DIFFERENTIAL EQUATION

EXERCISE # 1

Questions based on Order & Degree

Q.1 The order and degree of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ are-

(A) 1, 2 (B) 2, 1 (C) 1, 1 (D) 2, 2

Sol. **[A]**

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{y\sqrt{1-x^2}}$$

Clearly order one and degree one

Q.2 The order and degree of the differential equation $\left[4+\left(\frac{dy}{dx}\right)^2\right]^{2/3} = \frac{d^2y}{dx^2}$ are-

(A) 2, 2 (B) 3, 3 (C) 2, 3 (D) 3, 2

Sol. **[C]**

Clearly order one and degree two

Q.3 The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ is -

(A) 1 (B) 2 (C) 3 (D) None

Sol. **[C]**

$$\left[4+\left(\frac{dy}{dx}\right)^2\right]^2 = \left[\frac{d^2y}{dx^2}\right]^3$$

$\Rightarrow$ order two and degree three

Q.4 The order 'O' and degree D of the differential equation $y = 1 + x\left(\frac{dy}{dx}\right) + \frac{x^2}{2!}\left(\frac{dy}{dx}\right)^2 + \ldots.$

$\ldots.+ \frac{x^n}{n!}\left(\frac{dy}{dx}\right)^n + \ldots.. \infty$ are given -

(A) 'O' = 1 (B) D = 2

(C) D = 0 (D) D is not defined

Sol.[A] $y = 1 + x\frac{dy}{dx} + \frac{x^2}{2!}\left(\frac{dy}{dx}\right)^2 + \ldots.+ \frac{x^n}{n!}\left(\frac{dy}{dx}\right)^4 + \ldots \infty$

$\Rightarrow$ order 'O' = 1, degree D = ∞.

Q.5 If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{1/3} + x^2y^2 = \sin x$, then-

(A) $p > q$ (B) $\frac{p}{q} = \frac{1}{2}$ (C) $p = q$ (D) $p < q$

Sol. **[D]**

Order p = 2 and degree q = 6 $\Rightarrow p < q$

Questions based on Formation of Differential Equation

Q.6 The differential equation of the family of curves represented by the equation $x^2 + y^2 = a^2$ is-

(A) $x + y\frac{dy}{dx} = 0$ (B) $y\frac{dy}{dx} = x$

(C) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (D) None of these

Sol. **[A]**

$x^2 + y^2 = a^2$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow x + y\frac{dy}{dx} = 0$

Q.7 The differential equation of the family of curves $y^2 = 4a(x + a)$, where a is an arbitrary constant, is-

(A) $y\left[1+\left(\frac{dy}{dx}\right)^2\right] = 2x\frac{dy}{dx}$

(B) $y\left[1-\left(\frac{dy}{dx}\right)^2\right] = 2x\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

(D) $\left(\frac{dy}{dx}\right)^3 + 3\frac{dy}{dx} + y = 0$

Sol. **[B]**

$y^2 = 4a(x + a)$ …..(1)

$\Rightarrow 2y \frac{dy}{dx} = 4a$

$\Rightarrow y \frac{dy}{dx} = 2a$

from (1)

$y^2 = 2y \frac{dy}{dx} \left(x + \frac{y}{2}\frac{dy}{dx}\right)$

$\Rightarrow y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\left(1-\left(\frac{dy}{dx}\right)^2\right) = 2x\frac{dy}{dx}$

Q.8 The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (where a is a constant)-

(A) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(B) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$

(C) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(D) None of these

Sol. **[A]**

$(x-h)^2 + (y-k)^2 = a^2$ ....(1)

$\Rightarrow 2(x-h) + 2(y-k)\frac{dy}{dx} = 0$

$\Rightarrow (x-h) + (y-k)\frac{dy}{dx} = 0$ ....(2)

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 + (y-k)\frac{d^2y}{dx^2} = 0$ ....(3)

from (1), (2) and (3) we get

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

Q.9 The differential equation for all the straight lines which are at a unit distance from the origin is

(A) $\left(y-x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

(B) $\left(y+x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(C) $\left(y-x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(D) $\left(y+x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

Sol. **[C]**

line is $ax + by + \sqrt{a^2+b^2} = 0$ ....(1)

$\Rightarrow a + b\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{a}{b}$

from (1) $\frac{a}{b}x + y + \sqrt{\left(\frac{a}{b}\right)^2 + 1} = 0$

$\Rightarrow \left(y-x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

Q.10 The differential equation of all parabolas that have origin as vertex and y-axis as axis of symmetry is-

(A) $xy' = 2y$ (B) $2xy' = y$

(C) $yy' = 2x$ (D) $y'' + y = 2x$

Sol. **[A]**

Equation of parabola is

$x^2 = 4ay$ ......(i)

diff. w.r. to x we have

$2x = 4ay'$

$\Rightarrow x = 2ay'$

$\Rightarrow a = \frac{x}{2y'}$

From (i)

$\Rightarrow x^2 = 4\frac{x}{2y'}y$

$\Rightarrow xy' = 2y$

Q.11 Differential equation whose general solution is $y = c_1x + c_2/x$ for all values of c_1 and c_2 is-

(A) $\frac{d^2y}{dx^2} + \frac{x^2}{y} + \frac{dy}{dx} = 0$

(B) $\frac{d^2y}{dx^2} + \frac{y}{x^2} - \frac{dy}{dx} = 0$

(C) $\frac{d^2y}{dx^2} + \frac{1}{2x}\frac{dy}{dx} = 0$

(D) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

Sol. **[D]**

$$y = c_1x + \frac{c_2}{x}$$

$$y' = c_1 - \frac{c_2}{x^2}$$

$$y'' = \frac{2}{x^3}c_2$$

$$y = \left(y' + \frac{1}{x^2} \times \frac{x^3 y''}{2}\right)x + \frac{x^3 y''}{2x}$$

$$\Rightarrow y = xy' + \frac{x^2y''}{2} + \frac{x^2y''}{2}$$

$$\Rightarrow y = xy' + x^2y''$$

$$y'' + \frac{1}{x}y' - \frac{y}{x^2} = 0$$

Questions based on **Variable separable**

Q.12 The solution of the equation

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \text{ is-}$$

(A) $x\sqrt{1-y^2} - y\sqrt{1-x^2} = c$

(B) $x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$

(C) $x\sqrt{1+y^2} + y\sqrt{1+x^2} = c$

(D) None of these

Sol. **[B]**

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \sin^{-1} y = -\sin^{-1}x + c$$

$$\Rightarrow \sin^{-1} y + \sin^{-1}x = c$$

$$\Rightarrow \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = c$$

$$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$$

Q.13 Solution of the equation

$$\cos x \cos y \frac{dy}{dx} = -\sin x \sin y \text{ is-}$$

(A) $\sin y + \cos x = c$ (B) $\sin y - \cos x = c$

(C) $\sin y. \cos x = c$ (D) $\sin y = c \cos x$

Sol. **[D]**

$$\int \cot y \, dy = -\int \tan x \, dx$$

$$\Rightarrow \log \sin y = \log \cos x + \log c$$

$$\Rightarrow \sin y = c \cos x$$

Q.14 The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is

(A) $y + \cot\left(\frac{x-y}{2}\right) = C$ (B) $x + \cot\left(\frac{x-y}{2}\right) = C$

(C) $x + \tan\left(\frac{x-y}{2}\right) = C$ (D) none of these

Sol. **[B]**

$$\frac{dy}{dx} = \cos(x - y)$$

Let $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow 1 - \frac{dt}{dx} = \cos t$$

$$\Rightarrow \frac{dt}{dx} = 1 - \cos t \Rightarrow \frac{dt}{dx} = 2\sin^2\frac{t}{2}$$

$$\Rightarrow \int dx = \frac{1}{2}\int \text{cosec}^2\frac{t}{2}dt$$

$$\Rightarrow x = -\cot\frac{t}{2} + C$$

$$\Rightarrow x = -\cot\frac{t}{2} + C$$

$$\Rightarrow x + \cot\left(\frac{x-y}{2}\right) = C$$

Questions based on **Homogeneous**

Q.15 The solution of the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is-

(A) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$

(B) $\tan^{-1}\left(\frac{y}{x}\right) = -\log x + c$

(C) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$

(D) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

Sol. **[A]**

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = 1 + v + v^2$$

$$\Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$\Rightarrow \tan^{-1}v = \log x + c$

