 0$ & $x \neq \dfrac{1}{100}$

$\dfrac{2\log_{10}x+1}{-x} > 0 \Rightarrow \dfrac{2\log_{10}x+1}{x} < 0$

x is always a +ve number

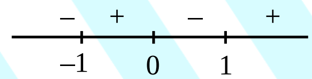
$\Rightarrow 2\log_{10}x < -1$

$\log_{10}x < 2^{-1} \Rightarrow x < \dfrac{1}{\sqrt{10}}$

$\therefore$ $x \in \left(0, \dfrac{1}{\sqrt{10}}\right) - \left\{\dfrac{1}{100}\right\}$

$\Rightarrow x \in \left(0, \dfrac{1}{100}\right) \cup \left(\dfrac{1}{100}, \dfrac{1}{\sqrt{10}}\right)$

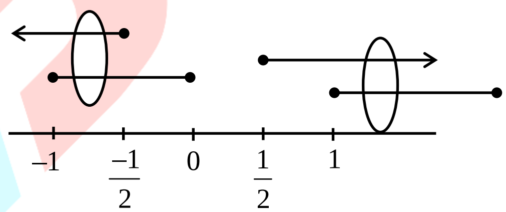
(vii) $x(x+1)(x-1) > 0$

$\Rightarrow$ 

$\Rightarrow$ $x \in (-1, 0) \cup (1, \infty)$

$4x^2-1 > 0 \Rightarrow 4x^2 > 1 \Rightarrow x^2 > \dfrac{1}{4}$

$\Rightarrow |x| > \dfrac{1}{2}$ $\Rightarrow x \in \left(-\infty, \dfrac{-1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$



$x \in \left(-1, \dfrac{-1}{2}\right) \cup (1, \infty)$

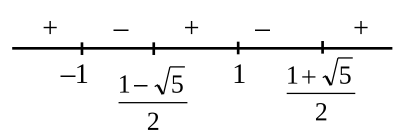
(viii) $\log_{1/2} \dfrac{x}{x^2-1} \geq 0$

since $\dfrac{1}{2} < 1 \Rightarrow \dfrac{x}{x^2-1} \leq 1$

$\dfrac{x-x^2+1}{x^2-1} \leq 0$

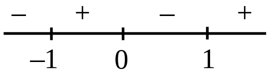
$\dfrac{x^2-x-1}{x^2-1} \geq 0$

$\dfrac{\left(x-\dfrac{1+\sqrt{5}}{2}\right)\left(x-\dfrac{1-\sqrt{5}}{2}\right)}{(x+1)(x-1)} \geq 0$



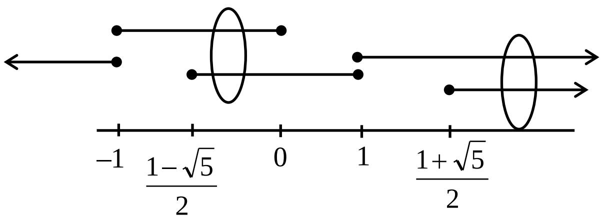
$$x \in (-\infty, -1) \cup \left[\frac{1-\sqrt{5}}{2}, 1\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right) \to \text{(i)}$$

also $\dfrac{x}{x^2-1} > 0$

$\Rightarrow$ 

$x \in (-1, 0) \cup (1, \infty) \to$ (ii)

(i) $\cap$ (ii)



$$\therefore\ x \in \left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$$

(ix) $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9-x^2}}$

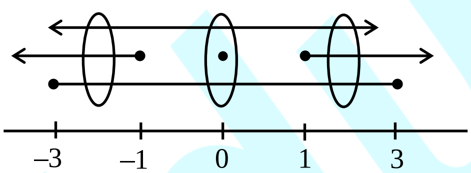
$9 - x^2 > 0 \Rightarrow x^2 < 9 \Rightarrow -3 < x < 3 \to$ (i)

$x^2 - |x| \geq 0 \Rightarrow |x|^2 - |x| \geq 0$

$\Rightarrow\ |x|\{|x| - 1\} \geq 0 \Rightarrow |x| \geq 0$

$\Rightarrow x = 0$ "OR" R

& $|x| - 1 \geq 0 \Rightarrow x \in (-\infty, -1] \cup [1, \infty)$



$\Rightarrow x \in [-3, -1] \cup \{0\} \cup [1, 3)$

(x) $f(x) = \sqrt{\log_x(\cos 2\pi x)}$

$\log_x \cos 2\pi x \geq 0$

If 0 < x < 1

(i) $\cos 2\pi x \leq 1$

$2\pi x \leq \cos^{-1}(1)$

$2\pi x \leq n\pi$

$x \leq \dfrac{n}{2}\ \ \forall\ n \in I$

No solution

If x > 1

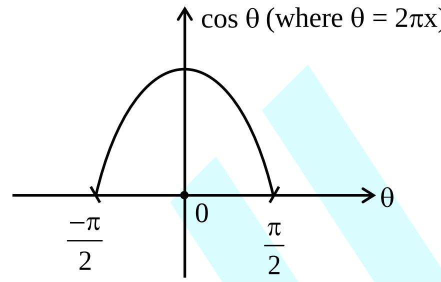
$\cos 2\pi x \geq 1 \Rightarrow\ 2\pi x \geq n\pi$

$\Rightarrow\ x \geq \dfrac{n}{2}\ \ \forall\ n \in I$

$\therefore\ x \in N$ where

$x \geq 2$

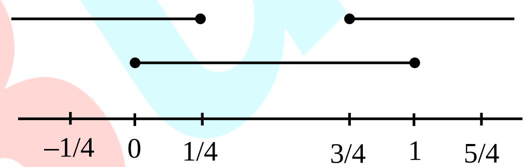
(ii) $0 < \cos 2\pi x$



$$\frac{-\pi}{2} < \theta < \frac{\pi}{2}$$

$$2n\pi - \frac{\pi}{2} < 2\pi x < \frac{\pi}{2} + 2n\pi$$

$$n - \frac{1}{4} < x < \frac{1}{4} + n$$



final

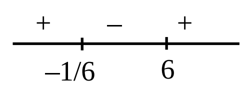
$$\therefore\ x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right) \cup [x : x \in N\ \forall\ n \geq 2]$$

(xi) $\cos x - \dfrac{1}{2} \geq 0 \Rightarrow \cos x \geq \dfrac{1}{2}$

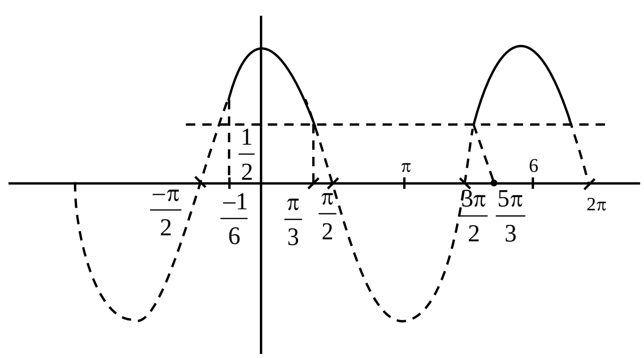
$6 + 35x - 6x^2 > 0$

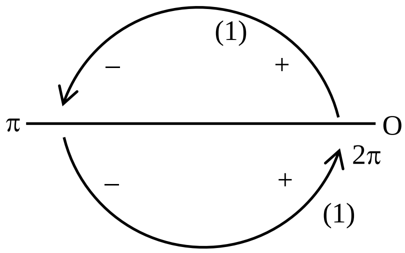
$6x^2 - 35x - 6 < 0$

$\left(x + \dfrac{1}{6}\right)(x - 6) < 0$

$\Rightarrow$ 

$x \in \left(\dfrac{-1}{6}, 6\right)$





$\Rightarrow x \in \left(\frac{-1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

(xii) $\log_{1/3} \log_4 [x]^2 - 5 \geq 0$

$0 < \log_4 [x]^2 - 5 \leq 1$

(I) $1 < [x]^2 - 5 \leq 4$

(II) $[x]^2 - 5$ {included in I}

from (I) $6 < [x]^2 \leq 9$

$\sqrt{6} < |[x]| \leq 3$

$[x] \in [-3, -\sqrt{6}) \cup (\sqrt{6}, 3]$

$-3 \leq x < -\sqrt{6}$ | $\sqrt{6} < x \leq 3$

$[x] = -3$ | $3 \leq x < 4$ ….(ii)

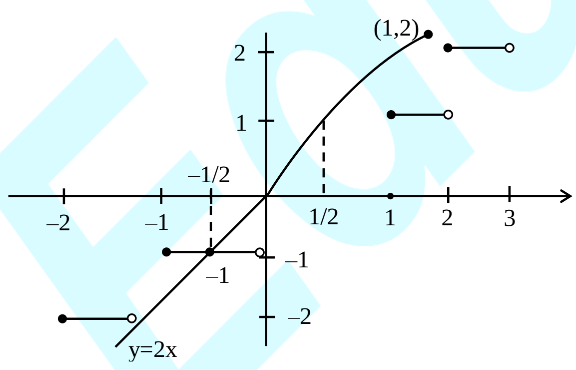
$-3 \leq x < -2$…(i)

(xiii) $f(x) = \frac{[x]}{2x - [x]}$

NOT defind when $2x - [x] = 0$

$\Rightarrow [x] = 2x$

graph intersects at $x = 0, -\frac{1}{2}$



$\Rightarrow$ Domain

$x \in R - \left\{\frac{-1}{2}, 0\right\}$

(xiv) $f(x) = \log_x \sin x$

$x > 0$ and $x \neq 1$

$\sin x > 0$

$2n\pi + 0 < x < \pi + 2n\pi$

$\therefore x \in (2n\pi, 2n\pi + \pi) - \{1\}$

where $n > 0$; $n \in I$

(xv) $\log_{\left[x+\frac{1}{x}\right]} |x^2 - x - 6| + {}^{16-x}C_{2x-1} + {}^{20-3x}P_{2x-5}$

(I) $16 - x \geq 2x - 1$ & $2x - 1 \geq 0$

$17 \geq 3x$ & $2x \geq 1$

$\Rightarrow x \leq \frac{17}{3}$ & $x \geq \frac{1}{2}$

(II) $20-3x \geq 2x-5$ & $2x-5 \geq 0$

$25 \geq 5x$ & $2x \geq 5$

$\Rightarrow x \leq 5$ & $x \geq \frac{5}{2}$

(III) $\left[x + \frac{1}{x}\right] \neq 1 \Rightarrow x + \frac{1}{x} \notin [1, 2)$

$\left[x + \frac{1}{x}\right] > 0$

$|x^2 - x - 6| > 0$

$(x + 3)(x - 2) > 0$

$x \geq 3$, $(x + 3)(x - 2) > 0$

$\Rightarrow x \in (-\infty, -3) \cup (2, \infty)$

Q.2 Find the domain and range of the following functions.

(Read the symbols [*] & {*} as greatest integers & fractional part functions respectively.)

(i) $y = \log_{\sqrt{5}} (\sqrt{2}(\sin x - \cos x) + 3)$

(ii) $y = \frac{2x}{1 + x^2}$

(iii) $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$

(iv) $f(x) = \frac{x}{1 + |x|}$

(v) $y = \sqrt{2 - x} + \sqrt{1 + x}$

(vi) $f(x) = \log_{(\operatorname{cosec} x - 1)} (2 - [\sin x] - [\sin x]^2)$

(vii) $f(x) = \frac{\sqrt{x + 4} - 3}{x - 5}$

(viii) $\cot^{-1} (2x - x^2)$.

(ix) $f(x) = \log_2 (\sqrt{x - 4} + \sqrt{6 - x})$

Sol. (i) $y = \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3)$

Range $-\sqrt{2+2} \le \sqrt{2}\,(\sin x - \cos x) \le \sqrt{2+2}$

$1 \le \sqrt{2}\,(\sin x - \cos x) + 3 \le 5$

$0 \le \log_{\sqrt{5}}(\sqrt{2}(\cos x + \sin x) + 3 \le 2\log_5 5)$

Range $\in [0, 2]$

Domain : $\sqrt{2}\,(\sin x - \cos x) + 3 > 0 \ \forall\ x \in R$

$\therefore$ domain $\in R$

(ii) $y = \dfrac{2x}{1+x^2}$; $1+x^2 \ne 0\ \forall x \in R$

domain : $x \in R$

$yx^2 + y = 2x \Rightarrow x^2y - 2x + y = 0$

$D \ge 0 \Rightarrow 4 - 4\,(xy^2) \ge 0 \Rightarrow 1 - y^2 \ge 0$

$y^2 \le 1 \Rightarrow y \in [-1, 1]$

(iii) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + x - 6} = \dfrac{(x-1)\,(x-2)}{(x+3)(x-2)}$

$\forall\ x \in R$

$y = \dfrac{x-1}{x-3} \Rightarrow x = \dfrac{3y+1}{1-y}$

where $y \in R - \{1\}$

at $x = 2$, $y = \dfrac{1}{5}$

$\Rightarrow$ range $\in R - \left\{\dfrac{1}{5}, 1\right\}$

domain $\in R - \{2, -3\}$

(iv) $f(x) = \dfrac{x}{1+|x|}$

$1 + |x| \ne 0$

$\Rightarrow |x| \ne -1$

$\Rightarrow \forall\ x \in R$

$\therefore$ domain = R

$y + y|x| - x = 0$

(a) when $x < 0$

$y - yx - x = 0$

$x = \dfrac{y}{y+1} \Rightarrow y \ne -1$ ....(i)

& $\dfrac{y}{y+1} < 0 \Rightarrow y \in (-1,0)$ ...(ii)

(b) when $x \ge 0$

$y + yx - x = 0$

$\Rightarrow x = \dfrac{y}{1-y}$

(iii) $y \ne 1$

(iv) $\dfrac{y}{1-y} \ge 0 \Rightarrow y \ge 0 \Rightarrow y \le 1$

from (i, ii, iii, iv) $\Rightarrow y \in (-1, 1)$

(v) $y = \sqrt{2-x} + \sqrt{1+x}$

$2 - x \ge 0 \Rightarrow x \le 2$ ...(i)

$1+x \ge 0 \Rightarrow x \ge -1$ (ii)

