

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. $f(x) = \sin^{-1} \left(\frac{1+x^3}{2x^{3/2}} \right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1)$

Domain : $3\{x\} + 1 \neq 1$ or $0 \Rightarrow x \notin \mathbb{I}$

and $-1 \leq \frac{1+x^3}{2x^{3/2}} \leq 1$

$-2x^{3/2} \leq 1+x^3 \leq 2x^{3/2}$

$1+x^3+2x^{3/2} \geq 0$

$(1+x^{3/2})^2 \geq 0 \Rightarrow x \in \mathbb{R}$

$1+x^3-2x^{3/2} \leq 0$ or $(1-x^{3/2})^2 \leq 0$

or $1-x^{3/2} = 0$ or $x = 1$

Hence domain $x \in \phi$

6. $f(x) = \sqrt{\log \frac{(5x-x^2)}{6}}$

$\log \frac{5x-x^2}{6} \geq 0$

$\Rightarrow \frac{5x-x^2}{6} \geq 1 \Rightarrow x^2-5x+6 \leq 0$

$\Rightarrow (x-2)(x-3) \leq 0 \Rightarrow 2 \leq x \leq 3$

So domain $\in [2, 3]$

7. (i) $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$

D : $x-1 \geq 0$ & $3-x \geq 0 \Rightarrow x \in [1, 3]$

Range : $f'(x) = \frac{1}{2\sqrt{x-1}} - \frac{1}{\sqrt{3-x}} = 0$

or $f'(x) = 0$ at $x = \frac{7}{5}$

$f'\left(\frac{7}{5}\right) > 0$ & $f'\left(\frac{7}{5}\right) < 0$

$\Rightarrow$ maxima at $x = \frac{7}{5}$

Range : $[\sqrt{2}, \sqrt{10}]$

9. $f(x) = (\sin^{-1}x + \cos^{-1}x)^3 - 3 \sin^{-1}x \cos^{-1}x (\sin^{-1}x + \cos^{-1}x)$

$= \frac{\pi^3}{8} - 3 \sin^{-1}x \left(\frac{\pi}{2} - \cos^{-1}x \right) \frac{\pi}{2}$

$= \frac{\pi^3}{8} - \frac{3\pi^2}{4} \sin^{-1}x + 3 \frac{\pi}{2} (\sin^{-1}x)^2$

$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1}x)^2 - \frac{\pi}{2} \sin^{-1}x + \frac{\pi^2}{16} \right] - \frac{3\pi^3}{32}$

$= \frac{\pi^3}{32} + \frac{3\pi}{2} \left(\sin^{-1}x - \frac{\pi}{4} \right)^2$

maximum value of $f(x)$ at $x = -1$

$f_{\text{maximum}} = \frac{\pi^3}{32} + \frac{3\pi}{2} \times \frac{9\pi^2}{16} = \frac{7\pi^3}{8}$

11. $f(x) = {}^{7-x}P_{x-3}$

For domain

$7-x \geq 0$, & $x-3 \geq 0$ & $7-x \geq x-3$

$x \leq 7$, & $x \geq 3$ & $2x \leq 10 \Rightarrow x \leq 5$

$x \in \{3, 4, 5\}$

Range $\in \{f(3), f(4), f(5)\}$

Range $\in \{1, 3, 2\}$

14. $f(xy+1) = f(yx+1)$

$f(x)f(y) - f(y) - x + 2 = f(y)f(x) - f(x) - y + 2$

$f(x) - f(y) = x - y$

Putting $y = 0$

$f(x) - 1 = x - 0$

$f(x) = x + 1$

16. (A) $f(x) = e^{1/2 \bullet nx} = \sqrt{x}$, D : $x > 0$

$g(x) = \sqrt{x}$, D : $x \geq 0$

(B) $\tan^{-1}(\tan x) = x$ D : $x \neq \pm(2n+1)\frac{\pi}{2}$

$\cot^{-1}(\cot x) = x$ D : $x \neq \pm n\pi$



$$\begin{aligned} \text{(C)} \quad f(x) &= \cos^2 x + \sin^4 x = \cos^2 x + (1 - \cos^2 x)^2 \\ &= 1 - \cos^2 x + \cos^4 x = \sin^2 x + \cos^4 x \\ g(x) &= \sin^2 x + \cos^4 x \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad f(x) &= \frac{|x|}{x}, \quad D: x \neq 0 \\ g(x) &= \operatorname{sgn}(x), \quad D: x \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} 19. \quad f(x+1) - f(x) &= 8x + 3 \\ f(0+1) - f(0) &= 3 \quad (\text{put } x=0) \\ \Rightarrow (b+c+d) - d &= 3 \\ \Rightarrow b+c &= 3 \quad \text{.....(i)} \\ \text{Also } f(-1+1) - f(-1) &= -8 + 3 \quad (\text{put } x=-1) \\ \Rightarrow f(0) - f(-1) &= -5 \Rightarrow d - (b-c+d) = -5 \\ \Rightarrow -b+c &= -5 \quad \text{.....(ii)} \\ \text{from (i) and (ii)} \\ b=4, \quad c &= -1 \end{aligned}$$

$$\begin{aligned} 20. \quad f(x) &= \frac{4a-7}{3} x^3 + (a-3)x^2 + x + 5 \\ f'(x) &= (4a-7)x^2 + 2(a-3)x + 1 \\ D \leq 0 \text{ for all } x \in \mathbb{R} \\ 4(a-3)^2 - 4(4a-7) &\leq 0 \\ a^2 + 9 - 6a - 4a + 7 &\leq 0 \\ a^2 - 10a + 16 &\leq 0 \\ (a-8)(a-2) &\leq 0 \quad \text{or} \quad a \in [2, 8] \\ f'(x) \text{ is always +ve for } a &\in [2, 8] \end{aligned}$$

$$\begin{aligned} 22. \quad f(x) &= x - [x] + (x+1) - [x+1] + \dots \\ &\quad (x+99) - [x+99] \\ &= x - [x] + x - [x] + \dots + x - [x] \\ &= 100(x - [x]) = 100\{x\} \\ f(\sqrt{2}) &= 100\{\sqrt{2}\} = 41 \end{aligned}$$

$$\begin{aligned} 25. \quad f\left(x + \frac{1}{3}\right) &= \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] + [x+1] - 3\left(x + \frac{1}{3}\right) + 15 \\ &= \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] + [x] - 3x + 15 \\ &= f(x) \\ \therefore \text{fundamental period is } &1/3 \end{aligned}$$

$$\begin{aligned} 26. \quad f(x) &= |x-1| \quad f: \mathbb{R}^+ \rightarrow \mathbb{R} \\ g(x) &= e^x, \quad g: [-1, \infty) \rightarrow \mathbb{R} \\ fog(x) &= f[g(x)] = |e^x - 1| \\ D &: [-1, \infty) \\ R &: [0, \infty) \end{aligned}$$

