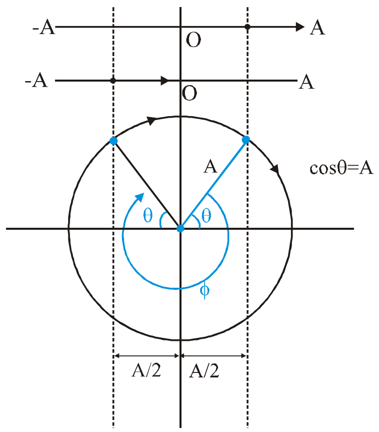
HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1.



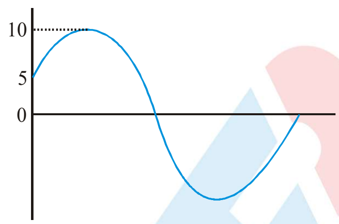
$$\cos\theta = \frac{A/2}{A} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Total phase difference between them

$$\phi = 2\theta + \pi = \frac{5\pi}{3}$$

2. $\omega^2 = \pi^2 \Rightarrow \omega = \pi \Rightarrow f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}\text{Hz}$

3.



From figure

maximum amplitude $A = 10$

Position of particle at $t = 0$, $x = 5$

let equation of SHM is

$x = A\sin(\omega t+\phi)$ At $t = 0$, $x = 5$

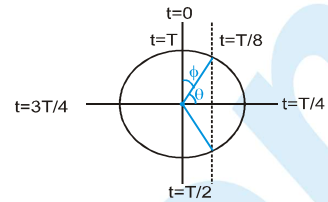
$5 = 10\sin\phi \Rightarrow \phi = \pi/6$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

Thus, equation of SHM $x = 10\sin\left(\pi t + \frac{\pi}{6}\right)$

4. $x = a\sin\omega t = a\sin\left(\frac{2\pi t}{T}\right)$

At $t = \frac{T}{8}$, $x = a\sin\left(\frac{2\pi\left(\frac{T}{8}\right)}{T}\right) = a\sin\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}}$

Or



$$\omega = \omega t;\ \phi = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

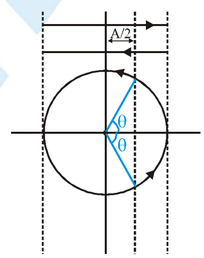
As $\cos\theta = \frac{x}{a}$ so $x = \frac{a}{\sqrt{2}}$

5. $x = A\sin\omega t = \frac{A}{2} \Rightarrow \omega t = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\Rightarrow$ Phase difference $= \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$ or $120°$

Or

from phaser



$$\cos\theta = \frac{A/2}{A} = \frac{1}{2} \Rightarrow \theta = 60°$$

phase difference $2\theta = 120°$

6. $x = a\sin(\omega t + \phi)$

at $t = 1$s, $x=0 = a\sin(\omega t+\phi) \Rightarrow \phi = -\omega$

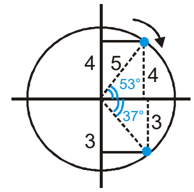
✈ $v = a\omega\cos(\omega t + \phi)$ ∴ At $t=2$s, $\frac{1}{4} = a\omega\cos(2\omega+\phi)$

$$\Rightarrow \frac{1}{4} = a\left(\frac{2\pi}{6}\right)\cos(\omega) = \frac{a\pi}{3}\cos\left(\frac{\pi}{3}\right) = \frac{a\pi}{3}\left(\frac{1}{2}\right)$$

$$\Rightarrow a = \frac{3}{2\pi}$$

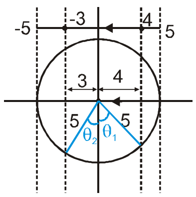
7. Minimum phase difference between two position

$= 53° + 37° = 90°$



$\Rightarrow$ Time taken $= \frac{T}{4} = \frac{20}{4} = 5s$

Or



$\omega = \frac{2\pi}{T} = \frac{2\pi}{20} = \frac{\pi}{10}$

from figure $\theta_1 + \theta_2 = 53° + 37° = 90°$ or $\frac{\pi}{2}$

$\theta = \omega t \Rightarrow \frac{\pi}{2} = \frac{\pi}{10} t \Rightarrow t = 5$ sec

8. $x_1 = a \sin \omega t, x_2 = a \sin(\omega t + \phi)$

Greatest distance

$= |x_2 - x_1|_{max} = 2a \sin\frac{\phi}{2} = \frac{3a}{2} \Rightarrow \sin\frac{\phi}{2} = \frac{3}{4}$

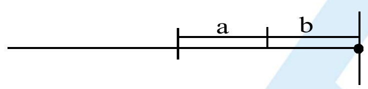
Now according to question $x_1 = x_2$

$\Rightarrow a \sin \omega t = a \sin(\omega t + \phi)$

$\Rightarrow \pi - \omega t = \omega t + \phi \Rightarrow \omega t = \frac{\pi - \phi}{2}$

$\Rightarrow x_1 = a \sin\left(\frac{\pi - \phi}{2}\right) = a \cos\frac{\phi}{2} = \frac{a\sqrt{7}}{4}$

9.



$x = A\cos\omega t \Rightarrow a = A\cos\omega$ and $a + b = A\cos 2\omega$

$\Rightarrow a + b = A[2\cos^2\omega - 1] = A\left[2 \cdot \frac{a^2}{A^2} - 1\right]$

$\Rightarrow \frac{2a^2}{A} - A = a + b \Rightarrow A^2 + (a+b)A - 2a^2 = 0$

$\Rightarrow A = \frac{-(a+b) + \sqrt{a^2 + b^2 + 2ab + 8a^2}}{2}$

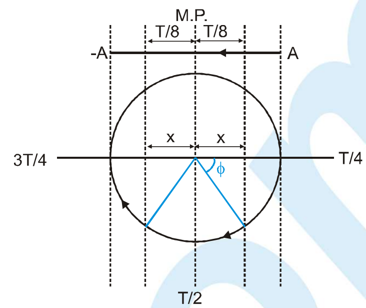
Note : Please correct the answer of the question

10. Maximum possible average velocity will be around mean position.

Average velocity in time

$\frac{T}{4} = \frac{2(A/\sqrt{2})}{T/4} = \frac{4\sqrt{2}A}{T}$

Or



$\phi = \omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$\cos\phi = \frac{x}{A} \Rightarrow \cos\frac{\pi}{4} = \frac{x}{A} \Rightarrow x = \frac{A}{\sqrt{2}}$

Average velocity

$= \frac{\text{total displacement}}{\text{total time}} = \frac{2x}{T/4} = \frac{2A/\sqrt{2}}{T/4} = \frac{4\sqrt{2}A}{T}$

11. $x = 2\sin\omega t$ &

$y = 2\sin\left(\omega t + \frac{\pi}{4}\right) = \sqrt{2}\sin\omega t + \sqrt{2}\cos\omega t$

$\Rightarrow x^2 + y^2 - \sqrt{2}xy = 2$

which represent oblique ellipse

12. $\because x = A\sin\omega t$

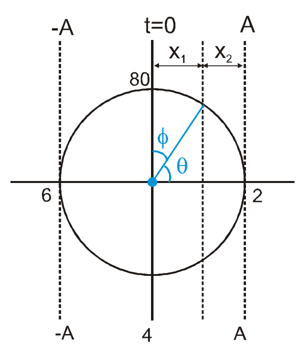
$\therefore x_1 = A\sin\omega$ & $x_1 + x_2 = A\sin(2\omega)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4} \Rightarrow x_1 = \frac{A}{\sqrt{2}}$ & $x_1 + x_2 = A$

$\Rightarrow x_2 = A - \frac{A}{\sqrt{2}}$. Therefore $\frac{x_1}{x_2} = \frac{1}{\sqrt{2} - 1}$

Or

Suppose amplitude be A and distance traveled in 1 sec be x_1 and in 2 sec be x_2.



$\phi = \omega t = \frac{2\pi}{8} 1 = \frac{\pi}{4}$

Therefore $\theta = \frac{\pi}{4}$

$\cos\theta = \cos\frac{\pi}{4} = \frac{x_1}{A}$

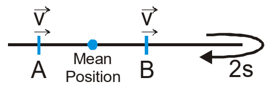
$\Rightarrow x_1 = \frac{A}{\sqrt{2}}$ & $x_1 + x_2 = A$

$\Rightarrow x_2 = A\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$. Therefore $\frac{x_1}{x_2} = \frac{1}{\sqrt{2} - 1}$

13. Maximum KE $= 2\times 5 = 10J$

Total energy $= 15 + 10 = 25J$

14. Time period $= 4(1+1) = 8$



15. Both the spring are in series

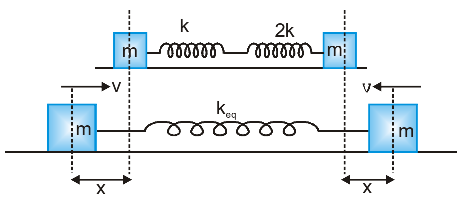
$$\therefore K_{eq} = \frac{K(2K)}{K+2K} = \frac{2K}{3}$$

Time period $T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ Here $\mu = \frac{m}{2}$

$$\therefore T = 2\pi\sqrt{\frac{m}{2}.\frac{3}{2K}} = 2\pi\sqrt{\frac{3m}{4K}}$$

Or



Total extension $= 2x$

By energy conservation

$$E = \frac{1}{2}K_{eq}(2x)^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$E = \frac{1}{2}\frac{2k}{3}4x^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{4}{3}kx^2 + mv^2$$

$$\frac{dE}{dt} = \frac{4}{3}k(2x)\frac{dx}{dt} + m(2v)\frac{dv}{dt}$$

there is no loss of energy

$$\frac{dE}{dt} = 0 \Rightarrow \frac{8}{3}kxv + 2mva = 0 \Rightarrow \frac{8kxv}{3} = -2mva$$

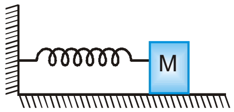
$$a = -\frac{4kx}{3m} \Rightarrow -\omega^2 x = -\frac{4kx}{3m} \Rightarrow \omega = \sqrt{\frac{4k}{3m}}$$

$$T = \frac{2\pi}{\omega} \Rightarrow 2\pi\sqrt{\frac{3m}{4k}}$$

16. For maximum displacement

$$Mg(x) = \frac{1}{2}k(2x)^2 \Rightarrow x = \frac{Mg}{2k} \Rightarrow x = \frac{Mg}{2k} - x_0$$

18.

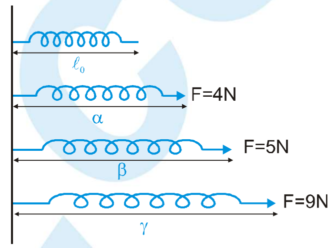


$$f = \frac{1}{m}\sqrt{\frac{k}{m}} \Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{f_1}{f_2} = \left(\frac{M+m}{M}\right)^{1/2} = \left(1+\frac{m}{M}\right)^{1/2}$$

19. Let the natural length of the spring $=$ ●$_0$

From figure



$4 = k(\alpha -$ ●$_0)$...**(i)** $\Rightarrow 5 = k(\beta -$ ●$_0)$...**(ii)**

$9 = k(\gamma -$ ●$_0)$ ...**(iii)**

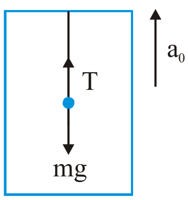
$$\text{eq. } \frac{(iii)-(i)}{(iii)-(ii)} \Rightarrow \frac{5}{4} = \frac{k(\gamma-\alpha)}{k(\gamma-\beta)} \Rightarrow \gamma = 5\beta - 4\alpha$$

20. $\omega_1 = \frac{2\pi}{2} = \pi$ and $\omega_2 = \left(\frac{2\pi}{3}\right)$

They will be in phase if $(\omega_1 - \omega_2)\,t = 0, 2\pi, 4\pi...$

$$\Rightarrow t = \frac{2\pi}{(\omega_1-\omega_2)} = \frac{2\pi}{\pi - \frac{2\pi}{3}} = 6\text{ sec}$$

21. $T_0 = 2\pi\sqrt{\frac{l}{g_{eff}}}$ ...(i)



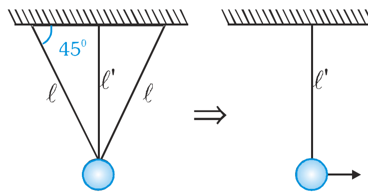
Now $\frac{T_0}{2} = 2\pi\sqrt{\frac{l}{g+a}}$ ...(ii)

$$\Rightarrow \frac{g+a}{g} = 4 \Rightarrow a = 3g(\text{upwards})$$

22. $T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

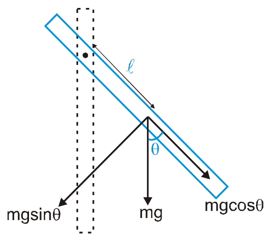
$$\frac{f_1}{f_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \left(\frac{n}{n+1}\right)^2 = \frac{l_2}{l_1} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n+1}{n}\right)^2$$

23.



$T = 2\pi\sqrt{\frac{l'}{g}}$ where $l' = \left(\frac{l}{\sqrt{2}}\right)$ So $T = 2\pi\sqrt{\frac{l}{\sqrt{2}g}}$

24.