$\Rightarrow \tan^{-1}\frac{y}{x} = \log x + c$

Q.16 The solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is-

(A) $y = xe^{cx}$ (B) $y + xe^{cx} = 0$
(C) $y + e^{x} = 0$ (D) None of these

Sol. **[A]**

$\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v(\log v + 1)$

$\Rightarrow \int\frac{dv}{v\log v} = \int\frac{dx}{x}$

$\Rightarrow \log\log v = \log x + \log c$

$\Rightarrow \log\frac{y}{x} = cx$

$\Rightarrow y = x e^{cx}$

Q.17 Solution of differential equation $(3y - 7x + 7)\,dx + (7y - 3x + 3)\,dy = 0$ is-

(A) $C = (2x + y - 1)^5 (y - 3x + 1)^2$
(B) $C = (x + y - 1)^5 (y - x + 1)^2$
(C) $C = (x + 2y - 1)^5 (2y - x + 1)^2$
(D) None of these

Sol. **[B]**

$\frac{dy}{dx} = -\frac{3y - 7x + 7}{7y - 3x + 3}$

Put $x = X + h,\ y = Y + k$

$\Rightarrow \frac{dy}{dx} = \frac{dY}{dx}$ we have

$\frac{dY}{dX} = -\frac{7x - 3Y + 7h - 3k - 7}{3x - 7Y + 3h - 7k - 3}$

take $7h - 3k - 7 = 0$ and $3h - 7k - 3 = 0$

we get $h = 1, k = 0$

$\Rightarrow \frac{dY}{dX} = -\frac{7X - 3Y}{3X - 7Y}$

Put $Y = vX \Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX}$

$\Rightarrow X\frac{dv}{dX} = \frac{7(v^2 - 1)}{3 - 7v}$

$\int\frac{3 - 7v}{v^2 - 1}dv = 7\int\frac{dX}{X}$

$\Rightarrow \frac{3}{2}\log\left(\frac{v-1}{v+1}\right) - \frac{7}{2}\log(v^2 - 1)$

$= 7\log X + \log c$

Solving we get

$c = (x + y - 1)^5 (y - x + 1)^2$

Q.18 The solution of the equation $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ is

(A) $(x - y)^2 + C = \log(3x - 4y + 1)$
(B) $x - y + C = \log(3x - 4y + 4)$
(C) $x - y + C = \log(3x - 4y - 3)$
(D) $x - y + C = \log(3x - 4y + 1)$

Sol. **[D]**

$\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$

Let $3x - 4y = v$

$\Rightarrow 3 - 4\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dv}{dx} = -\left(\frac{v+1}{v-3}\right)$

$\Rightarrow \int\frac{v-3}{v+1}dv = -\int dx$

$\Rightarrow v - 4\log(v + 1) = -x - c$

$\Rightarrow (3x - 4y) - 4\log(3x - 4y + 1) = -x - c$

$\Rightarrow (x - y) + c = \log(3x - 4y + 1)$

Questions based on Linear & Reducible to linear

Q.19 The solution of the differential equation $x\log x\frac{dy}{dx} + y = 2\log x$ is-

(A) $y = \log x + c$
(B) $y = \log x^2 + c$
(C) $y\log x = (\log x)^2 + c$
(D) $y = x\log x + c$

Sol. **[C]**

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$

I.F. $= e^{\int\frac{1}{x\log x}dx} = e^{\log\log x} = \log x$

$y \cdot \log x = \int\frac{2}{x} \cdot \log x\,dx$

$\Rightarrow y \log x = (\log x)^2 + c$

Q.20 The solution of differential equation

$\frac{dy}{dx} + 1 = e^{x-y}$ is-

(A) $e^y = e^x/2 + ce^{-x}$ (B) $e^y = e^x + ce^{-x}$

(C) $3e^y = e^x/2 + ce^{-x}$ (D) None of these

Sol. **[A]**

$e^y \frac{dy}{dx} + e^y = e^x$

Let $e^y = t \Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t = e^x$

I.F. $= e^{\int dx} = e^x$

$\Rightarrow t \,.\, e^x = \int e^{2x} dx$

$\Rightarrow t \,.\, e^x = \frac{e^{2x}}{2} + c$

$\Rightarrow e^y = \frac{e^x}{2} + ce^{-x}$

Q.21 The solution of differential equation sin y (dy/dx) = cos y (1 – x cos y) is-

(A) $\cos y = (x+1) - ce^x$ (B) $\sec y = (x+1) - ce^x$

(C) $\sec y = (x-1) - ce^x$ (D) None of these

Sol. **[B]**

$\tan y \sec y \frac{dy}{dx} - \sec y = -x$

Let $\sec y = t \Rightarrow \sec y \tan y \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} - t = -x$

I.F. $= e^{-\int dx} = e^{-x}$

$\Rightarrow t.e^{-x} = -\int xe^{-x}$

$\Rightarrow t.e^{-x} = xe^{-x} - \int e^{-x} dx - c$

$\Rightarrow t.e^{-x} = (x+1)\, e^{-x} - c$

$\Rightarrow \sec y = (x + 1) - ce^x$

Questions based on Trajectories and exact differentials

Q.22 The value of k such that the family of parabola $y = cx^2 + k$ is the orthogonal trajectory of the family of ellipses $x^2 + 2y^2 - y = c$.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

Sol. **[B]**

$x^2 + 2y^2 - y = c$

diff. w.r.to x we have

$2x + 4y\frac{dy}{dx} - \frac{dy}{dx} = 0$

For orthogonal trajectory

$2x - 4y\frac{dx}{dy} + \frac{dx}{dy} = 0$

$\Rightarrow 2x = (4y - 1)\frac{dx}{dy}$

$\Rightarrow \int \frac{2dy}{4y-1} = \int \frac{dx}{x}$

$\Rightarrow \frac{1}{2}\log(4y - 1) = \log x + \log c$

$\Rightarrow 4y - 1 = cx^2$

$\Rightarrow y = cx^2 + \frac{1}{4}$

So $k = \frac{1}{4}$

Q.23 $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$

(A) $\frac{y}{x} - \frac{1}{(x^2+y^2)} = c$ (B) $\frac{x}{y} - \frac{1}{(x^2+y^2)} = c$

(C) $\frac{y}{x} + \frac{1}{(x^2+y^2)} = c$ (D) $\frac{x}{y} + \frac{1}{(x^2+y^2)} = c$

Sol. **[A]**

$\Theta\ \frac{x\,dx + y\,dy}{(x^2+y^2)^2} = -\frac{x\,dy - y\,dx}{x^2}$

$\Rightarrow \frac{2x\,dx + 2y\,dy}{2(x^2+y^2)^2} = -\left(\frac{x\,dy - y\,dx}{x^2}\right)$

$\Rightarrow \frac{1}{2}\int d\left(\frac{-1}{x^2+y^2}\right) = -\int d\left(\frac{y}{x}\right)$

$\Rightarrow \frac{y}{x} - \frac{1}{2(x^2+y^2)} = c$

Q.24 $\frac{y + \sin x \cos^2(xy)}{\cos^2(xy)} dx + \left(\frac{x}{\cos^2(xy)} + \sin y\right) dy = 0$

(A) tan (xy) + cos x + cos y = c
(B) tan (xy) + cos x – cos y = c
(C) tan (xy) – cos x + cos y = c
(D) tan (xy) – cos x – cos y = c

Sol. **[D]**

$$\int \sec^2(xy)\,d(xy) + \int \sin x\,dx + \int \sin y\,dy = 0$$

$\Rightarrow \tan(xy) - \cos x - \cos y = c$

Questions based on **Miscellaneous**

Q.25 The solution of

$e^{x\frac{(y^2-1)}{y}}\{xy^2\,dy + y^3dx\} + \{ydx - xdy\} = 0$, is-

(A) $e^{xy} + e^{x/y} + c = 0$ (B) $e^{xy} - e^{x/y} + c = 0$
(C) $e^{xy} + e^{y/x} + c = 0$ (D) $e^{xy} - e^{y/x} + c = 0$

Sol. **[A]**

$e^{(xy - x/y)}\, y^2\,(x\,dy + y\,dx) + (y\,dx - x\,dy) = 0$

$\Rightarrow e^{xy}\,(x\,dy + y\,dx) + e^{x/y}\left(\dfrac{y\,dx - x\,dy}{y^2}\right) = 0$

$\Rightarrow d\,(e^{xy}) + d(e^{x/y}) = 0$

$\Rightarrow \displaystyle\int \frac{d}{dx}(e^{xy})dx + \int \frac{d}{dx}e^{x/y}\,dx = c$