$$27. \quad \text{Hint: } f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \begin{cases} \frac{e^x - e^{-x}}{e^x + e^{-x}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\text{and } \frac{e^x - e^{-x}}{e^x + e^{-x}} > 0 \quad \forall x > 0$$

$$\begin{aligned} 29. \quad f: \mathbb{R} &\rightarrow \mathbb{R}, \quad f(x) = x^3 + ax^2 + bx + c \\ f'(x) &= 3x^2 + 2ax + b \\ D \leq 0 \quad \text{or} \quad 4a^2 - 12b &\leq 0 \\ \text{or} \quad a^2 &\leq 3b \end{aligned}$$

$$\begin{aligned} 31. \quad f(x) &= \sin \sqrt{[a]} x \\ \text{period of } \sin x &= 2\pi \\ \Rightarrow \text{period of } f(x) &= \frac{2\pi}{\sqrt{[a]}} = \pi \\ \Rightarrow \sqrt{[a]} &= 2 \Rightarrow [a] = 4 \Rightarrow a \in [4, 5) \end{aligned}$$

$$\begin{aligned} 33. \quad \text{Put } y &= -x, \text{ we get } f(x) = -x \text{ also } f(0) = 0 \\ f(x+y) &= f(x) + f(y) \text{ is an odd function so it is} \\ &\text{symmetric about origin.} \end{aligned}$$

$$\begin{aligned} 34. \quad f(x+1) + f(x+3) &= K \quad \forall x \\ \text{put } x &= -1 \\ f(0) + f(2) &= K \quad \text{.....(i)} \\ \text{put } x &= 1 \quad f(2) + f(4) = K \quad \text{.....(ii)} \\ \text{from (i) \& (ii)} \\ f(4) &= f(0) = 0 \Rightarrow \text{period} = 4 \end{aligned}$$

$$\begin{aligned} 36. \quad f(x) &= 2^{x(x-1)} \\ \text{It is one-one onto function} \\ \log_2 y &= x(x-1) \\ \Rightarrow x^2 - x - \log_2 y &= 0 \\ x &= \frac{1 \pm \sqrt{1 + 4 \log_2 y}}{2} \\ f^{-1}(x) &= \frac{1 + \sqrt{1 + 4 \log_2 x}}{2} \end{aligned}$$

EXERCISE - 2

Part # I : Multiple Choice

3. $f(x) = \sin \log \left(\frac{\sqrt{4-x^2}}{1-x} \right)$

$4-x^2 > 0$ or $x \in (-2, 2)$

and $\frac{\sqrt{4-x^2}}{1-x} > 0$

$D: (-2, 1)$

$R: [-1, 1]$

8. $f(x) = \sin \left[\log \left(\frac{\sqrt{4-x^2}}{1-x} \right) \right]$

$f(x)$ will be defined if $\frac{\sqrt{4-x^2}}{1-x} > 0$ & $4-x^2 > 0$

$\Rightarrow -2 < x < 1$ & $-\infty < \log \frac{\sqrt{4-x^2}}{1-x} < \infty$

$-1 \leq \sin \left[\log \left(\frac{\sqrt{4-x^2}}{1-x} \right) \right] < 1$

So range of $f(x)$ is $[-1, 1]$

13. put $x = 1$

$2f(1) + 1f(1) - 2f\left(\left|\sqrt{2} \sin \frac{5\pi}{4}\right|\right) = -1$

$\Rightarrow 3f(1) - 2f(1) = -1 \Rightarrow f(1) = -1$

Now put $x = 2$

$2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4 \cos^2 \pi + 2 \cos \frac{\pi}{2}$

$\Rightarrow 2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4$

$\Rightarrow f(2) + f\left(\frac{1}{2}\right) = 1$ (i)

Now put $x = 1/2$ we get

$4f\left(\frac{1}{2}\right) + f(2) = 1$ (ii)

from (i) and (ii)

$f\left(\frac{1}{2}\right) = 0$ & $f(2) = 1$

14. $\log_{x^2}(x) \geq 0$ & $x > 0, x \neq \pm 1$

$\therefore x \in (0, 1) \cup (1, \infty)$

16. (A) $f(x) = e^{\ln(\sec^{-1} x)} = \sec^{-1} x,$

$x \in (-\infty, -1] \cup (1, \infty)$

$g(x) = \sec^{-1} x, x \in (-\infty, -1] \cup [1, \infty)$

non-identical functions

(B) $f(x) = \tan(\tan^{-1} x) = x, x \in \mathbb{R}$

$g(x) = \cot(\cot^{-1} x) = x, x \in \mathbb{R}$

identical functions

(C) $f(x) = \operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

$\Rightarrow g(x) = \operatorname{sgn}(\operatorname{sgn} x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

Identical functions

(D) $f(x) = \cot^2 x \cdot \cos^2 x,$

$x \in \mathbb{R} - \{n\pi\}, n \in \mathbb{I}$

$g(x) = \cot^2 x - \cos^2 x = \cot^2 x (1 - \sin^2 x)$

$= \cot^2 x \cdot \cos^2 x$

$x \in \mathbb{R} - \{n\pi\}, n \in \mathbb{I}$

Identical functions

17. $f(x) = \frac{1-x}{1+x}, 0 \leq x \leq 1$

$g(x) = 4x(1-x), 0 \leq x \leq 1$

$f \circ g(x) = \frac{1-g(x)}{1+g(x)} = \frac{1-4x(1-x)}{1+4x(1-x)} = \frac{1-4x+4x^2}{1+4x-4x^2}$

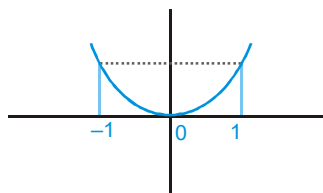
$g \circ f(x) = 4f(x) \cdot (1-f(x))$

$= 4 \left(\frac{1-x}{1+x} \right) \left(1 - \left(\frac{1-x}{1+x} \right) \right) = \frac{8x(1-x)}{(1+x)^2}$

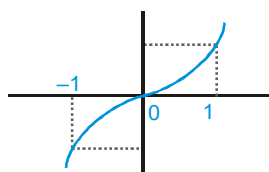


20. Domain $D \in [-1, 1]$

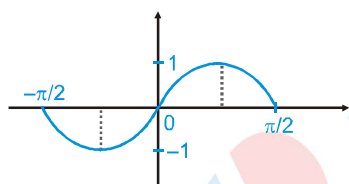
(A) $f(x) = x^2$ many - one



(B) $g(x) = x^3$ one - one



(C) $h(x) = \sin 2x$ many - one



(D) $k(x) = \sin\left(\frac{\pi x}{2}\right)$ one-one function

23. $f: \mathbb{R} \rightarrow [-1, 1]$

$$f(x) = \sin\left(\frac{\pi}{2}[x]\right) = \begin{cases} -1 & , -1 \leq x < 0 \\ 0 & , 0 \leq x < 1 \\ 1 & , 1 \leq x \leq 2 \end{cases}$$

