HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. Since sum of coefficients = 0

$\therefore$ It's one root is 1 and other root is $\dfrac{a-2b+c}{a+b-2c}$

4. For $(p^2-3p+2)x^2-(p^2-5p+4)x+p-p^2=0$ to be an identity

$p^2-3p+2=0 \Rightarrow p=1, 2$ ...(i)

$p^2-5p+4=0 \Rightarrow p=1, 4$ ...(ii)

$p-p^2=0 \Rightarrow p=0, 1$ ...(iii)

For (i), (ii) & (iii) to hold simultaneously
$p=1$.

5. x = 1 is root
Let other root = α

$\therefore$ Product of the roots $=(1)(\alpha)=\dfrac{a-b}{b-c}$

$\Rightarrow$ roots are $1, \dfrac{a-b}{b-c}$

6. $q^2-4p \ge 0$

$q=2 \Rightarrow p=1$

$q=3 \Rightarrow p=1, 2$

$q=4 \Rightarrow p=1, 2, 3, 4$

Hence 7 values of (p, q)
7 equations are possible.

14. for $x \ge 1$

$E=x^5(x^3-1)+(x-1)+1>0$

for $1<x<0$,

$E=(1-x)+x^2(1-x^3)+x^8>0$

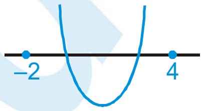
For $x<0$, all terms are positive $\Rightarrow$ >0 Hence A

16. $x^2-2mx+m^2-1=0$

(i) $f(-2)>0$

(ii) $f(4)>0$

(iii) $D \ge 0$



(iv) $-2<\dfrac{-b}{2a}<4$

Common solution $m \in (-1, 3)$

17. $(x^2+4)^2=(2x-3)^2$

$\Rightarrow x^2+4=\pm(2x-3)$

$\Rightarrow x^2+2x+1=0$ or $\underbrace{x^2-2x+7}_{D<0}=0$

$\Rightarrow (x+1)^2=0$ or No solution

$\Rightarrow x=-1$

Have only one solution.

25. $\dfrac{\sum\alpha\beta\gamma}{\alpha\beta\gamma\delta}=\dfrac{-4}{10}=\dfrac{-2}{5}$

26. Let roots of $x^3-Ax^2+Bx-C=0$ are α, β, γ

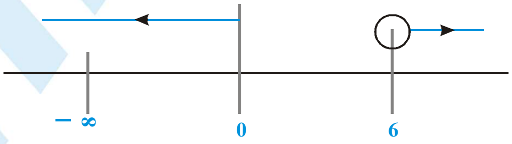
$\Rightarrow \alpha+\beta+\gamma=A, \ \Sigma\alpha\beta=B, \ \alpha\beta\gamma=C$

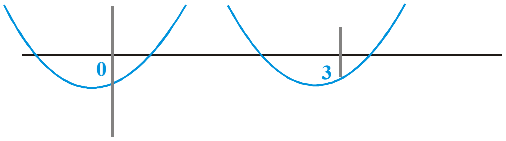
& $(\alpha+1)(\beta+1)(\gamma+1)=19$

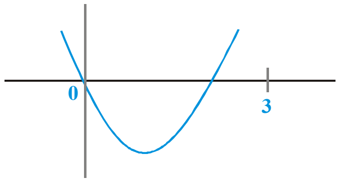
$\Rightarrow (\alpha+\beta+\gamma)+(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma+1=19$

$A+B+C=18$

28. $f(0) \,.\, f(3)<0$ check end points separately







29. For integral roots, D of equation should be perfect sq.

✈ $4D=4(1+n)$

By observation, for n N, D should be perfect sq. of even integer.

So $D=4(1+n)=6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$

No. of values of n = 8.

Part # II : Assertion & Reason

3. $ax^3+bx+c=0$ — α, β, γ $\quad \alpha \ge 0, \quad \beta \ge 0,$

$\gamma \ge 0 \qquad \alpha+\beta+\gamma=0 \qquad \Rightarrow \quad \alpha=0,$

$\beta=0\ \gamma=0 \ \Rightarrow \quad f(x)=ax^3$

4. $a_1+a_2+a_3+a_4+a_5=0$

$\Rightarrow$ 1 is one of the root of the equation & degree of equation is 4 & complex roots occur in conjugate

$\Rightarrow$ at least 2 real roots.