$\tau = -mg\sin\theta \bullet \Rightarrow I\alpha = -mg\bullet\sin\theta$

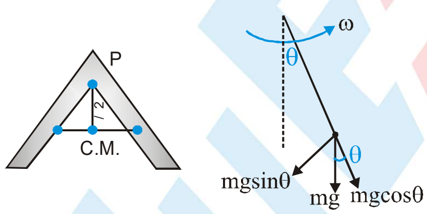
$\Rightarrow m(\bullet^2 + k^2)\alpha = -mg\bullet\theta(\sin\sim\theta)$

$$\Rightarrow \alpha = -\frac{gl\theta}{(l^2 + k^2)} = -\omega^2\theta \Rightarrow \omega = \sqrt{\frac{gl}{l + k^2}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l^2 + k^2}{gl}} = 2\sqrt{\frac{l^2 + k^2}{l}}\,[g = \pi^2]$$

$$\Rightarrow l^2 - \left(\frac{T^2}{4}\right)l + k^2 = 0$$

25. Center of mass 2m of a system is at a distance from peg P is $\frac{l}{2\sqrt{2}}$ and moment of inertia of the system is $\frac{2ml^2}{3}$



$$\tau = r \times F = -mg\sin\theta\frac{l}{2\sqrt{2}}$$

$$\Rightarrow I\alpha = -mg\sin\theta\frac{l}{2\sqrt{2}} \quad (\sin\theta\ ;\ \theta \text{ for small } \theta)$$

$$\Rightarrow \frac{2ml^2}{3}\alpha = -mg\theta\frac{l}{2\sqrt{2}} \Rightarrow \alpha = -\frac{3g\theta}{2\sqrt{21}} = -\omega^2\theta$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{2\sqrt{21}}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\sqrt{21}}{3g}}$$

26. For weightlessness

$mg = m\omega^2 a \Rightarrow g = (2\pi f)^2(0.5)$

$$\Rightarrow 2\pi f = \sqrt{2}\,g \Rightarrow f = \frac{\sqrt{2}g}{2\pi}$$

27. $U(x) = ax^2 + bx^4$

$$F = -\frac{\partial U}{\partial x} = -2ax - 4bx^3 \approx -2ax \text{ for small } x$$

$$\text{So } m\omega^2 = 2a \Rightarrow \omega = \sqrt{\frac{2a}{m}}$$

28. Time period for spring block system is $T = 2\pi\sqrt{\frac{m}{k}}$ does not effected. $T = 2\pi\sqrt{\frac{m}{k}}$

29. Here $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2x^2$

$$\Rightarrow x = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\text{cm}$$

30. at $x = \frac{\sqrt{3}}{2}A$

$$KE = \frac{1}{2}m\omega^2\left(A^2 - \frac{3}{4}A^2\right) = \frac{1}{8}m\omega^2A^2$$

KE is increased by an amount of $\frac{1}{2}m\omega^2A^2$. Let now amplitude be A_1 then total KE

$$KE_1 = \frac{1}{8}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2$$

$$= \frac{5}{8}m\omega^2A^2 = \frac{1}{2}m\omega^2\left(A_1^2 - \frac{3}{4}A^2\right) \Rightarrow A_1 = \sqrt{2}A$$

31. $x = 3\sin 2t + 4\cos 2t = 5\sin(2t + \phi)$

$\Rightarrow a = 5,\ v_{max} = a\omega = (5)(2) = 10$

32. Total energy

$$E = \frac{1}{2}m\omega^2a^2 \Rightarrow E \propto \frac{a^2}{T^2}$$

33. $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l^2 + k^2}{gl}}$

34. $\sin\left(\frac{13\pi}{6}\right) = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

Now $x = a\sin(\omega t) = \frac{a}{2}$

35. **For (A)** : at $t = \frac{3T}{4}$, particle at extreme position

$a = -\omega^2 x \quad \therefore F \neq 0$

For (B) at t = T/2, particle at mean position

$v = \omega A$(maximum)

For (C) : at t=T, particle at mean position

(t=T)

$\Rightarrow a = -\omega^2 x = 0$

For (D) : at t =T/2, particle at mean position

(t=T/2)

so $x = 0 \Rightarrow U = \frac{1}{2}kx^2 = 0$

36. For 1st condition $k_{eff} = \frac{k}{2}$

For 2nd condition $k_{eff} = \frac{(2k)(k)}{2k+k} = \frac{2}{3}k$

$\therefore T = 2\pi\sqrt{\frac{m}{k}} \qquad \therefore f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\frac{f_1}{f_2} = \sqrt{\frac{k/2}{2k/3}} = \frac{\sqrt{3}}{2}$

37. $\frac{\Delta T}{T} = \frac{\Delta\phi}{2\pi} \Rightarrow \frac{2}{8} = \frac{\Delta\phi}{2\pi} \Rightarrow \Delta\phi = \frac{\pi}{2}$

38. $\langle acceleration \rangle = \frac{\int_0^{T/2} \omega^2 a \sin\omega t\, dt}{\int_0^{T/2} dt} = \frac{2\pi\omega^2}{\pi}$

39. In an artificial satellite $g_{eff} = 0 \Rightarrow T = \infty$

40. KE at centre

$= \frac{1}{2}m\omega^2(A^2) = \frac{1}{2}m4\pi^2 f^2 A^2$

KE at distance x $= \frac{1}{2}m4\pi^2 f^2(A^2 - x^2)$

Difference $= \frac{1}{2}m \times 4\pi^2 f^2 x^2 = 2\pi^2 f^2 x^2 m$

41. Required time $= \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

42. $a = 8\pi^2 - 4\pi^2 x = -4\pi^2(x-2) \Rightarrow \omega = 2\pi$

Here a = 0 so mean position at x = 2

Let $x = A\sin(\omega t + \phi)$

As particle is at rest at x = –2 (extreme position)

and Ampliutde = 4 as particle start from extreme position.

Therefore

$x - 2 = -4\cos 2\pi t \Rightarrow x = 2 - 4\cos 2\pi t$

43. From the graph, equation of acceleration can be written

as $a = -a_{max}\cos\omega t$

$\therefore$ velocity can be written is

$v = -v_{max}\sin\omega t$.

$KE = \frac{1}{2}mv^2 = \frac{1}{2}mv^2_{max}\sin^2\omega t$

Hence the graph is as shown in A

44. $y = \sin\omega t + \sqrt{3}\cos\omega t = 2\sin\left(\omega t + \frac{\pi}{3}\right)$

To breaks off mg

$mg = m\omega^2_{min}A \Rightarrow g = 2\omega^2_{min} \Rightarrow \omega = \sqrt{\frac{g}{2}}$

moment it occurs first after t = 0

(t=0)

$2 = 2\sin\left(\omega t_1 + \frac{\pi}{3}\right) \Rightarrow \omega t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$

45. $x = A_0(1 + \cos 2\pi\gamma_2 t)\sin(2\pi\gamma_1 t)$

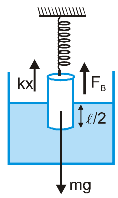
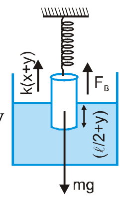
$= A_0[\sin 2\pi\gamma_1 t + \cos 2\pi\gamma_2 t + \sin 2\pi\gamma_1 t]$

$= A_0[\sin 2\pi\gamma_1 t + \frac{1}{2}\sin(2\pi(\gamma_1 + \gamma_2)t) - \frac{1}{2}\sin(2\pi(\gamma_1 - \gamma_2)t)]$

Required ratio $= \nu_1 : (\nu_1 - \nu_2) : (\nu_1 + \nu_2)$

EXERCISE - 2

Part # I : Multiple Choice

1. Initially  finally 

$kx + F_B = mg$

$kx = mg - \rho Vg$

$kx = mg - \frac{\rho A l g}{2}$ ...(i)

Let cylinder be displaced through y then restoring force,

$f_{net} = -[k(x+y) + F_B - mg]$

$$f_{net} = -\left[kx + ky + \rho A\left(\frac{1}{2}+y\right)g - mg\right]$$

$f_{net} = -[ky + \rho Agy] \Rightarrow ma = -[ky + \rho Agy]$

$$\Rightarrow \omega = \sqrt{\frac{k+\rho Ag}{m}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k+\rho Ag}{m}}$$

2. $v = A\omega\cos\omega t,\ a = -\omega^2 A\sin\omega t$

$$\Rightarrow \left(\frac{v}{A\omega}\right)^2 + \left(\frac{a}{\omega^2 A}\right)^2 = 1$$

$\Rightarrow$ Straight line in v^2 and a^2

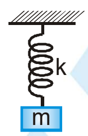
3. $\omega_1 = \sqrt{\frac{k}{m}},\ \omega_2 = \sqrt{\frac{k}{4m}} = \frac{\omega_1}{2}$

$(\omega_1 - \omega_2)\, t = 0, 2\pi, 4\pi, \ldots\ t = 0$

$$\Rightarrow t = \frac{2\pi}{\omega_1 - \omega_2} = 4 \Rightarrow 2\pi = 4(\omega_1 - \omega_2)$$

$$\Rightarrow \frac{\pi}{2} = (\omega_1 - \omega_2) \Rightarrow k = \pi^2\ N/m$$

4. At equilibrium $mg = kx_0$



$$\Rightarrow x_0 = \frac{mg}{k} = \frac{(0.2)(10)}{200} = 0.01\ m = 1\ cm$$

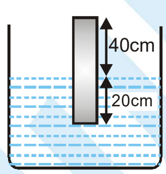
Amplitude of SHM = 1cm

$$\text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200}{0.2}} \approx 5\,Hz$$

5. When cylindrical block is partially immersed

$F_B = mg \Rightarrow 3\rho Ayg = \rho A(60\times 10^{-2})\, g$

$y = 20$ cm $\Rightarrow$ Maximum amplitude = 20 cm



Restoring force when it is slightly depressed by an amount of x.

$F = -(\Delta V\sigma g) = -(A\sigma g)x$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{A\sigma g}} = 2\pi\sqrt{\frac{\rho Ah}{A3\rho g}} = 2\pi\sqrt{\frac{h}{3g}}$$

$$= 2\pi\sqrt{\frac{60\times 10^{-2}}{3\times 9.8}} = \frac{2\pi}{7}\,s$$

6. $\vec{r} = x\hat{i} + y\hat{j} = (A\cos\omega t)\hat{i} + (2A\cos\omega t)\hat{j}$

$\Rightarrow x = A\cos\omega t,\ y = 2A\cos\omega t \Rightarrow y = 2x$

$\Rightarrow$ The motion of particle is on a straight line, periodic and simple harmonic

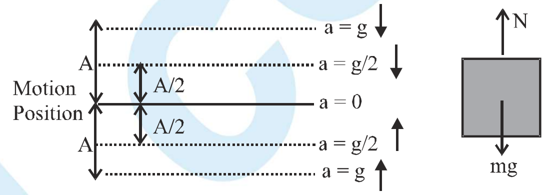
7. $x = 3\sin(100\pi t),\ y = 4\sin(100\pi t) \Rightarrow y = \frac{4}{3}x$

$\Rightarrow$ Motion of particle will be on a straight line with slope 4/3.

As $r = \sqrt{x^2+y^2} = 5\sin(100\pi t)$

so motion of particle will be SHM with amplitude 5.

8. The position of momentary rest in S.H.M. is extreme position where velocity of particle is zero.



As the block loses contact with the plank at this position i.e. normal force becomes zero, it has to be the upper extreme where acceleration of the block will be g downwards.

$$\therefore \omega^2 A = g \Rightarrow \omega^2 = \frac{10}{0.4} = 25 \Rightarrow \omega = 5\ rad/s$$

$$\text{Therefore period } T = \frac{2\pi}{\omega} = \frac{2\pi}{5}\,s$$

Acceleration in S.H.M. is given by $a = \omega^2 x$

From the figure we can see that, At lower extreme, acceleration is g upwards

$\therefore N - mg = ma \Rightarrow N = m(a+g) = 2mg$

At halfway up, acceleration is g/2 downwards

$$\therefore mg - N = ma \Rightarrow N = m\left(g - \frac{g}{2}\right) = \frac{1}{2}mg$$

At halfway down acceleration is g/2 upwards

$$\therefore N - mg = ma \Rightarrow N = m\left(g + \frac{g}{2}\right) = \frac{3}{2}mg$$

9. At maximum compression $\vec{v}_A = \vec{v}_B$ & kinetic energy of A–B system will be minimum

$$\text{so } v_A = v_B = \frac{v}{2} \Rightarrow K_{AB} = \frac{1}{4}mv^2$$

From energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx_m^2$$

$$\Rightarrow x_m = v\sqrt{\frac{m}{2k}}$$

10. $mgh = \frac{1}{2}(M+m)v^2 = \frac{1}{2}kx^2$

$mgh = \frac{1}{2}kx^2 \Rightarrow x = \left[\frac{2mgh}{k}\right]^{1/2}$

11. Let small angular displacement of cylinder be θ then restoring torque

$I\alpha = -k(R\theta)R$ where $I = \frac{3}{2}MR^2$

$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{kR^2}{\frac{3}{2}MR^2}\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \frac{2k}{3M}\theta = 0$

$\Rightarrow \omega^2 = \frac{2k}{3m}$

12. As $\beta < \alpha$ so $T = 2\pi\sqrt{\frac{L}{g}}$

13. $y = 10\sin(\omega t + \phi)$

Maximum KE $= \frac{1}{2}m\omega^2A^2$

$\Rightarrow \frac{64}{100} \times \frac{1}{2}m\omega^2A^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$\Rightarrow 64A^2 = (A^2 - x^2)100 \Rightarrow x = 0.6A$

$\Rightarrow \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2A^2 \times x$

$\Rightarrow A^2 - 0.25A^2 = A^2x$

$\Rightarrow x = 0.75$ means 75% of energy

14. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m+M}}$

15. $x = 3\sin 100t + 8\cos^2 50t$

$= 3\sin(100t) + 4(1 + \cos 100t)$

$= 4 + 5\sin(100t + 37°)$

Amplitude = 5

Maximum displacement = 4 + 5 = 9 cm

16. Maximum speed $= v_0 \Rightarrow A = \frac{v_0}{\omega_0}$

So equation of motion $x = \frac{v_0}{\omega_0}\sin(\omega_0 t)$

17. Velocity of 3kg block just before collision

$= \omega\sqrt{a^2 - x^2} = \sqrt{\left(\frac{k}{m}\right)(a^2 - x^2)}$

$= \sqrt{\left(\frac{900}{3}\right)(2^2 - 1^2)} = 30\ m/s$

Velocity of combined masses immediately after the collision $= \frac{(3)(30)}{3+6} = 10$ m/s

New angular frequency

$\omega' = \sqrt{\frac{k}{m}} = \sqrt{\frac{900}{9}} = 10$

Therefore $v' = \omega'\sqrt{a'^2 - x^2}$

$\Rightarrow 10 = 10\sqrt{a'^2 - 1^2} \Rightarrow a' = \sqrt{2}\,m$

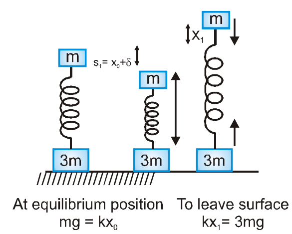
18. $a = -\omega^2 y$ and at $t = T$, y is maximum so acceleration is maximum at $t = T$.