Many - one function into function

$$\text{Also } f(x+4) = \sin\left(\frac{\pi}{2}[x+4]\right)$$

$$= \sin\left(2\pi + \frac{\pi}{2}[x]\right) = \sin\left(\frac{\pi}{2}[x]\right)$$

$= f(x)$ and hence periodic

EXERCISE - 3

Part # II : Comprehension

Comprehension # 3

$$f(2-x) = f(2+x)$$

$$\& \quad f(20-x) = f(x)$$

$$\Rightarrow f(2-(2-x)) = f(4-x)$$

$$\& \quad f(20-(x+16)) = f(x+16)$$

$$\Rightarrow f(x) = f(4-x)$$

$$\& \quad f(4-x) = f(x+16)$$

$$\Rightarrow f(x) = f(x+16)$$

1. $f(0) = f(4) = f(16)$

no. of values of $x = 22$

2. If graph is symmetric about $x = a$ then

$$f(a+x) = f(a-x)$$

$$f(16) = f(20) \Rightarrow \text{symmetric about } x = 18$$

$$f(4) = f(32)$$

3. $f(0) = f(1) = f(2) = f(3) = f(4) = f(5) = f(6)$

Hence period can't be one.

EXERCISE - 4

Subjective Type

1. (i) $[-1, 1]$

(ii) $\mathbb{R}$

(iii) $\bigcup_{n \in \mathbb{I}} \left[n\pi, n\pi + \frac{\pi}{4} \right]$

(iv) $(2, 3)$

(v) $\frac{f}{g}(x) = \frac{\sqrt{x^2 - 5x + 4}}{x + 3}$

$\Rightarrow x^2 - 5x + 4 \geq 0$

$(x - 4)(x - 1) \geq 0$

also $x \neq -3$

So $x \in (-\infty, -3) \cup (-3, 1] \cup [4, \infty)$

(vi) $\frac{1}{[x]} \Rightarrow x \notin [0, 1)$

and $\log_{1-\{x\}}(x^2 - 3x + 10)$

$x^2 - 3x + 10 > 0 \Rightarrow x \in \mathbb{R}$

$1 - \{x\} > 0 \Rightarrow x \in \mathbb{R}$

$1 - \{x\} \neq 1 \Rightarrow x \notin \mathbb{I}$

and $2 - |x| > 0 \Rightarrow |x| - 2 < 0$

$\Rightarrow x \in (-2, 2)$

and $\sec(\sin x) > 0$

$\Rightarrow -1 \leq \sin x \leq 1 \Rightarrow x \in \mathbb{R}$

$x \in (-2, -1) \cup (-1, 0) \cup (1, 2)$

2. (i) $(-\infty, 1]$

(ii) $\mathbb{R}^+$

(iii) $\left[\frac{1}{3}, 1 \right]$

(iv) $\left(-\infty, -\frac{1}{4} \right] \cup \left[-\frac{1}{20}, \infty \right)$

(v) $\left[\frac{1}{3}, 3 \right]$

(vi) $\left[0, \frac{3}{\sqrt{2}} \right]$

(vii) $[4, \infty)$

(viii) $[-11, 16]$

(ix) $\left[\frac{3}{4}, 1 \right]$

3. $f(3) = 1$

$f(3x) = x + f(3x - 3)$

put $x = 1$

$f(3) = 1 + f(0)$

$f(0) = 0$

$f(6) = 2 + f(3) = 3$

$f(9) = 3 + f(6) = 3 + 3 = 6$

$f(12) = 4 + 6 = 10$

Hence $f(300) = 1 + 3 + 6 + 1 + \dots$ 100th term

$S = 1 + 3 + 6 + 10 + \dots + T_n$

$S = 1 + 3 + 6 + \dots + T_n$

$T_n = 1 + 2 + 3 + 4 + \dots$ up 100 term

$= \frac{100}{2} \times 101 = 5050$

4. $f(x) = \frac{9^x}{9^x + 3}$

$f(1-x) = \frac{9^{1-x}}{9^{1-x} + 3} = \frac{3}{3 + 9^x}$

$f(x) + f(1-x) = 1$

$f\left(\frac{1}{2008}\right) + f\left(\frac{2007}{2008}\right) = 1$ (i)

$f\left(\frac{2}{2008}\right) + f\left(\frac{2006}{2008}\right) = 1$ (ii)

$f\left(\frac{1003}{2008}\right) + f\left(\frac{1005}{2008}\right) = 1$ (iii)

& $f\left(\frac{1004}{2008}\right) + f\left(\frac{1004}{2008}\right) = 1$

$\Rightarrow f\left(\frac{1004}{2008}\right) = \frac{1}{2}$ (iv)

add all we get

$f\left(\frac{1}{2008}\right) + f\left(\frac{2}{2008}\right) + f\left(\frac{3}{2008}\right) + \dots + f\left(\frac{2007}{2008}\right)$
 $= 1003.5$



5. (i) neither even nor odd (ii) even
(iii) odd (iv) even
(v) odd

6. (i) π (ii) 2 (iii) $\frac{2\pi}{3}$ (iv) 2π

- (v) $2^n \pi$ (vi) π

7. (9)

9. $f(x) = (a - x^n)^{1/n}$

$$f(f(x)) = (a - (f(x))^n)^{1/n} \\ = [a - \{(a - x^n)^{1/n}\}^n]^{1/n} = (a - a + x^n)^{1/n} = x$$

So $f \circ f(x) = x$

$$\Rightarrow f^{-1}(x) = f(x) = (a - x^n)^{1/n}$$

10. $f^{-1}(x) = x + (-1)^{x-1}, x \in \mathbb{N}$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

5. $y = \sin^{-1}[\log_3(x/3)] \Rightarrow -1 \leq \log_3(x/3) \leq 1$

$$\Rightarrow \frac{1}{3} \leq \frac{x}{3} \leq 3 \Rightarrow 1 \leq x \leq 9 \Rightarrow x \in [1, 9]$$

6. $f(x) = \log(x + \sqrt{x^2 + 1})$

and $f(-x) = -\log(x + \sqrt{x^2 + 1}) = -f(x)$

$f(x)$ is odd function.