$(i \cap ii) \Rightarrow x \in [-1, 2]$

$y^2 = 2 - x + 1 + x + 2\sqrt{(2-x)\,(1+x)}$

$= 3 + 2\sqrt{2 + x - x^2}$

$= 3 + 2\sqrt{2 - \left(x^2 - x + \dfrac{1}{4}\right) + \dfrac{1}{4}}$

$= 3 + 2\sqrt{\dfrac{9}{4} - \left(x - \dfrac{1}{2}\right)^2}$

$\therefore y^2_{max} = 3 + 2\sqrt{\dfrac{9}{4}} = 3 + 3 = 6$

$y^2_{min} = 3 \Rightarrow$ range $[\sqrt{3}, \sqrt{6}]$

(vi) $f(x) = \log_{\text{cosec}\,x-1}(2 - [\sin x] - [\sin x])^2$

$\text{cosec}\,x - 1 > 0$ &	$\text{cosec}\,x \ne 1$
$\text{cosec}\,x > 1$	$\text{cosec}\,x \ne 2$
	$\sin x \ne \dfrac{1}{2}$
$\Rightarrow x \in (0, \pi) - \{\pi/2\}$	$x \ne 2n\pi + \dfrac{\pi}{6}, 2n\pi + \dfrac{5\pi}{6}$
	...(ii) $\forall n \in I$

$x \in (2n\pi + 2n\pi + \pi) - \left\{(2n+1)\dfrac{\pi}{2}\right\} \forall\ n \in I$ ...(i)

domain : (i) $\cap$ (ii)

from domain, $2 - [\sin x] - [\sin x]^2 > 0$

$2 < 0$

$\Rightarrow f(x) = \dfrac{\log 2}{\log(\text{cosec}\,x - 1)}$

$\text{cosec}\,x - 1 \in R - \{0\}$

$\log(\text{cosec}\,x - 1) \in R - \{0\}$

(vii) $f(x) = \dfrac{\sqrt{x+4} - 3}{x - 5}$

$x \neq 5$ & $x + 4 \geq 0 \Rightarrow x \geq -4$

Domain $\Rightarrow x \in [-4, \infty) - \{5\}$

$$f(x) = \frac{x+4-9}{(x-5)(\sqrt{x+4}+3)} = \frac{1}{\sqrt{x+4}+3}, \ x \neq 5$$

$$f(x)_{min} = \frac{1}{\infty} = 0$$

$$f(x)_{max} = \frac{1}{3}$$

at $x = 5$, $f(x) = \dfrac{1}{6}$

$$\Rightarrow \text{range} \in \left(0, \frac{1}{3}\right] - \left\{\frac{1}{6}\right\}$$

(viii) $y = \cot^{-1}(2x - x^2)$

$$2x - x^2 \in \left[\frac{-(4)}{4(-1)}, \infty\right) \Rightarrow 2x - x^2 \in [1, \infty)$$

domain $x \in R$

(ix) $y = \log_2(\sqrt{x-4} + \sqrt{6-x})$

$= \log_2 t$

where $t = \sqrt{x-4} + \sqrt{6-x}$; $x \in [4, 6]$

$$\frac{dt}{dx} = 0 \Rightarrow \frac{1}{2\sqrt{x-4}} - \frac{1}{2\sqrt{x-6}} = 0$$

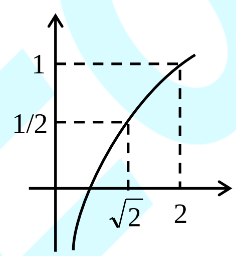
$6 - x = x - 4 \Rightarrow x = 5$

$t(5) = 2$, $t(4) = \sqrt{2}$, $t(6) = \sqrt{2}$

$\Rightarrow$ range $t \in [\sqrt{2}, 2]$

$y = \log_2 t$

$$\text{range} \in \left[\frac{1}{2}, 1\right]$$



Q.3 (a) Draw graphs of the following function, where [ ] denotes the greatest integer function.

(i) $f(x) = x + [x]$

(ii) $y = (x)^{[x]}$ where $x = [x] + (x)$ & $x > 0$ and $x \leq 3$

(iii) $y = \text{sgn}[x]$

(iv) $\text{sgn}(x - |x|)$

(b) Identify the pair(s) of functions which are identical? (where [x] denotes greatest integer and {x} denotes fractional part function)

(i) $f(x) = \text{sgn}(x^2 - 3x + 4)$ and $g(x) = e^{[\{x\}]}$

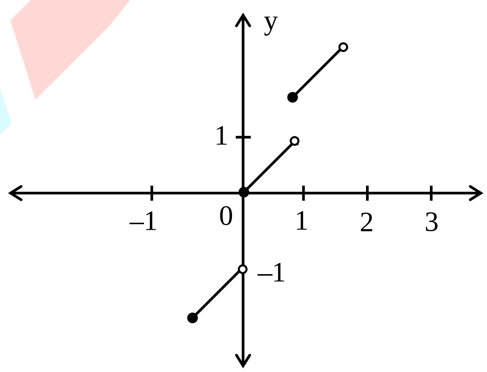
(ii) $f(x) = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$ and $g(x) = \tan x$

(iii) $f(x) = \lambda n(1 + x) + \lambda n(1 - x)$ and $g(x) = \lambda n(1 - x^2)$

(iv) $f(x) = \dfrac{\cos x}{1 - \sin x}$ & $g(x) = \dfrac{1 + \sin x}{\cos x}$

Sol. (a) $f(x) = x + [x]$

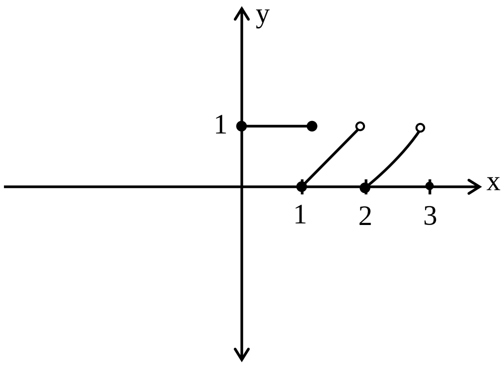
$$f(x) = \begin{cases} x+1 & 1 \leq x < 2 \\ x & 0 \leq x < 1 \\ x+2 & 2 \leq x < 3 \\ x-1, & -1 \leq x < 0 \\ x-2, & -2 \leq x < -1 \end{cases}$$



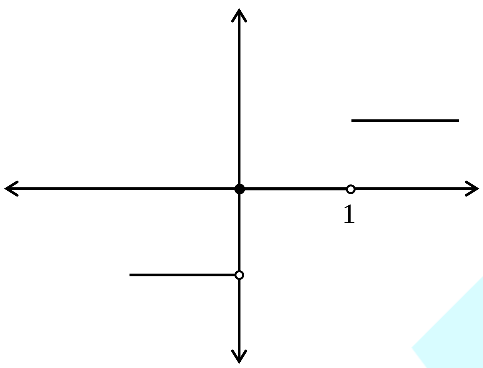
(b) $y = (x)^{[x]}$; $0 < x \leq 3$; $(x) = x - [x]$

$$y = \begin{cases} 1, & 0 < x < 1 \\ x-1, & 1 \leq x < 2 \\ (x-2)^2, & 2 \leq x < 3 \\ 0, & x = 3 \end{cases}$$

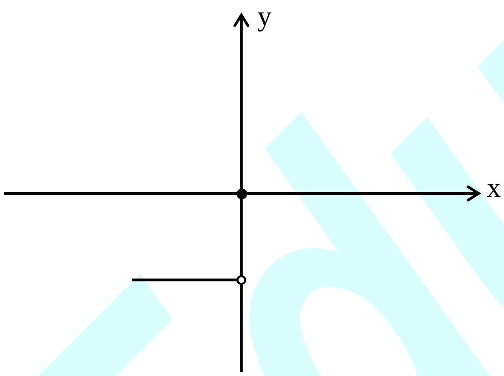
(ii)



(iii) y = sgn [x]



(iv) y = sgn (x – |x|)



(b) $f(x) = \text{sgn}(x^2 - 3x + 4)$ | $g(x) = e^{[\{x\}]}$

$x^2 - 3x + 4 > 0,\ x \in R$ | $x \in R$

range $\in \{1\}$ | range $\in \{1\}$

thus identical

(ii) $f(x) = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$

domain $\in R - \dfrac{n\pi}{2}$; $n \in I$

g(x) = tan x

range $\in R$

domain $\in R - \left(\dfrac{2n+1}{2}\right)\pi$; $n \in I$

$\therefore$ thus not identical

(iii) $f(x) = \ln(1+x) + \ln(1-x)$ $\quad$ $g(x) = \ln(1-x^2)$

domain : (–1, 1) $\quad$ domain : (–1, 1)

range $\in (-\infty, 0)$ $\quad$ range $\in (-\infty, 0)$

thus identical

(iv) $f(x) = \dfrac{\cos x}{1 - \sin x}$

domain $\in R - \left(\dfrac{2n+1}{2}\right)\dfrac{\pi}{2}$

$g(x) = \dfrac{1+\sin x}{\cos x}$

domain $\in R - \dfrac{(2n+1)\pi}{2}$; $n \in I$

Q.4 Let f be a function satisfying

$2f(xy) = \{f(x)\}^y + \{f(y)\}^x$ and $f(1) = k \neq 1$.

Prove that $(k-1)\sum_{n=1}^{n} f(n) = k^{n+1} - k$

Sol. Given

$2f(xy) = \{f(x)\}^y + \{f(y)\}^x$ and $f(1) = k \neq 1$

We have to Prove that $(k-1)\displaystyle\sum_{n=1}^{n} f(n) = k^{n+1} - k$

$(k-1)\displaystyle\sum_{n=1}^{n} f(n) = (k-1)\,[f(1) + f(2) + f(3) + f(4)$

$+ \ldots\ldots\ldots\ldots + f(n)]$

$2f(1.1) = 2f(1) = (f(1))^1 + (f(1))^1 = 2f(1) = 2k$

$2f(1.2) = 2f(2) = (f(1))^2 + (f(2))^1 = k^2 + f(2)$

$\Rightarrow f(2) = k^2$

$2f(1.3) = (f(1))^3 + (f(3))^1 = k^3 + f(3) \Rightarrow f(3) = k^3$

$2f(1.4) = (f(1))^4 + (f(4))^1 = k^4 + f(4) \Rightarrow f(4) = k^4$

$2f(1.n) = (f(1))^n + (f(n))^1 \Rightarrow f(n) = k^n$

$(k-1)\displaystyle\sum_{n=1}^{n} f(n) = (k-1)\,[k + k^2 + k^3 + k^4 +$

$\ldots\ldots + k^n]$

$= (k-1)\,\dfrac{k.[k^n - 1]}{(k-1)} = k^{n+1} - k$

$\therefore (k-1)\displaystyle\sum_{n=1}^{n} f(n) = k^{n+1} - k$

Hence, Proved

Q.5 Determine all functions f satisfying the functional relation $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$ where x is a real number, $x \neq 0$, $x \neq 1$.

Sol. Given

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)} \; ; x \in R \text{ and } x \neq 0, 1$$

Replace x by $\frac{1}{1-x}$ in (i), we get

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{1}{1-\frac{1}{1-x}}\right) = 2\,\frac{1-\frac{2}{1-x}}{\frac{1}{1-x}\left(1-\frac{1}{1-x}\right)}$$

$$= \frac{2(1-x-2)(1-x)}{(1-x-1)}$$

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{1-x}{1-x-1}\right) = \frac{2(-x-1)(1-x)}{(-x)}$$

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{x-1}{x}\right) = \frac{2(1+x)(1-x)}{x} \quad \text{...(ii)}$$

Again Replace x by $\frac{x-1}{x}$ in (i), we get

$$f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-\frac{x-1}{x}}\right) = \frac{2\left[1-2\frac{x-1}{x}\right]}{\frac{x-1}{x}\left(1-\frac{x-1}{x}\right)}$$

$$= \frac{2[x-2x+2]x}{(x-1)(x-x+1)}$$

$$f\left(\frac{x-1}{x}\right) + f\left(\frac{x}{x-x+1}\right) = \frac{2(2-x)x}{(x-1)(1)}$$

$$= \frac{2(x-2)x}{(1-x)}$$

$$f\left(\frac{x-1}{x}\right) + f(x) = \frac{2(x-2)x}{(1-x)} \quad \text{...(iii)}$$

subtracting equation (ii) from (iii) , we get

$$f(x) - f\left(\frac{1}{1-x}\right) = \frac{2(x-2)x}{(1-x)} - \frac{2(1+x)(1-x)}{x}$$

$$f(x) - f\left(\frac{1}{1-x}\right) = 2\left[\frac{x^2(x-2)-(1+x)(1-x)^2}{x(1-x)}\right]$$

$$f(x) - f\left(\frac{1}{1-x}\right) = \frac{2[x^3-2x^2-(1+x)(x^2+1-2x)]}{x(1-x)}$$

$$f(x) - f\left(\frac{1}{1-x}\right) = \frac{2[x^3-2x^2-x^2-1+2x-x^3-x+2x^2]}{x(1-x)}$$

$$f(x) - f\left(\frac{1}{1-x}\right) = \frac{2[-x^2-1+x]}{x(1-x)} \quad \text{..(iv)}$$

Adding equations (i) and (iv), we get

$$2f(x) = \frac{2(1-2x)}{x(1-x)} + \frac{2(x-1-x^2)}{x(1-x)}$$

$$2f(x) = \frac{2}{x(1-x)}[1-2x+x-1-x^2]$$

$$= \frac{2}{x(1-x)}[-x^2-x]$$

$$f(x) = \frac{x(x+1)}{x(x-1)} \Rightarrow f(x) = \frac{x+1}{x-1}$$

which is required result

Q.6 Let {x} and [x] denotes the fractional and integral part of a real number x respectively. Solve 4{x} = x + [x].

Sol. Given that 4{x} = x + [x]

where [x] = greatest integer $\leq$ x

{x} = fractional part of x

$\therefore$ x = [x] + {x}, for any $x \in R$

$\therefore$ Given equation becomes, 4{x}= [x] + {x} + [x]

$\Rightarrow$ 3{x} = 2[x]

$\Rightarrow$ $[x] = \frac{3}{2}\{x\}$ …(1)

Now $-1 < \{x\} < +1$

$\Rightarrow$ $-\frac{3}{2} < \frac{3}{2}\{x\} < \frac{3}{2}$

$\Rightarrow$ $-\frac{3}{2} < [x] < \frac{3}{2}$ [Using eq^n (1)]

$\Rightarrow$ [x] = – 1, 0, 1

If [x] = –1

$\Rightarrow$ $-1 = \frac{3}{2}\{x\}$ [Using eq^n (1)]

$\Rightarrow$ $\{x\} = -\frac{2}{3}$

$\therefore$ x = [x] + {x}

$\Rightarrow \quad x = -1 + \left(-\frac{2}{3}\right) = -\frac{5}{3}$

If $\quad [x] = -0$

$\Rightarrow \quad \frac{3}{2}\{x\} = 0 \qquad$ [Using eq^n (1)]

$\Rightarrow \quad \{x\} = 0$

$\therefore \quad x = 0 + 0 = 0$

If $\quad [x] = 1$

then $\quad \frac{3}{2}\{x\} = 1 \qquad$ [Using eq^n (1)]

$\Rightarrow \quad \{x\} = \frac{2}{3}$

$\Rightarrow \quad x = 1 + \frac{2}{3} = \frac{5}{3}$

Thus, $\quad x = -\frac{5}{3}, 0, \frac{5}{3}$

Q.7 Let $f(x) = x^2 + kx$; k is a real number. The set of values of k for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have same real solution set.