Also y=0 at $t = \frac{3T}{4}$, so force is zero at $t = \frac{3T}{4}$

$(t = \frac{3T}{4}\ y=0)$

At $t = \frac{T}{2}$, v=0 $\Rightarrow$ PE = oscillation energy

19. From energy conservation



$\frac{1}{2}k\delta_1^2 = mg(\delta_1 + x_1) + \frac{1}{2}kx_1^2$

$\Rightarrow \delta_1^2 - \frac{2mg}{k}\delta_1 - \frac{15m^2g^2}{k^2} = 0$

$\Rightarrow \delta_1 = \frac{5mg}{k} \Rightarrow \delta = \frac{4mg}{k}$

$\Rightarrow$ If $\delta \geq \frac{4mg}{k}$ the lower disk will bounce up.

$\delta \geq \frac{4mg}{k}$

Now If $\delta = \frac{2mg}{k}$ then maximum normal reaction from ground on lower disk $\delta = \frac{2mg}{k}$

$N = 3mg + k(x_0 + \delta) = 6\,mg$

Part # II : Assertion & Reason

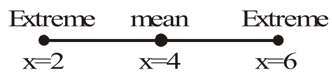
1. A **2.** A **3.** D **4.** B **5.** A **6.** D
7. A

EXERCISE - 3

Part # I : Matrix Match Type

Match the column

1.



$F = 8 - 2x = -2(x-4)$

2. $y = A\sin\omega t + A\sin\omega t\cos\frac{2\pi}{3} + A\cos\omega t\sin\frac{2\pi}{3}$

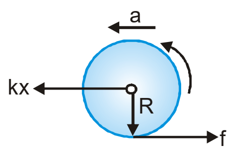
$$= \frac{A}{2}\sin\omega t + \frac{A\sqrt{3}}{2}\cos\omega t = A\sin\left(\omega t + \frac{\pi}{3}\right)$$

3. x is positive in II & IV; v is positive if $f_{ext} > 0$

Part # II : Comprehension

Comprehension # 1

1.



$a = \alpha R$

$$kx - f = ma,\ fR = \left(\frac{mR^2}{2}\right)\alpha \Rightarrow f = \frac{1}{3}kx$$

But $x = A\cos\omega t$
($\therefore$ the cylinder is starting from x=A)

So $f = \frac{kA}{3}\cos\omega t$

2. $\frac{1}{2}kA^2 = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) = \frac{1}{2}mv^2\left(1 + \frac{1}{2}\right) = \frac{3}{4}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2kA^2}{3m}} = \sqrt{\frac{2(10)(2)^2}{3(2)}} = \sqrt{\frac{40}{3}}\,ms^{-1}$$

Compreshension # 2

1. Max. acceleration of 1kg $= \frac{(0.6)(1)(10)}{1} = 6ms^{-2}$

Max. acceleration of 2kg $= \frac{(0.4)(3)(10)}{3} = 4ms^{-2}$

Therefore maximum acceleration of system can be 4 m/s^2

$$\Rightarrow \omega^2 A = 4 \Rightarrow A = \frac{4}{\omega^2} = \frac{4}{(k/m)} = \frac{4}{54/6} = \frac{4}{9}\,m$$

2. $\omega^2 A$ = constant $\Rightarrow A \propto \frac{1}{k}$

Comprehension # 3

1. Both the blocks remains in contact until the spring is in compression. In this time system complete half oscillation. By reduced mass concept time period of system

$$T = 2\pi\sqrt{\frac{\mu}{k}} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2s$$

$$\Rightarrow \text{Required time} = \frac{T}{2} = \frac{2}{2} = 1s$$

2.

Let velocity of rear 2kg be v_1 and front 2kg be v_2 then $20 = 2v_2 - 2v_1 \Rightarrow v_2 - v_1 = 10$

Now by conservation of mechanical energy

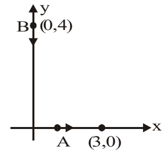
$$\frac{1}{2}(2)(10)^2 = \frac{1}{2}(2)v_1^2 + \frac{1}{2}(2)v_2^2 \Rightarrow v_1^2 + v_2^2 = 100$$

But $v_2^2 + v_1^2 - 2v_1v_2 = 100 \Rightarrow v_1v_2 = 0 \Rightarrow v_1 = 0$ as $v_2 \neq 0$

Comprehension # 4

1. As A is at its negative extreme at t=0

so $x - 3 = 2\sin(2\pi t + 3\pi/2) \Rightarrow x = 3 - 2\cos(2\pi t)$



2. As B is at its equilibrium position and moving towards negative extreme at t = 0

so $y - 4 = 0\ 2\sin(2\pi t + \pi) \Rightarrow y = 4 - 2\sin(2\pi t)$

3. Distance between A & B

$= \sqrt{x^2 + y^2}$

$= \sqrt{(3 - 2\cos 2\pi t)^2 + (4 - 2\sin 2\pi t)^2}$

$= \sqrt{9 + 4\cos^2 2\pi t - 12\cos 2\pi t + 16 + 4\sin^2 2\pi t - 16\sin 2\pi t}$

$= \sqrt{29 - 20\left(\frac{3}{5}\cos 2\pi t + \frac{4}{5}\sin 2\pi t\right)}$

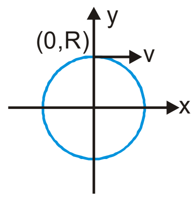
$= \sqrt{29 - 20\sin(2\pi t + 37°)}$

Maximum distance $= \sqrt{29 + 20} = \sqrt{49} = 7\text{cm}$

Minimum distance $= \sqrt{29 - 20} = \sqrt{9} = 3\text{cm}$

Comprehension # 5

1. $\omega = \frac{2\pi}{T} = \frac{2\pi}{(2\pi R / v)} = \frac{v}{R}$



2. At t=0, x=0, $v = v_0 = \omega R$

so $\cos(\omega t + \phi) = 0$ & $-\sin(\omega t + \phi) = +1 \Rightarrow \phi = \frac{3\pi}{2}$

EXERCISE - 4

Subjective Type

1. $S = A\cos\omega t - \frac{A}{2}\sin\omega t - \frac{A}{2}\cos\omega t + \frac{A}{8}\sin\omega t$

$= \frac{A}{2}\cos\omega t - \frac{3A}{8}\sin\omega t$

$= \frac{5A}{8}\left(\frac{4}{5}\cos\omega t - \frac{3}{5}\sin\omega t\right) = \frac{5A}{8}\cos(\omega t + 37°)$

$\Rightarrow A' = \frac{5A}{8}, \delta = 37°$

2. The maximum velocity of the particle at the mean position

$v_{max} = A\omega = A(2\pi n)$

$\Rightarrow A = \frac{v_{max}}{2\pi n} = \frac{3.14}{2 \times 3.14 \times 20} = 0.025\text{m}$

If at the instant t = 0, displacement be zero so displacement equation is

$y = A\sin\omega t = A\sin 2\pi nt = 0.025\sin(40\pi t)\text{ m}$

3. $x = A\sin(\pi t + \phi)$

At t = 0 x = 1 cm $\Rightarrow A\sin\phi = 1$ ...(i)

Velocity $v = \frac{dx}{dt} = \pi A\cos(\pi t + \phi)$

At t = 0 $\pi = \pi A\cos\phi \Rightarrow A\cos\phi = 1$ ...(ii)

from (i) & (ii) $A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$

$\Rightarrow A = \sqrt{2}$ cm and $\tan\phi = 1 \Rightarrow \phi = \frac{\pi}{4}$ rad

4. Maximum distance

$= 2a\sin\left(\frac{\phi}{2}\right) = (2a)(0.9) = 1.8a$

5. $y = A\sin\omega t = A\sin\frac{2\pi}{T}t$

$\therefore \frac{\sqrt{3}}{2}A = A\sin\left(\frac{2\pi}{T} \times 2\right) = A\sin\left(\frac{4\pi}{T}\right)$

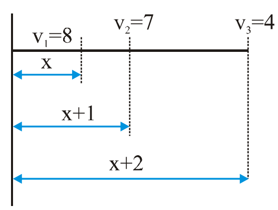
$\Rightarrow \frac{4\pi}{T} = \frac{\pi}{3} \Rightarrow T = 12\text{s}$

6. As $v = \omega\sqrt{a^2 - x^2}$ so $v_1 = n\sqrt{a^2 - \frac{3a^2}{4}} = \frac{na}{2}$

and $v_2 = \frac{3}{2}na = n\sqrt{A^2 - \frac{3a^2}{4}}$

$\Rightarrow A = \sqrt{3}a = 15\sqrt{3}\text{cm}$

7. Energy at all the three points are equal



$\frac{1}{2}mv_1^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}k(x+1)^2$

$\frac{1}{2}m(64) + \frac{1}{2}kx^2 = \frac{1}{2}m(49) + \frac{1}{2}k(x+1)^2$

$15 = \omega^2 + 2\omega^2 x$ ...(i)

$\frac{1}{2}mv_1^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + \frac{1}{2}k(x+2)^2$

$\frac{1}{2}m(64) + \frac{1}{2}kx^2 = \frac{1}{2}m(16) + \frac{1}{2}k(x+2)^2$

$12 = \omega^2 + \omega^2 x$ ...(ii)

from (i) & (ii) $\omega = 3$ & $x = 1/3$ then, total energy is equal for the maximum kinetic energy

$$\frac{1}{2}m(64)+\frac{1}{2}m\frac{1}{9}=\frac{1}{2}mv_{max}^2 \Rightarrow v_{max}=\sqrt{65}\text{ m/s}$$

OR

$$v=\omega\sqrt{a^2-x^2} \Rightarrow 8=\omega\sqrt{a^2-x^2}$$

$$\Rightarrow 7=\omega\sqrt{a^2-(x+1)^2},\ 4=\omega\sqrt{a^2-(x+2)^2}$$

After solving the equation

$$V_{max}=a\omega=\sqrt{65}\text{ m/s}$$

8. Velocity $v=\omega\sqrt{a^2-x^2}$

$$\therefore \frac{v_2}{v_1}=\frac{\sqrt{a^2-x_2^2}}{\sqrt{a^2-x_1^2}} \Rightarrow \frac{49}{100}=\frac{a^2-16}{a^2-9} \Rightarrow a=4.76\text{ cm}$$

$\Rightarrow$ length of path = 2a = 9.52 cm

9. $x = 12\sin\omega t - 16\sin^3\omega t$

$= 12\sin\omega t - 4(4\sin^3\omega t)$ $\quad (\because 4\sin^3\theta = 3\sin\theta - \sin3\theta)$

$= 12\sin\omega t - 4(3\sin\omega t - \sin3\omega t)$

$= 12\sin\omega t - 12\sin\omega t + 4\sin3\omega t = 4\sin3\omega t$

$\Rightarrow$ motion is SHM with angular frequency 3ω

So $a_{max} = 36\omega^2$

10. (i) At equilibrium position

$$F=-\frac{dU}{dx}=0 \Rightarrow 2x-4=0 \Rightarrow x=2\text{m}$$

(ii) $F=-\frac{dU}{dx}=-(2x-4)=-2(x-2)$

$\Rightarrow F \propto -x \Rightarrow$ SHM

Here $\omega^2 = 2 \Rightarrow T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{2}}=\sqrt{2}\pi\ s^{-1}$

(iii) $a\omega = 2\sqrt{6} \Rightarrow a=\frac{2\sqrt{6}}{\sqrt{2}}=2\sqrt{3}$ m

11. $\frac{1}{2}mv_{max}^2 = 8\times10^{-3}$

$\Rightarrow \frac{1}{2}m\omega^2a^2 = 8\times10^{-3}$

$\Rightarrow \omega = 4$ rad/s

Therefore equation of SHM

$$x=0.1\sin\left(4t+\frac{\pi}{4}\right)$$

12. (i) Maximum speed of oscillating body

$$v_{max}=A\omega=A\times\frac{2\pi}{T}$$

Here A = 1 metre, T = 1.57 s

$$\therefore v_{max}=\frac{2\times3.14\times1}{1.57}=4\text{ m/s}$$

(ii) Maximum kinetic energy

$$K_{max}=\frac{1}{2}mv_{max}^2=\frac{1}{2}\times1\times(4)^2=8\text{J}$$

(iii) Total energy of particle will be equal to maximum kinetic energy.