7. $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$. So, $4 - x^2 \neq 0$

$$\Rightarrow x \neq \pm\sqrt{4}$$

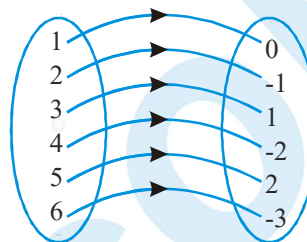
and $x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x > 0, x > 1$



$$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\} \text{ i.e., } D = (-1, 0) \cup (1, 2) \cup (2, \infty)$$

9. $f: \mathbb{N} \rightarrow \mathbb{I}$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$
and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

10. To define $f(x), 9 - x^2 > 0 \Rightarrow -3 < x < 3 \dots (i)$

$-1 \leq (x - 3) \leq 1 \Rightarrow 2 \leq x \leq 4 \dots (ii)$

From (i) and (ii), $2 \leq x < 3$ i.e., $[2, 3)$.

11. $f(3) = {}^{7-3}P_0 = 1, f(4) = {}^3P_1 = 3$ and $f(5) = {}^2P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$.

12. Using $-\sqrt{a^2 + b^2} \leq (a \sin x + b \cos x) \leq \sqrt{a^2 + b^2}$

$$-\sqrt{1 + (-\sqrt{3})^2} \leq (\sin x - \sqrt{3} \cos x) \leq \sqrt{1 + (-\sqrt{3})^2}$$

$$-2 \leq (\sin x - \sqrt{3} \cos x) \leq 2$$

$$-2 + 1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 2 + 1$$

$$-1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 3 \text{ i.e., range} = [-1, 3]$$

$\therefore$ For f to be onto $S = [-1, 3]$.

13. For $-1 < x < 1$, $\tan^{-1} \frac{2x}{1-x^2} = 2 \tan^{-1} x$

Range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Co-domain of function = $B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

14. $f(a - (x - a)) = f(A)f(x - a) - f(0)f(x) \dots (i)$

Put $x=0$, $y=0$; $f(0) = (f(0))^2 - [f(A)]^2 \Rightarrow f(A) = 0$

$[\rightarrow f(0) = 1]$. From (i), $f(2a - x) = -f(x)$.

15. Let $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$\Rightarrow 3(y-1)x^2 + 9(y-1)x + 7y - 17 = 0$

Since x is real, we have

$\{9(y-1)\}^2 - 4.3(y-1)(7y-17) \geq 0$

$\Rightarrow -3y^2 + 126y - 123 \geq 0$

$\Rightarrow (y-41)(y-1) \leq 0$

$\Rightarrow 1 \leq y \leq 41$

So, maximum value of y is 41.

16. $f(x)$ is defined if $-1 \leq \frac{x}{2} - 1 \leq 1$ and $\cos x > 0$

or $0 \leq x \leq 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$

19. For real x , $f(x) = x^3 + 5x + 1$

$\lim_{x \rightarrow \infty} f(x) = +\infty$

and $\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\therefore$ Range is $\mathbb{R}$ - $f(x)$ is onto

Now $f'(x) = 3x^2 + 5 > 0$

$\therefore f(x)$ is one-one

$f(x)$ is one-one onto.