5. Suppose α, β be non real

$\bar{\alpha}=\beta$

$\alpha=x+iy$

(imaginary roots appear in conjugate pairs)

$\beta=x-iy=\bar{\alpha}$

$$\frac{1}{\beta}=\frac{x+iy}{x^2+y^2}$$

$$\bar{\alpha}=\frac{1}{\beta}\Rightarrow x-iy=\frac{x+iy}{x^2+y^2}$$

$\Rightarrow$ $x=0$ or $y=0$

$\Rightarrow$ either both equation have real root or both imaginary roots

$\Rightarrow$ $D_1D_2\ge 0$ $\Rightarrow$ statement 1 is true

$\alpha\beta=q$ (from 1st given equation)

$\frac{\alpha}{\beta}=\frac{c}{a}$ (from 2nd given equation)

$\beta^2=\frac{aq}{c}$; also $p^2\neq 1$

$\Rightarrow$ $c\neq aq$;

$\alpha+\beta=-2p$; $\quad \alpha+\frac{1}{\beta}=-\frac{2b}{a}$

$$\frac{2b}{a}-2p=\beta-\frac{1}{\beta}=\frac{\beta^2-1}{\beta}\neq 0$$

$\Rightarrow$ $b\neq pa$ $\Rightarrow$ statement 2 is true

6. $a>b>c \quad \Rightarrow \quad$ a, b, c, are distinct real also

$a^3+b^3+c^3-3abc=0$

$$\left(\frac{a+b+c}{2}\right)[(a-b)^2+(b-c)^2+(c-a)^2]=0$$ as a, b, c are distinct

$\therefore \quad a+b+c=0$

hence $x=1$ is a root of $ax^2+bx+c=0$

a+b+c=0 and a >b>c $\Rightarrow$ a and c are of opposite sign

otherwise a+b+c $\neq$ 0 therefore $\frac{c}{a}$ negative.

9. $f(x)=a(x+1)(x-\beta)$ (as -1 is root)

$f(1)+f(2)=2a(1-\beta)+3a(2-\beta)=0$

$=a(8-5\beta)=0$ as $a\neq 0 \Rightarrow \beta=\frac{8}{5}$

$$=\left(x+\frac{1}{x}\right)^3-\left(x^3+\frac{1}{x^3}\right)=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)-\left(x^3+\frac{1}{x^3}\right)$$

$$=3\left(x+\frac{1}{x}\right);$$

hence $y_{min}=6$ as $x+\frac{1}{x}\ge 2$ for $\forall$ $x>0$

(C) Since P (x) divides into both of them

hence P (x) also divides

$(3x^4+4x^2+28x+5)-3(x^4+6x^2+25)$

$=\ -14x^2+28x-70=-14(x^2-2x+5)$

which is a quadratic.

Hence $P(x)=x^2-2x+5$

$\therefore\ P(1)=4$

Alternatively :

$x^4+6x^2+25=(x^4+10x^2+25)-4x^2$

$=(x^2-2x+5)(x^2+2x+5)$

Hence P (x) can be x^2-2x+5 or x^2+2x+5

by using long division we find that only x^2-2x+5 is a factor of $3x^4+4x^2+28x+5$ which is equal to

$(x^2-2x+5)(3x^2+6x+1)$

$\therefore\ P(x)=x^2-2x+5$

$\Rightarrow\ P(1)=4$]

EXERCISE - 4

Subjective Type

1. $x^2-3x+2=0$

3. $x^2+18x+45-2\sqrt{x^2+18+45}+1=16$

$\Rightarrow\ \left(\sqrt{x^2+18x+45}-1\right)^2=16$

$\Rightarrow\ \sqrt{x^2+18x+45}-1=\pm 4$

$\Rightarrow\ \sqrt{x^2+18x+45}=\pm 4+1=5,-3$

$\Rightarrow\ x^2+18x+45=25,$ (Reject -3)

$\Rightarrow\ x^2+18x+20=0$

Product of root $=+20$.

4. Since the equation has unequal real roots, the discriminant is positive, that is

$4(a+b)^2>4(a-b+8)$

$\Rightarrow\ a^2+2ab+b^2>a-b+8$

$\Rightarrow\ a^2+(2b-1)a+(b^2+b-8)>0$

$\therefore$ Discriminant should be negative

$\Rightarrow\ (2b-1)^2<4(b^2+b-8)$

$\Rightarrow\ 4b^2-4b+1<4b^2+4b-32$

$\Rightarrow\ 33<8b$

$\therefore\ b>\frac{33}{8}$

Hence, smallest natural number b = 5.