(iv) Time period of mass suspended by spring

$$T=2\pi\sqrt{\frac{m}{K}}$$

so force constant

$$K=\frac{4\pi^2m}{T^2}=\frac{4\times(3.14)^2\times1}{(1.57)^2}=16\text{ N/m}$$

13. Common velocity after collision be v then by COLM

$$2Mv = Mu \Rightarrow v=\frac{u}{2}$$

Hence, kinetic energy

$$=\frac{1}{2}(2M)\left(\frac{u}{2}\right)^2=\frac{1}{4}Mu^2$$

It is also the total energy of vibration because the spring is unstretched at this moment, hence if A is the amplitude, then

$$\frac{1}{2}KA^2=\frac{1}{4}Mu^2 \Rightarrow A=\sqrt{\frac{M}{2K}}$$

14. Frequency of oscillation

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m_1+m_2}}=\frac{1}{2\pi}\sqrt{\frac{600}{1.5}}=\frac{10}{\pi}\text{ Hz}$$

Let maximum amplitude be A then

$$v=\omega\sqrt{A^2-x^2}$$

where x = diference in equilibrium position

$$=\frac{(m_1+m_2)g}{k}-\frac{m_1g}{k}=\frac{1}{120}\text{m}$$

and $v=\frac{0.5\times3}{1.5}=1$ m/s

Therefore

$$1=20\sqrt{A^2-\left(\frac{1}{120}\right)^2} \Rightarrow A=\frac{5\sqrt{37}}{600}=\frac{5\sqrt{37}}{6}\text{ cm}$$

15. Centre of mass will be at rest as there is no external force acting on the system.
So effective length

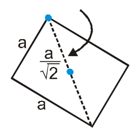
$$\ell_{eff} = \left(\frac{m_1}{m_1+m_2}\right)l$$

$$T = 2\pi\sqrt{\frac{l_{eff}}{g}} = 2\pi\sqrt{\frac{m_1 l}{(m_1+m_2)g}}$$

16. Additional force

$$\frac{mg}{4} = kA \Rightarrow A = \frac{g}{4} = 2.45\text{m}$$

17. $$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{ma^2}{6}+\frac{ma^2}{2}}{mg\left(\frac{a}{\sqrt{2}}\right)}} = 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$



18. Moment of inertia about hinged point
$I = mR^2 + mR^2 = 2mR^2$

$$f = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}} = \frac{1}{2\pi}\sqrt{\frac{mg\sqrt{R^2+\frac{4R^2}{\pi^2}}}{2mR^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g\sqrt{1+\frac{4}{\pi^2}}}{2R}}$$

19. $T = 2\pi\sqrt{\frac{I}{mgl}}$ where

$$I = \frac{mL^2}{3} + \left(\frac{mL^2}{12}+mL^2\right) = \frac{17mL^2}{12} \ \& \ \ell = \frac{3L}{4}$$

(Distance of centre of mass from hinge)

$$\Rightarrow T = 2\pi\sqrt{\frac{17mL^2}{12(2mg(3L/4))}} = 2\pi\sqrt{\frac{17L}{18g}}$$

20. **(i)** Let $x_0 = \frac{Mg}{k}$ = initial compression in spring

COME : $\frac{1}{2}k(x_0+b)^2 = \frac{1}{2}k(a-x_0)^2 + (m+M)g(b+a)$

$$\Rightarrow k = \frac{2mg}{b-a}$$

(ii) $k = \frac{2mg}{(b-a)} \Rightarrow (m+M)\omega^2 = \frac{2mg}{b-a}$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(m+M)}}$$

(iii) Let h = initial height of m over the pan
v = common velocity of (m+M) after collision

COLM : $m\sqrt{2gh} = (m+M)v$ ....(i)

COME: $\frac{1}{2}kx_0^2 + \frac{1}{2}(m+M)v^2 + (m+M)gb = \frac{1}{2}k(x_0+b)^2$

$$\Rightarrow h = \left(\frac{M+m}{m}\right)\left(\frac{ab}{b-a}\right)$$

21. Let block be displaced by x then displacement in springs be x_1, x_2, x_3 and x_4
Such that $x = 2x_1 + 2x_2 + 2x_3 + 2x_4$
Now let restoring force on m be F = kx then

$$2f = k_1x_1 = k_2x_2 = k_3x_3 = k_4x_4$$

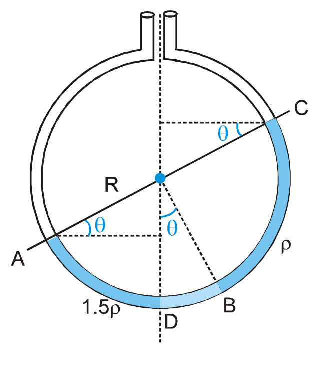
$$\Rightarrow \frac{F}{k} = \frac{4F}{k_1} + \frac{4F}{k_2} + \frac{4F}{k_3} + \frac{4F}{k_4}$$

$$\Rightarrow \frac{1}{k} = 4\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{1}{k_4}\right)$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{4m\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{1}{k_4}\right)}$$

22. **(a)** Let the liquid of density 1.5 ρ occupy the left portion AB and the liquid of density ρ occupy the right portion BC of the tube. The pressure at the lowest point D due to the liquid on the left is

$P_1 = (R - R\sin\theta)\,1.5\,\rho g$



The pressure due to the liquid on the right is $P_2 = (R\sin\theta + R\cos\theta)\rho g + (R - R\cos\theta)1.5\,\rho g$

Since the liquids are in equilibrium

$P_1 = P_2$ or $(R - R\sin\theta)\,1.5\,\rho g = (R\sin\theta + R\cos\theta)\rho g$
$+ (R - R\cos\theta)1.5\,\rho g$

Solving, we get

$\tan\theta = 0.2$ or $\theta = \tan^{-1}(0.2)$

(b) Let the whole liquid be given a small angular displacement α towards right. Then the pressure difference between the right and the left limbs is

$dP = P_2 - P_1$

$= [R\sin(\theta+\alpha) + R\cos(\theta+\alpha)\,\rho g + [R - R\cos(\theta+\alpha)\,1.5\rho g - [R - R\sin(\theta+\alpha)]\,1.5\,\rho g$

$= R\rho g\,[2.5\sin(\theta+\alpha) - 0.5\cos(\theta+\alpha)]$

$= R\rho g\,[2.5(\sin\theta\cos\alpha + \cos\theta\sin\alpha) - 0.5(\cos\theta\cos\alpha - \sin\theta\sin\alpha)]$

For small α

$\sin\alpha \approx \alpha, \cos\alpha \approx 1 \quad \therefore dP = R\rho g$

$[2.5\sin\theta + 2.5\,\alpha\cos\theta - 0.5\cos\theta + 0.5\,\alpha\sin\theta]$

As $\tan\theta = 0.2$, $\sin\theta \dfrac{0.2}{\sqrt{1.04}} \approx 0.2$, $\cos\theta = \dfrac{1}{\sqrt{1.04}} \approx 1$

$\therefore dP = R\rho g\,[2.5\times0.2 + 2.5\,\alpha - 0.5 + 0.5\times0.2\,\alpha]$

$= R\rho g\,[2.6\,\alpha] = 2.6\,\rho g$

where $y = R\alpha$, the linear displacement.

$\therefore$ Restoring force $F = 2.6\,\rho gAy$

This shows that $F \propto y$

Hence the motion is simple harmonic with force constant $k = 2.6\,\rho gA$

Now, total mass of the liquid

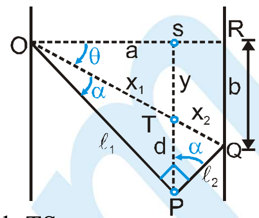
$$m = \frac{2\pi R}{4}A\rho + \frac{2\pi R}{4}A(1.5p) = \frac{5\pi RA\rho}{4}$$

$\therefore$ Time period

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5\pi rA\rho}{4\times2.6\rho gA}}$$

$$= \pi\sqrt{\frac{1.93\pi R}{9.8}} = 2.47\sqrt{R}\ \text{seconds.}$$

23. Given $a^2 + b^2 = \bullet_1^2 + \bullet_2^2$



$$\tan\theta = \frac{b}{a}$$

$$\tan\alpha = \frac{l_2}{l_1}$$

Let $OT = x_1$, $TQ = x_2$, $PT = d$, $TS = y$

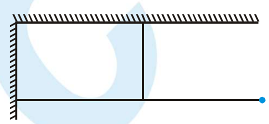
From geometry

$d + y = \bullet_1\sin(\alpha+\theta)$

and $d + \bullet_1\cos(\alpha+\theta)\tan\theta = \bullet_1\sin(\alpha+\theta)$

$$\Rightarrow d = \frac{l_1 l_2}{a} \qquad \therefore T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{l_1 l_2}{ag}}$$

24. $T\sin\theta l = I\alpha$

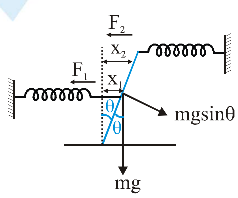


$$\Rightarrow -2mg\theta l = m(4l^2)\alpha \Rightarrow \alpha = -\frac{2g\theta}{4l}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{2g}{4l}\theta = 0 \Rightarrow \omega = \sqrt{\frac{2g}{4l}} = \sqrt{\frac{g}{2l}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2l}{g}}$$

25. (i)



$$\theta = \frac{x_1}{1/2},\ x_1 = \frac{\theta l}{2},\ f_1 = \frac{K\theta l}{2}$$

$$\theta = \frac{x_2}{1},\ x_2 = \theta\bullet,\ f_2 = k\theta\bullet$$

$$\tau = \frac{L}{2}\times F_1 + L\times F_2 \ge mg\sin\theta\frac{L}{2}$$

$$\Rightarrow \frac{L}{2}\frac{K\theta L}{2} + LK\theta L \ge mg\theta\frac{L}{2}$$

$$\Rightarrow \frac{5}{4}K\theta L^2 \ge mg\frac{\theta L}{2} \Rightarrow K \ge \frac{2mg}{5L}$$

(ii) If the rod is displaced through an angle θ, then

$$-(kL\theta)L - \left(\frac{kL}{2}\theta\right)\frac{L}{2} + Mg\left(\frac{L}{2}\right)\theta = \frac{ML^2}{3}\alpha \quad \left(k = \frac{Mg}{L}\right)$$

$$\Rightarrow -\frac{9k\theta}{4M} = \alpha \Rightarrow \omega = \frac{3}{2}\sqrt{\frac{k}{m}}$$

26. $T_0 = 2\pi\sqrt{\frac{L}{g}}$ (Lift is stationary)

$T_1 = 2\pi\sqrt{\frac{L}{g+a}}$ (Lift is accelerated upward)

$T_2 = 2\pi\sqrt{\frac{L}{g-a}}$ (Lift is deacelerated upward)

Let x = total upward distance travelled

$\Rightarrow \frac{x}{2} = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{x}{a}}$

$\therefore$ for upward accelerated motion

$$\Delta T = \left(t\frac{T_1}{T_0} - t\right)\times 2 = 2\left(\sqrt{\frac{x}{a}} - 1\right)\left(\sqrt{\frac{g}{g+a}}\right) \quad \text{...(i)}$$

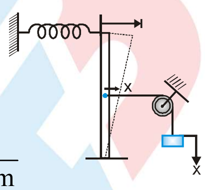
& for upward decelerated motion

$$\Delta T = \left(t_1\frac{T_2}{T_0} - t\right)\times 2 = 2\left(\sqrt{\frac{x}{a}} - 1\right)\left(\sqrt{\frac{g}{g-a}}\right) \quad \text{...(ii)}$$

$$\therefore \Delta T_{total} = 2\left(\sqrt{\frac{x}{a}}\right)\left(\sqrt{\frac{g}{g-a}} - \sqrt{\frac{g}{g+a}}\right)$$

$$= \sqrt{\frac{4x}{a}}\left(\sqrt{\frac{g}{g-a}} - \sqrt{\frac{g}{g+a}}\right)$$

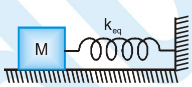
27. From energy equation



$$\frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{x}{a}b\right)^2 = E$$

$$\Rightarrow ma + \frac{kb^2}{a^2}x = 0 \Rightarrow T = 2\pi\frac{a}{b}\sqrt{\frac{m}{k}}$$

28. **(i)** This system can be reduced to



Where $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(0.1)(0.1)}{0.1 + 0.1} = 0.05$ kg

and $k_{eq} = k_1 + k_2 = 0.1 + 0.1 = 0.2\ Nm^{-1}$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{\mu}} = \frac{1}{2\pi}\sqrt{\frac{0.2}{0.05}} = \frac{1}{\pi}\text{Hz}$$

(ii) Compression in one spring is equal to extension in other spring $= 2R\theta = 2(0.06)\left(\frac{\pi}{6}\right) = \frac{\pi}{50}$ m

Total energy of the system

$$E = \frac{1}{2}k_1(2R\theta)^2 + \frac{1}{2}k_2(2R\theta)^2 = k(2R\theta)^2$$

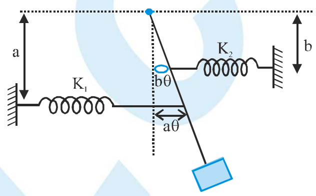
$$= (0.1)\left(\frac{\pi}{5}\right)^2 = 4\pi \times 10^{-5} J$$

(iii) From mechanical energy conservation

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = E$$

$\Rightarrow 0.1v^2 = 4\pi^2 \times 10^{-5} \Rightarrow v = 2\pi\times 10^{-2} ms^{-1}$

29.