Sol. $f(x) = x^2 + kx$.

$f(x) = 0 \Rightarrow \lambda^2 + kx = 0$

$\Rightarrow$ Solution : 0, – k

$= \lambda^4 + x^2(k^2 + k) + 2kx^3 + k^2x$

fof $(x) = 0 \Rightarrow (x^2 + kx)^2 + k(x^2 + kx) = 0$

$(x^2 + kx)(x^2 + kx + k) = 0$

Solution → (0, k)

Solutions: → imaganary $k^2 - 4k<0$ $k\in(0, 4)$

OR → $x = 0$, $k = 0$

OR → $x = -k$, $k = 0$

Thus answer: [0, 4)

Q.8 Let f(x) be defined on [–2, 2] and is given by

$$f(x) = \begin{cases} -1 & -2 \le x \le 0 \\ x-1 & 0 < x \le 2 \end{cases} \text{ and}$$

$g(x) = f(|x|) + |f(x)|$.Then find g(x).

Sol. $$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$$

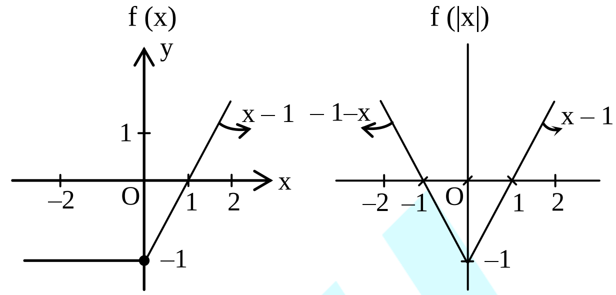
$g(x) = f(|x|) + |f(x)|$

| f(x) is not possible |

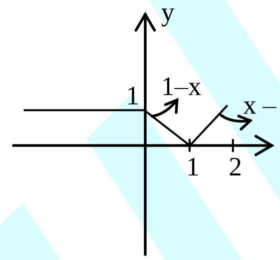
$$f(|x|) = \begin{cases} -1, & |x| = 0 \\ |x|-1, & 0 <|x| \le 2 \end{cases}$$

$$= \begin{cases} -1, & x = 0 \\ x-1, & 0 \le x \le 2 \end{cases}$$

f (x) $\qquad$ f (|x|)



|f(x)|



$$g(x) = \begin{cases} -1-x+1, & 2 \le x < 0 \\ x-1+1-x, & 0 \le x < 1 \\ 2(x-1), & 1 \le x \le 2 \end{cases}$$

$$= \begin{cases} -x, & 2 \le x < 0 \\ 0, & 0 \le x < 1 \\ 2(x-1), & 1 \le x \le 2 \end{cases}$$

Q.9 Solve the equation:

$|2x - 1| = 3[x] + 2\{x\}$ where $[\cdot]$ and $\{\cdot\}$ denotes greatest integer function and fractional part function respectively.

Sol. $|2x - 1| = 3[x] + 2\{x\}$

$\{x\} = x - [x]$

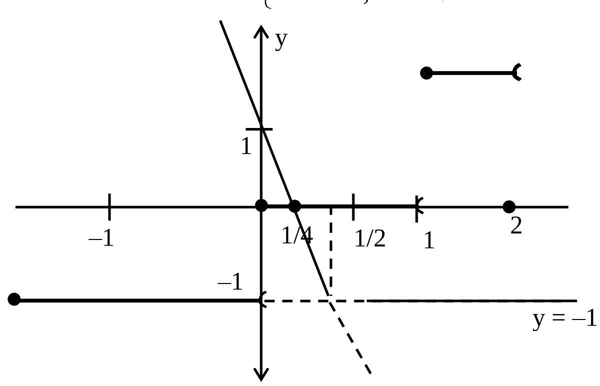
$|2x - 1| = 3[x] + 2x - 2[x]$

$|2x - 1| = [x] + 2x$

$|2x - 1| - 2x = [x]$

Hence, $y = [x]$ and $y = |2x - 1| - 2x$ can be solved graphically as

$$y = |2x - 1| = \begin{cases} -4x+1 \ ; & x < 1/2 \\ -1 \ ; & x \ge 1/2 \end{cases}$$

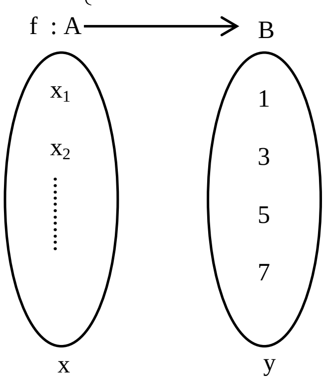


only solution at x = 1/4

Q.10 Let $f(x) = \begin{cases} 3^{x-1} + \dfrac{8}{3} & \text{for } 0 \le x \le 2 \\ 7 + \log_2(x-2) & \text{for } 2 < x \le 4 \\ x^2 - 9x + 21 & \text{for } 4 < x \le 6 \end{cases}$.

A set ' B' is formed by elements which are ' f ' images of the elements of set A. If B = {1, 3, 5, 7}, find A. Hence or otherwise state reasons whether it is possible to have a function, $f^{-1} : B \to A$ or not ?

Sol. $f(x) = \begin{cases} 3^{x-1} + \dfrac{8}{3} \text{ for } 0 \le x \le 2 \\ 7 + \log_2(x-2) \text{ for } 2 < x \le 4 \\ x^2 - 9x + 21 \text{ for } 4 < x \le 6 \end{cases}$



For, $y = 3^{x-1} + \dfrac{8}{3},\ 0 \le x \le 2$

$y = 1 = 3^{x-1} + \dfrac{8}{3} \Rightarrow 3^{x-1} = 1 - \dfrac{8}{3} = -5/3$

(not defined)

$y = 3 = 3^{x-1} + \dfrac{8}{3} \Rightarrow 3^{x-1} = \dfrac{9-8}{3} = \dfrac{1}{3}$

$\Rightarrow 3^x - 1 = 0 \Rightarrow x = 0 \qquad 0 \to 3$

$y = 5 = 3^{x-1} + \dfrac{8}{3} \Rightarrow 3^{x-1} = \dfrac{7}{3}$

$\Rightarrow 3^x = 7 \Rightarrow x = \log_3 7 \quad ; \quad \log_3 7 \to 5$

$y = 7 = 3^{x-1} + \dfrac{8}{3} \Rightarrow 3^{x-1} = \dfrac{21-8}{3} = \dfrac{13}{3}$

$\Rightarrow 3^x = 13 \Rightarrow x = \log_3 13 > 2$ (reject it)

For, $y = 7 + \log_2(x-2)\ ;\ 2 < x \le 4$

$y = 1 = 7 + \log_2(x-2) \Rightarrow \log_2(x-2) = -6$

$\Rightarrow x - 2 = 2^{-6} = \dfrac{1}{64} \Rightarrow x = 2 + \dfrac{1}{64} = \dfrac{129}{64}$

$\dfrac{129}{64} \to 1$

$y = 3 = 7 + \log_2(x-2) \Rightarrow \log_2(x-2) = -4$

$\Rightarrow x - 2 = 2^{-4} = \dfrac{1}{16} \Rightarrow x = 2 + \dfrac{1}{16} = \dfrac{33}{16}$

$\dfrac{33}{16} \to 3$

$y = 5 = 7 + \log_2(x-2) \Rightarrow \log_2(x-2) = -2$

$\Rightarrow (x-2) = 2^{-2} = \dfrac{1}{4} \Rightarrow x = 2 + \dfrac{1}{4} = 9/4$

$\dfrac{9}{4} \to 5$

$y = 7 = 7 + \log_2(x-2) \Rightarrow \log_2(x-2) = 0$

$\Rightarrow x - 2 = 1 \Rightarrow x = 3 \qquad 3 \to 7$

For $x^2 - 9x + 21\ ;\ 4 < x \le 6$

$y = 1 = x^2 - 9x + 21 \Rightarrow x^2 - 9x + 20 = 0$

$\Rightarrow (x-4)(x-5) = 0$

$\Rightarrow x = 4, 5$

x = 4 (reject)

x = 5 (accept)

because $4 < x \le 6$

$5 \to 1$

$y = 3 = x^2 - 9x + 21 \Rightarrow x^2 - 9x + 18 = 0$

$\Rightarrow x = \dfrac{9 \pm \sqrt{81-72}}{2} = \dfrac{9 \pm 3}{2} = 6, 3$

only x = 6 (accepted) $(6 \to 3)$

$y = 5 = x^2 - 9x + 21 \Rightarrow x^2 - 9x + 16 = 0$

$\Rightarrow x = \dfrac{9 \pm \sqrt{81-64}}{2} = \dfrac{9 \pm \sqrt{17}}{2} = \dfrac{9 \pm 4.1}{2}$

$\Rightarrow x = 6.55, 2.55$

since, $4 < x \le 6$

Hence both value must be rejected

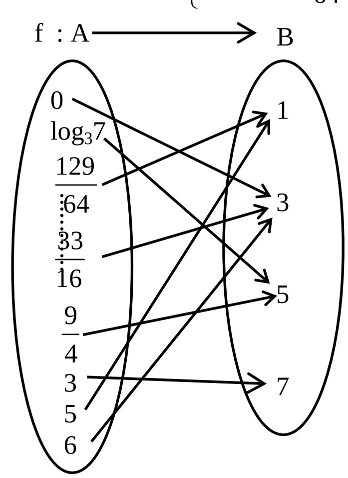
$y = 7 = x^2 - 9x + 21 \Rightarrow x^2 - 9x + 14 = 0$

$\Rightarrow x = \dfrac{9 \pm \sqrt{81-56}}{2} = \dfrac{9 \pm 5}{2}$

$\Rightarrow x = 7, 2$

Both value must be rejected because $4 < x \le 6$

Hence set, $A = \left\{0, \log_3 7, \dfrac{129}{64}, \dfrac{33}{16}, \dfrac{9}{4}, 3, 5, 6\right\}$



It is many one onto function

Hence, f^{-1} does not exist

Q.11 If $f(x) \in [1, 2]$ when $x \in R$ and for a fixed positive real number p,

$f(x + p) = 1 + \sqrt{2f(x) - \{f(x)\}^2}$ for all $x \in R$

then prove that f(x) is a periodic function.

Sol. Given $f(x + p) = 1 + \sqrt{2f(x) - \{f(x)\}^2}$ for all $x \in R$ and $f(x) \in [1, 2]$

$f(x + p) - 1 = \sqrt{2f(x) - \{f(x)\}^2}$; $f(x) \in [1, 2]$

squaring both sides, we get

$f^2(x + p) + 1 - 2f(x + p) = 2f(x) - f^2(x)$

or $f^2(x) - 2f(x) + [f(x + p) - 1]^2 = 0$

$$f(x) = \frac{2 \pm \sqrt{4 - 4[f(x+p) - 1]^2}}{2 \times 1}$$

$$f(x) = \frac{2 \pm 2\sqrt{1 - f^2(x+p) - 1 + 2f(x+p)}}{2}$$

$f(x) = 1 \pm \sqrt{2f(x+p) - f^2(x+p)}$; $f(x) \in [1, 2]$

$\therefore$ $f(x) = 1 + \sqrt{2f(x+p) - f^2(x+p)}$

we can write

$f(x + p) - \sqrt{2f(x) - f^2(x)}$

$= f(x) - \sqrt{2f(x+p) - f^2(x+p)}$

or $f(x + p) + \sqrt{2f(x+p) - f^2(x+p)}$

$= f(x) + \sqrt{2f(x) - f^2(x)}$

squaring both sides, we get

$f^2(x + p) + 2f(x + p) - f^2(x + p) + 2f(x + p).\sqrt{2f(x+p) - f^2(x+p)}$

$= f^2(x) + 2f(x) - f^2(x) + 2f(x)\sqrt{2f(x) - f^2(x)}$

or $f(x + p)\left[1 + \sqrt{2f(x+p) - f^2(x+p)}\right]$

$= f(x)\left[1 + \sqrt{2f(x) - f^2(x)}\right]$

or $f(x + p).\ f(x) = f(x).\ f(x + p)$

This holds only if $f(x + p) = f(x)$

Hence, f(x) is Periodic with Period P.

Q.12 If $f(a - x) = f(a + x)$ and $f(b - x) = f(b + x)$ for all real x where a, b(a > b) are constants then prove that f(x) is a periodic function.

Sol. Given $f(a - x) = f(a + x)$ ...(i)

$f(b - x) = f(b + x)$ ...(ii)

for all $x \in R$ (a > b)

Replace x by a –x in equation (i) and in equation (ii) x by b – x, we get

$f(a - a + x) = f(a + a - x) \Rightarrow f(x) = f(2a - x)$

$f(b - b + x) = f(b + b - x) \Rightarrow f(x) = f(2b - x)$

$f(2a - x) = f(2b - x)$

Replace x by 2b – x, we get

$f(2a - 2b + x) = f(2b - 2b + x)$

$f(2a - 2b + x) = f(x)$

Hence, f(x) is Periodic with Period 2(a – b).

Q.13 Let n be a positive integer and define

$f(n) = 1! + 2! + 3! + \ldots\ldots + n!$,

where $n! = n(n - 1)(n - 2) \ldots. 3.2.1.$

Find the polynomial P(x) and Q(x) such that

$f(n + 2) = P(n)\, f(n + 1) + Q(n)\, f(n)$, for all $n \geq 1$.

Sol. Given $f(n) = 1! + 2! + 3! + \ldots\ldots.. + n!$

$n! = n(n - 1)(n - 2) \ldots\ldots\ldots\ldots\ldots\ldots. 3.2.1$

$f(n + 2) = P(n).f(n + 1) + Q(n).\ f(n)$, for all $n \geq 1$

We have to find P(x) and Q(x).