5. $x\le\frac{-2}{3},\ \frac{1}{2}\le x\le 2$

6. Considering denominator $x^2-8x+32$

$D<0$ and $a>0$

So denominator is always positive

$\Rightarrow ax^2+2(a+1)x+9a+4<0$

$\Rightarrow a<0\ \&\ 4(a+1)^2-4a(9a+4)<0$

$\Rightarrow 4(a^2+2a+1-9a^2-4a)<0$

$\Rightarrow\ 4(-8a^2-2a+1)<0$

$8a^2+2a-1>0$

$(4a-1)(2a+1)>0$

$\Rightarrow\ a\in\left(-\infty,-\frac{1}{2}\right)$

Let other roots be β and δ

then $\alpha + \beta = -p$, $\alpha\beta = q$

$\alpha + \delta = -q$, $\alpha\delta = p$

$$\beta - \delta = q - p, \quad \frac{\beta}{\delta} = \frac{q}{p} \quad \Rightarrow \quad \frac{\beta - \delta}{\delta} = \frac{q - p}{p}$$

$$\frac{q-p}{\delta} = \frac{q-p}{p} \qquad \delta = p$$

$$\beta = q$$

Equation having β, δ as roots

$x^2 - (\beta + \delta)\,x + \beta\delta = 0$

$x^2 - (p + q)\,x + pq = 0$

$x^2 + x + pq = 0 \qquad [\,p + q = -1\,]$

25. $x^3 + px^2 + qx + r \begin{array}{l} \nwarrow \alpha \\ \leftarrow \beta \\ \swarrow \gamma \end{array}$

$\alpha\beta\gamma = -r$

$$\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right) =$$

$$\left(\alpha + \frac{\alpha}{r}\right)\left(\beta + \frac{\beta}{r}\right)\left(\gamma + \frac{\gamma}{r}\right) = \alpha\beta\gamma\left(1 + \frac{1}{r}\right)^3 = -r\,\frac{(r+1)^3}{r^3}$$

$$= -\frac{(r+1)^3}{r^2}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

2. $(x - a)\,(x - b) - c = (x - \alpha)\,(x - \beta)$

$(x - \alpha)\,(x - \beta) + c = (x - a)\,(x - b)$

so $(x - \alpha)\,(x - \beta) + c = 0$ have roots a, 6

4. Let roots α, 2α

$$3\alpha = \frac{3a - 1}{a^2 - 5a + 3}$$

$$2\alpha^2 = \frac{2}{a^2 - 5a + 3}$$

$$\frac{2\alpha^2}{9\alpha^2} = \frac{2}{a^2 - 5a + 3}\,\frac{(a^2 - 5a + 3)^2}{(3a - 1)^2}$$

$$\frac{2}{9} = \frac{2(a^2 - 5a + 3)}{(3a - 1)^2}$$

$9a^2 - 45a + 27 = 9a^2 - 6a + 1$

$39a = 26$

$$\boxed{a = \frac{2}{3}}$$

5. $\alpha + \beta = \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ (given)

$$\alpha + \beta = \frac{(\alpha^2 + \beta^2)}{\alpha^2\beta^2}$$

$$(\alpha + \beta) = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$-\,bc^2 = ab^2 - 2a^2c \quad \Rightarrow \quad bc^2 + ab^2 = 2a^2c$

$$\frac{c}{a} + \frac{b}{c} = \frac{2a}{b}$$

So

$\dfrac{c}{a}, \dfrac{a}{b}, \dfrac{b}{c}$ ... A.P.

$\dfrac{a}{c}, \dfrac{b}{a}, \dfrac{c}{b}$ ... H.P.

7. $x + \dfrac{1}{x} \geq 2$

✈ AM ≥ GM

$x + \dfrac{1}{x}$ is min at $x = 1$

From (i) $q = p + r$

$(p + r)^2 - 4pr = 0$

$(p - r)^2 = 0$

$p = r$

from eq. (i) $q = 2r$

So from eq. (ii) $4r - 4r + r = 2$

$r = 2$

So $4p + 2q + r = 4r + 4r + r = 9r = 18$

24. Given $e^{\sin x} - e^{-\sin x} = 4$

let $e^{\sin x} = y$

$$y - \frac{1}{y} = 4$$

$\Rightarrow$ $y^2 - 4y - 1 = 0$

$y = 2 \quad \sqrt{5}$

$e^{\sin x} = 2 + \sqrt{5}$ $\qquad$ $e^{\sin x} = 2 - \sqrt{5}$

but we know that

$e^{-1} \le e^{\sin x} \le e^{1}$

so $e^{\sin x} \ne 2 + \sqrt{5}$ and $2 - \sqrt{5}$

so No real solution of given equation.