For small angular displacement θ, net torque towards mean position is

$\tau = (k_1a\theta)\,a + (k_2b\theta)\,b \Rightarrow I\alpha = (k_1a^2 + K_2b^2)\,\theta$

$\Rightarrow (mL^2 + \frac{1}{3}ML^2)\,\alpha = (k_1a^2 + k_2b^2)\,\theta$

$$\Rightarrow \alpha = \frac{k_1a^2 + k_2b^2}{L^2\,(m + \frac{M}{3})}\theta \qquad \therefore \omega^2 = \frac{k_1a^2 + k_2b^2}{L^2\,(m + \frac{M}{3})}$$

Hence frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1a^2 + k_2b^2}{L^2(m + \frac{M}{3})}}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Time period of a mass loaded spring

$T = 2\pi\sqrt{\frac{m}{k}}$ So $T \propto \frac{1}{\sqrt{k}}$ ...(i)

Spring constant (k) is inversely proportional to the length of the spring, i.e.

$$k \propto \frac{1}{l}$$

$$\frac{k_{\text{complete spring}}}{k_{\text{cut spring}}} = \frac{(1/l)}{\left(\frac{1}{l/n}\right)} = \frac{1}{n}$$

$$\Rightarrow k_{\text{cut spring}} = n\left(k_{\text{complete spring}}\right)$$

$$\Rightarrow \frac{T_{\text{cut spring}}}{T_{\text{complete spring}}} = \sqrt{\frac{k_{\text{complete spring}}}{k_{\text{cut spring}}}} = \frac{1}{\sqrt{n}} \quad \text{[From (i)]}$$

$$\Rightarrow T_{\text{cut spring}} = \frac{T_{\text{complete spring}}}{\sqrt{n}}$$

2. In a simple harmonic oscillator the potential energy is directly proportional to the square of displacement of the body from the mean position; at the mean position the displacement is zero so the PE is zero but speed is maximum; hence KE is maximum.

3. The time period of the swing is

$$T = 2\pi\sqrt{\frac{l_{eff}}{g}}$$

Where l_{eff} is the distance from point of suspension to the centre of mass of child. As the child stands up; the l_{eff} decrease hence T decreases.

4. $T = 2\pi\sqrt{\frac{M}{k}}$ and $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

On dividing $\frac{5T}{3T} = \sqrt{\frac{M+m}{M}}$

$$\Rightarrow \frac{25}{9} = \frac{M+m}{M} \Rightarrow \frac{m}{M} = \frac{16}{9}$$

5. As maximum value of

$$A\sin\theta + B\cos\theta = \sqrt{A^2 + B^2}$$

so amplitude of the particle

$$= 4\sqrt{1^2 + 1^2} = 4\sqrt{2}$$

6. A harmonic oscillator crosses the mean position with maximum speed hence kinetic energy is maximum at mean position (i.e., x=0)

Total energy of the harmonic oscillator is a constant.

PE is maximum at the extreme position.

7. Maximum velocity $v_{max} = \omega A$

Given $(v_{max})_1 = (v_{max})_2 \Rightarrow \omega_1 A = \omega_2 A_2$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{m} \times \frac{m}{k_1}} = \sqrt{\frac{k_2}{k_1}}$$

8. The time period of a simple pendulum of density ρ when held in a surrounding of density σ is

$$t_{medium} = t_{air}\sqrt{\frac{\rho}{\rho - \sigma}}$$

$$\Rightarrow t_{water} = t_0\sqrt{\frac{\frac{4}{3}\times 10^{-3}}{\left(\frac{4}{3}-1\right)\times 10^{-3}}} = t_0\sqrt{\frac{4}{3}\times\frac{3}{1}} = 2t_0$$

9. $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T^2 \propto \frac{1}{k}$

$$t_1^2 \propto \frac{1}{k_1} \;\&\; t_2^2 \propto \frac{1}{k_2} \Rightarrow t_1^2 + t_2^2 \propto \frac{1}{k_1} + \frac{1}{k_2}$$

But $\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k} \propto T^2 \Rightarrow t_1^2 + t_2^2 = T^2$

10. The total energy of a harmonic oscillator is a constant and it is expressed as $E = \frac{1}{2}m\omega^2$ (Amplitude)^2

E is independen to x instantaneous displacement.

11. Natural frequency of oscillator = ω_0

Frequency of the applied force = ω

Net force acting on oscillator at a displacement x

$= m(\omega_0^2 - \omega^2)x$ ...(i)

Given that $F \propto \cos\omega t$ ...(ii)

From eqs. (i) and (ii) we get

$m(\omega_0^2 - \omega^2)\; x \propto \cos\omega t$ ...(iii)

Also, $x = A\cos\omega t$ ...(iv)

From eqs. (iii) and (iv), we get

$$m(\omega_0^2 - \omega^2)A\cos\omega t \propto \cos\omega t \Rightarrow A \propto \frac{1}{m\left(\omega_0^2 - \omega^2\right)}$$

12. Initial acceleration

$$= \frac{F}{m} = \frac{(15)(0.20)}{0.3} = 10\text{ ms}^{-2}$$

13. $y = \sin^2\omega t = \frac{1-\cos 2\omega t}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

⇒ motion is SHM with time period

$$= \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

14. $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$

$$v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$$

$$= 10\pi\cos\left(100\pi t + \frac{\pi}{3}\right) \Rightarrow y_2 = 0.1\cos 100\pi t$$

$$\Rightarrow v_2 = -0.1\ \pi\sin 100\pi t = 0.1\pi\cos\left(100\pi t + \frac{\pi}{2}\right)$$

Phase difference between v_1 and v_2

$$= \left(100\pi t + \frac{\pi}{3}\right) - \left(100\pi t + \frac{\pi}{2}\right) = -\frac{\pi}{6}$$

15. $\frac{d^2x}{dt^2} + \alpha x = 0$

On comparing the above equation with the equation of SHM

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \Rightarrow \omega^2 = \alpha \Rightarrow \omega = \sqrt{\alpha} \Rightarrow T = \frac{2\pi}{\sqrt{\alpha}}$$

16. The expression of time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l_{eff}}{g_{eff}}}$$

Where l_{eff} is the distance between point of suspension and centre of gravity of bob. As the hole is suddenly unplugged, l_{eff} first increases then decrease because of shifting of CM due to which the time period first increases and then decreases to the original value.

17. Maximum velocity of a particle during SHM is

$$v_{max} = \omega A \Rightarrow 4.4 = \frac{2\pi}{T}\left(7 \times 10^{-3}\right) \Rightarrow T = 0.01s$$

18. $KE = \left(\frac{75}{100}\right) TE$

$$\Rightarrow \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{3}{4}\left[\frac{1}{2}m\omega^2A^2\right] \Rightarrow x = \frac{A}{2}$$

Now from $x = A\sin\omega t$ we have

$$\frac{A}{2} = A\sin(\omega t) \Rightarrow \left(\frac{2\pi}{T}\right)t = \frac{\pi}{6}s \Rightarrow t = \frac{1}{6}$$

19. $\because$ $x = 2 \times 10^{-2}\cos\pi t$

$\therefore$ $v = (2 \times 10^{-2})(\pi)(-\sin\pi t) = -2\pi \times 10^{-2}\sin\pi t$

Speed will be maximum if

$$\pi t = \frac{\pi}{2} \Rightarrow t = \frac{1}{2} = 0.5s$$

20. $\because x = x_0\cos\left(\omega t - \frac{\pi}{4}\right)$

$$\therefore v = \frac{dx}{dt} = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$\Rightarrow a = \frac{dv}{dt} = -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0\omega^2\cos\left(\pi + \omega t - \frac{\pi}{4}\right)$$

$$\Rightarrow x_0\omega^2\cos\left(\pi + \omega t - \frac{\pi}{4}\right) = A\cos(\omega t + \delta)$$

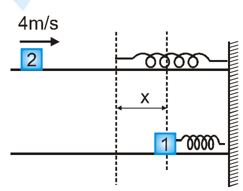
$$\Rightarrow A = x_0\omega^2 \text{ and } \delta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

21. $W.D_f = K_f + U_f - K_i - U_i$

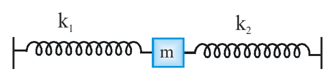
$$\Rightarrow -f.x = \frac{1}{2}kx^2 + 0 - \frac{1}{2}mv^2 - 0$$

$$\Rightarrow -15x = 5000x^2 - 16 = 0$$

$$\Rightarrow 5000x^2 + 15x - 16 = 0 \Rightarrow x = 5.5\text{ cm}$$



22.



The original frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

On increasing the k_1 and k_2 by 4 times, then f' becomes

$$f' = \frac{1}{2\pi}\sqrt{\frac{4(k_1 + k_2)}{m}} = 2f$$

23. Mass = m, amplitude = a, frequency = ν

$$(KE)_{av} = \frac{1}{4}m(2\pi\nu)^2 a^2 = \pi^2 m\nu^2 a^2$$

24. $a = -\omega^2 x$, $v = \omega\sqrt{A^2 - x^2}$

$$a^2T^2 + 4\pi^2v^2 = \omega^4x^2T^2 + 4\pi^2\omega^2(A^2 - x^2)$$

$$= \frac{4\pi^2}{T^2}\omega^2x^2T^2 + 4\pi^2\omega^2(A^2 - x^2)$$

$$= 4\pi^2\omega^2A^2 = \text{constant}$$

$$\frac{aT}{x} = -\frac{\omega^2 xT}{x} = -\omega^2 T = \text{constant}$$

25. $n_1 = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$ ..... (i)

and

$n_2 = \frac{1}{2\pi}\sqrt{\frac{k}{M+m}}$ ..... (ii)

according to conservation of linear momentum

$Mv_1 = (M+m)v_2 \Rightarrow M_1A_1\omega_1 = (M+m)A_2\omega_2$

From equation (i) & (ii)

$$\frac{A_1}{A_2} = \left(\frac{M+m}{M}\right).\frac{\omega_2}{\omega_1} = \left(\frac{M+m}{M}\right)\sqrt{\frac{M}{M+m}} = \sqrt{\frac{M+m}{M}}$$

26. $X_1 = A\sin\omega t$

$X_2 = X_0 + A\sin(\omega t + \phi)$

$X_2 - X_1 = X_0 + A\sin(\omega t+\phi) - A\sin\omega t$

Become $|X_2 - X_1|_{max} = X_0 + A$

$|A\sin(\omega t+\phi) - A\sin\omega t|_{max} = A \Rightarrow \phi = \frac{\pi}{3}$

27. By using $T = 2\pi\sqrt{\frac{m}{A\rho g}}$

Where $m = \bullet^3 d$ and $A = \bullet^2$

$T = 2\pi\sqrt{\frac{1^3 d}{1^2\rho g}} \Rightarrow T = 2\pi\sqrt{\frac{1d}{\rho g}}$

28. $k_1\bullet_1 = k_2\bullet_2 = k\bullet.\ k_1 = \frac{5k}{2}$

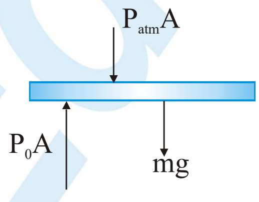
29. Equation of damped simple pendulum

$\frac{d^2x}{dt^2} = -bv + g\sin\theta \Rightarrow \frac{d^2x}{dt^2} = -bv + \frac{g}{1}x = 0$

By solving above equation $x = A_0 e^{-\frac{b}{2}t}\sin(\omega t+\phi)$

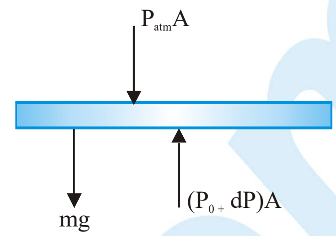
At $t = \tau$, $A = \frac{A_0}{2}$ so $\tau = \frac{2}{b}$

30. FBD of piston at equilibrium



$\Rightarrow P_{atm}A + mg = P_0A$ ...(i)

FBD of piston when piston is pushed down a distance x



$P_{atm} + mg - (P_0 + dP)A = m\frac{d^2x}{dt^2}$ ...(ii)

