## EXERCISE - 2

### Part # I : Multiple Choice

**3.** Total number of required possibilities

$^5C_3 \,.\, ^8C_7 + {}^5C_4 \,.\, ^8C_6 + {}^5C_5 \,.\, ^8C_5 \,.\, ^5C_5$

$= {}^5C_3 \,.\, ^8C_7 + {}^5C_4 \,.\, ^8C_6 + {}^8C_6 = {}^{13}C_{10} - {}^5C_3 = 276$

**6.** $\dfrac{^{200}C_2 . ^{198}C_2 . ^{196}C_2 ...... ^2C_2}{100!} = \dfrac{200!}{2^{100}.100!}$

$= \dfrac{101.102.103.....200}{2^{100}}$

$= \left(\dfrac{100}{2}\right).\left(\dfrac{102}{2}\right).\left(\dfrac{103}{2}\right).....\left(\dfrac{100}{2}\right)$

and $\dfrac{1.2.3.4.5.6.7.8.....200}{2^{100}.100!}$

$= \dfrac{(1.3.5.7.....199)(2.4.6.8......200)}{2^{100}.100!}$

$= \dfrac{(1.3.5......199).2^{100}.100!}{2^{100}.100!} = 1.3.5\,.199$

**9.** $x_1 + x_2 + x_3 + x_4 \le n$

$\Rightarrow \; x_1 + x_2 + x_3 + x_4 + y = n$

(where y is known as pseudo variable)

Total no. of required solution is $= {}^{n+5-1}C_n$

$= {}^{n+4}C_n$ or ${}^{n+4}C_4$

**11.** We have arrange all the letter except 'ccc' is $\dfrac{12!}{5!.3!.2!}$

new there all 13 place where 'i' can be placed $^{13}C_3$

Hence required number of ways is

$= \dfrac{12!}{5!3!2!}\,{}^{13}C_3 = 11\,.\,\dfrac{13!}{6!}$

**12.** Here given no. be 1,2,3,.........n

Let common difference = r

Total way of selection = (1, 1 + r, 1+2r),

(2, 2 + r, 2 + 2r), ..(n – 2r, n – r, n)

Total numbers are = (n – 2r)

Here $r_{min.} = 1$ and $r_{max.} = (n - 1)/2$

Case- I When n is odd

$\therefore \; r_{max} = \dfrac{(n-1)}{2}$ & total no. of selection is

$= \sum_{r=1}^{(n-1)/2} (n-2r) = \dfrac{n(n-1)}{2} - \dfrac{2\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)}{2}$

$= \left(\dfrac{n-1}{2}\right)^2$

Case - II when n is even = $r_{max} = \dfrac{n-2}{2}$

so total no. selection is

$= \sum_{r=1}^{(n-2)/2} (n-2r) = \dfrac{n(n-2)}{2} - \dfrac{2\left(\frac{n-2}{2}\right)\frac{n}{2}}{2}$

$= \left(\dfrac{n-2}{2}\right)\left(n - \dfrac{n}{2}\right) = \dfrac{n(n-2)}{4}$

**14.** Total no. of visits that a teacher goes is = $^{25}C_5$

(selection of 5 different kids each time & teacher goes every time)

Number of visits of a boy = select one particular boy & 4 from rest 24 = $^{24}C_4$

So extra visits of a teacher from a boy is

$= {}^{25}C_5 - {}^{24}C_4 = {}^{24}C_5$

**17.** Number of ways he can fail is either one or two, three or four subject then total of ways.

$^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$

**19.** Total number of required quadrilateral

$^7C_4 + {}^7C_3 \times {}^5C_1 + {}^7C_2 \times {}^5C_2$

$= \dfrac{7\times6\times5\times4}{1\times2\times3\times4} + \dfrac{7\times6\times5}{1\times2\times3}.5 + \dfrac{7\times6}{1\times2}\times\dfrac{5\times4}{1\times2}$

$= 35 + 175 + 210 = 420 = 2\,.{}^7p_3$

### Part # II : Assertion & Reason

**3.** **Statement -1:** Two circles intersect in 2 points.

$\therefore$ Maximum number of points of intersection

= 2 × number of selections of two circles from 8 circles.