$\Rightarrow e^{xy} + e^{x/y} + c = 0$

Q.26 The degree and order of the differential equation of the family of all parabolas whose axis is x- axis are respectively-

(A) 2, 1 (B) 1, 2
(C) 3, 2 (D) 2, 3

Sol. **[B]**

Equation is $y^2 = 4a\,(x - h)$

differentiate we get

$2y\dfrac{dy}{dx} = 4a$

Again differentiate then

$y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$

Clearly degree 1 and order 2

Q.27 Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$ is-

(A) 0 (B) 1
(C) 2 (D) more than 2

Sol. **[C]**

$\Theta\ y = e^{mx}$

$\therefore Dy = me^{mx},\ D_y^2 = m^2e^{mx},\ D_y^3 = m^3e^{mx}$

Then $D^3y - 3D^2y - 4Dy + 12y = 0$ is

$m^3e^{mx} - 3m^2e^{mx} - 4me^{mx} + 12e^{mx} = 0$

$\Rightarrow m^3 - 3m^2 - 4m + 12 = 0$

$\Rightarrow (m - 2)\,(m + 2)\,(m - 3) = 0$

$\Rightarrow m = -2, 2, 3$

But $m \in N$

$\Rightarrow m = 2, 3$ two values

Q.28 The solution to the differential equation $y\,\lambda ny + xy' = 0$, where $y(1) = e$, is-

(A) $x(\lambda ny) = 1$ (B) $xy\,(\lambda ny) = 1$
(C) $(\lambda ny)^2 = 2$ (D) $\lambda ny + \left(\dfrac{x^2}{2}\right)y = 1$

Sol. **[A]**

$y\,\lambda ny + x\dfrac{dy}{dx} = 0$

$\Rightarrow \displaystyle\int \frac{dy}{y\,\lambda ny} = -\int \frac{dx}{x}$

$\Rightarrow \lambda n\,(\lambda n\,y) = -\,\lambda n\,x + \lambda n\,c$

$\Rightarrow x\,\lambda n\,y = c$

$\Theta\ y(1) = e \Rightarrow c = 1$

$\Rightarrow x\,\lambda n\,y = 1$

Q.29 A curve passing through (2, 3) and satisfying the differential equation $\displaystyle\int_0^x ty(t)\,dt = x^2y\ (x)$, $(x > 0)$ is-

(A) $x^2 + y^2 = 13$ (B) $y^2 = \dfrac{9}{2}x$
(C) $\dfrac{x^2}{8} + \dfrac{y^2}{18} = 1$ (D) $xy = 6$

Sol. **[D]**

$\Theta\ \displaystyle\int_0^x t\,y(t)\,dt = x^2y\,(x)$

on differentiation

$\Rightarrow xy(x) = 2xy(x) + x^2y^1(x)$

$\Rightarrow -\,xy(x) = x^2\dfrac{dy(x)}{dx}$

$\Rightarrow \dfrac{dy(x)}{y(x)} = -\,\dfrac{dx}{x}$

Integrating we get

$\lambda n\,y(x) = -\,\lambda n\,x + \lambda n\,c$

$\Rightarrow xy(x) = c$

Passing through (2, 3)

$\Rightarrow c = 6$

curve is xy = 6

Q.30 Family $y = Ax + A^3$ of curve represented by the differential equation of degree-

(A) three (B) two
(C) one (D) None of these

Sol. **[A]**

$y = Ax + A^3$

$\Rightarrow \frac{dy}{dx} = A$

So $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$

$\Rightarrow$ degree 3

Q.31 If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then function y is-

(A) $e^{(1-x)^2/2}$ (B) $e^{(1+x)^2/2} - 1$
(C) $\log_e(1 + x) - 1$ (D) 1 + x

Sol. **[B]**

$\because \frac{dy}{dx} = (1 + x)(1 + y)$

$\Rightarrow \int \frac{dy}{1+y} = \int (1+x)\,dx$

$\Rightarrow \log(1 + y) = x + \frac{x^2}{2} + c$

$\because y(-1) = 0 \Rightarrow c = \frac{1}{2}$

$\Rightarrow \log(1 + y) = \frac{(x+1)^2}{2}$

$\Rightarrow y = e^{(1+x)^2/2} - 1$

Q.32 Solution of differential equation xdy – ydx = 0 represents-

(A) rectangular hyperbola
(B) straight line passing through origin
(C) parabola whose vertex is at origin
(D) circle whose centre is at origin

Sol. **[B]**

$\because x\,dy = y\,dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow \log y = \log x + \log c$

$\Rightarrow y = cx$

This is a equation of straight line passing through origin.

➤ True / False type questions

Q.33 The differential equation $y^3dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$.

Sol. **[True]**

$\frac{dy}{dx} = -\frac{(x+y^2)}{y^3}$

$y^2 = t$

$2y \cdot y' = \frac{dt}{dx} \Rightarrow 2y \times \frac{-(x+y^2)}{y^3} = \frac{dt}{dx}$

$\Rightarrow \frac{-2(x+t)}{t} = \frac{dt}{dx}$

homogenous

Q.34 The degree of the differential equation $2\left(\frac{d^2y}{dx^2}\right)^2 + \frac{d^3y}{dx^3} - \frac{dy}{dx} - \sin^2 y + \sin\left(\frac{dy}{dx}\right) = 0$ is 2.

Sol. **[False]**

Degree not defined.

Q.35 The differential equation of the family of parabola whose axis is parallel to y-axis has order 3 & degree 1.

Sol. **[True]**

$(x - b)^2 = 4a(y - k)$

order = 3

$2(x - h) = 4ay'$

$1 = 2ay''$

Q.36 If the solution of $\frac{dy}{dx} = \frac{ax+3}{2y+1}$ represents a circle, then the value of a is – 4.

Sol. **[False]**

$\int (2y+1)\,dy = \int (ax+3)\,dx$

$$\frac{(2y+1)^2}{2} = \frac{(ax+3)^2}{a} + c$$

$\Rightarrow a = -2$

Fill in the blanks type questions

Q.37 Solution of differential equation

$$x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!}$$

$+ \ldots$ is ………

Sol. $x^2 = e^{\frac{y}{x}\frac{dy}{dx}}$

$$\Rightarrow \frac{1}{2}\int 2(\lambda n x^2)dx = \int y\, dy$$

$$\frac{1}{2}x^2(\lambda n\, x^2 - 1) = \frac{y^2}{2} + c$$

$x^2 \lambda n\, x^2 - x^2 = y^2 + 2c$

Q.38 The orthogonal trajectory of system of curve $y = ax^2$ which does not passes through origin, is……………

Sol. $y' = 2ax$

$$y' = 2x \cdot \frac{y}{x^2}$$

$$-\frac{1}{y'} = \frac{2y}{x}$$

$$-\int x\, dx = 2\int y\, dy$$

$$-\frac{x^2}{2} = y^2 + c$$

$c \neq 0$

$$y^2 + \frac{x^2}{2} = k$$

ellipse

Q.39 If gradient of a curve at any point P(x, y) is $\frac{x+y+1}{2y+2x+1}$ and it passes through origin, then curve is……………………

Sol. $$\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+1}$$

$x + y = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 1 + \frac{x+y+1}{2x+2y+1} = \frac{dt}{dx}$$

$$= 1 + \frac{t+1}{2t+1} = \frac{dt}{dx}$$

$$\Rightarrow \frac{3t+2}{2t+1} = \frac{dt}{dx}$$

$$\Rightarrow \int dx = \frac{2}{3}\int \frac{3t + \frac{3}{2}}{3t+2} dt$$

$$x + c = \frac{2}{3}t - \frac{1}{2}\lambda n\,|3t+2|$$

$$c = -\frac{1}{2}\lambda n\, 2$$

Q.40 Solution of differential equation $\frac{3dy}{dx} + \frac{2y}{x+1} = \frac{x^3}{y^2}$ is ……………… .