27. $x^2 - 5x + 5 = 1$

$\Rightarrow$ $x = 1, 4$

or $x^2 - 5x + 5 = -1$

$\Rightarrow$ $x = 2, 3$

or $x^2 + 4x - 60 = 0$

$\Rightarrow$ $x = -10, 6$

$\therefore$ $x = 3$ will be rejected as L.H.S. becomes -1

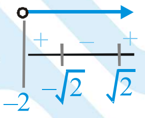
So, sum of value of $x = 1 + 4 + 2 - 10 + 6 = 3$

Part # II : IIT-JEE ADVANCED

2. $x^2 - |x + 2| + x > 0$

Case-I : $x + 2 \ge 0$ $\Rightarrow$ $x^2 - x - 2 + x > 0$

$\Rightarrow$ $x^2 - 2 > 0$



$\Rightarrow$ $x \in \left[-2, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

Case-II : $x + 2 < 0$

$x^2 + x + 2 + x > 0$ $\Rightarrow$ $x^2 + 2x + 2 > 0$

$\Rightarrow$ $x < -2$ is solution

$\Rightarrow$ $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

6. **(B)** $x^2 - 10cx - 11d = 0 \begin{array}{l} \nearrow a \\ \searrow b \end{array}$

$x^2 - 10ax - 11b = 0 \begin{array}{l} \nearrow c \\ \searrow d \end{array}$

$a + b = 10c$ ......**(i)**

& $c + d = 10a$ ......**(ii)**

add **(i)** & **(ii)**

$\Rightarrow$ $a + b + c + d = 10(a + c)$

subract **(i)** & **(ii)**

$(a - c) + (b - d) = 10(c - a)$

$\Rightarrow$ $b - d = 11(c - a)$ ......**(iii)**

also $a^2 - 10ca - 11d = 0$ ......**(iv)**

$c^2 - 10ac - 11b = 0$ ......**(v)**

from **(iv)** & **(v)**

$\Rightarrow$ $a^2 - c^2 = 11(d - b)$

$(a - c)(a + c) = 11(d - b)$

$\Rightarrow$ $(a + c) = 121$ (from **(iii)**)

and $a + b + c + d = 10\,(a + c)$

$= 121 \times 10 = 1210$

7. **(A)** $x^2 - px + r = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$ $\quad x^2 - qx + r \begin{array}{l} \nearrow \alpha/2 \\ \searrow 2\beta \end{array}$

$\alpha + \beta = p, \quad \frac{\alpha}{2} + 2\beta = q \quad \Rightarrow \quad \alpha + 4\beta = 2q$

$\alpha\beta = r$

$\Rightarrow$ $3\beta = (2q - p)$

$\Rightarrow$ $\beta = \frac{2q - p}{3}$

and $\alpha = p - \frac{(2q - p)}{3} = \frac{4p - 2q}{3}$

$r = \alpha\beta = \frac{2}{9}\,(2p - q)\,(2q - p)$

8. $x^2 + 2px + q = 0$

then $\alpha + \beta = -2p$ & $\alpha\beta = q$

and $ax^2 + 2bx + c = 0$

$\alpha + \frac{1}{\beta} = -\frac{2b}{a}$ & $\frac{\alpha}{\beta} = \frac{c}{a}$

4. (C)

Given $bx^2 + cx + a = 0$ has imaginary roots

$\Rightarrow$ $c^2 - 4ab < 0$

$\Rightarrow$ $c^2 < 4ab$

$\Rightarrow$ $-c^2 > -4ab$ ....(i)

Let $f(x) = 3b^2x^2 + 6bcx + 2c^2$

Here, $3b^2 > 0$

So, the given expression has a minimum value

$\therefore$ Minimum value $= \dfrac{-D}{4a}$

$$= \frac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)} = -\frac{12b^2c^2}{12b^2} = -c^2 > -4ab$$

[From eq. (i)]

5. $(x^2 + bx + c).P(x) = 3x^4 + 18x^2 + 75$ ....(i)