$f(n + 2) = 1! + 2! + 3! + \ldots\ldots\ldots\ldots. + n! + (n + 1)! + (n + 2)!$

$= 1! + 2! + 3! + \ldots\ldots\ldots\ldots. + n! + (n + 1)! + (n + 2)(n + 1)!$

$f(n + 2) = f(n) + (n + 3)(n + 1)!$

$f(n + 1) = 1! + 2! + 3! + \ldots\ldots\ldots. + n! + (n + 1)!$

$f(n + 1) = f(n) + (n + 1)!$

$\therefore$ $[f(n) + (n + 3)(n + 1)!] = P(n)\, [f(n) + (n + 1)!] + Q(n).\ f(n)$

Equating coefficients of f(n) and (n + 1)!, we get

$1 = P(n) + Q(n)$ and $P(n) = n + 3$

Replace n by x, we get

$P(x) + Q(x) = 1$ and $P(x) = x + 3$

$\therefore$ $Q(x) = 1 - P(x) = 1 - x - 3 = -x - 2$

$\therefore$ $P(x) = x + 3$ and $Q(x) = -x - 2$.

Q.14 Let $f : R \to R$ be a function given by $f(x + y) + f(x - y) = 2f(x)\, f(y)$ for all x, y $\in$ R and $f(0) \neq 0$. Prove that f(x) is an even function.

Sol. Given $f : R \to R$

$f(x + y) + f(x - y) = 2f(x).f(y)$ ...(i)

for all $x, y \in R$ and $f(0) \neq 0$

Put x = 0 and y = 0,

$f(0 + 0) + f(0 - 0) = 2f(0).f(0)$

$\Rightarrow 2f(0) = 2f^2(0) \Rightarrow 2f(0)[f(0) - 1] = 0$

$\therefore f(0) \neq 0$ Hence, $f(0) = 1$

Replace y by x in equation (1) we get
$f(2x) + f(0) = 2f(x).f(x) = 2f^2(x)$ ..(ii)
Replace y by $-x$ in equation (i) we get
$f(0) + f(2x) = 2f(x)\, f(-x)$ ...(iii)
from (ii) and (iii), we get
$2f^2(x) = 2f(x).f(-x)$
$2f(x)[f(x) - f(-x)] = 0 \Rightarrow f(x)= f(-x)$ and $f(x)\neq 0$
Hence, f(x) is an even function

Q.15 Let $f : [-2, 2] \to R$ be a function if, for $x \in [0, 2]$

$$f(x) = \begin{cases} x\tan x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}[x], & \frac{\pi}{2} < x \le 2 \end{cases}$$

define f for $x \in [-2, 0]$ when
(i) f is odd function
(ii) f is an even function
(where $[\cdot]$ is the greatest integer function)

Sol. $f : [-2, 2] \to R$, for $x \in [0, 2]$

$$f(x) = \begin{cases} x\tan x, & 0 < x \le \pi/2 \\ \frac{\pi}{2}[x], & \pi/2 < x \le 2 \end{cases}$$

(i) When f is odd function ; $f \in [-2, 0]$

$$f(x) = \begin{cases} -x\tan x \;; & -\frac{\pi}{2} < x \le 0 \\ -\frac{\pi}{2}[-x] \;; & -2 \le x < -\pi/2 \end{cases}$$

(ii) When f is even function ; $f \in [-2, 0]$

$$f(-x) = \begin{cases} x\tan x \;; & -\frac{\pi}{2} < x \le 0 \\ \frac{\pi}{2}[-x] \;; & -2 \le x < -\pi/2 \end{cases}$$

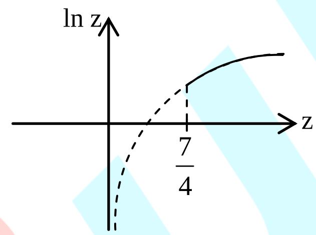
Q.16 If $f(x) = -1 + |x - 2|,\ 0 \le x < 4$
$g(x) = 2 - |x|,\ -1 \le x \le 3$
Then find fog (x) , gof (x) fof (x) & gog (x). Draw rough sketch of the graphs of fog (x) & gof (x).

Sol. Class illustration

Q.17 If $f(x) = \ln(x^2 - x + 2)$; $R^+ \to R$ and
$g(x) = \{x\} + 1$; $[1, 2] \to [1, 2]$,
where $\{x\}$ denotes fractional part of x find the domain and range of f(g(x)) when defined.

Sol. $f(x) = \ln(x^2 - x + 2)$; domain $\to$ k , range $\to$ R
$g(x) = \{x\} + 1$; domain$\to [1, 2]$, range$\to [1, 2]$

$fog(x) = (\ln(g(x)^2 - g(x) + 2)$; $0 < g(x) < \infty$
$= \ln((\{x\}+1)^2 - \{x\} - 1 + 2)$; $0 < \{x\} + 1 < \infty$
But $1 \le \{x\} + 1 \le 2$
$= \ln\left((\{x\})^2 + \{x\} + 2\right)$
Since ln x is monotonically increasing put end points of domain for range.
$\Rightarrow fog(x) \in [\ln 2, \ln 4)$
Let $\{x\} = t$; $t \in [0, 1)$
$fog(x) = \ln \underbrace{(t^2 + t + 2)}_{z\in\left[\frac{7}{4},\infty\right)}$



Q.18 Examine whether the following functions are even or odd or none.

(i) $f(x) = \dfrac{(1+2^x)^7}{2^x}$

(ii) $f(x) = \dfrac{\sec x + x^2 - 9}{x\sin x}$

(iii) $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$

(iv) $f(x) = \begin{cases} x|x| & x \le -1 \\ [1+x] + [1-x] & -1 < x < 1 \\ -x|x| & x \ge 1 \end{cases}$

(v) $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\frac{x + 2\pi}{\pi}\right] - 3}$ where [.] denotes greatest integer function.

Sol. i) $f(x) = \dfrac{(1+2^x)}{2^x}$

$f(-x) = \dfrac{(1+2^{-x})^7}{2^{-x}}$

ii) $f(x) = \dfrac{\sec x + x^2 - 9}{x\sin x}$

$f(-x) = \dfrac{\sec x + x^2 - 9}{(-x)(-\sin x)} = \dfrac{\sec x + x^2 - 9}{x\sin x}$

$\Rightarrow f(x) = f(-x)$
$\therefore$ even function

iii) $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$

$$f(-x) = \frac{-x}{e^{-x}-1} - \frac{x}{2} + 1$$

$$f(x) - f(-x) = \frac{x}{e^{x}-1} + \frac{x}{2} + 1$$

$$\frac{(-x)}{e^{-x}-1} + \frac{x}{2} - 1$$

$$\frac{x}{e^{x}-1} + \frac{xe^{x}}{1-e^{x}} + x$$

$$\Rightarrow \frac{x(1-e^{x})}{e^{x}-1} = -x$$

$= 0 \quad \therefore$ even function

(iv) $f(x) = \begin{cases} x|x|, & x \le -1 \\ [1+x]+[1-x], & -1 < x < 1 \\ -x|x|, & x \ge 1 \end{cases}$

$$f(x) = \begin{cases} x|x|, & x \le -1 \\ [x]+[-x]+2, & -1 < x < 1 \\ -x|x|, & x \ge 1 \end{cases}$$

$$f(-x) = \begin{cases} -x|-x|, & -x \ge 1 \\ [-x]+[x]+2, & -1 < -x < 1 \\ -|-x|, & x \ge 1 \end{cases}$$

$f(x) = f(-x)$

$\therefore$ even function

(v) $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+2\pi}{\pi}\right]-3} = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x}{\pi}\right]+4-3}$

$$f(-x) = \frac{2(-x)[\sin(-x)+\tan(-x)]}{2\left[\dfrac{-x}{\pi}\right]+1}$$

$$= \frac{2x[\sin x + \tan x]}{2\left[\dfrac{-x}{\pi}\right]+1}$$

$[x] + [-x] = 0 \quad \forall x \in I$

$= -1 \quad \forall x \notin I$

$[-x] = -[x] \ \forall x \in I$

$= -1-[x] \ \forall x \notin I$

Q.19 Find the period of the following functions.

(i) $f(x) = 1 - \dfrac{\sin^2 x}{1+\cot x} - \dfrac{\cos^2 x}{1+\tan x}$

(ii) $f(x) = \tan\dfrac{\pi}{2}[x]$ where [.] denotes greatest integer function.

(iii) f(x) = λog (2 + cos 3x)

(iv) $f(x) = e^{\ln \sin x} + \tan^3 x - \text{cosec}\,(3x-5)$

(v) $f(x) = \sin x + \tan\dfrac{x}{2} + \sin\dfrac{x}{2^2} + \tan\dfrac{x}{2^2}$ $+ \sin\dfrac{x}{2^3} + \ldots\ldots + \sin\dfrac{x}{2^{n-1}} + \tan\dfrac{x}{2^n}$

(vi) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

Sol. (i) $f(x) = 1 - \dfrac{\sin^2 x}{1+\cot x} - \dfrac{\cos^2 x}{1+\tan x}$

$\sin^2 x \to \pi$, $\cos^2 x \to \pi$

$1 + \cot x \to \pi$, $1 + \tan x \to \pi$

LCM $\to \pi$

(ii) $f(x) = \tan\dfrac{\pi}{2}[x]$

f(x + T) = f(x)

$\tan\dfrac{\pi}{2}[x+T] = \tan\dfrac{\pi}{2}[x]$

$\dfrac{\pi}{2}[x+T] = n\pi + \dfrac{\pi}{2}[x]$

[x + T] = 2n + [x]

Least +ve value at n = 1

$[x + T] = [x + 2] \Rightarrow T = 2$

(iii) f(x) = log (2 + cos 3x)

period $\dfrac{2\pi}{3}$

(iv) $f(x) = e^{\lambda n \sin x} + \tan^3 x - \text{cosec}\,(3x-5)$

$$\downarrow, \quad \downarrow, \quad \downarrow$$

$$T = \frac{\text{L.C.M.}}{\text{H.C.F.}} \quad 2\pi \quad \pi \quad \frac{2\pi}{3}$$

$$T = \frac{2\pi}{1}$$

(v) $f(x) = \sin x + \tan\dfrac{x}{2} + \sin\dfrac{x}{2^2} + \tan\dfrac{x}{2^2}$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

L.C.M. $\quad 2\pi \quad 2\pi \quad 8\pi \quad 4\pi$

$+\ldots + \sin\dfrac{x}{2^{n-1}} + \tan\dfrac{x}{2^n}$

$$\downarrow \quad \downarrow$$

$$2^n\pi \quad 2^n$$

$T = 2^n\pi$

(vi) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$f(x) = \frac{2\sin(2x)\cos(x)}{2\cos(2x)\cos(x)} = \tan 2x \to T = \frac{\pi}{2}$$

Q.20 A function f, defined for all x, y $\in$ R is such that f(1) = 2 ; f(2) = 8 and f(x + y) – k xy = f(x) + $2y^2$, where k is some constant . Find f(x) & show that $f(x+y).f\left(\frac{1}{x+y}\right) = k$ for $x + y \neq 0$.

Sol. $f(x + y) - kxy = f(x) + 2y^2$ $\quad$ f(1) = 2
$f(x + y) - f(x) = kxy + 2y^2$ $\quad$ f(2) = 8
Let y = 1 & x = 1
f (2) – k = f (1) + 2
$8 - k = 2 + 2 \Rightarrow k = 4$
Let x = 0, y = 1 $\Rightarrow$ f (1) = f (0) + 2 $\rightarrow$ f(0) = 2
Now y = –x
$\Rightarrow f(x) + 2x^2 = 4x^2$
$\Rightarrow f(x) = 2x^2$

Q.21 Suppose p(x) is a polynomial with integer coefficients. The remainder when p(x) is divided by x –1 is 1 and the remainder when p(x) is divided by x –4 is 10. If r(x) is the remainder when p(x) is divided by (x –1) (x – 4), find the value of r(2006).

Sol. Given, P (1) = 1, P(4) = 10
P (x) = (x –1) (x – 4) Q(x) + R (x)
$\therefore$ R (1) = 1, R (4) = 10
From pattern, R (x) = 3x – 2
R (2006) = 6018 – 2
= 6016

Part-B Passage based objective questions

Passage I (Question 22 to 24)

Let $f(x) = x^2 - 3x + 2$, g(x) = f(|x|)
h(x) = |g(x)| and I(x) = |g(x)| – [x]
are four function, where [x] is the integral part of real x.

Q.22 Find the value of 'a' such that equation g(x) – a = 0 has exactly 3 real roots-
(A) 2 (B) 1
(C) 0 (D) None of these

Sol. **[A]**
g(x) – a = 0
$\Rightarrow$ f(|x|) – a = 0
$\Rightarrow |x|^2 - 3|x| + 2 - a = 0$
$\Rightarrow x^2 - 3|x| + 2 - a = 0$

$$\Rightarrow x^2 - 3|x| + 2 - a = \begin{cases} x^2 - 3x + 2 - a; \ x \geq 0 \\ x^2 + 3x + 2 - a; \ x < 0 \end{cases}$$

When $x \geq 0$,
$x^2 - 3x + 2 - a = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(2-a)}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{1 + 4a}}{2}$$

when x < 0,
$x^2 + 3x + 2 - a = 0$

$$x = \frac{-3 \pm \sqrt{9 - 4(2-a)}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{1 + 4a}}{2}$$

If we take (1 + 4a) > 0, then equation has four real roots and at $a = -\frac{1}{2}$, It gives two roots. Our condition is that given equation has exactly 3 real roots. It is only possible when
a = 2

$$x = \frac{3 \pm 3}{2} = 0, 3$$

$$x = \frac{-3 \pm 3}{2} = -3, 0$$

i.e. x = 0, 3, –3 at a = 2
$\therefore$ Option (A) is correct answer.