20. $f(x) = (x+1)^2 - 1$; $x \geq -1$
 $f(x) = 2(x+1) \geq 0$ for $x \geq -1$

$\therefore f(x)$ is bijection

Statement (2) is correct

Now $f^{-1}(x) = f(x)$

To solve put $y = x$ in $f(x)$

$x = (x+1)^2 - 1$

$x+1 = (x+1)^2$

$x = -1, x = 0$

$x = \{0, -1\}$ Statement (1) is also correct

21. $f(x) = \frac{1}{\sqrt{|x|} - x}$

For domain of real function

$|x| - x > 0$

$|x| > x$

$x \in (-\infty, 0)$

22. $f(x) = (x-1)^2 + 1$; $(x \geq 1)$

and $f'(x) = 2(x-1) \geq 0$ for $x \geq 1$

$\therefore f(x)$ is one-one and onto

$\Rightarrow f(x)$ is Bijection

and $f^{-1}(x) = 1 + \sqrt{x-1}$

Statement-2 is true

Now $f(x) = f^{-1}(x)$

$\Rightarrow (x-1)^2 + 1 = \sqrt{x-1} + 1$

$\Rightarrow x = 1, 2$

$\therefore$ Statement-1 is true

23. $[x]$ is continuous at $\mathbb{R} - \mathbb{I}$

$\therefore f(x)$ is continuous at $\mathbb{R} - \mathbb{I}$

Now At $x = \mathbb{I}$

LHL = $\lim_{h \rightarrow 0} [I - h] \cos \frac{(2(I-h)-1)}{2} \pi$

$\lim_{h \rightarrow 0} (I - 1) \cos [2I - 2h - 1] \frac{\pi}{2}$



$$= (I - 1) \cos (2I - 1) \frac{\pi}{2} = 0$$

similarly,

$$\text{RHL} = 0$$

$$\text{and } f(i) = 0$$

$\therefore$ Function is continuous everywhere

$$25. f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$$

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

$$3f(x) = \frac{6}{x} - 3x$$

$$f(x) = \frac{2}{x} - x$$

$$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x$$

$$\frac{4}{x} = 2x$$

$$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

$$26. g(x) = f(f(x))$$

$$\Rightarrow g'(x) = f'(f(x)) f'(x)$$

$$\Rightarrow g'(0) = f'(f(0)) f'(0)$$

$$\text{For } x \rightarrow 0, \log 2 > \sin x$$

$$\therefore f(x) = \log 2 - \sin x$$

$$\therefore f'(x) = -\cos x$$

$$\Rightarrow f'(0) = -1$$

$$\text{Also } x \rightarrow \log 2, \log 2 > \sin x$$

$$\therefore f(x) = \log 2 - \sin x$$

$$\therefore f'(x) = -\cos x$$

$$\Rightarrow f'(\log 2) = -\cos(\log 2)$$

$$\therefore g'(0) = (-\cos(\log 2))(-1) = \cos(\log 2)$$

Part # II : IIT-JEE ADVANCED

$$3. g(x) = 1 + \{x\}$$

$$\Rightarrow 0 + 1 \leq g(x) \leq 1 + 1$$

$$\Rightarrow 1 \leq g(x) < 2$$

$$f(g(x)) = 1 \quad (\because g(x) > 0)$$

$$6. n(\text{into} + \text{onto}) = 2^4$$

$$n(\text{into}) = 2$$

$$n(\text{onto}) = 16 - 2 = 14$$

$$7. f(x) = \frac{\alpha x}{x+1}, x \neq -1$$

$$\text{Now } f(f(x)) = x \Rightarrow f(x) = f^{-1}(x)$$

$$\text{Let } y = \frac{\alpha x}{x+1} \Rightarrow xy + y = \alpha x$$

$$\Rightarrow x(y - \alpha) = -y \Rightarrow x = \frac{-y}{y - \alpha}$$

$$f^{-1}(x) = \frac{-x}{x - \alpha}$$

$$\text{Now } \frac{\alpha x}{x+1} = \frac{-x}{x - \alpha}$$

on solving we get $\alpha = -1$

$$13. \phi(x) = f(x) - g(x)$$

$$= \begin{cases} -x & x \in Q \\ x & x \notin Q \end{cases}$$

It is one-one onto function

$$14. \text{Given } f(x) = x^2; g(x) = \sin x$$

$$f \circ g \circ g \circ f(x) = \sin^2(\sin x^2)$$

$$\text{and } g \circ g \circ f(x) = \sin(\sin x^2)$$

$$\text{given } f \circ g \circ g \circ f(x) = g \circ g \circ f(x)$$

$$\Rightarrow \sin^2(\sin x^2) = \sin(\sin x^2)$$

$$\Rightarrow \sin(\sin x^2) = 0 \text{ or } 1 \text{ (rejected)}$$

$$\sin(\sin x^2) = 0$$

$$\Rightarrow x^2 = n\pi$$

$$\Rightarrow x = \pm\sqrt{n\pi}; x \in \{0, 1, 2, 3, \dots\}$$

15. $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

$\rightarrow f(x)$ is non monotonic in $x \in [0, 3]$

$\Rightarrow f(x)$ is not one-one

$f(x)$ is increasing in $x \in [0, 2]$ and decreasing in $x \in (2, 3]$

$f(0) = 1, f(2) = 29 \text{ \& } f(3) = 28$

$\therefore$ Range of $f(x)$ is $[1, 29]$

$\Rightarrow f(x)$ is onto.

16. $\rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Now $f(\cos 4\theta) = \frac{2}{2 - \sec^2 \theta}$

$= \frac{1 + \cos 2\theta}{\cos 2\theta} = 1 + \frac{1}{\cos 2\theta} \quad \dots\dots(i)$

Let $\cos 4\theta = \frac{1}{3} \Rightarrow 2 \cos^2 2\theta - 1 = \frac{1}{3}$

$\Rightarrow \cos 2\theta = \pm \sqrt{\frac{2}{3}}$

$\Rightarrow$ From (i), $f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

$\Rightarrow$ (A, B) are correct

21. $f(x) = x^3 + 3x + 2, \quad f(1) = 6, g(6) = 1$

$g(f(x)) = x \Rightarrow g'(f(x)) \times f'(x) = 1$

put $x = 0, \quad g'(f(0)) \cdot f'(0) = 1$

$g'(2) \frac{1}{f'(0)} = \frac{1}{3}$

$f(3) = 38$

$\therefore g(38) = 3$

$\therefore h(g(3)) = h(g(g(38))) = 38$

$f(2) = 16$

$\Rightarrow g(16) = 2$

$\therefore h(g(g(16))) = h(g(2)) = h(0)$

$\therefore 16 = h(g(g(16))) = h(0)$

$\therefore$ (C) is correct.

$f(x) = 3x^2 + 3$

$f(6) = 111, \quad f(1) = 6 \Rightarrow g'(6) = \frac{1}{6}$

$h(g(g(x))) = x$

$\Rightarrow h'(g(g(x))) \times g'(g(x)) \times g'(x) = 1$

Put $x = 236, h'(g(g(236))) \times g'(g(236)) \times g'(236) = 1$

$\Rightarrow h'(g(6)) g'(6) \times \frac{1}{f'(6)} = 1$

$\Rightarrow h'(1) = 666 \quad \text{But } g(1) \neq 1$



MOCK TEST

1. (B)

Put $x = y = 1$, $(f(1))^2 = 3f(1) - 2$

$\Rightarrow f(1) = 1$ or 2

Let $f(1) = 1$, then put $y = 1$

$f(x) \cdot f(1) = f(x) + f(1) + f(x) - 2$

$\Rightarrow f(x) = 1$

constant function but $f(x)$ is not constant function

$\therefore f(1) \neq 1$, hence $f(1) = 2$

2. $f(x) = \sqrt{-\log_{\frac{x+4}{2}} \left(\log_2 \frac{2x-1}{3+x} \right)}$

For domain: $\log_{\frac{x+4}{2}} \left(\log_2 \frac{2x-1}{3+x} \right) \leq 0$

Case-I

$0 < \frac{x+4}{2} < 1 \Rightarrow -4 < x < -2$ A

then $\log_{\frac{x+4}{2}} \left(\log_2 \frac{2x-1}{3+x} \right) \leq 0$

$\Rightarrow \log_2 \frac{2x-1}{3+x} \geq 1 \Rightarrow \frac{2x-1}{3+x} \geq 2$

$\Rightarrow x < -3$ B

$\Rightarrow$ on $A \cap B$ $x \in (-4, -3)$ (i)

Case-II

$\frac{x+4}{2} > 1$ or $x > -2$ A

$\log_{\frac{x+4}{2}} \left(\log_2 \frac{2x-1}{3+x} \right) \leq 0$

$\Rightarrow 0 < \log_2 \frac{2x-1}{3+x} \leq 1 \Rightarrow 1 < \frac{2x-1}{3+x} \leq 2$

$\Rightarrow x \in (4, \infty)$ (ii)

$\therefore$ (i) $\cup$ (ii) Domain $x \in (-4, -3) \cup (4, \infty)$

3. $f(x) = ax^2 + bx + c$

$f(0) = c \Rightarrow c \in I$

$f(1) = a + b + c \Rightarrow (a + b + c) \in I$

$\Rightarrow (a + b) \in I$

4. $y = \frac{\sin^2 x + 4\sin x + 4}{2\sin^2 x + 8\sin x + 8} + \frac{1}{2\sin^2 x + 8\sin x + 8}$

$= \frac{1}{2} + \frac{1}{2(\sin x + 2)^2}$

$y_{\max} = \frac{1}{2} + \frac{1}{2(-1+2)^2} = 1$

$\Rightarrow y_{\min} = \frac{1}{2} + \frac{1}{2(1+2)^2} = \frac{5}{9}$

$\therefore \text{range} = \left[\frac{5}{9}, 1 \right]$

5. (C)

$f(x) = x + \tan x$

$f(f^{-1}(x)) = f^{-1}(x) + \tan(f^{-1}(x))$

$x = g(x) + \tan(g(x))$

$1 = g'(x) + \sec^2(g(x)) g'(x)$

$g'(x) = \frac{1}{2 + \tan^2(g(x))}$

$g'(x) = \frac{1}{2 + (x - g(x))^2}$

6. (A)