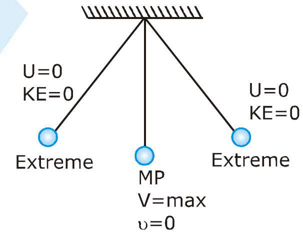
Process is adiabatic

$\Rightarrow PV^\gamma = C \Rightarrow -dP = \frac{\gamma PdV}{V}$

Using 1, 2, 3 we get $f = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

31. 2 **32.** 1

33. KE_{max} at mean position.



PE_{min} at mean position

34. $\frac{dT}{T} = \frac{1}{2}\frac{dL}{L} - \frac{1}{2}\frac{dg}{g}$

$\frac{90}{100} + \frac{1}{100}$

$\frac{1}{2}\frac{dg}{g} = \frac{1}{2}\frac{dL}{L} - \frac{dT}{T}$

$\frac{1}{2}\times\frac{0.1}{20} = \frac{1/100}{90/100} = \frac{1}{400} + \frac{1}{90}$

$\frac{1}{2}\frac{dg}{g} = \frac{1}{400} + \frac{1}{90}$

$\frac{dg}{g} = \left(\frac{490}{400\times 90}\right)\times 2 = \left(\frac{490}{200\times 90}\right) = 0.20272$

$= dg/g \times 100 \approx 2.72\% \approx 3\%$

35. $T_{mean} = \dfrac{90+91+95+92}{4} = 92$

$\Delta_1 = 90 - 92 = 2$

$\Delta_2 = 91 - 92 = 1$

$\Delta_3 = 95 - 92 = 3$

$\Delta_4 = 92 - 92 = 0$

$\Delta_{mean} = \dfrac{2+1+3+0}{4} = 1.5$

$\therefore$ T = 92 ± 2 s (Using significant Figure)

36. $v = \omega\sqrt{A^2 - x^2}$; $v = \omega\sqrt{A^2 - \dfrac{4A^2}{9}} = \dfrac{\omega\sqrt{5}A}{3}$

New SHM

$3v = \omega\sqrt{A_N^2 - x_N^2}$; $\dfrac{3\omega\sqrt{5}A}{3} = \omega\sqrt{A_N^2 - \dfrac{4A^2}{9}}$

$5A^2 = A_N^2 - \dfrac{4A^2}{9}$

$A_N^2 = \dfrac{49A^2}{9} \Rightarrow A_N = \dfrac{7A}{3}$

Part # II : IIT-JEE ADVANCED

1. $U(x) = k|x|^3$

$\therefore \; [k] = \dfrac{[U]}{[x^3]} = \dfrac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Now, time period may depend on

$T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$

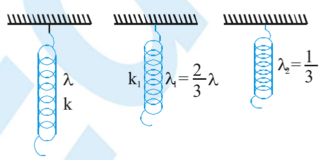
$\Rightarrow [M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z = [M^{x+z}L^{y-z}T^{-2z}]$

Equating the powers, we get

$-2z = 1$ or $z = -1/2$ $y - z = 0$ or $y = z = -1/2$

Hence, $T \propto (\text{amplitude})^{-1/2} \Rightarrow T \propto (a)^{-1/2} \Rightarrow T \propto \dfrac{1}{\sqrt{a}}$

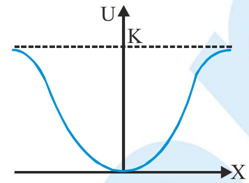
2. $●_1 = 2●_2 \;\therefore ●_1 = \dfrac{2}{3}●$



Force constant $k \propto \dfrac{1}{\text{length of spring}}$ $\therefore k_1 = \dfrac{3}{2}k$

3. $U(x) = k(1 - e^{-x^2})$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U versus x graph will be as shown.



From the graph it is clear that at origin.Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is also zero because

$F = \dfrac{-dU}{dx} = -(\text{slope of U–x graph}) = 0.$

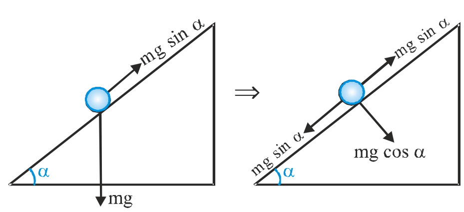
Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about x=0 for small displacements. Therefore, correct option is (d). (a), (b) and (c) options are wrong due to following reasons :

(a) At equilibrium positoin $F = \dfrac{-dU}{dx} = 0$ i.e, slope of U–x graph should be zero and from the graph we can see that slope is zero at x=0 and x=±∞.
Now among these equilibriums stable equilibrium position is that where U is minimum (Here x=0). Unstable equilibrium position is that where U is maximum (Here none).
Neutral equilibrium position is that where U is constant (Here x =± ∞). Therefore, option (a) is wrong.

(b) For any finite non–zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, option (b) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (c) is also wrong.

4. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows:



$\therefore$ Net force on the bob is

$F_{net} = mg\cos\alpha$ (figure b)

or

Net acceleration of the bob is $g_{eff} = g\cos\alpha$

$T = 2\pi\sqrt{\dfrac{L}{g_{eff}}} \Rightarrow T = 2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$

5. In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from O to A/2 will be less than the time taken to go it from A/2 to A, or $T_1 < T_2$.

6. Potential energy is minimum (in this case zero) at mean position (x=0) and maximum at extreme positions (x=±A). At time t=0, x=A. Hence, PE should be maximum. Therefore, graph I is correct. Further is graph III, PE is minimum at x=0. Hence, this is also correct.

7. Block Q oscillates but does not slip on P. It means that acceleration is same for Q and P both. There is a force of friction between the two blocks while the horizontal plane is frictinless. The spring is connected to upper block. The (P–Q) system oscillates with angular frequency ω. The spring is stretched by A.

$$\therefore\ \omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

∴ Maximum acceleration in SHM = $\omega^2 A$

$$a_m = \frac{kA}{2m} \qquad ....(i)$$

Now consider the lower block.

Let the maximum force of friction = f_m

$$\therefore\ f_m = ma_m \Rightarrow f_m = m \times \frac{kA}{2m} \Rightarrow f_m = \frac{kA}{2}$$

8. $y = Kt^2 \Rightarrow \frac{d^2y}{dt^2} = 2K \Rightarrow a_y = 2\,m/s^2 \quad (\text{as } K = 1\ m/s^2)$

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{l}{g+a_y}}$$

$$\therefore\ \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$$

9. $\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2 \Rightarrow \frac{y}{x} = \frac{1}{2}$

10. Displacement equation from graph

$$X = \sin t\left[\because\ \omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\right]$$

Acceleration $a = -\frac{\pi^2}{16}\sin\left(\frac{\pi}{4}t\right)$

At $t = \frac{4}{3}$ s, $a = -\frac{\sqrt{3}\pi^2}{32}$ cm/s²

11. In series spring force remain same; if extension in k_1 and k_2 are x_1 and x_2 respectively.

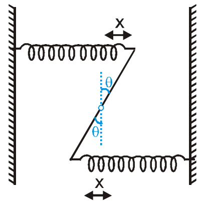
Then $k_1x_1 = k_2x_2 \Rightarrow x_1 + x_2 = A \Rightarrow x_1 + \frac{k_1x_1}{k_2} = A$

$$\Rightarrow x_1\left(\frac{k_1+k_2}{k_2}\right) = A \quad \Rightarrow x_1 = \frac{k_2A}{(k_1+k_2)}$$

Amplitude of point P will be the max. ext. in k_1.

So amplitude of point P is $\frac{k_2A}{(k_1+k_2)}$.

12. $x = \theta\frac{L}{2}$



$$F = -k\left(\theta\frac{L}{2}\right)$$

$$\tau_1 = -k\left(\theta\frac{L}{2}\right) \times \frac{L}{2}$$

$$\tau_2 = -k\left(\theta\frac{L}{2}\right) \times \frac{L}{2}$$

$$\tau_{net} = -k\theta\frac{L^2}{2} \Rightarrow \alpha = -\left(\frac{kL^2}{2}\theta\right) \times \frac{12}{mL^2}$$

$$\Rightarrow -\left(\frac{k6}{m}\right)\theta \Rightarrow \omega = \left(\frac{k6}{m}\right)^{1/2} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

MCQ

1. From superposition principle :

$y = y_1 + y_2 + y_3$

$= a\sin\omega t + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$

$= a[\sin\omega t + \sin(\omega t + 90°)] + a\sin(\omega t + 45°)$

$= 2a\sin(\omega t + 45°)\cos 45° + a\sin(\omega t + 45°)$

$= (\sqrt{2}+1)a\sin(\omega t + 45°) = A\sin(\omega t + 45°)$

Therefore, resultant motion is simple harmonic of amplitude. $A = (\sqrt{2}+1)a$ and which differ in phase by 45° relative to the first.

Energy in SHM ∝ (amplitude)²

$$\therefore\ \frac{E_{resultant}}{E_{single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\therefore\ E_{resultant} = (3+2\sqrt{2})E_{single}$$

2. For $A = -B$ and $C = 2B$

$$X = B\cos 2\omega t + B\sin 2\omega t = \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This is equation of SHM of amplitude $\sqrt{2}$ B

If $A = B$ and $C = 2B$, then $X = B + B\sin 2\omega t$

This is also equation of SHM about the point X=B. Function oscillates between X=0 and X=2B with amplitude B.

4. A **5.** A,B,D

Comprehension

1. For linear motion of disc

$$F_{net} = Ma = -2kx + f$$

where f = frictional force

For rolling motion

$$fR = -\left(\frac{MR^2}{2}\right)(\alpha) = -\left(\frac{Ma}{2}\right)R \Rightarrow f = -\frac{Ma}{2} = -\frac{F_{ext}}{2}$$

Therefore $F_{ext} = -2kx - \frac{F_{ext}}{2} = -\frac{4kx}{3}$

2. Total energy of system

$$E = \frac{1}{2}Mv^2\left(1+\frac{1}{2}\right) + 2\times\frac{1}{2}kx^2 = \frac{3}{4}Mv^2 + kx^2$$

$$\frac{dE}{dt} = 0 \Rightarrow \frac{3}{4}M(2v)\frac{dv}{dt} + 2k\times\left(\frac{dx}{dt}\right) = 0$$

$$\Rightarrow \frac{dv}{dt} + \left(\frac{4k}{3M}\right)x = 0 \Rightarrow \omega = \sqrt{\frac{4k}{3M}}$$

3. Using energy conservation law

$$\frac{1}{2}Mv_0^2\left(1+\frac{1}{2}\right) = 2\times\frac{1}{2}kx_1^2$$

$$2kx_1 - f_{max} = Ma \;\&\; f_{max}R = \left(\frac{MR^2}{2}\right)\alpha$$

$$\text{But } f_{max} = \mu Mg \Rightarrow x_1 = \frac{3\mu Mg}{2K}$$

$$\Rightarrow \frac{3}{4}Mv_0^2 = Kx_1^2 = \frac{1}{K}\left(\frac{9\mu^2M^2g^2}{4}\right) \Rightarrow v_0 = \mu g\sqrt{\frac{3M}{K}}$$

Subjective

1. **(i)** Two masses m_1 and m_2 are connected by a spring of length $\bullet_0$. The spring is in compressed position. It is held in this position by a string. When the string snaps, the spring force is brought into operation. The spring force is an internal force w.r.t. masses–spring system. No external force is applied on the system. The velocity of centre of mass will not change.

Velocity of centre of mass = v_0

∴ Location/x–coordinate of centre of mass at time t=v_0t

$$\therefore\ \bar{x} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$\Rightarrow v_0t = \frac{m_1[v_0t - A(1-\cos\omega t)] + m_2x_2}{m_1 + m_2}$$

$$\Rightarrow (m_1 + m_2)v_0t = m_1[v_0t - A(1-\cos\omega t)] + m_2x_2$$

$$\Rightarrow m_1v_0t + m_2v_0t = m_1v_0t - m_1A(1-\cos\omega t)] + m_2x_2$$

$$\Rightarrow m_2x_2 = m_2v_0t + m_1A(1-\cos\omega t)$$

$$\Rightarrow x_2 = v_0t + \frac{m_1A}{m_2}(1-\cos\omega t) \quad \text{....(i)}$$

(ii) To express $\bullet_0$ in terms of A.

$$\therefore x_1 = v_0t - A(1-\cos\omega t) \quad \therefore \frac{dx_1}{dt} = v_0 - A\omega\sin\omega t$$

$$\therefore \frac{d^2x_1}{dt^2} = -A\omega^2\cos\omega t \quad \text{....(ii)}$$

x_1 is displacement of m_1 at time t.

$\therefore \frac{d^2x_1}{dt^2}$ = acceleration of m_1 at time t.

When the spring attains its natural length $\bullet_0$, then acceleration is zero and $(x_2 - x_1) = \bullet_0$

$\therefore x_2 - x_1 = \bullet_0$. Put x_2 from (i)

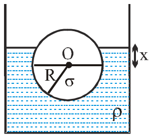
$$\Rightarrow \left[v_0t + \frac{m_1A}{m_2}(1-\cos\omega t)\right] - [v_0t - A(1-\cos\omega t)] = l_0$$

$$\Rightarrow \bullet_0 = \left(\frac{m_1}{m_2} + 1\right)A(1-\cos\omega t)$$

When $\frac{d^2x_1}{dt^2}$=0, $\cos\omega t = 0$ from (ii).

$$\therefore \bullet_0 = \left(\frac{m_1}{m_2} + 1\right)A.$$

2. A sphere of radius R is half sumerged in a liquid of density ρ.



For equilibrium of sphere

Weight of sphere = Upthrust of liquid on sphere.