Q.23 Find the set of values of 'b' such that equation h(x) – b = 0 has exactly 8 real solution
(A) b $\in$ [0, 1/4]
(B) b $\in$ [0, 1/4)
(C) b $\in$ (0, 1/4)
(D) None of these

Sol. **[B]**
h(x) – b = 0 has exactly 8 real roots.
h(x) = |g(x)| = |f(|x|)|
h(x) – b = |f(|x|)| – b = 0
$\Rightarrow$ |f(|x|)| = b
$\Rightarrow$ f(|x|) = $\pm$ b
Case I : When f(|x|) = b
$\Rightarrow |x|^2 - 3|x| + 2 = b$
$\Rightarrow x^2 - 3|x| + 2 - b = 0$
$\Rightarrow x^2 - 3x + 2 - b = 0 ; \ x \geq 0$
$x^2 + 3x + 2 - b = 0 ; x < 0$

$$x = \frac{3 \pm \sqrt{9 - 4(2-b)}}{2} \ ; \ x \geq 0$$

$$x = \frac{-3 \pm \sqrt{9-4(2-b)}}{2} \; ; \; x < 0$$

i.e., $x = \dfrac{3 \pm \sqrt{1+4b}}{2}$; $x \ge 0$

$$x = \frac{-3 \pm \sqrt{1+4b}}{2} \; ; \; x < 0$$

For real roots, it must be $1 + 4b > 0$

$\Rightarrow b > -1/4$

At the same time for $x \ge 0$

$$\frac{3 \pm \sqrt{1+4b}}{2} \ge 0$$

$\Rightarrow \quad \pm \sqrt{1+4b} \ge -3$

$\Rightarrow \quad 1 + 4b \le 9$

$\Rightarrow \quad 4b \le 8 \Rightarrow \quad b \le 2$

$$-\frac{1}{4} < b \le 2$$

Case II : When $f(|x|) = -b$

$|x|^2 - 3|x| + 2 = -b$

$x^2 - 3|x| + 2 + b = 0$

$x^2 - 3x + 2 + b = 0$; $x \ge 0$

$x^2 + 3x + 2 + b = 0$; $x < 0$

i.e. $x = \dfrac{3 \pm \sqrt{9-4(2+b)}}{2}$; $x \ge 0$

$$x = \frac{-3 \pm \sqrt{9-4(2+b)}}{2} ; \; x < 0$$

i.e $x = \dfrac{3 \pm \sqrt{1-4b}}{2}$; $x \ge 0$

$x = \dfrac{-3 \pm \sqrt{1-4b}}{2}$; $x < 0$

For real roots, $1 - 4b > 0 \Rightarrow b < 1/4$

At the same time, for $x \ge 0$

$$\frac{3 \pm \sqrt{1-4b}}{2} \ge 0$$

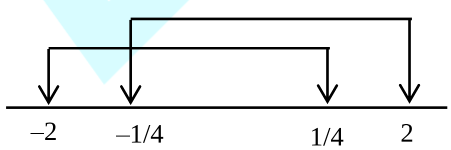
$\Rightarrow \pm \sqrt{1-4b} \ge -3$

$\Rightarrow 1 - 4b \le 9$

$\Rightarrow \; -4b \le 8$

$\Rightarrow \; b \ge -2$

$\Rightarrow \; -2 \le b < 1/4$



$$-\frac{1}{4} < b < \frac{1}{4}$$

At b = 0, equation has 8 roots also.

Hence required values of b $\in [0, \frac{1}{4})$

$\therefore$ Option (B) is correct Answer.

Q.24 Which statement is true for $I(x) = 0$ -

(A) Two values of x is satisfied for $I(x) = 0$

(B) One value of x is satisfied for $I(x) = 0$ and that x lie between 1 and 2

(C) One value of x is satisfied for $I(x) = 0$ and that x lie between 3 and 4

(D) None of these

Sol. **[C]**

$I(x) = |g(x)| - [x] = 0$

$|g(x)| = [x]$

$g(x) = \pm [x]$

$f(|x|) = \pm [x]$

$x^2 - 3|x| + 2 = \pm [x]$

$x^2 - 3x + 2 = \pm [x]$; $x \ge 0$

$x^2 + 3x + 2 = \pm [x]$; $x < 0$

when $x \ge 0$, $[x] \le x$

$x^2 - 3x + 2 = \pm x$

with +ve sign, $x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$x = 2 \pm \sqrt{3} = 3.7$ or 0.3

with –ve sign; $x^2 - 2x + 2 = 0$

$$x = \frac{2 \pm \sqrt{4-8}}{2}$$

x have Imaginary roots

only roots for $x \ge 0$, $x = 0.3$, 3.7

when $x < 0$, $[x] = x - 1$

$x^2 + 3x + 2 = \pm (x - 1)$

with +ve sign ; $x^2 + 3x + 2 = x - 1$

$x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{4-12}}{2} = \text{Imaginary}$$

with –ve sign, $x^2 + 3x + 2 = -x + 1$

$x^2 + 4x + 1 = 0$

$$x = \frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$$

$x = -3.7$ or -0.3

$I(x) = 0$ has total four roots

One of them lies between 3 and 4

$\therefore$ Option (C) is correct Answer.

Passage II (Question 25 to 27)

If $f(x) = 0$; if $x \in Q$

$= 1$; if $x \notin Q$.

then answer the following questions-

Q.25 $f(x)$ is -

(A) an even function

(B) an odd function

(C) Neither even nor odd function

(D) one-one function

Sol. **[A]**

If f(x) = 0 ; if $x \in Q$

= 1 ; if $x \notin Q$

$f(-x) = 0 = f(x)$; if $x \in Q$

$= 1 = f(x)$; if $x \notin Q$

$\therefore$ f(x) is an-even function

$\therefore$ Options (A) is correct Answer.

Q.26 $f(f(x))$ is-

(A) a constant function

(B) an even function

(C) an odd function

(D) many one function

Sol. **[A,B,D]**

$f(f(x)) = 0$; if $x \in Q$

= 1 ; if $x \notin Q$

It is a constant, an even function and many one function.

$\therefore$ Options (A), (B), (D) are correct Answer.

Q.27 Domain of $g(x) = ln\,(\text{sgn}\, f(x))$ is-

(A) R

(B) set of all rational numbers

(C) set of all irrational number

(D) R^+

Sol. **[C]**

Domain of g(x) = λn (sgnf(x))

g(x) to be defined if

sgn f(x) > 0

$$\Rightarrow \quad \frac{|f(x)|}{f(x)} > 0$$

$\Rightarrow$ 1 > 0 ; $x \notin Q$

= 0 ; $x \in Q$

i.e. set of all irrational numbers

$\therefore$ Option (C) is correct Answer.

Passage III (Question 28 to 30)

Consider the function

$$f(x) = \begin{cases} x - [x] - \frac{1}{2}; & \text{if} \quad x \notin I \\ 0 \quad ; & \text{if} \quad x \in I \end{cases}$$

where [.] denotes greatest integer function.

If $g(x) = \max.\{x^2, f(x), |x|\}$; $-2 \le x \le 2$, then.

Q.28 Range of f(x) is-

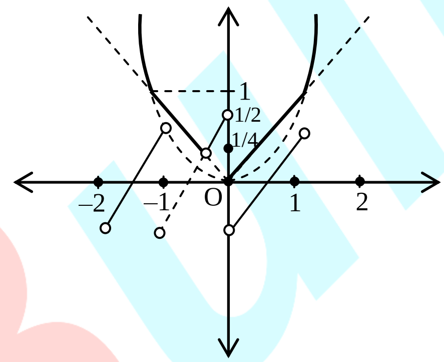
(A) [0, 1) (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right)$

Sol. **[C]**

$$f(x) = \begin{cases} \{x\} - \frac{1}{2} & ; x \notin I \\ 0 & ; x \in I \end{cases}$$

$g(x) = \max\{x^2, f(x), |x|\}$; $x \in [-2, 2]$



range of $f(x) \in \left(\frac{-1}{2}, \frac{1}{2}\right)$

Q.29 f(x) is-

(A) non periodic

(B) periodic with period 1

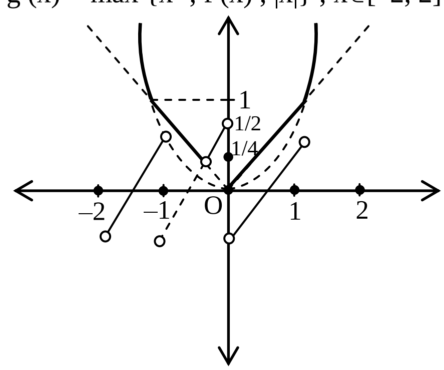
(C) periodic with period 2

(D) periodic with period 1/2

Sol. **[B]**

$$f(x) = \begin{cases} \{x\} - \frac{1}{2} & ; x \notin I \\ 0 & ; x \in I \end{cases}$$

$g(x) = \max\{x^2, f(x), |x|\}$; $x \in [-2, 2]$



Period of {x} = 1

Q.30 The set of values of a, if g(x) = a has three real and distinct solutions, is -

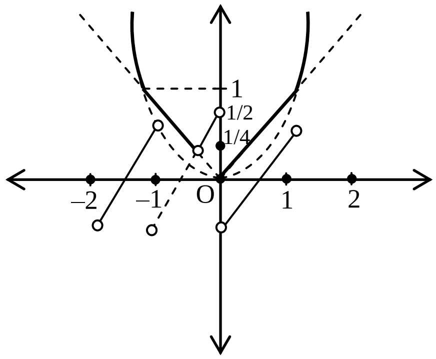
(A) (0, 1/2) (B) (0, 1/4)

(C) (1/4, 1/2) (D) (0, 1)

Sol. **[C]**

$$f(x) = \begin{cases} \{x\} - \dfrac{1}{2} & ; x \notin I \\ 0 & ; x \in I \end{cases}$$

$g(x) = \max\{x^2, f(x), |x|\}$; $x \in [-2, 2]$



$$\{x\} - \frac{1}{2} = -x$$

(between $-1 \le x < 0$)

For 3 distinct solution,

$$g(x) \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

Passage IV (Question 31 to 33)

Consider the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x \le 1 \\ \lambda nx, & 1 < x \le e \end{cases}$$

Let $f_1(x) = f(|x|)$

$f_2(x) = |f(|x|)|$

$f_3(x) = f(-x)$

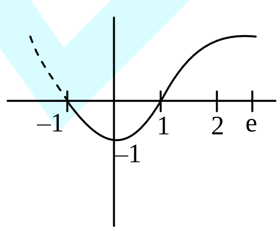
Now answer the following questions.

Q.31 Number of positive solutions of the equation $2f_2(x) - 1 = 0$ is-

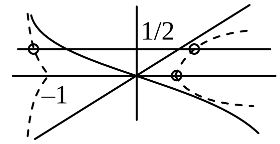
(A) 4 (B) 3 (C) 2 (D) 1

Sol. **[C]**

$$f(x) = \begin{cases} x^2 - 1 & ; -1 \le x \le 1 \\ \ln x & ; 1 < x \le e \end{cases}$$



$f_1(x) = f(|x|)$



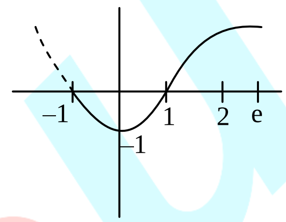
$$f(|x|) = \frac{1}{2}$$

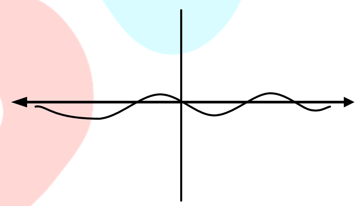
Q.32 Number of integral solution of the equation $f_1(x) = f_2(x)$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol. **[D]**

$$f(x) = \begin{cases} x^2 - 1 & ; -1 \le x \le 1 \\ \ln x & ; 1 < x \le e \end{cases}$$





$f_2(x) = |f(|x|)|$

i) $x^2 - 1 = 1 - x^2$; $-1 \le x < 1$

$2x^2 = 2 \Rightarrow x = \pm 1 \Rightarrow x = -1$

$\Rightarrow \boxed{x = -1}$

ii) $x^2 - 1 = x^2 - 1$; $0 \le x \le 1$

$1 = 1 \Rightarrow x = 0, 1$

$\ln x = |\ln x|$; $1 \le x \le e$

$x = 1, 2$

$\therefore$ x = –1, 0, 1, 2} 4 solutions

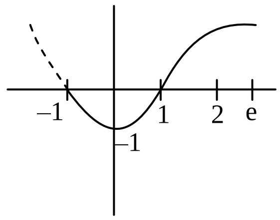
Q.33 If $f_4(x) = \log_{27}(f_3(x) + 2)$, then range of $f_4(x)$ is

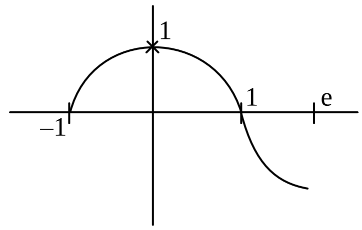
(A) [1, 9] (B) $\left[\frac{1}{3}, \infty\right)$

(C) $\left[0, \frac{1}{3}\right]$ (D) [1, 27]

Sol. **[C]**

$$f(x) = \begin{cases} x^2 - 1 & ; -1 \le x \le 1 \\ \ln x & ; 1 < x \le e \end{cases}$$





$f_4(x) = \log_{27}(f(-x) + 2)$

$$f_4(x) = \begin{cases} \log_{27}(x^2+1) & ; \quad -1 \le x \le 1 \\ \log_{27}(\ln(-x)) & ; \quad -e \le x < -1 \end{cases}$$

$$y(-1) = \frac{1}{3}\log_3(-1+1+2)$$

$$= \frac{1}{3}\log_3 2$$

$$y(1) = \frac{1}{3}\log_3 2$$

$$y'(x) = \frac{1}{x^2+1} \cdot 2x = 0 \Rightarrow x = 0$$

$$y(-e) = \frac{1}{3}\log_3(1+2) = \frac{1}{3}$$

$$y \in \left[0, \frac{1}{3}\right]$$

Passage V (Question 34 & 35)

If notation [x] denotes integer less than or equal to x and (.) denotes integer greater than or equal to x, then

Q.34 The solution set of the equation

$(x)^2 + [x]^2 = [x-1]^2 + (x+1)^2$ is -

(A) $\{x; x \in R\}$ (B) $\{x; x \in R - Z\}$

(C) $\{x; x \in Z\}$ (D) $\{x; x \in \phi\}$

Sol. **[B]**

$(x)^2 + [x]^2 = [x-1]^2 + (x+1)^2$

$(x) = 2.5 = 3$

$[x]^2 + (x)^2 = (x-1)^2 + [x+1]^2$

Subs tile on integer & real no.