$= 2 \times {}^8C_2 = 2 \times 28 = 56$

**Statement -2:** 4 lines intersect each other in $^4C_2 = 6$ points.

4 circles intersect each other in $2 \times {}^4C_2 = 12$ points.

Further, one lines and one circle intersect in two points.

So, 4 lines will intersect four circles in 32 points.

$\therefore$ Maximum number of points = 6 + 12 + 32 = 50.

**5.** $x_1 x_2 x_3 x_4 = 2 \times 5 \times 7 \times 11 \;\Rightarrow\; N = 4^4$

## EXERCISE - 4
### Subjective Type

**1.** Selecting 3 horses out of ABC A'B'C' is ${}^6C_3$ ways
When AA' is always selected among (ABC A'B'C')
Remaining (BB'CC') can be selected in ${}^4C_1$ ways similarly, when BB' and CC' is selected
$\therefore$ Undesirable ways will be $({}^4C_1) \times 3$
using, total ways–undesirable ways
= desired ways we get
$({}^6C_3 - ({}^4C_1)3) \rightarrow$ This is selection of 3 horses among (ABC A'B'C') under given condition.
Remaining 3 can be selected in ${}^{10}C_3$ ways.
Hence, desired ways will be $[{}^6C_3 - {}^4C_1 \times 3]{}^{10}C_3 = 792$

**Method II :** Select one horse each from AA', BB' and CC' hence ${}^2C_1 \times {}^2C_1 \times {}^2C_1$ ways. Now select 3 horses from remaining 10 horses in ${}^{10}C_3$ ways.
Total ways = ${}^{10}C_3 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1$

**2.** Total no. of M are = 1 Total no. of I are = 4
Total no. of P are = 2 Total no. of S are = 4
First we arrange all the words other than I's are

$$\frac{7!}{2!\,4!} = \frac{7\times6\times5}{1\times2} = 105$$

Now, there are 8 places which can be fulfilled by I's i.e. the number of ways is ${}^8C_4$

$$\text{Total required no.} = 105 \times {}^8C_4 = \frac{105\times8\times7\times6\times5}{1\times2\times3\times4}$$
$$= 105 \times 70 = 7350$$

**3.** Step 1st : Select 2 lines out of n lines in ${}^nC_2$ ways to get a point (say p).

Step-2nd : Now select another 2 lines in ${}^{n-2}C_2$ ways, to get another point (say Q)

Step-3rd : When P and Q are joined we get a fresh line.


But when we select P first then Q and Q first then P we get same line.

$$\therefore \frac{{}^nC_2 \times {}^{n-2}C_2}{2} \text{ Fresh lines}$$

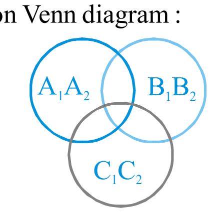
**4.** First we select n grand children from 2n grand children is ${}^{2n}C_n$
Now arrangement of both group is n! × n!
Now Rest all (m + 1) place where we occupy the grandfather and m sons but grandfather refuse the sit to either side of grand children so the out of m – 1 seat one seat can be selected
Now required number of sitting in
${}^{2n}C_n \times n! \times n! \times {}^{(m-1)}C_1 \,.\, m!$

$$= \frac{12n}{n!\times n!} \times n! \times n! \times {}^{(m-1)}C_1 \,.\, m! = 2n!\,.\,m!\,.\,(m-1)$$

**6.** Number of ways of distributing 200 objects into 20 groups each containing 10 objects

$$= \frac{200!}{(10!)^{20}\,.20!} \times 20 = \frac{200!}{(10!)^{20}\,.19!} \text{ which must be an integer.}$$