$g(f(x)) = \tan \left(x - \frac{\pi}{4} \right) = \frac{\tan x - 1}{\tan x + 1}$

$\Rightarrow g(x) = \frac{x-1}{x+1}$

$f(g(x)) = \tan \left(\frac{x-1}{x+1} \right)$

7. $f(x) = \sin^{-1} \left[x^2 + \frac{1}{2} \right] + \cos^{-1} \left[x^2 - \frac{1}{2} \right]$

Domain : $-1 \leq \left[x^2 - \frac{1}{2} \right] \leq 1$

$\Rightarrow x \in \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}} \right)$ and $-1 \leq \left[x^2 + \frac{1}{2} \right] \leq 1$

$\Rightarrow x \in \left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}} \right)$

$\Rightarrow$ domain is

$x \in \left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}} \right)$ or $x^2 \in \left[0, \frac{3}{2} \right]$

if (i) $x^2 \in \left[0, \frac{1}{2} \right]$, then $f(x) = \pi$

if (ii) $x^2 \in \left[\frac{1}{2}, 1 \right]$, then $f(x) = \pi$

if (iii) $x^2 \in \left[1, \frac{3}{2} \right]$, then $f(x) = \pi$

$\Rightarrow$ range = $\{\pi\}$

8. $f(x) + 5 \leq f(x+5) \leq f(x+4) + 1 \leq f(x+3) + 2 \leq f(x+2) + 3 \leq f(x+1) + 4 \leq f(x) + 5$

$\Rightarrow$ In all steps there is equality only

$\Rightarrow f(x+1) = f(x) + 1$

Now $f(1) = 1$

$\Rightarrow f(2) = 2$

$f(3) = 3$

$f(4) = 4$

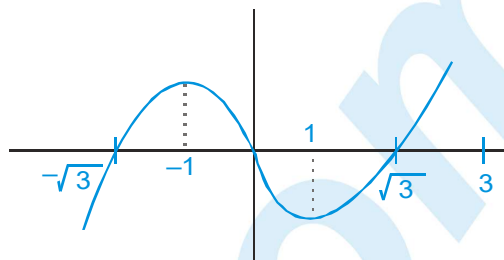


$f(2013) = 2013$

$\Rightarrow g(2013) = 2013 + 1 - 2013 = 1$

9. (D)

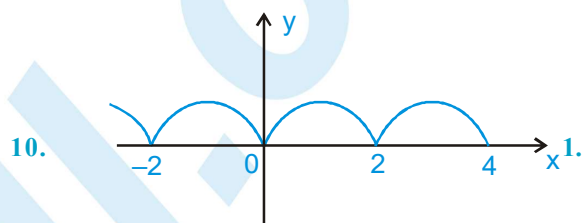
$f(x) = 4x(x^2 - 3)$



$f'(x) = 12x^2 - 12 = 0$

or $x = \pm 1$

$f(x) \in [f(1), \max(f(-1), f(3))] = [-8, 72]$



$y = f(x+2)$ is drawn by shifting the graph by 2 units horizontally.

11. $f[g(x)] = a(bx+a) + b = abx + a^2 + b$ (i)

$g[f(x)] = b(ax+b) + a$

$= abx + b^2 + a$ (ii)

(i) - (ii) $f[g(50)] - g[f(50)] = a^2 - b^2 + b - a$

$\therefore (a^2 - b^2) + (b - a) = 28$

$(a-b)(a+b-1) = 28 = (1 \times 28) \text{ or } (2 \times 14) \text{ or } (4 \times 7)$

let $a-b=1$ and $a+b-1=28$

and $2a-2=28 \Rightarrow a=15; b=14$

$\therefore ab=210$

if $a-b=2$ and $a+b-1=14$ (not possible)

if $a-b=4$ and $a+b-1=7$

$2a-1=11 \Rightarrow a=6$ and $b=2$

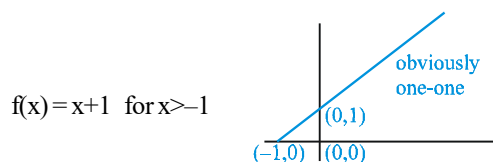
$\therefore ab=12$

12. $f(-x) = \begin{cases} 0 & x=0 \\ -x^2 \sin\left(\frac{\pi}{x}\right) & x \in (-1, 1) - \{0\} \\ -x & |x| > 1 \end{cases} = -f(x)$

odd function

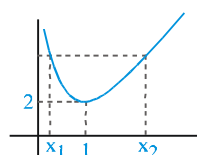


13. $f(x) = x + 1$



$g(x) = x + \frac{1}{x}$ for $x > 0$

Many-one function



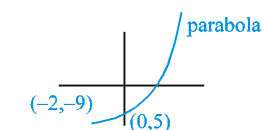
$h(x) = x^2 + 4x - 5$ for $x > 0$

Hence One - one

$= (x + 2)^2 - 9$

$f(x) = e^{-x}$ for $x \geq 0$

Obviously one-one



14. $y = f(x) \Rightarrow x = f^{-1}(x)$

$x = \frac{y}{a} - \frac{b}{a}$

$f^{-1}(y) = \frac{y}{a} - \frac{b}{a}$

$f^{-1}(x) = \frac{x}{a} - \frac{b}{a}$

and $f(x) = ax + b$

now in order that (i) and (ii) coincide

$a = \frac{1}{a}$

$\frac{b}{a} = -b$

from (1), $a^2 = 1$

if $a = -1$, $b = b$

if $a = +1$, then $2b = 0$

hence $(-1, R), (1, 0)$

now $y = ax + b$

.....(i)

.....(ii)

.....(i)

.....(ii)

$\Rightarrow a = 1$ or -1

$\Rightarrow b \in R$

$\Rightarrow b = 0$

15. Let, $2x + y = 3x - y$

$\Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$

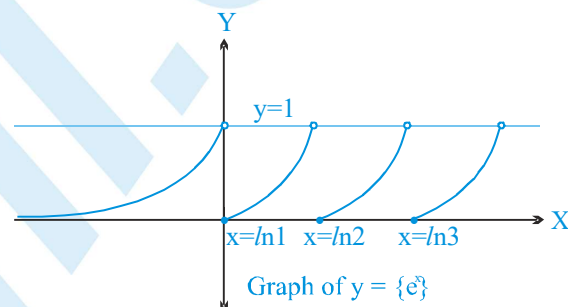
$\therefore$ put $y = \frac{x}{2}$

$f(x) + f\left(\frac{5x}{2}\right) + \frac{5x^2}{2} = f\left(\frac{5x}{2}\right) + 2x^2 + 1$

$f(x) = 1 - \frac{x^2}{2} \Rightarrow A, B$

16. (D)

$y = \{e^x\} = \begin{cases} e^x & ; -\infty < x < 0 \\ e^x - 1 & ; 0 \leq x < \ln 2 \\ e^x - 2 & ; \ln 2 \leq x < \ln 3 \end{cases}$



and so on

Clearly $f(x)$ is aperiodic on R .