$\therefore\ V\sigma g = \frac{V}{2}(\rho)g$

where σ= denisty of sphere $\Rightarrow\ \sigma = \frac{\rho}{2}$ ...(i)

From this position, the sphere is slightly pushed down. Upthrust of liquid on the sphere will increase and it will act as the restoring force.

$\therefore$ Restoring force

F = Upthrust due to extra–immession

$\Rightarrow$ F = –(extra volume immersed) × ρg

$\Rightarrow$ (mass of sphere m) × acc(a) = $-\pi R^2 \rho g x$

$\Rightarrow$ $F = -\pi R^2 x \times \rho g$

$\Rightarrow\ \frac{4}{3}\pi R^3 \times \sigma \times a = -\pi R^2 \rho g x \Rightarrow a = -\frac{3g\rho}{4R\sigma}x$

$\Rightarrow\ a = -\frac{3g \times 2}{4R}x$, [from (i)] $= -\frac{3g}{2R}x$

$\Rightarrow$ a is proportional to x.

Hence the motion is simple harmonic.

$\therefore$ Frequency of oscillation

$$= \frac{1}{2\pi}\sqrt{\frac{a}{x}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}.$$

3. A small body of mass attached to one end of a vertically hanging spring performs SHM.

Angular frequency = ω

Amplitude = a

Under SHM, velocity $v = \omega\sqrt{a^2 - y^2}$

After detaching from spring, net downward acceleration of the block = g.

$\therefore$ Height attained by the block = h

$\therefore\ h = y + \frac{v^2}{2g} \Rightarrow h = y + \frac{\omega^2(a^2 - y^2)}{2g}$

For h to be maximum

$\frac{dh}{dy} = 0,\ y = y^*.$

$\therefore\ \frac{dh}{dy} = 1 + \frac{\omega^2}{2g}(-2y^*) \Rightarrow 0 = 1 - \frac{2\omega^2 y^*}{2g}$

$\Rightarrow\ \frac{\omega^2 y^*}{g} = 1 \Rightarrow y^* = \frac{g}{\omega^2}$

Since $a\omega^2 > g$ (given) $\therefore\ a > \frac{g}{\omega^2}\ \therefore\ a > y^*$. y* from mean position

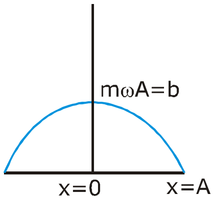
Hence $y^* = \frac{g}{\omega^2}$.

4. $v = \omega\sqrt{A^2 - x^2}$

or $v^2 = \omega^2(A^2 - x^2)$

or $v^2 = \omega^2 A^2 - \omega^2 x^2$

or $v^2 + \omega^2 x^2 = \omega^2 A^2$



or $\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$

as A = a $\therefore\ m\omega_1 a = b$

$\frac{a}{b} = \frac{1}{m\omega_1} = n^2$ (given)

$\frac{a}{R} = n$ (given)

$E_1 = \frac{1}{2}m\omega_2^2 A_2^2 \Rightarrow E_2 = \frac{1}{2}m\omega_2^2 R^2$

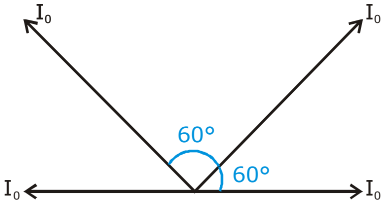
$m\omega_1 A_1 = b \Rightarrow m\omega_1 a = b \Rightarrow \omega_1 = \frac{b}{am} = \frac{1}{mn^2}$

$m\omega_2 A_2 = R \Rightarrow m\omega_2 R = R \Rightarrow \omega_2 = \frac{1}{m}$

$\frac{\omega_1}{\omega_2} = \frac{1}{mn^2} \times m = \frac{1}{n^2}$

$\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

5.

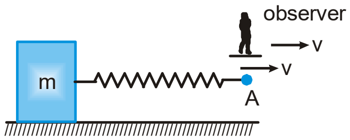


$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

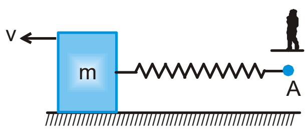
$= 2I_0 + 2I_0\cos 60° = 3I_0 \Rightarrow n = 3$

MOCK TEST

1. Consider an observer moving with speed v with point A in the same direction.



In the frame of observer, block will have initial velocity v towards left.



During maximum extension, the block will come to rest with respect to the observer.
Now, by energy conservation,

$$\frac{1}{2}mv^2 = \frac{1}{2}k\,x_{max}^2 \quad \therefore \quad x_{max} = \sqrt{\frac{mv^2}{k}}$$

2. Both the spring are in series

$$\therefore \quad K_{eq} = \frac{K(2K)}{K+2K} = \frac{2K}{3}$$

Time period

$$T = 2\pi\sqrt{\frac{\mu}{K_{eq}}} \quad \text{where} \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

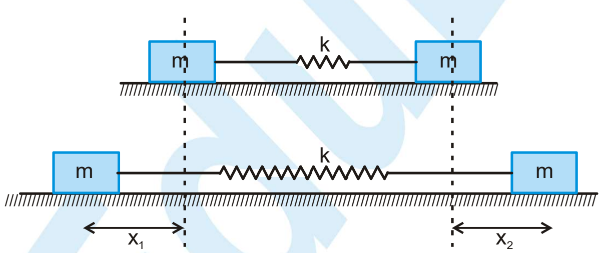
Here $\mu = \frac{m}{2}$ $\quad \therefore T = 2\pi\sqrt{\frac{m}{2}\times\frac{3}{2K}} = 2\pi\sqrt{\frac{3m}{4K}}$

Method II

$\therefore \quad mx_1 = mx_2 \Rightarrow \quad x_1 = x_2$

force equation for first block;

$$\frac{2k}{3}(x_1 + x_2) = -m\frac{d^2x_1}{dt^2}$$



Put $x_1 = x_2 \Rightarrow \frac{d^2x_1}{dt^2} + \frac{4k}{3m}\times x_1 = 0 \Rightarrow \omega^2 = \frac{4k}{3m}$

$$\therefore \quad T = 2\pi\sqrt{\frac{3m}{4K}}$$

3. $x_1 = a\sin(\omega t + \phi_1)$

$x_2 = a\sin(\omega t + \phi_2) \Rightarrow |x_1 - x_2| = 2a\sin(\omega t + \frac{\phi_1+\phi_2}{2})\cos\left(\frac{\phi_1-\phi_2}{2}\right)$

To maximize $|x_1 - x_2|$:

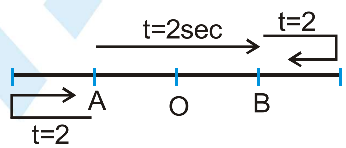
$$\sin(\omega t + \frac{\phi_1+\phi_2}{2}) = 1$$

$$\Rightarrow \quad a\sqrt{2} = 2a \times 1 \times \cos\left(\frac{\phi_1-\phi_2}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1-\phi_2}{2}\right) \Rightarrow \frac{\pi}{4} = \frac{\phi_1-\phi_2}{2}$$

$\Rightarrow \phi_1 - \phi_2 = \frac{\pi}{2}$. Hence (B).

4. From the given information it can be inferred that points A and B are equidistant from mean position
Hence from diagram it is clear that time period of oscillation is $= 2 + 2 \times 2 + 2 = 8$ second.



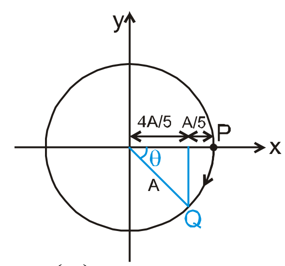
5. **Method : I**
Particle is starting from rest, i.e. from one of its extreme position.

As particle moves a distance $\frac{A}{5}$, we can represent it on a circle as shown.

$\cos\theta = \frac{4A/5}{A} = \frac{4}{5}$

$\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$\omega t = \cos^{-1}\left(\frac{4}{5}\right)$



$$t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

Method : II As starts from rest i.e. from extreme position

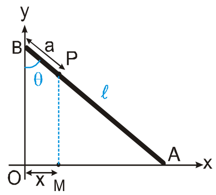
$x = A\sin(\omega t + \phi)$

At $t = 0$; $x = A \Rightarrow \phi = \frac{\pi}{2}$

$\therefore A - \frac{A}{5} = A\cos\omega t \; \frac{4}{5} = \cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$

$$\Rightarrow t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

6. $\frac{d\theta}{dt} = 2 \quad \therefore \theta = 2t$



Let BP = a

$\therefore x = OM = a \sin\theta = a \sin(2t)$

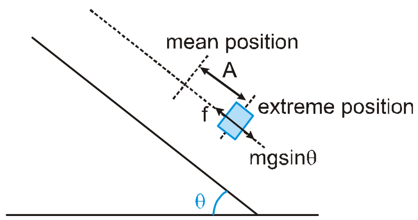
Hence M executes SHM

within the given time period and its acceleration is opposite to 'x' that means towards left

7. The maximum static frictional force is

$f = \mu mg \cos\theta = 2 \tan\theta \, mg \cos\theta = 2\, mg \sin\theta$

Applying Newton's second law to upper block at lower extreme position

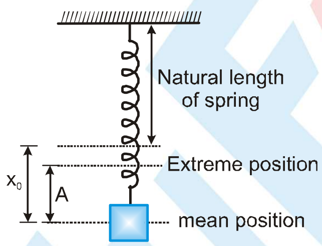


$f - mg\sin\theta = m\omega^2 A \Rightarrow f = m\omega^2 A + mg \sin\theta$

or $\omega^2 A = g \sin\theta$

or $A = \frac{3\, mg \sin\theta}{k}$

8. The spring is never compressed. Hence spring shall exert least force on the block when the block is at top-most position.



$$F_{least} = kx_0 - kA = mg - m\omega^2 A = mg - 4\frac{\pi^2}{T^2} mA$$

9. At t = 4 x = 0

i.e. particle must pass through the mean position so curve (3) is not correct

The slope of x–t graph is the speed

Among curves (1) and (2), slope of (1) greater at

t = 4 so (A) is correct

t = 4, x = 0

10. Due to impulse, the total energy of the particle becomes :

$$\frac{1}{2} m\omega^2 A^2 + \frac{1}{2} m\omega^2 A^2 = m\omega^2 A^2$$

Let ; A' be the new amplitude. (Apply energy conservation law)

$$\therefore \frac{1}{2} m\omega^2 (A')^2 = m\omega^2 A^2 \Rightarrow A' = \sqrt{2}\,A. \textbf{Ans.}$$

11. Conserving momentum : $2V = 3V'$

$$\Rightarrow V' = \frac{2}{3}V. \Rightarrow \omega' A' = \frac{2}{3}\omega A$$

$$\Rightarrow \sqrt{\frac{K}{3}} \times A' = \frac{2}{3}\sqrt{\frac{K}{2}} \times A \Rightarrow A' = \sqrt{\frac{2}{3}}\,A$$

12. The extension developed in the string due to small values of 'θ' is :

$x = h \sin\theta \cong h\theta$

Torque about 'O' :

$\tau_0 = (Mg \sin\theta) L + (kx)h$

or, $\tau_0 \cong mg\,\theta L + kh^2\theta$

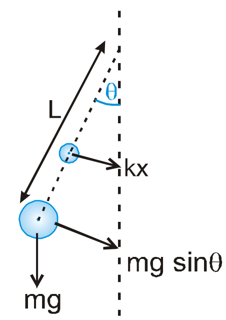
$= (mgL + kh^2)\theta$ ☹ (1)

Also;

$\tau_0 = I_0\alpha = mL^2\alpha$ ☹ (2)

From (1) and (2) :

$mL^2 \alpha = (mg L + kh^2)\theta$



or $\alpha = \frac{1}{L^2}\left(gL + \frac{kh^2}{m}\right)\theta$

Now : $T = 2\pi\sqrt{\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{\theta}{\frac{1}{L^2}\left(gL + \frac{kh^2}{m}\right)\theta}}$

$$\Rightarrow \upsilon = \frac{1}{T} = \frac{1}{2\pi L}\sqrt{\left(gL + \frac{kh^2}{m}\right)}$$ Hence (D).

13. $f_0 = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$

where, • is distance between point of suspension and centre of mass of the body.

Thus, for the stick of length L and mass m :

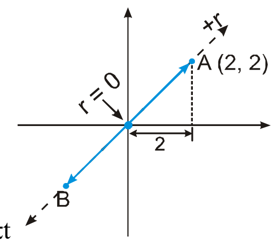
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{m.g.\frac{L}{2}}{(mL^2/3)}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$$

when bottom half of the stick is cut off

$$f_0' = \frac{1}{2\pi}\sqrt{\frac{\frac{m}{2}.g.\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{3}}} = \frac{1}{2\pi} \times \sqrt{\frac{3g}{L}} = \sqrt{2}\, f_0 \textbf{Ans.}$$

14. Torque about hinge
$2.5\,g.\,(0.4)\cos\theta - 1g.(1)\cos\theta = 0$
So rod remains stationary after the release.

15. Let the line joining AB represents axis 'r'. By the conditions given 'r' coordinate of the particle at time t is



$r = 2\sqrt{2}\cos\omega t$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$\therefore r = 2\sqrt{2}\cos\pi t$

$x = r\cos 45° = \frac{r}{\sqrt{2}} = 2\cos\pi t$

$\therefore\ a_x = -\omega^2 x = -\pi^2\, 2\cos\pi t$

$\therefore\ F_x = ma_x = -4\pi^2\cos\pi t$

16. $K_1 = m_1\frac{g}{x}$; $k_2 = m_2\frac{g}{x}$

Thus, Using $T = 2\pi\sqrt{\frac{m}{k}}$

$T_1 = 2\pi\sqrt{\frac{m_1}{(m_1 g/x)}} \Rightarrow T_2 = 2\pi\sqrt{\frac{m_2}{(m_2 g/x)}} \Rightarrow T_1 = T_2$

Energy of oscillation :

$E = \frac{1}{2}m\omega^2 A^2$

Since ω and A are same for both and $m_1 > m_2$
$\Rightarrow E_1 > E_2$.