$(x^2 + bx + c).Q(x) = 3x^4 + 4x^2 + 28x + 5$ ....(ii)

equation (i) – (ii)

$(x^2 + bx + c)P(x) - Q(x) = 14x^2 - 28x + 70$

$= 14(x^2 - 2x + 5)$

$x^2 + bx + c = x^2 - 2x + 5$

hence $f(x) = x^2 - 2x + 5$

$= (x - 1)^2 + 4$

$\min(f(x)) = 4$

6. (C)

$$\frac{ax^2}{(1-x)^2} + \frac{bx}{1-x} + c = 0 \qquad \text{.........(i)}$$

If $t = \alpha$, then $t = \dfrac{x}{1-x}$ $\Rightarrow$ $\alpha = \dfrac{x}{1-x}$

$\Rightarrow$ $x = \dfrac{\alpha}{\alpha+1}$

$\Rightarrow$ roots of (i) are $\left(\dfrac{\alpha}{1+\alpha}, \dfrac{\beta}{1+\beta}\right)$

7. Dis. of $x^2 + px + 3q$ is $p^2 - 12q \equiv D_1$

Dis. of $-x^2 + rx + q$ is $r^2 + 4q \equiv D_2$

Dis. of $-x^2 + sx - 2q$ is $s^2 - 8q \equiv D_3$

Case 1 : If $q < 0$, then $D_1 > 0$, $D_3 > 0$ and D_2 may or may not be positive

Case 2 : If $q > 0$, then $D_2 > 0$ and D_1, D_3 may or may not be positive

Case 3 : If $q = 0$, then $D_1 \geq 0$, $D_2 \geq 0$ and $D_3 \geq 0$

from Case 1, Case 2 and Case 3 we can say that the given equation has at least two real roots.

8. (B)

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

$$x_1 = \frac{\alpha+\beta}{2}, x_2 = \frac{2\alpha\beta}{\alpha+\beta}$$

So equation whose roots are x_1 and x_2 is

$$x^2 - x\left(\frac{\alpha+\beta}{2} + \frac{2\alpha\beta}{\alpha+\beta}\right) + \alpha\beta = 0$$

$\Rightarrow$ $x^2 + x\left(\dfrac{b^2 + 4ac}{2ab}\right) + \dfrac{c}{a} = 0$

$\Rightarrow$ $2ab\,x^2 + (b^2 + 4ac)\,x + 2bc = 0$

9. Given equation can be written as

$$\frac{x-a}{b} - \frac{b}{x-a} + \frac{x-b}{a} - \frac{a}{x-b} = 0$$

$\Rightarrow$ $\dfrac{(x-a)^2 - b^2}{b(x-a)} + \dfrac{(x-b)^2 - a^2}{a(x-b)} = 0$

$\Rightarrow$ $(x - a - b)\left[\dfrac{x-a+b}{b(x-a)} + \dfrac{x-b+a}{a(x-b)}\right] = 0$

$\Rightarrow$ $(x - a - b)$

$$\left\{\frac{a\left[x^2 - bx - ax + ab + bx - b^2\right] + b\left[x^2 - ax - bx + ab + ax - a^2\right]}{ab(x-a)(x-b)}\right\}$$

$= 0$

$\Rightarrow$ $(x - a - b)(ax^2 - a^2x + a^2b - ab^2 + bx^2 - b^2x + ab^2 - a^2b)$

$= 0$

$\Rightarrow$ $x(x - a - b)\{x(a + b) - (a^2 + b^2)\} = 0$

$\therefore$ roots will be $x = 0, a + b, \dfrac{a^2 + b^2}{a + b}$

Let $x_1 = a+b$, $x_2 = \dfrac{a^2 + b^2}{a + b}$ and $x_3 = 0$

[✈ given $x_1 > x_2 > x_3$]

✈ $x_1 - x_2 - x_3 = c$ (given)

$\therefore$ $(a + b) - \dfrac{a^2 + b^2}{a + b} - 0 = c$

$\Rightarrow$ $\dfrac{(a+b)^2 - (a^2 + b^2)}{a + b} = c$

$\Rightarrow$ $\dfrac{2ab}{a + b} = c$

i.e. a, c, b are in H.P.