**7.** **(i)** Total ways = 10!
undesirable cases : when 2 Americans are together $(A_1A_2)$ or two British are together $(B_1B_2)$ or two Chinese are together $(C_1C_2)$
we plot them on Venn diagram :




we use,
$n(A_1A_2 \cup B_1B_2 \cup C_1C_2)$
$= n(A_1A_2) + n(B_1B_2) + n(C_1C_2) - n[(A_1A_2) \cup (B_1B_2)]$
$- n[(B_1B_2) \cup (C_1C_2)] - n[(C_1C_2) \cup (A_1A_2)]$
$+ n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)]$

where $n(A_1A_2)$ denotes $\rightarrow$ when 2 Americans are together = 9! 2! correspondingly for $B_1B_2$ & $C_1C_2$
$n[(A_1A_2) \cup (B_1B_2)]$ denotes when 2 Americans and 2 Britishmen are together
$= 8! \times 2! \times 2!$
correspondingly same for others.
$n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)]$ denotes when 2 Americans, 2 Britishmen and 2 Chinese are together
$= 7! \times 2! \times 2! \times 2! = 86$

**15.** 2 clerks who prefer Bombay are to be sent to 2 different companies in Bombay, and Out of remaining 5 clerks (excluding 3 clerks who prefer for outside) 2 clerks are chosen in ${}^5C_2$ ways.

Now these 4 can be sent to 2 different companies into 2 groups of 2 each in ${}^4C_2$ ways

$\Rightarrow$ ${}^5C_2 \times {}^4C_2$

Now for outside companies we have 6 clerks remaining we select them as (2 for each company)

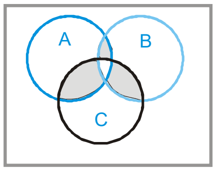
${}^6C_2 \times {}^4C_2 \times {}^2C_2$

Desired ways = $({}^5C_2 \times {}^4C_2)\,({}^6C_2 \times {}^4C_2 \times {}^2C_2) = 5400$ ways.

**16.** Total students $n(A \cup B \cup C) = 100$

Students reading Business India = n(A) = 80
Students reading Business World = n(B) = 50
Students reading Business Today = n(C) = 30

Students reading all the three magazines = $n(A \cap B \cap C) = 5$




Hence students reading exactly two magazines

$= n(A) + n(B) + n(C) - n(A \cup B \cup C) - 2 \times n(A \cap B \cap C)$

$= 80 + 50 + 30 - 100 - 2 \times 5 = 50$

**19.** **(a)** Selection of r things out of n +1 different things = Selection of r things out of n +1 different things, when a particular thing is excluded + a particular thing is included.

**(b)** Selection of r things out of not m + n different Things can be made by selecting x thing from m and y thing from such that x +y = r

& $(x, y) = (0, r), (1, r-1), (2, r-2), .......(r, 0)$

**21.** Step 1st : Arrange 5 boys in 5! ways

Step 2nd : Select 2 gaps from 6 gaps for 4 girls (2 girls for each gap) in ${}^6C_2$ ways.

Step 3rd : Select 2 girls to sit in one of the gaps and other 2 in remaining selected gaps = ${}^4C_2$ ways

Step 4th : Arrange 1st, 2 girls in 2! and other 2 in 2! ways

Hence, total ways $\rightarrow 5! \times {}^6C_2 \times {}^4C_2 \times 2 \times 2 = 43200$

**22.** Ordered pair = total – $(A \cup B = X) = 4^n - 3^n$

Subsets of X = $2^n$ will not repeat in both but here the whole set X has not been taken