Sol. $$3y^2\frac{dy}{dx} + \frac{2y^3}{(1+x)} = x^3$$

Let $y^3 = t \Rightarrow 3y^2\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow \frac{dt}{dx} + \frac{2t}{1+x} = x^3$$

$$\text{IF.} = e^{\int \frac{2}{1+x}} = e^{2\lambda n(1+x)} = (1+x)^2$$

$$\Rightarrow t(1+x)^2 = \int x^3(1+x)^2 dx$$

$$\Rightarrow y^3(1+x)^2 = \int (x^5 + 2x^4 + x^3)dx$$

$$\Rightarrow y^3(1+x)^2 = \frac{x^6}{6} + \frac{2}{5}x^5 + \frac{x^4}{4} + C$$

EXERCISE # 2

Part-A Only single correct answer type Questions

Q.1 The order of the differential equation whose general solution is

$y = C_1e^x + C_2e^{2x} + C_3e^{3x} + C_4e^{x+C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constant, is-

(A) 5 (B) 4 (C) 3 (D) none

Sol.[C] $y = (C_1 + C_4 e^{C_5}) e^x + C_2 e^{2x} + C_3 e^{3x}$

$= ae^x + C_2 e^{2x} + C_3e^{3x}$

$\because$ Three arbitrary constants

$\Rightarrow$ order = 3.

Q.2 The order and the degree of the differential equation whose general solution is, $y = c(x – c)^2$, are respectively -

(A) 1, 1 (B) 1, 2 (C) 1, 3 (D) 2, 1

Sol.[C] $y = c(x – c)^2$

$\Rightarrow$ One arbitrary constant $\Rightarrow$ order = 1

for degree diff. w.r.to x

$$\frac{dy}{dx} = 2c(x-c)$$

$$\left(\frac{dy}{dx}\right)^2 = 4cy \quad \Rightarrow \quad C = \frac{1}{4y}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = 2\frac{1}{4y}\left(\frac{dy}{dx}\right)^2\left(x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right)$$

$\Rightarrow$ degree = 3.

Q.3 The order of the differential equation formed by differentiating and eliminating the constants from $y = a \sin^2x + b \cos^2x + c \sin 2x + d \cos2x$. Where a, b, c, d are arbitrary constants; is -

(A) 1 (B) 2 (C) 3 (D) 4

Sol.[C] $y = a \sin^2x + b \cos^2 x + c \sin 2x + d \cos 2x$

diff. w.r.t. to x

$$\frac{dy}{dx} = a \sin 2x – b \sin 2x + 2c \cos 2x – 2d \sin 2x$$

$= P \sin2x + 2c\cos 2x \quad [P = a – b – 2d]$

$$\Rightarrow \frac{d^2y}{dx^2} = 2P \cos 2x – 4 c \sin 2x$$

$$\Rightarrow \frac{d^3y}{dx^3} = – 4 (P \sin 2x + 2c \cos 2x)$$

$$\Rightarrow \frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 0 \text{ ; order} = 3$$

Q.4 The degree of differential equation satisfying the relation

$\sqrt{1+x^2} + \sqrt{1+y^2} = n(x\sqrt{1+y^2} – y\sqrt{1+x^2})$ is

(A) 1 (B) 2 (C) 3 (D) none

Sol.[A] Put $x = \tan \theta$ and $y = \tan \phi$ then $\sqrt{1+x^2} = \sec\theta$ and $\sqrt{1+y^2} = \sec \phi$

equation becomes

$\sec \theta + \sec \phi = n (\tan \theta \sec \phi – \tan \phi \sec \theta)$

$\Rightarrow \cos \phi + \cos \theta = n (\sin \theta – \sin \phi)$

$$\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2n\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2} = n \Rightarrow \theta – \phi = 2\cot^{-1} n$$

$\Rightarrow \tan^{-1} x – \tan^{-1} y = 2 \cot^{-1} n$

Differentiating we get

$$\frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = 0$$

Which is a differential equation at degree 1.

Q.5 The solution curves of the given differential equation $xdx – dy = 0$ are given by a family of-

(A) parabola (B) hyperbola

(C) circles (D) ellipses

Sol.[A] $xdx – dy = 0$

By integrating we get

$$\Rightarrow \int xdx = \int dy$$

$$\Rightarrow \frac{x^2}{2} = y + c$$

$x^2 = 2(y + c)$

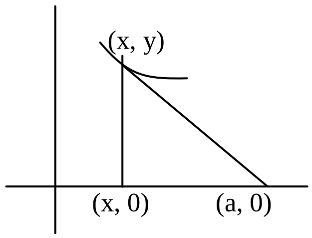
This is equation of parabola.

Q.6 The equation of the curve, passing through (2, 5) and having the area of, triangle formed by the x-axis, the ordinate of a point on the curve and the tangent at the point as 5 sq. units-

(A) $xy = 10$ (B) $x^2 = 10y$

(C) $y^2 = 10x$ (D) $xy^{1/2} = 10$

Sol.[A]



Area = 5

$\frac{1}{2}(a - x)\,y = 5$

$a = \frac{10}{y} + x$

Then point $(a, 0) \equiv \left(\frac{10}{y} + x, 0\right)$

Slop of tangent

$\Rightarrow \frac{dy}{dx} = \frac{-y}{\frac{10}{y}} = -\frac{y^2}{10}$

$\Rightarrow -10\int \frac{dy}{y^2} = \int dx$

$\Rightarrow \frac{10}{y} = x + c$

Curve passes through (2, 5) $\Rightarrow$ c = 0

So, equation of curve is

xy = 10

Q.7 The solution of the differential equation

$\log\left(\frac{dy}{dx}\right) = 4x - 2y - 2$, y = 1 when x = 1 is -

(A) $2e^{2y+2} = e^{4x} + e^2$ (B) $2e^{2y-2} = e^{4x} + e^4$

(C) $2e^{2y+2} = e^{4x} + e^4$ (D) $3e^{2y+2} = e^{3x} + e^4$

Sol.[C] $\frac{dy}{dx} = e^{4x} \cdot e^{-(2y+2)}$

$\Rightarrow \int e^{2y+2}dy = \int e^{4x}dx$

$\Rightarrow \frac{1}{2}e^{2y+2} = \frac{1}{4}e^{4x} + C$

$\because$ y = 1, x = 1

$C = \frac{1}{4}e^4$

$\Rightarrow 2e^{2y+2} = e^{4x} + e^4$

Q. 8 Solution of the differential equation $(x^2 + 1)y_1 + 2xy = 4x^2$ is-

(A) $y(1 + x^2) = \frac{4x^3}{3} + C$

(B) $y(1 - x^2) = x^3 + C$

(C) $y(1 - x^2) = \frac{x^3}{2} + C$

(D) none of these

Sol.[A] $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$

$I.F. = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1 + x^2$

$\Rightarrow y \cdot IF = \int\left(\frac{4x^2}{1+x^2}.I.F\right)dx$

$\Rightarrow y(1 + x^2) = \int 4x^2 dx$

$\Rightarrow y(1 + x^2) = \frac{4}{3}x^3 + C$

Q.9 y = f(x) satisfies the differential equation $\frac{dy}{dx} - y = \cos x - \sin x$ with the condition that y is bounded when $x \to +\infty$ the longest interval in which f(x) is increasing in the interval $[0, \pi]$ is-

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (B) $\left(0, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ (D) $\left(0, \frac{\pi}{6}\right)$

Sol. **[B]**

Q.10 The solution of the differential equation $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\frac{\pi}{2}}$ is-

(A) $\sin x^2y^2 = e^{x-1}$ (B) $\sin(x^2y^2) = x$

(C) $\cos^2xy^2 + x = 0$ (D) $\sin(x^2y^2) = e^x$

Sol. **[A]**

Q.11 The solution of differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)} \text{ is -}$$

(A) $x\phi\left(\frac{y}{x}\right) = k$ (B) $\phi\left(\frac{y}{x}\right) = kx$

(B) $y\phi\left(\frac{y}{x}\right) = k$ (D) $\phi\left(\frac{y}{x}\right) = ky$

Sol.[B] $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$

Let y = vx ⇒ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

⇒ $v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$

⇒ $\int \frac{\phi'(v)}{\phi(v)} dv = \int \frac{dx}{x}$

⇒ $\lambda n\, \phi(v) = \lambda nx + \lambda nk$

⇒ $\phi\left(\frac{y}{x}\right) = xk$

Q.12 The equation of curve through point (1, 0) and whose slope is $\frac{y-1}{x^2+x}$ is-

(A) (y – 1) (x + 1) + 2x = 0

(B) 2x (y – 1) + x + 1 = 0

(C) $y = \frac{1-x}{1+x}$

(D) none of these

Sol.[A] $\frac{dy}{dx} = \frac{y-1}{x(x+1)}$

⇒ $\int \frac{dy}{y-1} = \int\left(\frac{1}{x} - \frac{1}{x+1}\right) dx$

⇒ $\lambda n\,(y-1) = \lambda n \frac{x}{x+1} + \lambda nC$

⇒ $(y-1) = \frac{Cx}{x+1}$

Curve passes through (1, 0)

⇒ C = – 2

⇒ (y – 1) (x + 1) = – 2x

⇒ (y – 1) (x + 1) + 2x = 0

Q.13 The solution of the differential equation $xdy + ydx - \sqrt{1-x^2y^2}\, dx = 0$ is-

(A) $\sin^{-1} xy = C - x$

(B) xy = sin (x + C)

(C) $\log(1 - x^2y^2) = x + C$

(D) y = x sin x + C

Sol.[B] $xdy + ydx - \sqrt{1-x^2y^2}\, dx = 0$

⇒ $\frac{xdy}{dx} + y = \sqrt{1-x^2y^2}$

Let xy = t ⇒ $\frac{xdy}{dx} + y = \frac{dt}{dx}$

⇒ $\frac{dt}{dx} = \sqrt{1-t^2}$

⇒ $\int \frac{1}{\sqrt{1-t^2}} dt = \int dx$

⇒ $\sin^{-1} t = x + C$

⇒ $\sin^{-1} xy = x + C$

⇒ xy = sin (x + C)

Q.14 A curve passes through the point $\left(1, \frac{\pi}{4}\right)$ and its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation y = ...