16. (D)

Statement-I : Let $f(x) = (x-p)(x-r) + \lambda(x-q)(x-s)$,

$f(p) = \lambda(p-q)(p-s)$, $f(q) = (q-p)(q-r)$,

$f(s) = (s-p)(s-r)$

and $f(r) = \lambda(r-q)(r-s)$

If $\lambda > 0$ then $f(p) > 0$, $f(q) < 0$, $f(r) < 0$ and $f(s) > 0$

$\Rightarrow$ $f(x) = 0$ has one real root between p and q and other real root between r and s

Statement-II : Obviously true

17. (B)

Statement-I : Given equation $x^2 - bx + c = 0$

Let α, β two roots such that $|\alpha - \beta| = 1$

$\Rightarrow$ $(\alpha+\beta)^2 - 4\alpha\beta = 1$

$\Rightarrow$ $b^2 - 4c = 1$

Statement-II : Given equation

$4abc\,x^2 + (b^2-4ac)\,x - b = 0$

$D = (b^2-4ac)^2 + 16\,ab^2c$

$D = (b^2+4ac)^2 > 0$

Hence roots are real and unequal

18. (A)

$x = \sqrt{5} - \sqrt{2}$ squaring both sides

$x^2 = 5 + 2 - 2\sqrt{10}$

$(x^2-7)^2 = 40$

$x^4 - 14x^2 + 49 = 40$

$x^4 - 14x^2 + 9 = 0$

For polynomial equation with rational co-efficients irrational roots occurs in pairs.

19. (A)

In **Statement – 1 :** $a - b + b - c + c - a = 0$

$\Rightarrow$ If quadratic equation $ax^2 + bx + c = 0$ have one root $x = 1$ then $a + b + c = 0$

$\Rightarrow$ sum of co-efficients = 0

20. (D)

Statement 1 : $a^2 - 3a + 2 = 0$

$\Rightarrow$ $a = 1, 2$, $a^2 - 5a + 6 = 0$

$\Rightarrow$ $a = 2, 3$, $a^2 - 4 = 0$

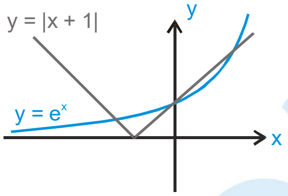
$\Rightarrow$ $a = \pm 2$

a = 2 is the only solution. Hence statement 1 is false

Statement 2 : is true by definition.

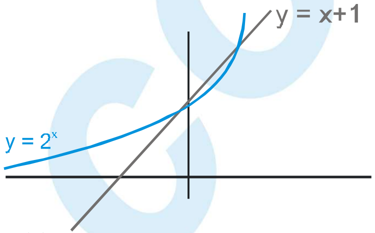
21. (A)$\rightarrow$(q), (B)$\rightarrow$(p), (C)$\rightarrow$(t), (D)$\rightarrow$(r)

(A)



Number of solutions is 3

(B) $2^x - x - 1 = 0$



consider $2^x = x + 1$

$\therefore$ there are two solutions

$x = 0, 1$ (both are non-negative)

(C) $p + q = \alpha - 2$

$pq = -\alpha - 1$

$\therefore$ $p^2 + q^2 = (\alpha-2)^2 - 2(-\alpha-1)$

$= \alpha^2 - 4\alpha + 4 + 2\alpha + 2$

$= \alpha^2 - 2\alpha + 6 = (\alpha-1)^2 + 5$

$\therefore$ least value of $p^2 + q^2 = 5$

(D) $\alpha + \beta = -\frac{7}{2}$, $\alpha\beta = \frac{c}{2}$

$$\therefore \quad \frac{7}{4} = |\alpha^2 - \beta^2| = |(\alpha+\beta)|\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \frac{7}{2}$$

$$\sqrt{\frac{49}{4} - 2c} = \frac{7}{4}\sqrt{49-8c}$$

$49 - 8c = 1 \quad \Rightarrow \quad c = 6$

22. (A)$\rightarrow$(p, q, r, t) ; (B)$\rightarrow$(r, t) ; (C)$\rightarrow$(p) ; (D)$\rightarrow$(p)