17. (C)

Statement-I : L.C.M. of $(2\pi, \pi) = 2\pi$

Statement-II : $f(x) = |\sin x| + |\cos x|$, fundamental

period is $\frac{\pi}{2}$

18. (A)

(i) $y = f(x)$ is symmetric about $y = x$

$\Rightarrow x = f(y)$

$\therefore f(f(x)) = f(y) = x$

$\therefore$ statement 1 is true

(ii) $f(x) = \begin{cases} x & , x \text{ is rational} \\ 1-x & , x \text{ is irrational} \end{cases}$ is

symmetric about $y = x$

$\therefore f(f(x)) = x$

19. (A)

$$f(g(h(1))) = f(g(3)) = f(-g(-3)) = f(-2) = 1$$

$$g(h(f(3))) = g(h(-5)) = g(-h(5)) = g(-1) = -g(1) = -1$$

$$h(f(g(-1))) = h(f(-g(1))) = h(f(-1)) = h(f(1)) = h(0)$$

$$\text{as } h \text{ is odd} \Rightarrow h(x) + h(-x) = 0$$

$$h(0) + h(0) = 0 \Rightarrow h(0) = 0$$

$\Rightarrow$ sum of composite functions is zero.

20. (D)

Statement-I : Every function can be written as the sum of even and odd function

Statement-II : $f(x) = e^x$

$$f(-x) = e^{-x}$$

Here neither $f(x) = f(-x)$ nor $f(-x) = -f(x)$

So e^x is neither even nor odd function.

21. (A) $\rightarrow$ (p, r), (B) $\rightarrow$ (p, s), (C) $\rightarrow$ (q, s), (D) $\rightarrow$ (q, s)

$$(A) f'(x) = \frac{1}{2\sqrt{\sin x}} \cos x$$

$$f'(x) \text{ is positive if } x \in \left[0, \frac{\pi}{3}\right]$$

f is one to one function

$$\text{Since } 0 \leq x \leq \frac{\pi}{3}$$

$$0 \leq \sin x \leq \frac{\sqrt{3}}{2}$$

$$0 \leq \sqrt{\sin x} \leq \sqrt{\frac{\sqrt{3}}{2}} < 1$$

f is into function

$$(B) f(x) = \frac{x+3}{x-1}$$

$$f'(x) = \frac{(x-1) \cdot 1 - (x+3) \cdot 1}{(x-1)^2}$$

$$f'(x) = \frac{-4}{(x-1)^2}$$

$f'(x) < 0$ Hence $f(x)$ is one to one

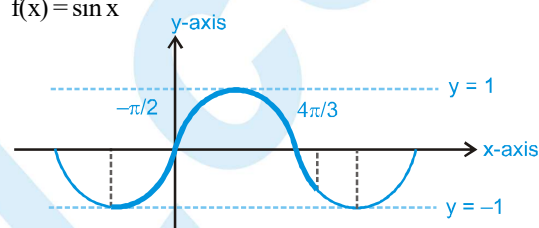
Since $x > 1$

$$\therefore \text{Range of } y = \frac{x+3}{x-1} \text{ is } (1, \infty)$$

f is onto function

$$(C) -\frac{\pi}{2} \leq x \leq \frac{4\pi}{3}$$

$$f(x) = \sin x$$



from graph $f(x)$ is many-one and onto

$$(D) f(x) = \frac{x^2}{x-2}$$

$$f'(x) = \frac{(x-2) \cdot 2x - x^2}{(x-2)^2}$$

$$f'(x) = \frac{x^2 - 4x}{(x-2)^2}$$

$$\therefore f'(x) < 0 \text{ if } 2 < x < 4$$

$$f'(x) > 0 \text{ if } x > 4$$

$f(x)$ is many-one

$f(4) = 8$ (is the least value of $f(x)$)

$$\therefore \text{range} = [8, \infty)$$

$\therefore f(x)$ is onto.

22. (A) $\rightarrow$ (s); (B) $\rightarrow$ (q); (C) $\rightarrow$ (r); (D) $\rightarrow$ (p)

$$f \circ g = \frac{2x^2 + 6x + 5}{x^2 + 3x + 2}; \quad g \circ f = \frac{x^2 + x + 1}{(x+1)^2};$$

$$f \circ f = \frac{x^4 + 3x^2 + 1}{x(x^2 + 1)}; \quad g \circ g = \frac{2x + 3}{3x + 5}$$

23.

1. $f(x) = (x-1)^2 - 2$ $a=1, b=-2$
 $a+b=-1$

2. $g(x) = x^2 + x - 2 = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$

$$\Rightarrow g|x| = \left(|x| + \frac{1}{2}\right)^2 - \frac{9}{4}$$

$$g_{\min} = g(0) = -2$$

3. $f: [1, \infty) \rightarrow [-2, \infty)$, then $f^{-1}: [-2, \infty) \rightarrow [1, \infty)$;

$$f(x) = y \Rightarrow x^2 - 2x - (1+y)$$

$$x = \frac{2 \pm \sqrt{4+4(1+y)}}{2}; \quad x = 1 \pm \sqrt{2+y};$$

$$f^{-1}(y) = 1 + \sqrt{2+y}; \quad f^{-1}(x) = 1 + \sqrt{2+x}$$

24.

1. (A)

Since period of $f(x)$ is $2(10-2) = 16$

$$\therefore f(0) = f(16) = f(32) = \dots = f(160) = 5$$

$\therefore$ there are atleast 11 values of x for which $f(x) = 5$

$$f(0) = f(4) = f(16)$$

due to symmetry in one period length $f(x) = 5$

has solution other than 0, 16, 32, _____

$\therefore$ minimum possible number of values of x is

$$10 + 11 = 21$$

2. (A) Obvious by definition

3. (C)

If 1 is a period, then $f(x) = f(x+1), \forall x \in \mathbb{R}$

$$\Rightarrow f(2) = f(3) = f(4) = f(5) = f(6)$$

which contradicts the given hypothesis that $f(2) \neq f(6)$

$\therefore$ 1 cannot be period of $f(x)$

25.