Q.35 Let $f(x) = x + (x)$; $x < 0$

$3x - 2(x)$; $x \ge 0$

Range of sgn f(x) is -

(A) $\{-1, 0, 1\}$ (B) $\{-1, 1\}$

(C) $\{1, 0\}$ (D) $\{-1, 0\}$

Sol. **[A]**

$(x)^2 + [x]^2 = [x-1]^2 + (x+1)^2$

$(x) = 2.5 = 3$

EXERCISE # 4

Old IIT-JEE Questions

Q.1 Let f (x) = $(1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is - **[IIT 2001]**

(A) [0, 1] (B) [0, 1/2]

(C) [1/2, 1] (D) (0, 1]

Sol. **[D]**

$f(x) = (1 + b^2)x^2 + 2bx + 1$

Differentiating w.r.t. x, we get

$f'(x) = (1 + b^2)\, 2x + 2b$ ...(i)

Again, differentiating wrt x, we get

$f''(x) = 2(1+b^2)$ ...(ii)

$f''(x) = 2(1+b^2) = +\text{ve}$ for all $b \in R$

$\therefore$ f(x) to be minimum

$f'(x) = 2(1 + b^2)x + 2b = 0$

$$\Rightarrow x = \frac{-b}{1+b^2}$$

$$\therefore f(x) / x = \frac{-b}{1+b^2} = m(b) = (1+b^2)\frac{b^2}{(1+b^2)^2}$$

$$+2b\frac{(-b)}{(1+b^2)} + 1$$

$$m(b) = \frac{b^2}{1+b^2} - \frac{2b^2}{1+b^2} + 1$$

$$m(b) = \frac{-b^2}{1+b^2} + 1 = \frac{-b^2+1+b^2}{1+b^2}$$

$$m(b) = \frac{1}{1+b^2}$$

Since $1+b^2$ is +ve greater than unity for all $b \in R$

$\therefore$ m(b) $\in$ (0, 1]

$\therefore$ Option (D) is correct Answer.

Q.2 Let E = {1, 2, 3, 4} and F = {1, 2}. Then the number of onto functions from E to F is- **[IIT 2001]**

(A) 14 (B) 16

(C) 12 (D) 8

Sol. **[A]**

We can take general case : If A and B are two sets having m and n elements respectively such that $1 \le n \le m$; then number of onto functions from A to B is $\sum_{r=1}^{n}(-1)^{n-r}\, {}^nC_r(r)^m$

But in our case : E = {1, 2, 3, 4} $\Rightarrow$ m = 4

F = {1, 2} $\Rightarrow$ n = 2

$\therefore$ No. of onto functions from E to F is

$$\sum_{r=1}^{2}(-1)^{2-r}\, {}^2C_r\,(r)^4 = (-1)^1\, {}^2C_1(1)^4 + (-1)^0\, {}^2C_2(2)^4$$

$$= -1\,\frac{2!}{1!1!} + \frac{2!}{2!0!}(16) = -2 + 16 = 14$$

$\therefore$ Option (A) is correct Answer.

Q.3 Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, then for what value of α, f {f (x)} = x. **[IIT Scr. 2001]**

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) 1 (D) –1

Sol. **[D]**

$$f(x) = \frac{\alpha x}{x+1},\ x \neq -1$$

f(f(x)) = x

$$f\left(\frac{\alpha x}{x+1}\right) = x$$

$$\frac{\alpha.\frac{\alpha x}{x+1}}{\frac{\alpha x}{x+1}+1} = \frac{\alpha^2 x}{\alpha x + x + 1} = x$$

$$\Rightarrow \frac{\alpha^2 x}{(\alpha+1)x+1} = x$$

$\Rightarrow \alpha^2 = (\alpha + 1)x + 1$

$\Rightarrow (\alpha - 1)(\alpha + 1) - (\alpha + 1)x = 0$

$\Rightarrow (\alpha + 1)[(\alpha - 1) - x] = 0 \Rightarrow \alpha = -1$

$\therefore$ option (D) is correct Answer.

Q.4 The domain of definition of

$f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is – **[IIT Scr. 2001]**

(A) R – { –2, + 2} (B) (– 2, ∞)

(C) R– {–1, –2, –3} (D) (–3, ∞) / {–1, –2}

Sol. **[D]**

$$f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$$

f(x) to be defined if

$x + 3 > 0$ and $x^2 + 3x + 2 \neq 0$

$\Rightarrow$ $x > -3$ and $(x + 1)(x + 2) \neq 0$

$\Rightarrow$ $x > -3$ and $x \neq -1, -2$

$\Rightarrow$ $x \in (-3, \infty) - \{-1\} - \{2\}$

$\therefore$ Option (D) is correct Answer

Q.5 If $f : [1, \infty) \to [2, \infty)$ is given by

$f(x) = x + \dfrac{1}{x}$ then $f^{-1}(x)$ equals -

[IIT Scr. 2001]

(A) $\dfrac{x + \sqrt{x^2 - 4}}{2}$ (B) $\dfrac{x}{1 + x^2}$

(C) $\dfrac{x - \sqrt{x^2 - 4}}{2}$ (D) $1 + \sqrt{x^2 - 4}$

Sol. **[A]**

If $f : [1, \infty) \to [2, \infty)$

$f(x) = x + \dfrac{1}{x}$

$y = \dfrac{x^2 + 1}{x}$ $\Rightarrow$ $x^2 - yx + 1 = 0$

$\Rightarrow x = \dfrac{y \pm \sqrt{y^2 - 4}}{2 \times 1}$ $\Rightarrow f^{-1}(x) = \dfrac{x \pm \sqrt{x^2 - 4}}{2}$

Since, $f : [1, \infty) \to [2, \infty)$

$\therefore$ $f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

$\therefore$ Option (A) is correct answer.

Q.6 Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$

Then for all x, f (g(x) is equal to-

[IIT Scr. 2001]

(A) x (B) f (x)

(C) 1 (D) g (x)

Sol. **[C]**

$g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$

$f(g(x)) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$

$f(g(x)) = 1$, for $x > 0$

Q.7 Suppose $f(x) = (x + 1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line $y = x$, then g(x) equals– **[IIT Scr. 2002]**

(A) $-\sqrt{x} - 1, x \geq 0$ (B) $\dfrac{1}{(x+1)^2}, x > -1$

(C) $\sqrt{x + 1}, x \geq -1$ (D) $\sqrt{x} - 1, x \geq 0$

Sol. $f(x) = (x + 1)^2$ for $x \geq -1$

$y = (x + 1)^2$

$\Rightarrow (x + 1) = \sqrt{y}$

$\Rightarrow$ $x = \sqrt{y} - 1$

$\therefore$ $f^{-1}(x) = \sqrt{x} - 1 = g(x)$

$\Rightarrow g(x) = \sqrt{x} - 1 ; x \geq 0$

$\therefore$ Option (D) is correct answer.

Q.8 Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is–

[IIT Scr. 2002]

(A) one to one and onto

(B) one to one but not onto

(C) onto but not one to one

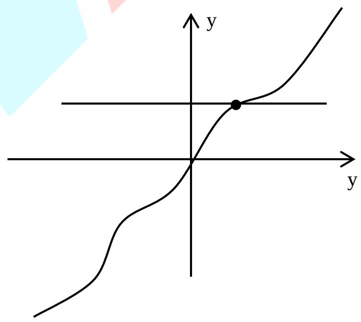
(D) neither one to one nor onto

Sol. **[A]**

$f : R \to R$

$f(x) = 2x + \sin x$ for $x \in R$

We can draw graph of f(x) as –



Any line parallel to x-axis cuts only at one point. Hence f(x) is one-one function.

$x = 0 ; y = 0$

$x = \dfrac{\pi}{6}$; $y = \dfrac{\pi}{3} + \dfrac{1}{2}$

$x = \dfrac{\pi}{4}$; $y = \dfrac{\pi}{2} + \dfrac{1}{\sqrt{2}}$

$x = \dfrac{\pi}{3}$; $y = \dfrac{2\pi}{3} + \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{2}$; $y = \pi + 1$ $\Rightarrow$ $x \to \infty, y \to \infty$

$x = -\dfrac{\pi}{6}$; $y = -\dfrac{\pi}{3} - \dfrac{1}{2}$

$x = -\frac{\pi}{4}$; $y = -\frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{3}$; $y = -\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

$x = \pi/2$; $y = -\pi - 1 \Rightarrow x \to -\infty,\ y \to -\infty$

$x = 0, y = 0$

Hence, it is also onto

$\therefore$ Option (A) is correct Answer.

Q.9 Let $f(x) = \frac{x}{1+x}$ defined as $[0, \infty) \to [0, \infty)$, f(x) is– **[IIT Scr.2003]**

(A) one- one & onto

(B) one–one but not onto

(C) not one–one but onto

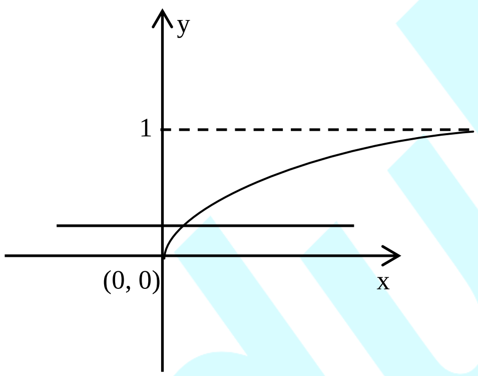
(D) neither one–one nor onto

Sol. **[A]**

$f : [0, \infty) \to [0, \infty)$

$f(x) = \frac{x}{1+x}$

$y = \frac{x}{1+x}$



$x = 0$; $y = 0$

$x = 1$; $y = \frac{1}{2} = 0.5$

$x = 2$; $y = \frac{2}{3} = 0.67$

$x = 3$; $y = \frac{3}{4} = 0.75$

$x = 4$; $y = \frac{4}{5} = 0.8$

$x = 5$; $y = \frac{5}{6} = 0.833$

$x \to \infty, y \to 1$

Any line Parallel to x-axis cuts curve only at one point. Hence, it is one-one. It is also onto.

$\therefore$ Option (A) is correct Answer.

Q.10 If $f(x) = \sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$, $\alpha \in (0, \pi/2)$, $x > 0$ then value of f(x) is greater than or equal to– **[IIT Scr.2003]**

(A) 2 (B) 2 tan α

(C) 5/2 (D) sec α

Sol. **[B]**

$f(x) = \sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$, $\alpha \in (0, \pi/2)$ $x > 0$

Using A.M. $\geq$ G.M.

$$\frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \geq \sqrt{\sqrt{x^2+x}\cdot\frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

$\Rightarrow \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$

$\Rightarrow f(x) = \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$

$\Rightarrow f(x) \geq 2\tan\alpha$

$\therefore$ Option (B) is correct Answer.

Q.11 Find the range of $f(x) = \frac{x^2+x+2}{x^2+x+1}$ is – **[IIT Scr.2003]**

(A) $(1, \infty)$ (B) $\left(1, \frac{11}{7}\right)$

(C) $\left(1, \frac{7}{3}\right)$ (D) $\left(1, \frac{7}{5}\right)$

Sol. **[C]**

$f(x) = y = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow x^2y + yx + y = x^2 + x + 2$

$\Rightarrow x^2(y-1) + x(y-1) + (y-2) = 0$

$\Rightarrow x = \frac{-(y-1) \pm \sqrt{(y-1)^2 - 4(y-1)(y-2)}}{2\times(y-1)}$; $y \neq 1$

$\Rightarrow x = \frac{(1-y) \pm \sqrt{(y-1)^2 + 4(1-y)(y-2)}}{2\times(y-1)}$; $y \neq 1$

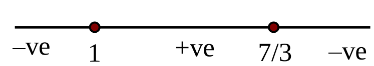
$(y-1)^2 + 4(1-y)(y-2) > 0$

$(y-1)[(y-1) - 4(y-2)] > 0$

$(y-1)[y - 1 - 4y + 8] > 0$

$\Rightarrow (y-1)(7-3y) > 0$

$\Rightarrow (y-1)(7-3y) > 0$



$\Rightarrow 1 < y < 7/3$

$\therefore$ Range $\in (1, 7/3)$

$\therefore$ Option (C) is correct Answer.

Q.12 Domain of $f(x) = \sqrt{\sin^{-1}(2x) + \pi/6}$ is–

[IIT Scr.2003]

(A) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (D) $\left[-\frac{1}{2}, \frac{1}{4}\right]$

Sol. **[A]**

$f(x) = \sqrt{\sin^{-1}(2x) + \pi/6}$

f(x) to be defined if

$\sin^{-1}(2x) + \pi/6 \geq 0$ and $-1 \leq 2x \leq 1$

$\Rightarrow 2x \geq -\frac{1}{2}$ and $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$\Rightarrow x \geq -\frac{1}{4}$ and $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

$\therefore$ Option (A) is correct Answer.

Q.13 Let f(x) = sin x + cos x and g(x) = $x^2 - 1$, then g (f(x)) will be invertible for the domain-

[IIT Scr.2004]

(A) $x \in [0, \pi]$ (B) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(C) $x \in \left[0, \frac{\pi}{2}\right]$ (D) $x \in \left[-\frac{\pi}{2}, 0\right)$

Sol. **[B]**

f(x) = sinx + cosx and g(x) = $x^2 - 1$

$g(f(x)) = (\sin x + \cos x)^2 - 1$

$= 1 + \sin 2x - 1 = \sin 2x$

$g(f(x)) = \sin 2x$

g(f(x)) to be defined if $-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

$\therefore$ Option (B) is correct Answer.

Q.14 $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \notin Q \end{cases}$; $g(x) = \begin{cases} 0 & x \in Q \\ x & x \notin Q \end{cases}$

then (f – g) is **[IIT Scr.2005]**

(A) one-one, onto

(B) neither one-one, nor onto

(C) one-one but not onto

(D) onto but not one-one

Sol. **[A]**

$f(x) = \begin{cases} x, & x \in Q \\ 0, & x \notin Q \end{cases}$; $g(x) = \begin{cases} x, & x \in Q \\ 0, & x \notin Q \end{cases}$

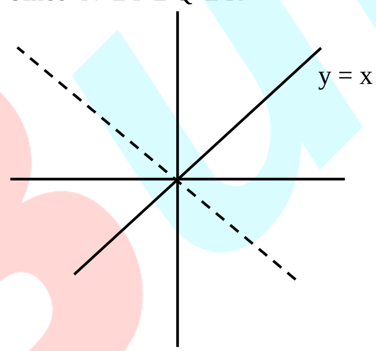
$f(x) - g(x) = (f - g)(x) = \begin{cases} x \; ; \; x \in Q \\ -x \; ; \; x \notin Q \end{cases}$

Q is set of Rational Numbers

$x \notin Q$ i.e., $x \in Q^C$

$Q^C \rightarrow$ Set of Irrational Numbers

Since $N \subset I \subset Q \subset R$



For $x \in Q^C$, y = –x

would be like points, not a continuous graph. Hence, consider only y = x. Draw any line parallel to x-axis meet only at one point. Hence, it is one-one as well as onto

$\therefore$ Option (A) is correct Answer.