(A) Let $f(x) = x^3 - 6x^2 + 9x + \lambda$

$\therefore$ $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

$\therefore$ $f'(x) < 0$ in $(1, 3)$

But $f(1) = 4 + \lambda$ and $f(3) = \lambda$

for $f(x) = 0$ to have exactly one root in $(1, 3)$

$\Rightarrow$ f(1) and f(3) should have oppposite signs

$\therefore$ $f(1)\,f(3) < 0$

$\Rightarrow$ $\lambda(\lambda+4) < 0$ $\Rightarrow$ $-4 < \lambda < 0$

$\therefore$ $-3 < \lambda + 1 < 1$

$\Rightarrow$ $[\lambda + 1] = -3, -2, -1, 0$

2 (B) Minimun value of $y=\frac{x^2}{2\sqrt{2}}-2\sqrt{2}$ is at $x=0$

i.e $-2\sqrt{2}$

3 (C) roots of $f(x)=0$

i.e $\frac{x^2}{2\sqrt{2}}-2\sqrt{2}=0$ are $x=\pm2\sqrt{2}$

∴ number of integral value of k for which $\frac{k}{2}$ lies

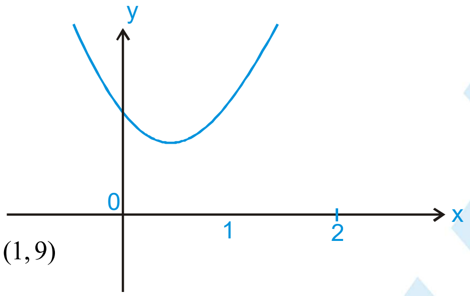
in $\left(-2\sqrt{2},2\sqrt{2}\right)$ are 11.

26. $f(x)=x^2-(m-3)x+m>0 \quad \forall x\in[1,2]$

Here $D=(m-3)^2-4m=m^2-10m+9=(m-1)(m-9)$

All possible graphs are

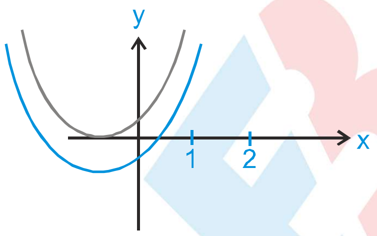
Case 1 :



$D<0$

$\Rightarrow m\in(1,9)$

Case 2 :



(i) $f(1)>0 \quad \Rightarrow \quad 4>0$ always true

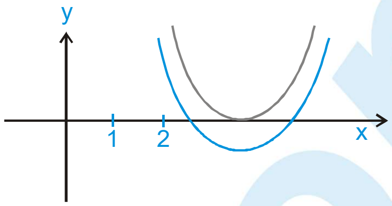
(ii) $-\frac{b}{2a}<1 \quad \Rightarrow \quad m<5$

(iii) $D\geq0 \quad \Rightarrow \quad m\in(-\infty,1]\cup[9,\infty)$

∴ **(i)** ∩ **(ii)** ∩ **(iii)**, we get $m\in(-\infty,1]$

Case 3 :

(i) $f(2)>0 \quad \Rightarrow \quad m<10$



(ii) $\frac{-b}{2a}>2 \Rightarrow m>7$

(iii) $D\geq0 \quad \Rightarrow \quad m\in(-\infty,1]\cup[9,\infty)$

∴ **(i)** ∩ **(ii)** ∩ **(iii)**, we get $m\in[9,10)$

Now final Answer is (Case 1) ∪ (Case 2) ∪ (Case 3)

we get $m\in(-\infty,10)$

27. ✈ α,β are the roots of $x^2-34x+1=0$

$\Rightarrow \alpha+\beta=34$ and $\alpha\beta=1$

✈ $\left(\alpha^{\frac{1}{4}}-\beta^{\frac{1}{4}}\right)^2=\sqrt{\alpha}+\sqrt{\beta}-2(\alpha\beta)^{1/4}$

✈ $\alpha\beta=1$

∴ $\left(\alpha^{\frac{1}{4}}-\beta^{\frac{1}{4}}\right)^2=\sqrt{\alpha}+\sqrt{\beta}-2$ ....(i)

✈ $\left(\sqrt{\alpha}+\sqrt{\beta}\right)^2=\alpha+\beta+2\sqrt{\alpha\beta}$

✈ $\alpha+\beta=34$ and $\alpha\beta=1$

∴ $\left(\sqrt{\alpha}+\sqrt{\beta}\right)^2=36$

✈ we consider the principal value

∴ $\sqrt{\alpha}+\sqrt{\beta}=6$ put in (i), we get.

$\left(\alpha^{\frac{1}{4}}-\beta^{\frac{1}{4}}\right)^2=4$

∴ $\alpha^{\frac{1}{4}}-\beta^{\frac{1}{4}}=\pm2$