17. $x = 3\sin 100\pi t$
$y = 4\sin 100\pi t$
Equation of path is

$\frac{y}{x} = \frac{4}{3}$ i.e. $y = \frac{4}{3}x$

which is equation of a straight line having slope $\frac{4}{3}$

Equation of resulting motion is

$\vec{r} = x\hat{i} + y\hat{j} = (3\hat{i} + 4\hat{j})\sin 100\pi t$

Amplitude is $\sqrt{3^2 + 4^2} = 5$

18. $v^2 = 108 - 9x^2$

$\frac{2vdv}{dx} = -18x \Rightarrow$ acc. $A = -9x$ (non-uniform)
at $x = 3$cm
$a = -27$ or $|a| = 27$cm/s^2
also $a = -9x$ is a S.H.M equation so particle perform S.H.M about the give fixed point
V is maximum at $x = 0$ and V is Zero at $x = \sqrt{12}$

So Amplitude $= 2\sqrt{3}$ cm

19. Given $A = 0.4$m, and $a = g$

So $\omega^2 A = g \Rightarrow \omega^2 = \frac{10}{0.4} = 25$

$\Rightarrow \omega = 5$ $T = \frac{2\pi}{\omega} = 2\pi/5$ sec.

At lowest position acceleration.
$= \omega^2 A + g = g + g = 2g$
So weight $= m(2g) = 2mg$
at half distance $a = g/2$
So weight at upper half distance $= m(g - g/2) = mg/2$

and weight at lower half distance

$= m(g + g/2) = \frac{3mg}{2}$

actual weight at equilibrium position (maximum v)

20. At $t = 0$

Displacement $x = x_1 + x_2 = 4\sin\frac{\pi}{3} = 2\sqrt{3}$ m.
Resulting Amplitude

$A = \sqrt{2^2 + 4^2 + 2(2)(4)\cos\pi/3} = \sqrt{4 + 16 + 8} = \sqrt{28}$

$= 2\sqrt{7}$ m

Maximum speed $= A\omega = 20\sqrt{7}$ m/s

Maximum acceleration $= A\omega^2 = 200\sqrt{7}$ m/s^2

Energy of the motion $= \frac{1}{2}m\omega^2 A^2$ = **28 J Ans.**

21. $x = 3\sin 100t + 8\cos^2 50t$

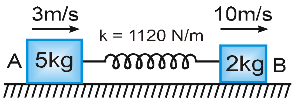
$= 3\sin 100t + \frac{8[1 + \cos 100t]}{2}$

$x = 4 + 3\sin 100t + 4\cos 100t$
$(x - 4) = 5\sin(100t + \phi)$ $\left\{\tan\phi = \frac{4}{3}\right\}$
Amplitude = 5 units
Maximum displacement = 9 units.

22. At max. extension both should move with equal velocity.
$\therefore$ By momentum conservation,
$(5 \times 3) + (2 \times 10) = (5 + 2)V$
$V = 5$ m/sec.
Now, by energy conservation

$\frac{1}{2}5 \times 3^2 + \frac{1}{2} \times 2 \times 10^2 = \frac{1}{2}(5 + 2)V^2 + \frac{1}{2}kx^2$

Put V and k



$\therefore\ x_{max} = \frac{1}{4}m = 25$ cm.

Also first maximum compression occurs at ;

$$t = \frac{3T}{4} = \frac{3}{4}2\pi\sqrt{\frac{\mu}{k}} = \frac{3}{4}2\pi\sqrt{\frac{10}{7\times1120}} = \frac{3\pi}{56} \text{ sec.}$$

(where $\mu \Rightarrow$ reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2}$).

23. PE is related to reference. Only when PE at mean position is taken zero, the assertion is true.

24. Statement-2 itself explains statement-1.

25. The mean position of the particle in statement-1 is $x = -\frac{b}{a}$ and the force is always proportional to displacement from this mean position. The particle executes SHM about this mean position. Hence statement-1 is false

26. When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on block.

27. At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by kx = mg ....(1)

From conservation of energy

$$mg(L+x) = \frac{1}{2}kx^2 + \frac{1}{2}mv^2_{max} \quad ....(2)$$

from (1) and (2) we get $v_{max} = \frac{3}{2}\sqrt{gL}$.

28. $V_{max} = \frac{3}{2}\sqrt{gL}$ and $\omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$

$\therefore\ A = \frac{V_{max}}{\omega} = \frac{3}{4}L$

Hence time taken t, from start of compression till block reaches mean position is given by

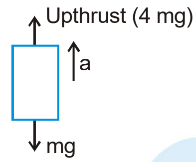
$x = A\sin\omega t_0$ where $x = \frac{L}{4}$

$\therefore\ t_0 = \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

Time taken by block to reach from mean position to bottom most position is $\frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{4}\sqrt{\frac{L}{g}}$

Hence the required time = $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

29.



$4\,mg - mg = ma \Rightarrow a = 3g$

30. The density of liquid is four times that of cylinder, hence in equilibrium position one fourth of the cylinder is submerged.

So as the cylinder is released from initial position, it moves by $\frac{3l}{4}$ to reach its equilibrium position. The upward motion in this time is SHM.

$a = 3g = \omega^2 A = \omega^2 \times \frac{3l}{4}$, so $\omega = \sqrt{\frac{4g}{l}}$.

Therefore required velocity is $v_{max} = \omega A$. $\omega = \sqrt{\frac{4g}{l}}$ and $A = \frac{3\lambda}{4}$. Therefore $v_{max} = \frac{3}{2}\sqrt{g\lambda}$

31. The require time is one fourth of time period of SHM.

Therefore $t = \frac{T}{4} = \frac{\pi}{2\omega} = \frac{\pi}{4}\sqrt{\frac{l}{g}}$

32. $F = 8 - 2x$

$= -2(x-4)$

At equilibrium position, $F = 0 \Rightarrow x = 4$ m

As particle is released at rest from x = 6 m, i.e. it is one of the extreme position.

Hence, Amplitude A = 2 m.

Here, force constant k = 2 N/m $\Rightarrow m\omega^2 = 2$

or $\omega = 1$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi$

Time to go from x = 2 m to x = 4 m (i.e. from extreme position to mean position) $= \frac{T}{4} = \frac{\pi}{2}$

Energy of S.H.M. $= \frac{1}{2}kA^2 = \frac{1}{2}\times 2\times 4$ N-m = 4 J

As particle has started it's motion from positive extreme

$\therefore$ Phase constant $= \frac{\pi}{2}$

33. $V_{max} = A\omega$

$\Rightarrow \quad A = \frac{V_{max}}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20\text{m}$

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2 \text{ kg}$

At t = 0.1, acc. is maximum

$\Rightarrow a_{max} = -\omega^2 A = -\left(\frac{2\pi}{0.2}\right)^2 \times 0.2 = -200 \text{ m/s}^2$

Maximum energy $= \frac{1}{2} mV_{max}^2 = 4$ J

34. **(A)** $x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$

$\Rightarrow x = \sqrt{2}\sin(\omega t - \frac{\pi}{4})$ is periodic with SHM.

(B) $x = \sin^3 \omega t$ can not be written

as $x = A\sin(\omega' t + \phi)$ so it is not SHM

but periodic motion.

(C) Linear combination of different periodic function is also periodic function.

$\frac{d^2x}{dt^2}$ is not directly proportional to x i.e. this motion is not SHM

(D) x continuously decreases with time.

So x is not periodic function.

35. **(A)** If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B $= \frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

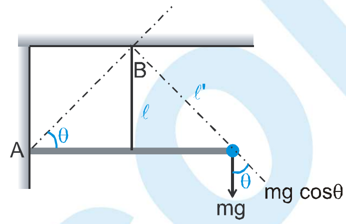
(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension and maximum compression of spring.

36. The bob will execute SHM about a stationary axis passing through AB. If its effective length is ●' then

$T = 2\pi\sqrt{\frac{\lambda'}{g'}}$

●' = ●/ $\sin\theta = \sqrt{2}\lambda$



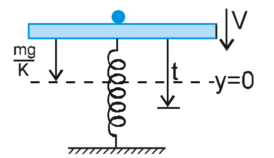
(because θ = 45°)

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\lambda}{g}} = 2\pi\sqrt{\frac{2 \times 0.2}{10}} = \frac{2\pi}{5}$ s.

37. Velocity of the particle just before collision

$u = \sqrt{2g \times \frac{4.5\,mg}{K}} \Rightarrow u = 3g\sqrt{\frac{m}{K}}$



Now it collides with the plate.

Now just after collision velocity (V) of system of "plate + particle"

$mu = 3mV \Rightarrow V = \frac{u}{3} = g\sqrt{\frac{m}{K}}$

Now system perform's SHM with time period $T = 2\pi\sqrt{\frac{3m}{K}}$

and mean position as $\frac{mg}{K}$ distance below the point of collision.

Let the equation of motion be

$y = A\sin(\omega t + \phi)$

for t = 0 $\Rightarrow$ y = mg/K

$\frac{mg}{K} = A\sin\phi$ ...(1)

Now for amplitude $V = \omega\sqrt{A^2 - x^2}$

$g\sqrt{\frac{m}{K}} = \sqrt{\frac{K}{3m}}\sqrt{A^2 - \frac{m^2g^2}{K^2}}$

$$\Rightarrow \left(\sqrt{3}\,\frac{mg}{K}\right)^2 = A^2 - \frac{m^2g^2}{K^2} \Rightarrow A = \frac{2mg}{K} \quad ...(2)$$

By (1) & (2)

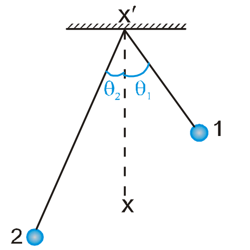
$x = \frac{A}{2}$ to $x = 0 \quad \Rightarrow \quad t = \frac{T}{12}$

$x = 0 \quad$ to $x = A \quad \Rightarrow \quad t = T/4$

total time $= \frac{T}{12} + \frac{T}{4} = \frac{2\pi}{3}\sqrt{\frac{3m}{K}}$

Using values, $t = 20\,\pi$ms.

38. The angular position of pendulum 1 and 2 are (taking angles to the right of reference line xx′ to be positive)



$\theta_1 = \theta \cos\left(\frac{4\pi}{T}t\right)$

[where $T = 2\pi\sqrt{\frac{l}{g}}$]

$\theta_2 = -\,\theta \cos\left(\frac{2\pi}{T}t\right)$

$= \cos\left(\frac{2\pi}{T}t + \pi\right)$

For the strings to be parallel for the first time

$\theta_1 = \theta_2$

or $\quad \cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$

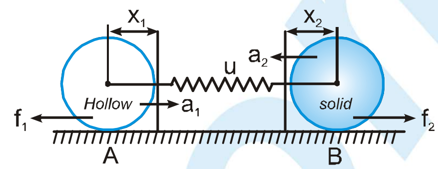
$\therefore \frac{4\pi}{T}t = 2n\,\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$

for $n = 0$, $t = \frac{T}{2}$ $\quad$ for $n = 1$, $t = \frac{T}{6}, \frac{3T}{2}$

$\therefore$ Both the pendulum are parallel to each other for the first time after $t = \frac{T}{6} = \frac{\pi}{3}\sqrt{\frac{l}{g}}$ **Ans.**

39. x_1 & x_2 be the displacement from equilibrium position. Now for hollow sphere, applying $\tau_A = I_A\alpha$

$k(x_1 + x_2)\,r = \frac{5}{3}\,m r^2\,\frac{a_1}{r} \quad ......(1)$



By angular momentum conservation (about A) of the system, $\frac{5}{3}\,m v_1 r = \frac{7}{5}\,m v_2 r$

$\Rightarrow 25\,v_1 = 21\,v_2 \quad ......(2)$

$\Rightarrow 25\,x_1 = 21\,x_2 \quad ......(3)$

Using (1) and (3) we get, $k\left(x_1 + \frac{25}{21}x_1\right) = \frac{5}{3}\,m a_1$

$\Rightarrow a_1 = \frac{46}{35}\,\frac{k}{m}\,x_1$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{46\,k}{35\,m}}$ Ans.

Now for amplitude, $A_1 + A_2 = x_0 \quad ......(4)$

From equation (3) we get $A_2 = \frac{25}{21}A_1 \quad ......(5)$

By (4) and (5) we get, $A_1 = \frac{21}{46}x_0$ $A_1 = \frac{21}{46}x_0$;

$A_2 = \frac{25}{46}x_0$; $f = \frac{1}{2\pi}\sqrt{\frac{46\,k}{35\,m}}$ **Ans.**