Q.15 If X and Y are two non-empty sets where $f : X \rightarrow Y$ is function is defined such that

$f(C) = \{f(x): x \in C\}$ for $C \subseteq X$

and $f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$

for any $A \subseteq Y$ and $B \subseteq Y$ then- **[IIT 2005]**

(A) $f^{-1}(f(A)) = A$

(B) $f^{-1}(f(A)) = A$ only if $f(X) = Y$

(C) $f(f^{-1}(B)) = B$ only if $B \subseteq f(x)$

(D) $f(f^{-1}(B)) = B$

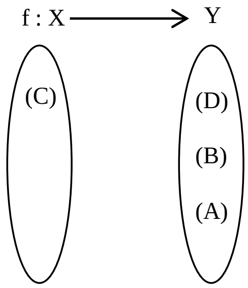
Sol. **[C]**

$f : X \rightarrow Y$

$f(C) = \{f(x) : x \in C\}$ for $C \subseteq X$

and $f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$

for any $A \subseteq Y$ and $B \subseteq Y$



According to given statement, f(A) does not exist
f^{-1}(B) exist
So, only possibility (C) or (D)
But (C) is most appropriate answer according to statement.

Q.16 Find the range of values of t for which

$$2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}\ ;\ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

[IIT 2005]

(A) $\frac{3\pi}{10} \le t \le \frac{\pi}{2}$ (B) $\frac{-3\pi}{10} \le t \le \frac{\pi}{2}$

(C) $\frac{\pi}{2} \le t \le \frac{\pi}{10}$ (D) None of these

Sol.[A] $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}\ ;\ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$6\sin t\, x^2 - 4\sin t\, x - 2\sin t = 1 - 2x + 5x^2$

$(6\sin t - 5)x^2 + x(2 - 4\sin t) - (1 + 2\sin t) = 0$

$(6\sin t - 5)x^2 - (4\sin t - 2)x - (1 + 2\sin t) = 0$

$$x = \frac{(4\sin t-2) \pm \sqrt{(4\sin t-2)^2 + 4(6\sin t-5)(1+2\sin t)}}{2\times(6\sin t-5)}\ ;\ \sin t \ne \frac{5}{6}$$

x to be defined if

$(4\sin t - 2)^2 + 4(6\sin t - 5)(1 + 2\sin t) \ge 0$

$(2\sin t - 1)^2 + (6\sin t - 5)(1 + 2\sin t) \ge 0$

$4\sin^2 t + 1 - 4\sin t + 6\sin t - 5 + 12\sin^2 t - 10\sin t \ge 0$

$16\sin^2 t - 8\sin t - 4 \ge 0$

$4\sin^2 t - 2\sin t - 1 \ge 0$

$\sin t = \frac{2 \pm \sqrt{4+16}}{2\times 4}$

$\sin t = \frac{1\pm\sqrt{5}}{4}$

$\Rightarrow \left(\sin t - \frac{1-\sqrt{5}}{4}\right)\left(\sin t - \frac{1+\sqrt{5}}{4}\right) \ge 0$

Either $\sin t \le \frac{1-\sqrt{5}}{4}$ or $\sin t \ge \frac{1+\sqrt{5}}{4}$

i.e., $t \le -\frac{\pi}{10}$ or $t \ge \frac{3\pi}{10}$

i.e. $-\frac{\pi}{2} \le t \le -\frac{\pi}{10}$ or $\frac{3\pi}{10} \le t \le \frac{\pi}{2}$

Q.17 Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is **[IIT 2011]**

(A) $\pm\sqrt{n\pi},\ n \in \{0, 1, 2 \ldots\}$

(B) $\pm\sqrt{n\pi},\ n \in \{1, 2 \ldots\}$

(C) $\frac{\pi}{2} + 2n\pi,\ n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

(D) $2n\pi,\ n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Sol. [A] $gof(x) = gf(x) = g(x^2) = \sin x^2$
$go(gof(x)) = g(\sin x^2) = \sin(\sin x^2)$
$fo(gogof(x)) = f(\sin(\sin x^2)) = (\sin(\sin x^2))^2$
$\therefore (\sin(\sin x^2))^2 = \sin(\sin x^2)$
$\sin(\sin x^2)(\sin(\sin x^2) - 1) = 0$

$\sin(\sin x^2) = 0$ or $\sin(\sin x^2) = 1$

$\sin x^2 = n\pi$	$\sin x^2 = 2n\pi + \frac{\pi}{2}$
At $n = 0$	At $n = 0$
$\sin x^2 = 0$	$\sin x^2 = \frac{\pi}{2}$
$x^2 = n\pi$	Not possible

$x = \pm\sqrt{n\pi}$; $n \in \{0, 1, 2, \ldots\}$

Q.18 The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **[IIT 2012]**

(A) one-one and onto.

(B) onto but not one-one.

(C) one-one but not onto.

(D) neither one-one nor onto.

Sol. [B] Given $f : [0, 3] \to [1, 29]$
$f(x) = 2x^3 - 15x^2 + 36x + 1$
$f'(x) = 6x^2 - 30x + 36$
$= 6(x - 2)(x - 3)$
$f'(x) > 0$ if $x \in (0, 2)$

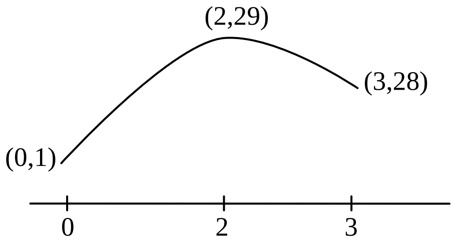
& $f'(x) < 0$ if $x \in (2, 3)$

$\therefore$ Function is many one & continuous

Now $f(0) = 1$

$f(2) = 29$

$\therefore$ Range = co-domain



Hence function is onto.

Q.19 Let $f : (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is (are) **[IIT 2012]**

(A) $1 - \sqrt{\dfrac{3}{2}}$ (B) $1 + \sqrt{\dfrac{3}{2}}$

(C) $1 - \sqrt{\dfrac{2}{3}}$ (D) $1 + \sqrt{\dfrac{2}{3}}$

Sol. [A,B]

$\cos 4\theta = \dfrac{1}{3}$

$2\cos^2 2\theta - 1 = \dfrac{1}{3}$

$\cos 2\theta = \pm\sqrt{\dfrac{2}{3}}$

On solving

$\cos^2\theta = \dfrac{1 \pm \sqrt{2/3}}{2}$

$\sec^2\theta = \dfrac{2}{1 \pm \sqrt{2/3}}$

On putting this

$f\left(\dfrac{1}{3}\right) = \pm\sqrt{\dfrac{3}{2}} + 1$

EXERCISE # 5

Q.1 Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is ....& out of these........ are onto functions. **[IIT-1985]**

Sol. Set A has n distinct elements.
Then to define a function from A to A, we need to associate each element of set A to any one of the n elements of set A. So total number of functions from set A to set A is equal to the number of ways of doing n jobs where each job can be done in n ways. The total number of such ways is $n \times n \times n \times \ldots \times n$ n-times.
Hence, the total number of functions from A to A is n^n.
Now for an onto function from A to A, we need to associate each element of A to one and only one element of A. So total number of onto functions from set A to A is equal to number of ways of arranging n elements in range (set A) keeping n elements fixed in domain (set A). n elements can be arranged in n ! ways.
Hence, the total number of onto function from A to A is n !.

Q.2 Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function 'f' satisfies the relation f(x + y) = f(x) f(y) for all natural numbers x, y and further f(1) = 2. **[IIT-1992]**

Sol. Let $f(n) = 2^n$ for all positive integers n
Now, for n = 1

$$f(1) = 2 = 2!$$

$\Rightarrow$ it is true for n = 1
Again let f(k) is true
$\Rightarrow f(k) = 2^k$ for some $k \in N$
Again f (k + 1) = f (k) . f(1) (by definition)
$= 2^k . 2$ (from induction assumption)
$= 2^{k+1}$
Therefore, the result is true for n = k + 1. Hence by principle of mathematical induction.
$f(n) = 2^n \ \forall\ n \in N$

Now $\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a) f(k) = f(a) \sum_{k=1}^{n} 2^k$

$= f(a) . \dfrac{2(2^n - 1)}{2-1}$

$= 2^a . 2(2^n - 1) = 2^{a+1}(2^n - 1)$

But $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1) = 2^4 . 2^n - 1$

Therefore. $a + 1 = 4 \Rightarrow a = 3$

Q.3 If a, b are positive real numbers such that a – b = 2, then find the smallest value of the constant L for which $\sqrt{x^2 + ax} - \sqrt{x^2 + bx} < L$ for all x > 0.

Sol. **[1]**

Q.4 A function f : R → R, where R, is the set of real numbers, is defined by $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

Find the interval of values of α for which f(x) is onto. Is the functions one-to-one for α = 3? Justify your answer. **[IIT 1996]**

Sol. $f : R \to R$

$f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

For onto, Range of f = Co-domain of f
Domain $\in$ R, Co-domain = Range $\in$ R

Let $y = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

$y\alpha + 6yx - 8yx^2 = \alpha x^2 + 6x - 8$
$(-8y - \alpha)x^2 + x(6y - 6) + y\alpha + 8 = 0$
$(8y + \alpha)x^2 - (6y - 6)x - (y\alpha + 8) = 0$
$(8y + \alpha)x^2 - 6(y - 1)x - (y\alpha + 8) = 0$

$$x = \frac{6(y-1) \pm \sqrt{36(y-1)^2 + 4(8y+\alpha)(y\alpha+8)}}{2 \times (8y+\alpha)}$$

$x \in R, \Rightarrow 8y + \alpha \neq 0$
$36(y - 1)^2 + 4(8y + \alpha)(y\alpha + 8) \geq 0$
$36(y^2 + 1 - 2y) + 4(8\alpha y^2 + y\alpha^2 + 64y + 8\alpha) \geq 0$
$36(y^2 - 2y + 1) + 4(8\alpha y^2) + 4y(\alpha^2 + 64) + 32\alpha \geq 0$
$y^2(36 + 32\alpha) + y[4\alpha^2 + 256 - 72] + (32\alpha + 36) \geq 0$
$(8\alpha + 9)y^2 + y[\alpha^2 + 64 - 18] + (8\alpha + 9) \geq 0$
$(8\alpha + 9)y^2 + y(\alpha^2 + 46) + (8\alpha + 9) \geq 0$
For $Ax^2 + Bx + C = 0$
if $A > 0$, $D \leq 0$
i.e., $B^2 - 4AC \leq 0$
$(\alpha^2 + 46)^2 - 4(8\alpha + 9). (8\alpha + 9) \leq 0$
i.e. $(\alpha^2 + 46)^2 \leq 4(8\alpha + 9)^2$
or $\alpha^2 + 46 \leq 2(8\alpha + 9)$
or $\alpha^2 + 46 \leq 16\alpha + 18$
or $\alpha^2 - 16\alpha + 28 \leq 0$

or $(\alpha - 2)(\alpha - 14) \le 0$

$2 \le \alpha \le 14$

$2 \in [2, 14]$

For $\alpha = 3$, $f(x) = \dfrac{3x^2 + 6x - 8}{3 + 6x - 8x^2}$

At $x = -1$, $f(-1) = \dfrac{3-6-8}{3-6-8} = 1$

At $x = 1$, $f(1) = \dfrac{3+6-8}{3+6-8} = 1$

$\therefore$ It is not one-one

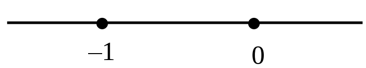
Q.5 Let $f(x) = [x] \sin\left(\dfrac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function. Then find the domain of f. **[IIT 1996]**

Sol. $f(x) = [x] \sin\left(\dfrac{\pi}{[x+1]}\right)$

$[x] \le x \Rightarrow [x + 1] \le x + 1$

$[x + 1] \ne 0$

$x \ne -1$



Domain of f is $x \in R$ except $[-1, 0)$

Q.6 If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ are.......,.......,.......... and........................... **[IIT-1996]**

Sol. Since f is an even function,

$f(-x) = f(x)\ \forall\ n \in (-5, 5)$

We are given that, $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ ……(1)

$f(-x) = f\left(\dfrac{-x+1}{-x+2}\right)$

$\Rightarrow f(-x) = f\left(\dfrac{-x+1}{-x+2}\right)$ [Θ $f(-x) = f(x)$]

taking f^{-1} of both sides

$x = \dfrac{-x+1}{-x+2}$

$\Rightarrow -x^2 + 2x = -x + 1$

$\Rightarrow x^2 - 2x = x - 1$

$\Rightarrow x^2 - 3x + 1 = 0$

$\Rightarrow x = \dfrac{3 \pm \sqrt{9-4}}{2} = \dfrac{3 \pm \sqrt{5}}{2}$

Also, $f(x) = f\left(\dfrac{x+1}{x+2}\right)$

$\Rightarrow f(-x) = f\left(\dfrac{x+1}{x+2}\right)$

Θ $f(-x) = f(x)$

taking f^{-1} of both sides, we get

$-x = \dfrac{x+1}{x+2} \Rightarrow -x^2 - 2x = x + 1$

$x^2 + 3x + 1 = 0$

$\Rightarrow x = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$

Therefore, four values of x are $\dfrac{\pm 3 \pm \sqrt{5}}{2}$

Q.7 Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if f(x) is an integer whenever x is an integer, then the number 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A + B and C are all integers then f(x) is an integer whenever x is an integer. **[IIT-1998]**

Sol. Suppose $f(x) = Ax^2 + Bx + C$ is an integer whenever x is an integer

$\therefore$ f(0), f(1), f(–1) are integers.

$\Rightarrow$ C, A + B + C, A – B + C are integers.

$\Rightarrow$ C, A + B, A – B are integers.

$\Rightarrow$ C, A + B, (A + B) – (A – B) = 2A are integers.

Conversely suppose 2A, A + B and C are integers.

Let n be any integer. We have

$f(n) = An^2 + Bn + C$

$= 2A\left[\dfrac{n(n-1)}{2}\right] + (A + B)n + C.$

Since n is an integer, n (n – 1) / 2 is an integers.

Also 2A, A + B and C are integers.

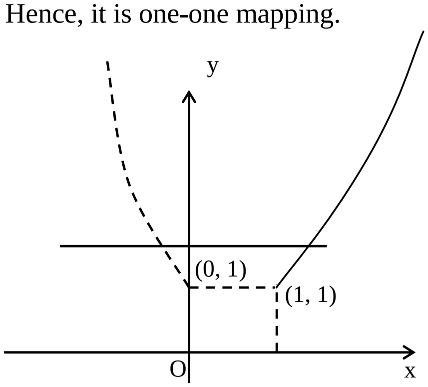
We get f (n) is an integer for all integer n.