So subsets of x which are not repeated $(2^n - 1)$

Hence unordered pair = $\dfrac{(4^n - 3^n) - (2^n - 1)}{2} + (2^n - 1)$

**23.** Total No. of bowlers = 6

Now,

**(i)** If 4 bowlers are including the no. of ways selecting 11 players out of 15 players

$= {}^6C_4 \times {}^9C_7 = 15 \times 36 = 540$

**(ii)** If 5 bowlers are selected $= {}^6C_5 \times {}^9C_6 = 6 \times 84 = 504$

**(iii)** If all 6 bowlers are selected $= {}^6C_6 \times {}^9C_5 = 1 \times 126 = 126$

Hence total no. of ways = 540 + 504 + 126 = 1170

**25.** $1980 = 2^2 . 3^2 . 5 . 11$,

number of divisions of 1980 = 36

**(i)** 3.3.2 = 18

sum $= 11.(1 + 2 + 2^2)$

$.(1 + 3 + 3^2)$

$.(1 + 5)$

**(ii)** 3.2 + 1.1.2 = 8

## Part # II : IIT-JEE ADVANCED

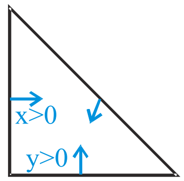
**4.** $x+y<21$

$x+y\le 20$

$x+y\le 18$ ($\because x>0$ & $y>0$)

Introducing new variable t

$x+y+t=18$




Now dividing 18 identical things among 3 persons.

$$= {}^{18+3-1}C_{3-1} = \frac{\lfloor 18+3-1}{\lfloor 18\ \lfloor 3-1} = 190$$

**5.** Total number of ways of distributing $n^2$ objects into n groups, each containing n objects

$$= \frac{(n^2)!}{(n!)^n\, n!}.n! = \frac{(n^2)!}{(n!)^n} = \text{integer}$$

(Since number of ways are always integer)

**7.** Since, r, s, t are prime numbers.

$\therefore$ Selection of p and q are as under

| p | q | number of ways |
|---|---|---|
| $r^0$ | $r^2$ | 1 way |
| $r^1$ | $r^2$ | 1 way |
| $r^2$ | $r^0, r^1, r^2$ | 3 ways |

$\therefore$ Total number of ways to select r = 5

| | | |
|---|---|---|
| $s^0$ | $s^4$ | 1 way |
| $s^1$ | $s^4$ | 1 way |
| $s^2$ | $s^4$ | 1 way |
| $s^3$ | $s^4$ | 1 way |
| $s^4$ | $s^0, s^1, s^2, s^3, s^4$ | 5 ways |

$\therefore$ Total number of ways to select s = 9.

Similarly total number of ways to select t

= 5 number of ways = 5 × 9 × 5 = 225.

**11. (D)**

**Case- I :** The number of elements in the pairs can be 1,1; 1,2; 1,3,; 2,2

$$= {}^4C_2 + {}^4C_1 \times {}^3C_2 + {}^4C_1 \times {}^3C_3 + \frac{{}^4C_2 . {}^2C_2}{2} = 25$$

**Case- II :** Number of pairs with $\phi$ as one of subsets $= 2^4 = 16$

$\therefore$ Total pairs = 25 + 16 = 41

**12** Balls can be distributed as 1, 1, 3 or 1, 2, 2 to each person.

When 1, 1, 3 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,1!\,3!}.\frac{1}{2!}.3! = 60$$

When 1, 2, 2 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,2!\,2!}.\frac{1}{2!}.3! = 90$$

$\therefore$ total = 60 + 90 = 150

**Paragraph for Question 13 and 14 :** For $a_n$

The first digit should be 1

For $b_n$

1 _ _ _ _ . . . _ _ 1
(n–2 Places)

Last digit is 1. so $b_n$ is equal to number of ways of $a_{n-1}$ (i.e. remaining (n – 1) places)