(A) $y = x\tan^{-1}\left(\lambda n \frac{e}{x}\right)$

(B) $y = x\tan^{-1}(\lambda n + 2)$

(C) $y = \frac{1}{x}\tan^{-1}\left(\lambda n \frac{e}{x}\right)$

(D) none of these

Sol.[A] Given $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$

Let y = vx

⇒ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

⇒ $\int \sec^2 v dv = -\int \frac{dx}{x}$

⇒ tan v = – λnx + C

at (1, π/4) ⇒ C = 1

⇒ $\tan\frac{y}{x} = -\lambda n\, x + \lambda ne$

⇒ $y = x\tan^{-1}\left(\lambda n \frac{e}{x}\right)$

Q.15 If x intercept of any tangent is 3 times the x-coordinate of the point of tangent, then the equation of the curve given that it passes through (1, 1) is-

(A) $y = \frac{1}{x}$ (B) $y = \frac{1}{x^2}$

(C) $y = \frac{1}{\sqrt{x}}$ (D) none of these

Sol.[C] Let point of tangent is (x, y)

point of x intercept is (3x, 0)

slope of tangent is

$$\frac{dy}{dx} = -\frac{y}{2x}$$

$$2\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$\Rightarrow$ $2\lambda ny = -\lambda nx + c$

Curve passes through (1, 1) $\Rightarrow$ c = 0

$\Rightarrow$ $y^2 = \frac{1}{x}$ $\Rightarrow$ $y = \frac{1}{\sqrt{x}}$

Q.16 A particle moves in a straight path such that its velocity is always 4 times its acceleration. If its velocity at time t = 0 is 2m/sec, what is its velocity at t = 2 sec?

(A) $\sqrt{2e}$ m/sec (B) $2\sqrt{e}$ m/sec

(C) $\frac{\sqrt{e}}{2}$ m/sec (D) $\frac{2}{\sqrt{e}}$ m/sec

Sol. **[B]**

$$v = 4\frac{dv}{dt}$$

$$\int_0^2 dt = 4\int_2^v \frac{dv}{v}$$

$$2 = 4\,\lambda n\,\frac{v}{2}$$

$$v = 2\,e^{1/2}$$

Q.17 A normal is drawn at a point P(x, y) of a curve. It meets the x-axis and y-axis in the points A and B respectively such that $\frac{1}{OA} + \frac{1}{OB} = 1$, where 'O' is the origin. The equation of such a curve passing through (5, 4) denotes -

(A) a line (B) a circle

(C) a parabola (D) pair of straight line

Sol.[B] Equation of normal at (x, y) is

$$(X - x) + (Y - y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{X}{x + y\frac{dy}{dx}} + \frac{Y}{\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}} = 1$$

$$\Rightarrow OA = x + y\frac{dy}{dx},\ OB = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Given that $\frac{1}{OA} + \frac{1}{OB} = 1$

$$\Rightarrow 1 + \frac{dy}{dx} = x + y\frac{dy}{dx}$$

$$\Rightarrow (y - 1)\frac{dy}{dx} = 1 - x$$

$$\Rightarrow (y - 1)\frac{dy}{dx} + (x - 1) = 0$$

$$\Rightarrow (y - 1)^2 + (x - 1)^2 = C$$

Curve passes through (5, 4) so C = 25 curve is $(y - 1)^2 + (x - 1)^2 = 25$

Which denotes equation of circle

Q.18 The equation of a curve for which the product of the abscissa of a point P and the intercept made by a normal at P on the x-axis equals twice the square of the radius vector of the point P is (curve passes through (1, 0))-

(A) $x^2 + y^2 = x^4$ (B) $x^2 + y^2 = 2x^4$

(C) $x^2 + y^2 = 4x^4$ (D) none of these

Sol.[A] Equation of normal at P(x, y) is

$$(X - x) + (Y - y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{X}{x + y\frac{dy}{dx}} + \frac{Y}{\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}} = 1$$

Θ radius vector of point P = $\sqrt{x^2 + y^2}$ from

Given condition

$$x\left(x + y\frac{dy}{dx}\right) = 2(x^2 + y^2)$$

$$\Rightarrow x^2 + xy\frac{dy}{dx} = 2x^2 + 2y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+2y^2}{xy}$$

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+2v^2}{v}$$

Solving we get $\int \frac{v}{1+v^2}dv = \int \frac{dx}{x}$

$\Rightarrow \lambda n(1 + v^2) = 2\lambda nx + C$

Curve passes through (1, 0) $\Rightarrow$ C = 0

Put $v = \frac{y}{x}$ we get $x^2 + y^2 = x^4$.

Q.19 The latus-rectum of the conic passing through the origin and having the property that normal at each point (x, y) intersects the x-axis at ((x + 1), 0) is -

(A) 1 (B) 2

(C) 4 (D) none

Sol.[B] Normal at (x, y) meets x-axis at ((x + 1), 0)

So, Slope of tangent $\frac{dy}{dx} = -\frac{y}{1}$

$\Rightarrow$ Slope of normal at (x,y)

$$\frac{dy}{dx} = \frac{1}{y}$$

$\Rightarrow \int ydy = \int xdx$

$\Rightarrow y^2 = 2x + C$

Curve passes through (0, 0)

$\Rightarrow$ C = 0

$\Rightarrow y^2 = 2x$

Latus rectum = 4a = 2

Q.20 The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at x = 0 as 3 is -

(A) $y = x^2 + 3x + 2$ (B) $y^2 = x^2 + 3x + 1$

(C) $y = x^3 + 3x + 1$ (D) none of these

Sol.[C] $y_2(x^2 + 1) = 2xy_1$

$$\Rightarrow \frac{y_2}{y_1} = \frac{2x}{x^2+1}$$

Integrate both side we get

$\lambda ny_1 = \lambda n(x^2 + 1) + C$

at x = 0, $y_1 = 3 \Rightarrow C = \lambda n 3$

$\Rightarrow \lambda ny_1 = \lambda n(x^2 + 1) + \lambda n 3$

$\Rightarrow y_1 = 3(x^2 + 1)$

$\Rightarrow y = x^3 + 3x + k$

It is passes through the point (0, 1)

$\Rightarrow$ k = 1

$\Rightarrow y = x^3 + 3x + 1$

Q.21 If $y = \frac{x}{\lambda n|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is -

(A) $\frac{x^2}{y^2}$ (B) $-\frac{x^2}{y^2}$ (C) $\frac{y^2}{x^2}$ (D) $-\frac{y^2}{x^2}$

Sol.[D] $y = \frac{x}{\lambda n|cx|}$ ....(i)

(i) is the solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right) \quad ....(ii)$$

From (i) $\frac{dy}{dx} = \frac{\lambda n|cx|-1}{(\lambda n|cx|)^2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{x}{y}-1}{\left(\frac{x}{y}\right)^2} = \frac{y}{x} - \left(\frac{y}{x}\right)^2 \quad ....(iii)$$