Q.8 If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then find the value of $f^{-1}(x)$. **[IIT 99]**

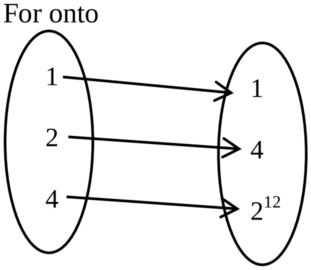
Sol. $f : [1, \infty) \to [1, \infty)$

$f(x) = 2^{x(x-1)} = 2^{x^2-x}$

If we draw any line ($y \geq 1$) Parallel to x-axis, then only one point of Intersection obtained Hence, it is one-one mapping.



For onto



one-one onto

$x = 1$; $y = 1$

$x = 2$; $y = 2^{4-2} = 4$

$x = 3$; $y = 2^{9-3} = 2^6 = 64$

$\therefore$ f^{-1} exists

$x^2 - x = \log_2 y \Rightarrow x^2 - x - \log_2 y = 0$

$$x = \frac{1 \pm \sqrt{1 + 4\log_2 y}}{2}$$

we take only +ve root,

$$x = \frac{1 + \sqrt{1 + 4\log_2 y}}{2}$$

$$f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_2 x}}{2}$$

Q.9 Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$-

(A) ≥ 0 only when $\theta \geq 0$

(B) ≤ 0 for all real θ

(C) ≥ 0 for all real θ

(D) ≤ 0 only when $\theta \leq 0$ **[IIT 2000]**

Sol. **[C]**

$f(\theta) = \sin\theta(\sin\theta + \sin3\theta)$

$f(\theta) = \sin^2\theta + \sin\theta(3\sin\theta - 4\sin^3\theta)$

$f(\theta) = \sin^2\theta + 3\sin^2\theta - 4\sin^4\theta$

$f(\theta) = 4\sin^2\theta - 4\sin^4\theta = 4\sin^2\theta(1 - \sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta$

$f(\theta) = (\sin2\theta)^2$

$-1 \leq \sin2\theta \leq 1 \Rightarrow \sin^2 2\theta \leq 1$

$f(\theta) = \sin^2 2\theta \leq 1$ for all real θ

$\Rightarrow f(\theta) \geq 0$ for all real θ

$\therefore$ Option (C) is correct Answer.

Q.10 Let $[x]$ = the greatest integer less than or equal to x. If all the values of x such that the product $\left[x - \frac{1}{2}\right]\left[x + \frac{1}{2}\right]$ is prime, belongs to the set $[x_1, x_2) \cup [x_3, x_4)$, find the value of $x_1^2 + x_2^2 + x_3^2 + x_4^2$.

Sol. **[11]**

Q.11 The set of real values of 'x' satisfying the equality $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$ (where [ ] denotes the greatest integer function) belongs to the interval $\left(a, \frac{b}{c}\right]$ where $a, b, c \in N$ and $\frac{b}{c}$ is in its lowest form. Find the value of $a + b + c + abc$.

Sol. **[20]**

Q.12 Let $f : R \to R - \{3\}$ be a function with the property that there exist $T > 0$ such that $f(x + T) = \frac{f(x) - 5}{f(x) - 3}$ for every $x \in R$. Prove that $f(x)$ is periodic.

Q.13 In a function

$$2f(x) = xf\left(\frac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\left(\pi\left(x + \frac{1}{4}\right)\right)\right|\right)$$

$$= 4\cos^2\frac{\pi x}{2} + x\cos\frac{\pi}{x}.$$ Prove that

(i) $f(2) + f(1/2) = 1$ (ii) $f(2) + f(1) = 0$

Q.14 Verify if $f(x) = \frac{x^2 - 8x + 18}{x^2 + 4x + 30}$ is an one-one function.

Sol. $f(x) = \frac{x^2 - 8x + 18}{x^2 + 4x + 30}$

Let $f(x) = f(y)$

$$\Rightarrow \frac{x^2 - 8x + 18}{x^2 + 4x + 30} = \frac{y^2 - 8y + 18}{y^2 + 4y + 30}$$

$\Rightarrow (x^2 - 8x + 18)(y^2 + 4y + 30)$

$= (x^2 + 4x + 30)(y^2 - 8y + 18)$

$\Rightarrow x^2y^2 - 8xy^2 + 18y^2 + 4x^2y - 32xy + 72y + 30x^2 - 240x + 540$

$= x^2y^2 + 4xy^2 + 30y^2 - 8x^2y - 32xy - 240y + 18x^2 + 72x + 540$

$\Rightarrow -8xy^2 + 8x^2y + 18y^2 - 18x^2 + 4x^2y - 4xy^2 + 72y - 72x + 30x^2 - 30y^2 - 240x + 240y = 0$

$\Rightarrow 8xy(x - y) + 18(y + x)(y - x) + 4xy(x - y) + 72(y - x) + 30(x - y)(x + y) + 240(y - x) = 0$

$\Rightarrow (x - y)[8xy - 18(y + x) + 4xy - 72 + 30(x + y)] = 0$

$\Rightarrow (x - y)[12xy + 12(x + y) - 72] = 0$

Either $x - y = 0$ and $12xy + 12(x + y) - 72 = 0$

or $x - y = 0$ and $12xy + 12(x + y) - 72 \neq 0$

or $x - y \neq 0$ and $12xy + 12(x + y) - 72 = 0$

Hence, by above method function may or may not be one-one

We can get exact result by drawing graph of given curve as –

$y = \dfrac{x^2 - 8x + 18}{x^2 + 4x + 30}$ as $x^2 + 4x + 30 \neq 0$

$x = 0$; $y = 3/5 = 0.6$

$x = 1$; $y = 11/35 = 0.31$

$x = 2$; $y = 6/42 = 1/7 = 0.14$

$x = 3$; $y = 1/17 \cong 0.06$

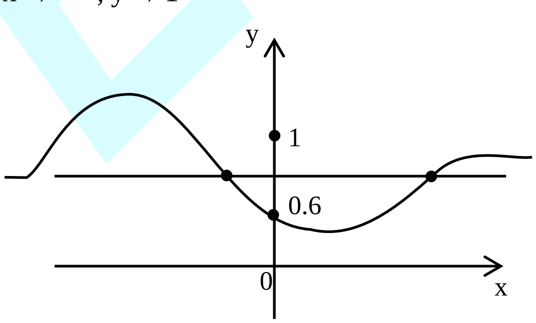
As $x \to +\infty$; $y \to 1$

$x = -1$; $y = \dfrac{27}{27} = 1$

$x = -2$; $y = \dfrac{19}{13} > 1$

$x = -3$; $y = \dfrac{17}{9} \cong 2$

$x \to -\infty$, $y \to 1$



If we draw any line parallel to x-axis. It cuts at more than one points. Hence, it is not one-one function.

Q.15 Find the domain of the function,

$f(x) = \dfrac{1}{|x-1| + |7-x| - 6}$ [·] is greatest integer function.

Sol. $f(x) = \dfrac{1}{[|x-1|] + [|7-x|] - 6}$

f(x) to be defined if $[|x - 1|] + [|7 - x|] - 6 \neq 0$

$\Rightarrow [|x - 1|] + [|7 - x|] \neq 6$

When $x < 1$

$x - 1 < 0$

$|x - 1| = -(x - 1) = 1 - x$

$x < 1 \Rightarrow -x > -1 \Rightarrow 7 - x > 7 - 1 \Rightarrow 7 - x > 6$

$\therefore |7 - x| = (7 - x)$

$[1 - x] + [7 - x] \neq 6$

$[x] \leq x$ & $x < 1$

$\therefore [x] < 1$

$\therefore [1 - x] + [7 - x] \neq 6$

$-x + 6 - x \neq 6 \Rightarrow x \neq 0$

$x \in (-\infty, 1) - \{0\}$

when $1 \leq x \leq 7 \Rightarrow |x - 1| = x - 1$

$x \leq 7 \Rightarrow -x \geq -7 \Rightarrow 7 - x \geq 0$

$|7 - x| = 7 - x$

$[x - 1] + [7 - x] \neq 6$; $1 \leq x \leq 7$

i.e. $0 \leq x - 1 \leq 6$

$[x - 1] = 0$

$x - 7 \leq 0 \Rightarrow 7 - x \geq 0$

$[7 - x] = (7 - x)$

$0 + 7 - x \neq 6$

$\Rightarrow x \neq 1$

when $x > 7$, $[x - 1] + [x - 7] \neq 6$

$x - 1 + x - 7 \neq 6 \Rightarrow 2x - 8 \neq 6$

$\Rightarrow 2x \neq 14$

$\Rightarrow x \neq 7$

$x \in (-\infty, 1] \cup [7, \infty) - \{0\} - \{1, 7\}$

$x \in R - \{0\} - \{1, 7\}$

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	D	C	D	A	C	A	D	C,D	D	B	A	B	C	A	A	B	A	D	A
Q.No.	**21**	**22**	**23**	**24**	**25**															
Ans.	A	D	B	A	A															

(26) False **(27)** False **(28)** False **(29)** True **(30)** 106!

EXERCISE # 2

(Part-A)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	D	B	B	D	C	D	C	B	B	D	A	A	D	B	B	A	B	D	A

(Part-B)

Q.No.	20	21	22	23	24	25	26	27
Ans.	A,B,D	A,B,C,D	A,B	A,B,C	A,B,C,D	A,B	A	B,C,D

(Part-C)

Q.No.	28	29	30	31
Ans.	A	D	A	B

(Part-D)

(32) $A \to R$, $B \to P$, $C \to S$, $D \to Q$

(33) $A \to Q$, $B \to S$, $C \to P$, $D \to R$

(34) $A \to S$, $B \to S$, $C \to S$, $D \to R$

(35) $A \to R$, $B \to Q,R$, $C \to Q$, $D \to S$

EXERCISE # 3

(1) (i) $\left[-\frac{5\pi}{4}, \frac{-3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ (ii) $\left(-4, -\frac{1}{2}\right) \cup (2, \infty)$ (iii) $(-\infty, -3]$

(iii) $(-\infty, -3]$ (iv) $(-\infty, -1) \cup [0, \infty)$ (v) $(3 - 2\pi < x < 3 - \pi) \cup (3 < x \le 4)$

(vi) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$ (vii) $(-1 < x < -1/2) \cup (x > 1)$ (viii) $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

(ix) $(-3, -1] \cup \{0\} \cup [1, 3)$ (x) (x) $(0, 1/4) \cup (3/4, 1) \cup \{x : x \in N, x \ge 2\}$

(xi) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$ (xii) $[-3, -2) \cup [3, 4)$ (xiii) $R - \left\{-\frac{1}{2}, 0\right\}$

(xiv) $2K\pi < x < (2K+1)\pi$ but $x \ne 1$ where K is non-negative integer.

(xv) $x \in \{4, 5\}$

(2) (i) D : $x \in R$ R : [0, 2] (ii) D = R ; range [–1, 1]

(iii) D : $\{x \mid x \in R; x \neq -3 ; x \neq 2\}$ R : $\{f(x) \mid f(x) \in R, f(x) \neq 1/5 ; f(x) \neq 1\}$

(iv) D : R ; R : (–1, 1) (v) D : $-1 \le x \le 2$; R : $[\sqrt{3}, \sqrt{6}]$

(vi) D : $x \in (2n\pi, (2n+1)\pi - \{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in I\}$ and

R : $\log_a 2$; $a \in (0, \infty) - \{1\} \Rightarrow$ Range is $(-\infty, \infty) - \{0\}$

(vii) D : $[-4, \infty) - \{5\}$; R : $\left(0, \frac{1}{6}\right) \cup \left(\frac{1}{6}, \frac{1}{3}\right]$ (viii) $[\pi/4, \pi)$ (ix) [1/2, 1]

(3) **(**b) (i), (iii) are identical. **(5)** $f(x) = \frac{x+1}{x-1}$ **(6)** (0, 5/3) **(7)** [0, 4)

(8) $\begin{cases} -x ; -2 \le x < 0 \\ 0 ; 0 \le x < 1 \\ 2(x-1) ; 1 \le x \le 2 \end{cases}$ **(9)** {1/4}

(10) $A = \left\{0, \log_3 7, \frac{129}{64}, \frac{33}{16}, \frac{9}{4}, 3, 5, 6\right\}$ and since f (x) is not bijective therefore $f^{-1} : B \to A$ is not possible.

(13) P(x) = x + 3, Q (x) = – (x + 2)

(15) $f_o(x) = -x \tan x$, $-\pi/2 \le x \le 0$; $-\pi/2\,[-x]$; $-2 \le x < -\pi/2$; $f_e(x) = x \tan x$; $-\pi/2 \le x \le 0$;
$\pi/2\,[-x]$, $-2 \le x < -\pi/2$

(16) $fog(x) = \begin{cases} -(1+x) & , -1 \le x \le 0 \\ x-1 & , 0 < x \le 2 \end{cases}$; $gof(x) = \begin{cases} x+1 & , 0 \le x < 1 \\ 3-x & , 1 \le x \le 2 \\ x-1 & , 2 < x \le 3 \\ 5-x & , 3 < x \le 4 \end{cases}$; $fof(x) = \begin{cases} x & , 0 \le x \le 1 \\ 4-x & , 3 \le x \le 4 \end{cases}$; $gog(x) = \begin{cases} -x & , -1 \le x \le 0 \\ x & , 0 < x \le 2 \\ 4-x & , 2 < x \le 3 \end{cases}$

(17) Domain : [1, 2]; Range : [λn 2, λn 4)

(18) (i) neither even nor odd (ii) even (iii) even (iv) even (v) odd

(19) (i) π (ii) 2 (iii) $\frac{2\pi}{3}$ (iv) 2π (v) $2^n \pi$ (vi) π **(20)** $f(x) = 2x^2$ **(21)** 6016

Q.No.	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**
Ans.	A	C	C	A	A,B,C,D	C	C	B	C	C	D	C	B	A

EXERCISE # 4

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	D	A	D	D	A	C	D	A	B	B
Q.No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	
Ans.	C	A	B	A	C	A	A	B	A,B	

EXERCISE # 5

(1) n^n, n! **(2)** a = 3 **(3)** 1 **(4)** $\alpha \in \phi$

(5) $\{x \in R \mid x \notin [-1, 0)\}$ **(6)** $\frac{\pm 3 \pm \sqrt{5}}{2}$ **(8)** $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$ **(9)** C **(10)** 11

(11) 20 **(14)** No **(15)** $R - (0, 1) \cup \{1, 2, 3, 4, 5, 6, 7\} \cup (7, 8)$