$b_n = a_{n-1}$

For $c_n$

Last digit is 0 so second last digit must be 1

So $c_n = a_{n-2}$

$b_n + c_n = a_n$

So $a_n = a_{n-1} + a_{n-2}$

Similarly $b_n = b_{n-1} + b_{n-2}$

**13. (B)**

$a_1 = 1$, $a_2 = 2$

So $a_3 = 3$, $a_4 = 5$ $a_5 = 8$

$\Rightarrow$ $b_6 = a_5 = 8$

**14. (A)**

$a_n = a_{n-1} + a_{n-2}$

put n = 17

$a_{17} = a_{16} + a_{15}$ (A) is correct

$c_n = c_{n-1} + c_{n-2}$

So put n = 17

$c_{17} = c_{16} + c_{15}$ (B) is incorrect

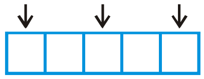
$b_n = b_{n-1} + b_{n-2}$

put n = 17

$b_{17} = b_{16} + b_{15}$ (C) is incorrect

$a_{17} = a_{16} + a_{15}$

while (D) says $a_{17} = a_{15} + a_{15}$ (D) is incorrect

**10.** 

There are 2M, 2T, 2A and 1 H, E, I, C, S
First find the number of ways if odd's no. position place be filled is ${}^{5}p_3 = 60$
Now Case I If even place words is same i.e no. of ways = 3
Case II If even place words is different i.e no. of ways = ${}^{3}c_2 \times 2! = 6$
Hence total no. of arragment is $60 \times (3 + 6) = 540$

**11.** First find no. of '2' at the end of (125)! is

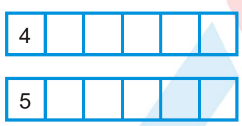
**(A)** $\left[\frac{125}{2}\right]+\left[\frac{125}{2^2}\right]+\left[\frac{125}{2^3}\right]+\left[\frac{125}{2^4}\right]+\left[\frac{125}{2^5}\right]+\left[\frac{125}{2^6}\right]+\left[\frac{125}{2^7}\right]$

$= 62 + 31 + 15 + 7 + 3 + 1 + 0 = 119$
Find the number of '5' at the end of (125)!

is $\left[\frac{125}{5}\right]+\left[\frac{125}{5^2}\right]+\left[\frac{125}{5^3}\right]+\ldots\ldots$

$= 25 + 5 + 1 = 31$
Hence no. of zero is 31

**(B)** Total no. of signals can made by each arm = 10 so total number of different signals can be formed $= 10^{10} - 1$
(here – 1 is because if all arms are at the position of rest, then no signal will pass away)

**(C)** 


$\frac{5!}{2!} = 60$ $\qquad \frac{5!}{2!.2!} = 30$

Total number of arrangement = 90

**(D)** Let number of player is n
then total number of games is ${}^{n}c_2 = 5050$
$\Rightarrow$ n = 101

**12.** Here the sum of the numbers are vanishes of six cards i.e
Case I : If selected 3 cards each of number –1 or 1

i.e The number of arrangement $= \frac{6!}{3!\,3!} = 20$

Case II : If selected 2 cards each of no. –1, 0 or 1 i.e

number of arrangement $= \frac{6!}{2!\,2!\,2!} = 90$

Case III : If selected one card each of number –1 and 1 and 4 cards of no. 0.

so no. of arrangement is $\frac{6!}{1!\,1!\,4!} = 30$

Case IV : If all cards selected fram the no. 0

So no. of arrangement is $\frac{6!}{6!} = 1$

Hence total no. of arrangement is
$20 + 90 + 30 + 1 = 141$

**13.** **(A)**

| | | |
|---|---|---|
| two 2's | five 1's | |
| two 2's | four 1's | one 3 |
| two 2's | three 1's | two 3's |
| two 2's | two 1's | three 3's |
| two 2's | one1 | four 3's |
| two 2's | five 3's | |

$= 2\left(\frac{7!}{2!\,5!}+\frac{7!}{2!\,4!}+\frac{7!}{2!\,3!\,2!}\right) = 672$

**14.** **(D)**
D P M L can be arranged in 4 ! ways & the two gaps out of 5 gaps can be selected in ${}^{5}C_2$ ways. {A A and E E} or {A E and A E} can be placed in 6 ways.
Total = $4!.\,{}^{5}C_2.\,6 = 1440$