1. (A)

$$f: (0, \infty) \rightarrow (0, \infty)$$

$$f(x f(y)) = x^2 y^a \quad (a \in \mathbb{R})$$

Put $x=1$, we get $f(f(y)) = y^a$

Put $f(y) = \frac{1}{x}$, we get $f(1) = \frac{1}{(f(y))^2} \cdot y^a$ and put $y=1$ we

$$\text{get } (f(1))^3 = 1$$

$$\therefore f(1) = 1$$

for $y=1$, we have $f(x(f(1))) = x^2$

$$\therefore f(x) = x^2$$

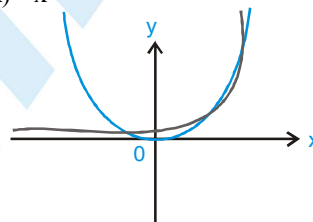
thus $a=4$

2. (C)

$$\begin{aligned} \sum_{r=1}^n f(r) {}^n C_r &= \sum_{r=1}^n r^2 {}^n C_r = \sum_{r=1}^n (r(r-1) + r) {}^n C_r \\ &= n(n-1) 2^{n-2} + n \cdot 2^{n-1} \end{aligned}$$

3. (C)

Since $f(x) = x^2$



$$\therefore 2x^2 = e^x$$

26. Let $x=y=1$

$$f(x) + f(y) + f(xy) = 2 + f(x) \cdot f(y)$$

$$3f(1) = 2 + (f(1))^2 \Rightarrow f(1) = 1, 2. \text{ But given that}$$

$$f(1) \neq 1 \text{ so } f(1) = 2$$

$$\text{Now put } y = \frac{1}{x}$$

$$f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) \cdot f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$$

$$\text{so } f(x) = \pm x^n + 1$$

$$\text{Now } f(4) = 17$$

$$\Rightarrow \pm (4)^n + 1 = 17 \Rightarrow n = 2$$

$$f(x) = +(x)^2 + 1.$$

$$\therefore f(5) = 5^2 + 1 = 26$$

27. (9)

Let $x = y = 1$

$$f(x) + f(y) + f(xy) = 2 + f(x) \cdot f(y)$$

$$3f(1) = 2 + (f(1))^2 \Rightarrow f(1) = 1, 2. \text{ But given that}$$

$$f(1) \neq 1 \text{ so } f(1) = 2$$

$$\text{Now put } y = \frac{1}{x}$$

$$f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) \cdot f\left(\frac{1}{x}\right)$$

$$\text{so } f(x) = \pm x^n + 1$$

$$\text{Now } f(4) = 17$$

$$\Rightarrow \pm (4)^n + 1 = 17 \Rightarrow n = 2$$

$$f(x) = (x)^2 + 1$$

$$\Rightarrow f(5) = 126$$

28. Put $y = -x$

$$f(x) + f(-x) = f(0) = 0$$

$$f(-x) = -f(x) \Rightarrow f(x) \text{ is an odd function.}$$

$$\text{Now put } y = x \Rightarrow 2f(x) = f(2x\sqrt{1-x^2})$$

$$\text{Now put } y = 2x\sqrt{1-x^2} \text{ in}$$

$$f(x) + f(y) = f(x\sqrt{1-y^2} + y\sqrt{1-x^2})$$

$$f(x) + f(2x\sqrt{1-x^2}) = f(x\sqrt{1-4x^2+4x^4} + 2x(1-x^2))$$

$$f(x) + 2f(x) = f(x(1-2x^2) + 2x-2x^3)$$

$$3f(x) = f(3x-4x^3) = -f(4x^3-3x)$$

$$\Rightarrow f(4x^3-3x) + 3f(x) = 0$$

29. (5)

Domain of $\sin^{-1}(\sin x)$ is whole of $\mathbb{R}$

$$-\log_{\frac{x+4}{2}} \log_2 \left(\frac{2x-1}{3+x} \right) > 0$$

$$\text{i.e. } \log_{\frac{x+4}{2}} \left(\log_2 \left(\frac{2x-1}{3+x} \right) \right) < 0$$

$$\text{Case-I If } 0 < \frac{x+4}{2} < 1 \text{ i.e. } -4 < x < -2$$

$$\text{then } \log_2 \frac{2x-1}{3+x} > 1 \text{ i.e. } \frac{2x-1}{3+x} > 2$$

$$\text{i.e. } \frac{2x-1-6-2x}{3+x} > 0$$

$$\text{i.e. } x+3 < 0 \text{ i.e. } x < -3$$

$$\therefore -4 < x < -3 \text{(i)}$$

$$\text{Case-II If } \frac{x+4}{2} > 1 \text{ i.e. } x > -2$$

$$\text{then } 0 < \log_2 \frac{2x-1}{3+x} < 1 \text{ i.e. } 1 < \frac{2x-1}{3+x} < 2$$

$$\text{i.e. } \frac{2x-1-3-x}{x+3} > 0 \text{ and } \frac{2x-1-6-2x}{x+3} < 0$$

$$\text{i.e. } \frac{x-4}{x+3} > 0 \text{ and } \frac{-7}{x+3} < 0$$

$$\text{i.e. } \{x < -3 \text{ or } x > 4\} \text{ and } x > -3$$

$$\text{i.e. } x > 4 \text{(iii)}$$

$$\text{from (i) and (ii) } x \in (-4, -3) \cup (4, \infty)$$

$$\therefore a = -4, b = -3, c = 4 \text{ and so } a + b + 3c = 5$$

30. (1)

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2}{x} - \frac{2}{1-x}$$

$$\text{Put } x = \frac{1}{1-t} \quad 1-x = 1 - \frac{1}{1-t} = -\frac{t}{1-t}$$

$$\therefore f\left(\frac{1}{1-t}\right) + f\left(\frac{t-1}{t}\right) = 2(1-t) - \frac{2(1-t)}{-t}$$

$$\therefore f\left(\frac{1}{1-x}\right) + f\left(\frac{x-1}{x}\right) = 2(1-x) + 2\left(\frac{1-x}{x}\right)$$

$$\text{Put } \frac{x-1}{x} = t$$

$$x-1 = xt \Rightarrow x = \frac{1}{1-t}$$



$$1-x = 1 - \frac{1}{1-t} = \frac{-t}{1-t}$$

$$\frac{1}{1-x} = \frac{t-1}{t}$$

$$\therefore f\left(\frac{t-1}{t}\right) + f(t) = \frac{2t}{t-1} + \left(\frac{2t}{t-1}\right)\left(\frac{1-t}{1}\right)$$

$$\therefore f\left(\frac{x-1}{x}\right) + f(x) = \frac{2x}{x-1} - 2x$$

$$\therefore 2f(x) = \frac{2}{x} - \frac{2}{1-x} - 2(1-x) - \frac{2(1-x)}{x} + \frac{2x}{x-1} - 2x$$

$$= \frac{2}{x} - \frac{2}{1-x} - 2 + 2x - \frac{2}{x} + 2 + \frac{2x}{x-1} - 2x$$

$$= \frac{2x}{x-1} + \frac{2}{x-1} = 2\left(\frac{x+1}{x-1}\right)$$

$$\therefore f(x) = \frac{x+1}{x-1}$$