From (ii) and (iii) we get

$$\phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$$

Q.22 The solution of $y^5 x + y - x\frac{dy}{dx} = 0$ is

(A) $x^4/4 + 1/5\,(x/y)^5 = C$

(B) $x^5/5 + (1/4)\,(x/y)^4 = C$

(C) $(x/y)^5 + x^4/4 = C$

(D) $(xy)^4 + x^5/5 = C$

Sol. **[B]**

$$x\,dx + \frac{y\,dx - x\,dy}{y^3 . y^2} = 0$$

$$\int x^4 dx + \int \frac{x^3}{y^3} d\left(\frac{x}{y}\right) = 0$$

$$\frac{x^5}{5} + \frac{1}{4}\left(\frac{x}{y}\right)^4 = c$$

Q.23 The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is

(A) y = x cot (c – x)

(B) $\cos^{-1} y/x = -x + c$

(C) y = x tan (c – x)

(D) $y^2/x^2 = x \tan(c - x)$

Sol. **[C]**

$$\int \frac{x\,dy - y\,dx}{x^2\left(1+\frac{y^2}{x^2}\right)} = \int -dx$$

$$\int \frac{d(y/x)}{1+(y/x)^2} = -x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = -x + c$$

Q.24 Let $A = \begin{bmatrix} x^2 & 0 \\ 0 & \left(\frac{dy}{dx}\right)^{1/3} \end{bmatrix}$ and

$B = \begin{bmatrix} y^{2/3} & 0 \\ 0 & x^{1/3} \end{bmatrix}$. Equation

$tr(AB) = \frac{dy}{dx}$ is a differential equation of order 'm' and degree 'n' then (m + n) is equal to

(A) 2 (B) 3

(C) 4 (D) 5

Sol. **[C]**

Part-B One or more than one correct answer type Questions

Q.25 The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal -

(A) is linear

(B) is homogeneous of first degree

(C) has separable variables

(D) is second order

Sol. **[A, B]**

y = yy'

$$\frac{dy}{dx} = 1$$

Q.26 The differential equation of all circles in a plane must be $\left(y_1 = \frac{dy}{dx}, y_2 = \frac{d^2y}{dx^2} \text{ etc.}\right)$

(A) $y_3(1 + y_1^2) - 3y_1 y_2^2 = 0$

(B) of order 3 and degree 1

(C) of order 3 and degree 2

(D) $y_3^2(1 - y_1^2) - 3y_1 y_1^2 = 0$

Sol.[A,B] Equation of all circles in a plane is

$x^2 + y^2 + 2gx + 2fy + C = 0$

diff. w. r. to x

$2x + 2y\,y_1 + 2y + 2fy_1 = 0$

Again diff. we have

$1 + yy_2 + y_1^2 + fy_2 = 0$ ....(1)

Again diff. we get

$yy_3 + y_1y_2 + 2y_1y_2 + fy_3 = 0$ ....(2)

from (1) & (2) we get

$y_3(1 + y_1^2) - 3y_1y_2^2 = 0$

Which is of order 3 and degree 1

option A, B are correct.

Q.27 The differential equation

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \sin y + x^2 = 0$ is the following type

(A) linear (B) homogeneous

(C) order two (D) degree one

Sol.[C,D]

Clearly the equation is not linear and not homogeneous.

But equation is of order two and degree one

⇒ option C, D are correct.

Q.28 The solution of $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ is-

(A) $y - \frac{c}{1+\cos x} = 0$ (B) $y = \frac{c}{1-\cos x}$

(C) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$ (D) $x = 2\cos^{-1}\sqrt{\frac{c}{2y}}$

Sol.[A,B,C,D]

Solving for $\frac{dy}{dx}$, we obtain

$$\frac{dy}{dx} = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$$

$$\frac{dy}{dx} = y(-\cot x \pm \text{cosec } x)$$

Thus we have

$$\int \frac{dy}{y} = \int (-\cot x + \text{cosec}\, x)\,dx$$

$\lambda ny = -\lambda n \sin x + \lambda n \tan \frac{x}{2} + \lambda nC$

$y = \frac{C \tan \frac{x}{2}}{\sin x} = \frac{C}{2\cos^2 \frac{x}{2}} = \frac{C}{1+\cos x}$

and $\cos^2 \frac{x}{2} = \frac{C}{2y} \Rightarrow x = 2\cos^{-1}\sqrt{\frac{c}{2y}}$

Again solving $\int \frac{dy}{y} = -\int (\cot x + \operatorname{cosec} x)\,dx$

We get $y = \frac{C}{2\sin^2 \frac{x}{2}} = \frac{C}{\cos x - 1}$

or $\sin^2 \frac{x}{2} = \frac{C}{2y}$

$x = 2\sin^{-1}\sqrt{\frac{C}{2y}}$

A, B, C, D all are correct.

Q.29 The solution of $v = u\frac{dv}{du} + \left(\frac{dv}{du}\right)^2$, where $u = y$ and $v = xy$ are-

(A) $y = 0$ (B) $y = -4x$

(C) $xy = cy + c^2$ (D) $x^2y = cy + c^2$

Sol.[A,C] $v = u\frac{dv}{du} + \left(\frac{dv}{du}\right)^2$

Put $\frac{dv}{du} = P$

$v = uP + P^2$ ....(1)

Solution $v = Cu + C^2$

$\because$ $v = xy, u = y$

$\Rightarrow$ $xy = Cy + C^2$

From (1)

$\frac{dv}{du} = P + u\frac{dP}{du} + 2P\frac{dP}{du}$

$\Rightarrow$ $(u + 2P)\frac{dP}{du} = 0$ $\because$ $\frac{dv}{du} = P$

$\Rightarrow$ $\frac{dP}{du} = 0$ or $u + 2P = 0$ ....(2)

$\because$ $P = -\frac{u}{2}$ ....(3)

From (1) & (3) we get

$v = -\frac{u^2}{2} + \frac{u^2}{2} = 0$

$xy = 0$

$\Rightarrow$ $x = 0$ or $y = 0$

$\Rightarrow$ option A, C are correct.

Q.30 The solution of $\frac{dy}{dx} + x = xe^{(n-1)y}$ is-

(A) $\frac{1}{n-1}\log\left(\frac{e^{(n-1)y}-1}{e^{(n-1)y}}\right) = \frac{x^2}{2} + C$

(B) $e^{(n-1)y} = Ce^{(n-1)y+(n-1)x^2/2} + 1$

(C) $\log\left(\frac{e^{(n-1)y}-1}{(n-1)e^{(n-1)y}}\right) = x^2 + C$

(D) $e^{(n-1)y} = Ce^{(n-1)x^2/2+x} + 1$

Sol.[A,B] $\frac{dy}{dx} = x\,(e^{(n-1)y} - 1)$

Let $(n-1)y = t$

$\Rightarrow$ $\frac{dy}{dx} = \frac{1}{n-1}\frac{dt}{dx}$

$\Rightarrow$ $\frac{1}{n-1}\frac{dt}{dx} = x\,(e^t - 1)$

$\Rightarrow$ $\frac{1}{n-1}\int \frac{e^{-t}}{1-e^{-t}}dt = \int x\,dx$

$\Rightarrow$ $\frac{1}{n-1}\lambda n\,(1 - e^{-t}) = \frac{x^2}{2} + C$

$\Rightarrow$ $\frac{1}{n-1}\lambda n\left(\frac{e^{(n-1)y}-1}{e^{(n-1)y}}\right) = \frac{x^2}{2} + C$

This equation we can write

$\lambda n\,\frac{e^{(n-1)y}-1}{e^{(n-1)y}} = (n-1)\frac{x^2}{2} + \lambda nC$

$\Rightarrow$ $\frac{e^{(n-1)y}-1}{Ce^{(n-1)y}} = e^{(n-1)\frac{x^2}{2}}$

$\Rightarrow$ $e^{(n-1)y} = Ce^{(n-1)y+(n-1)\frac{x^2}{2}} + 1$

$\Rightarrow$ Option A, B are correct.

