INTRODUCTION TO TRIGONOMETRY

TRIGONOMETRIC IDENTITIES

EXERCISE

Q.1 Prove the following identities :

(i) $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

(ii) $(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

Q.2 If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ prove that each of the side is equal to ± 1.

Q.3 If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Q.4 If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Q.5 If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$

Q.6 If $\sec\theta + \tan\theta = p$, show that $\frac{p^2-1}{p^2+1} = \sin\theta$.

Q.7 If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$ show that $(m^2 + n^2)\cos^2\beta = n^2$.

Q.8 If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

Q.9 If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

Q.10 Prove that : $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

Q.11 If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

Q.12 Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$

Q.13 Express the ratios cos A, tan A and sec A in terms of sin A.

Q.14 Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

ANSWER KEY

14. $\cos A = \sqrt{1 - \sin^2 A}$

$\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

Sec $A = \frac{1}{\sqrt{1 - \sin^2 A}}$