**15.** **(A,D)**
All AAAAA BBB D EEF can be arranged in

$\frac{12!}{5!\,3!\,2!}$ ways

Between the gaps C can be arranged in ${}^{13}C_3$ ways

Total ways $= {}^{13}C_3 \times \frac{12!}{5! \times 3! \times 2!}$

Number of ways
= without considering separation of C – in which all C's are together – in which exactly two C's are

together $= \frac{15!}{5!(3!)^2\,2!} - \frac{13!}{5!\,3!\,2!} - \frac{12!}{5!\,3!}\,{}^{13}C_2$

**16.** **(D)**

If $\left[\frac{1}{3}+\frac{\lambda}{50}\right] = 0$

$\Rightarrow \quad 0 \le \frac{1}{3}+\frac{\lambda}{50} < 1$

$$=\frac{1}{15}\left\{\frac{(1+2+........+9)^2-(1^2+2^2+...+9^2)}{2}+(1^2+2^2+...+9^2)\right\}$$

$$=\frac{1}{30}\left\{\left(\frac{9\times10}{2}\right)^2+\frac{9\times10\times19}{6}\right\}=77$$

**22.** **(A) → (r) ; (B) → (s) ; (C) → (q) ; (D) → (p)**

**(A)** Select m out of n = $^nC_m$
number of ways of arranging in increasing order = 1
Hence $^nC_m$

**(B)** A monkey has m choice

∴ $\underbrace{m\times m\times .m}_{n\text{ times}}=m^n$

**(C)** Arrange (m –1) green balls then out of m gaps select n positions for red balls and arrange red balls = 1. $^mC_n$ . 1
= $^mC_n$

**(D)** $\underbrace{n\times n\times n}_{m\text{ times}}=n^m$

**23.**

**1** **(B)**
Since there are 5 even places and 3 pairs of repeated letters therefore at least one of these must be at an odd place.

∴ the number of ways = $\frac{11!}{2!\ 2!\ 2!}$

**2** **(A)**
Make a bundle of both M's and another bundle of T's. Then except A's we have 5 letters remaining so M's, T's and the letters except A's can be arranged in 7 ! ways
∴ total number of arrangements = 7 ! × $^8C_2$

**3** **(A)**

Consonants can be placed in $\frac{7!}{2!2!}$ ways

Then there are 8 places and 4 vowels

∴ Number of ways = $\frac{7!}{2!2!}\cdot{}^8C_4\ \frac{4!}{2!}$

**24.**

**1.** **(D)**

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 1 | ⇒ | $^3C_1$ |
| 2 | 2 | 1 | ⇒ | $^3C_1$ |

∴ $^3C_1+{}^3C_1=6$

**2.** **(B)**

□ □ □

3 1 1 ⇒ $\frac{5!}{3!1!1!2!}=10$

2 2 1 ⇒ $\frac{5!}{1!2!2!2!}=15$

∴ 10 + 15 = 25

**3** **(A)**

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 1 | ⇒ | 1 |
| 2 | 2 | 1 | ⇒ | 1 |

∴ 1 + 1 = 2

**25.**

**1.** **(B)**
Since n = 4
∴ Number of matched arrangements

$$=\frac{8!}{4!\ 5!}=\frac{^8C_4}{5}$$

**2.** **(A)**
n = 3
∴ Number of matched arrangements

$$=\frac{6!}{3!4!}$$

Total number of arrangements = $\frac{6!}{3!3!}$

∴ probability = $\frac{1}{4}$

**3.** **(B)**
Since first pair is matched and it can be done in 1 way
∴ for mismatched pairs n = 4
∴ Number of mismatched pairs = $^8C_5$

**26.** Here, we note the following.
1. A door will open if it face odd no of changes.
2. No. of changes faced by any door will be equal to no. of factors of the door no.
3. So only those door will open, whose number is perfect square so ans is $[\sqrt{n}]$,
[where [ ] denotes the G.I.F.]