Q.31 The differential equation $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$ which is not with separated variables, can be transformed into one which is with separated variable; by the substitution -

(A) $2x + 3y = v$ (B) $4x + 6y + 5 = v$

(C) $2x + 3y + 4 = v$ (D) $3x + 2y = v$

Sol. **[A, B, C]**

Q.32 The function f(x) satisfying the equation,

$f^2(x) + 4f'(x).\, f(x) + [f'(x)]^2 = 0$, is-

(A) $f(x) = c.\ e^{(2-\sqrt{3})x}$

(B) $f(x) = c.\ e^{(2+\sqrt{3})x}$

(C) $f(x) = c.\ e^{(\sqrt{3}-2)x}$

(D) $f(x) = c.\ e^{-(2+\sqrt{3})x}$

Sol. **[CD]**

Q.33 The graph of the function y = f(x) passing through the point (0, 1) and satisfying the differential equation $\frac{dy}{dx} + y\cos x = \cos x$, is such that

(A) it is a constant function

(B) it is periodic

(C) it is neither an even nor an odd function

(D) it is continuous and differentiable for all x

Sol. **[A, B, D]**

$\text{I.F.} = e^{\int \cos x\, dx} = e^{\sin x}$

Its solution

$y \,.\, e^{\sin x} = \int \cos x\, e^{\sin x} dx = e^{\sin x} + c$

$c = 0 \Rightarrow y = 1$

Q.34 Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality k > 0 depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and k = 1/15 then the time to drain the tank if the water is 4 meter deep to start with is-

(A) 30 min (B) 45 min

(C) 60 min (D) 80 min

Sol. **[C]**

$-\frac{dy}{dt} = k\sqrt{y}$

$-\int_H^0 \frac{dy}{\sqrt{y}} = k\int_0^t dt$

$2\sqrt{H} = kt$

$t = \frac{2\sqrt{H}}{k} = \frac{2\times 2\times 15}{1} = 60$ min

Q.35 The solution of the differential equation,

$x^2 \frac{dy}{dx}.\cos\frac{1}{x} - y\sin\frac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$, is-

(A) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$ (B) $y = \frac{x+1}{x\sin\frac{1}{x}}$

(C) $y = \cos\frac{1}{x} + \sin\frac{1}{x}$ (D) $y = \frac{x+1}{x\cos\frac{1}{x}}$

Sol. **[A]**

$dy\left(\cos\frac{1}{x}\right) - \int \sin\frac{1}{x}\frac{1}{x^2}dx = -\frac{1}{x^2}dx$

$\int d\left(\cos\frac{1}{x}.y\right) = \int -\frac{1}{x^2}dx$

Q.36 If $\int_a^x t\,y(t)\,dt = x^2 + y(x)$ then y as a function of x is

(A) $y = 2 - (2 + a^2)\ e^{\frac{x^2-a^2}{2}}$

(B) $y = 1 - (2 + a^2)\ e^{\frac{x^2-a^2}{2}}$

(C) $y = 2 - (1 + a^2)\ e^{\frac{x^2-a^2}{2}}$

(D) None of these

Sol. **[A]**

Diff. w.r.t. to x

x y = 2x + y'

$\Rightarrow x(y-2) = \frac{dy}{dx}$

$\Rightarrow \frac{x^2}{2} = \lambda n\,|\,y-2\,| + c$

at x = a, y = $-a^2$

$x^2 = \lambda n (y-2)^2 + 2c$

$$c = \frac{a^2 - \lambda n(a^2+2)^2}{2}$$

Q.37 A function f(x) satisfying $\int_0^1 f(tx)\,dt = nf(x)$, where x > 0, is

(A) $f(x) = c.\ x^{\frac{1-n}{n}}$ (B) $f(x) = c.\ x^{\frac{n}{n-1}}$

(C) $f(x) = c.\ x^{\frac{1}{n}}$ (D) $f(x) = c.\ x^{(1-n)}$

Sol. **[A]**

$$\int_0^1 f(tx)\,dt = nf(x)$$

let tx = u

$$\int_0^x f(u)\frac{du}{x} = nf(x)$$

$$\int_0^x f(u)\,du = x \,.\, f(x)\, n$$

Diff. wrt x

f(x) = n . xf ' (x) + nf(x)

$$y(1-n) = n\,x\,\frac{dy}{dx}$$

$$\int \frac{dx}{x} = \frac{n}{1-n}\int \frac{dy}{y}$$

$$\lambda n\, c\, x = \frac{n}{1-n}\ \lambda n\, y$$

$$c\,x = y^{\frac{n}{1-n}}$$

Part-C Assertion-Reason type Questions

The following questions consist of two statements each, printed as Statement-1 and Statement-2. While answering these questions you are to choose any one of the following four responses.

(A) If both Statement-1 and Statement-2 are true and the Statement-2 is correct explanation of the Statement-1.

(B) If both Statement-1 and Statement-2 are true but Statement-2 is not correct explanation of the Statement-1.

(C) If Statement-1 is true but the Statement-2 is false.

(D) If Statement-1 is false but Statement-2 is true.

Q.38 **Statement-1**: The area of the ellipse $2x^2 + 3y^2 = 6$ will be more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

Statement-2: The length of the semi-major axis of ellipse $2x^2 + 3y^2 = 6$ is more than the radius of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$.

Sol. **[B]**

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

$A = \pi\sqrt{6}$

$x^2 + y^2 - 2x + 4y + 4 = 0$

$A = \pi$

$A_{ellipse} > A_{circle}$

Q.39 **Statement-1**: The differential equation $y^3dy + (x + y^2)dx = 0$ becomes homogeneous if we put $y^2 = t$.

Statement-2: All differential equation of first order and first degree becomes homogeneous if we put y = tx.

Sol. **[C]**

$$\frac{dy}{dx} = \frac{-(x+y^2)}{y^3}$$

$y^2 = t$

$$2y\,.\,\frac{dy}{dx} = \frac{dt}{dx}$$

$$2y\,.\,\frac{(-x+t)}{t\,y} = \frac{dt}{dx}$$

homogeneous.

Q.40 **Statement-1 :** The orthogonal trajectory to the curve $(x-a)^2 + (y-b)^2 = r^2$ is y = mx + b – am where a and b are fixed numbers and r & m are parameters.

Statement-2 : Any line that passes through the centre of circle is normal to the circle.

Sol. **[D]**

$2(x-a) + 2(y-b) \cdot y' = 0$

$$y' = -\frac{(x-a)}{y-b}$$

$y = mx + b - am$

$y' = m$

$y = y'x + b - ay'$

$$\frac{y-b}{x-a} = y'$$

Q.41 **Statement-1 :** $\sin x \dfrac{d^2y}{dx^2} + \cos x \dfrac{dy}{dx} + \tan x = 0$ is not a linear differential equation.

Statement-2 : A differential equation is said to be linear if dependent variable and its differential coefficients occurs in first degree and are not multiplied together.

Sol. **[A]**

Q.42 **Statement-1 :** The equation of the curve passing through (3, 9) which satisfies differential equation $\dfrac{dy}{dx} = x + \dfrac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$.

Statement-2 : The solution of differential equation $\left(\dfrac{dy}{dx}\right)^2 - \dfrac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ is $y = c_1e^x + c_2e^{-x}$

Sol. **[B]**

$$\int dy = \int\left(x + \frac{1}{x^2}\right)dx$$

$$y = \frac{x^2}{2} - \frac{1}{x} + c$$

$$9 = \frac{9}{2} - \frac{1}{3} + c$$

$$9 - \frac{9}{2} + \frac{1}{3} = c$$

$$\frac{29}{6} = c$$

$$y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$$

$6xy = 3x^2 + 29x - 6$

Statement 2

$(y')^2 - e^x y' - e^{-x} y' + 1 = 0$

$y'(y' - e^x) - e^{-x}(y' - e^x) = 0$

$y' = e^{-x}$ or $y' = e^x$

Part-D Column Matching type Questions

Q. 43

	Column I	**Column II**
(A)	A curve passing through (2, 3) having the property that length of the radius vector of any of its point P is equal to the length of the tangent drawn at this point, can be	(P) straight line
(B)	A curve passing through (1, 1) having the property that any tangent intersects the y-axis at the point which is equidistant from the point of tangency and the origin, can be	(Q) circle
(C)	A curve passing through (1, 0) for which the length of normal is equal to the radius vector, can be	(R) parabola
(D)	A curve passes through the point (2, 1) and having the property that the segment of any of its tangent between the point of tangency and the x- axis is bisected by the y- axis, can be	(S) hyperbola

Sol. **A → P, S; B → Q; C → Q, S; D → R**

(A) $\dfrac{y}{y'}\sqrt{1+(y')^2} = \sqrt{x^2+y^2}$

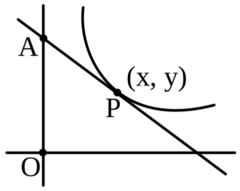
$$\frac{y^2}{(y')^2}(1 + (y')^2) = x^2 + y^2$$

$y^3 + y^2 (y')^2 = x^2 (y')^2 + y^2 (y')^2$

$$(y')^2 = \frac{y^2}{x^2}$$

$y' = \dfrac{y}{x}$	$y' = -\dfrac{y}{x}$
$\lambda n\, y = \lambda n\, cx$	$\lambda n \dfrac{1}{y} = \lambda n\, cx$
$y = cx$	$xy = c$

(B) OA = PA



$Y - y = y'(X - x)$

$A \equiv (0, y - xy')$

$PA = \sqrt{x^2 + x^2(y')^2}$

$y - xy' = x\sqrt{1 + (y')^2}$

$y^2 + x^2(y')^2 - 2xy\,y' = x^2 + x^2(y')^2$

$\dfrac{y^2 - x^2}{xy} = 2xy\,y'$

$y^2dx - x^2dx = 2xy\,dy$

$y^2\,dx - 2xy\,dy = x^2\,dx$

$-\dfrac{(2xy\,dy - y^2dx)}{x^2} = x^2dx$

$\int d(y^2/x) = -\int dx$

$\Rightarrow \dfrac{y^2}{x} = -x + c$

$\Rightarrow x^2 + y^2 - cx = 0$

c = 2

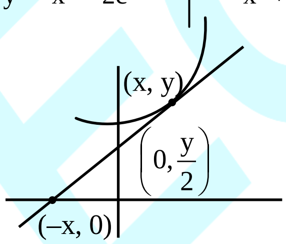
circle

(C) $y\sqrt{1 + (y')^2} = \sqrt{x^2 + y^2}$

$y^2 + y^2(y')^2 = x^2 + y^2$

$y' = \dfrac{x}{y}$	$y' = -\dfrac{x}{y}$
$y^2 - x^2 = 2c$	$x^2 + y^2 = 2c$

(D)



$y' = \dfrac{y}{2x}$

$\dfrac{dy}{y} = \dfrac{dx}{2x}$

$\lambda n\, y^2 = \lambda n\, cx$

$y^2 = cx$

Q.44 Match the following

Column I	**Column II**
(A) Solution of $y - \dfrac{xdy}{dx} = y^2 + \dfrac{dy}{dx}$ is	(P) $xy^2 = 2y^5 + c$
(B) Solution of $(2x - 10y^3)\dfrac{dy}{dx} + y = 0$ is	(Q) $\sec y = x + 1 + ce^x$
(C) Solution of $\sec^2 y\,dy + \tan y\,dx = dx$ is	(R) $(x + 1)(1 - y) = cy$
(D) Solution of $\sin y\dfrac{dy}{dx} = \cos y(1 - x\cos y)$ is	(S) $\tan y = 1 + ce^{-x}$

Sol. **A → R; B → P; C → S; D → Q**

(A) $y - y^2 = (x + 1)\dfrac{dy}{dx}$

$\int \dfrac{dx}{x+1} = -\int\left(-\dfrac{1}{y} + \dfrac{1}{y-1}\right)dy$

$= \int\left(\dfrac{1}{y} - \dfrac{1}{y-1}\right)dy$

$\lambda n(x + 1)c = \lambda n\left(\dfrac{y}{y-1}\right)$

$\dfrac{y}{y-1} = (x + 1)\,c$

$(x + 1)(y - 1) = ky$

(B) $2(x - 5y^3)\dfrac{dy}{dx} + \dfrac{y}{2} = 0$

$\dfrac{dx}{dy} + \dfrac{2x}{y} = 10\dfrac{y^3}{y}$

$I.F. = e^{\int \frac{2}{y}dy} = y^2$

Its solution

$x\,y^2 = 10\int y^4dy = 10 \times \dfrac{y^5}{5} + c$

$xy^2 = 2y^5 + c$

(C) $\sec^2 y\,.\,\dfrac{dy}{dx} + \tan y = 1$

$\tan y = v$

$\frac{dv}{dx} + v = 1$

I.F. = e^x

$v\,e^x = e^x + c$

$v = 1 + c\,e^{-x}$

$\tan y = 1 + c\,e^{-x}$

(D) $\tan y \,.\, \sec y \frac{dy}{dx} = \sec y - x$

$\sec y = v$

$\frac{dv}{dx} - v = -x$

I.F. = e^{-x}

Its solution

$v\,e^{-x} = -\int xe^{-x}dx$

$v\,e^{-x} = xe^{-x} + e^{-z} + c$

$v = x + 1 + c\,e^{x}$

$\sec y = x + 1 + ce^{x}$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Solve : $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

Sol. $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

$$\text{I.F.} = e^{\int \frac{1}{(1-x)\sqrt{x}}dx} = e^{\int \frac{1}{1-t^2}dt} \quad \Theta \quad \sqrt{x} = t$$

$$= e^{\lambda n \frac{1+t}{1-t}} = \frac{1+t}{1-t} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$y \cdot \frac{1+\sqrt{x}}{1-\sqrt{x}} = \int (1+\sqrt{x})\,dx$$

$$\Rightarrow \quad y\,\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{2}{3}x^{3/2} + C$$

Q.2 Find the solution of the differential equation satisfying the given initial conditions

$$y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0;\ y|_{x=0} = \sqrt{5}$$

Sol. $y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + x^2 - x^2 - y^2 = 0$

$$\left(y\frac{dy}{dx} + x\right)^2 = x^2 + y^2$$

$$y\frac{dy}{dx} + x = \pm\sqrt{x^2 + y^2}$$

$$\frac{dy}{dx} + \frac{x}{y} = \pm\sqrt{\left(\frac{x}{y}\right)^2 + t}$$

Q.3 Show that a particle x, y which moves so that, $\frac{dy}{dt} = ax + hy + g$, $\frac{dx}{dt} = -(hx + by + f)$, will always lie upon a conic section.

Sol. $\frac{dy}{dx} = \frac{ax + hy + g}{-(hx + by + f)}$

$$\frac{dy}{dx} = \frac{-(ax + hy)}{(hx + by)}$$

Y = Xv

$$\frac{dY}{dX} = X\frac{dv}{dX} + v$$

$$\Rightarrow \frac{-a - hv}{h + bv} - v = X\frac{dv}{dX}$$

$$\Rightarrow \frac{-a - 2hv - bv^2}{h + bv} = X\frac{dv}{dX}$$

$$\Rightarrow \int \frac{dX}{X} = \int \frac{-(h + bv)}{bv^2 + 2hv + a}\,dv$$

Q.4 Solve : (x + tan y) dy = sin 2y dx

Sol. $x\sqrt{\cot y} = c + \sqrt{\tan y}$

Q.5 Find the orthogonal trajectories of the family of semi-cubical parabolas $ay^2 = x^3$, where a is a variable parameter.

Sol. $ay^2 = x^3$ ....(1)

differentiate with respect to x we have

$$2ay\frac{dy}{dx} = 3x^2$$

replace $\frac{dy}{dx}$ by $-\frac{1}{\frac{dy}{dx}}$ we get

$$2ay = -3\,\frac{x^2dy}{dx}$$

$$\Rightarrow \quad a = -\frac{3}{2}\,\frac{x^2}{y}\,\frac{dy}{dx}$$

Put the value of a in equation (1) we get

$$-\frac{3}{2}\,\frac{x^2}{y}\,.\,y^2\,\frac{dy}{dx} = x^3$$

$\Rightarrow$ 3y dy = – 2x dx

$\Rightarrow$ $3y^2 + 2x^2 = C$

Q.6 The tangent at any point (x, y) of a curve makes an angle $\tan^{-1}$ (2x + 3y) with x axis. Find the equation of the curve if it passes through (1, 2)

Sol. $\theta = \tan^{-1}(2x + 3y)$

$\Rightarrow$ $\tan\theta = 2x + 3y$

Θ $\tan\theta = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 2x + 3y$

$\Rightarrow \frac{dy}{dx} - 3y = 2x$

I.F. $= e^{-\int 3dx} = e^{-3x}$

$ye^{-3x} = \int 2xe^{-3x}dx$

$\Rightarrow ye^{-3x} = -\frac{2}{3}xe^{-3x} + \int \frac{2}{3}e^{-3x}dx$

$\Rightarrow ye^{-3x} = -\frac{2}{3}xe^{-3x} - \frac{2}{9}e^{-3x} + C$

$\Rightarrow y = -\frac{2}{3}x - \frac{2}{9} + ce^{3x}$

This curve passes through (1, 2)

$\Rightarrow c = \frac{26}{9}e^{-3}$

$\Rightarrow y = \frac{1}{9}[26\,e^{3x-3} - 6x - 2]$

Q.7 A particle P moves so that its velocities parallel to the axes of x and y are respectively –ky and kx where k is a constant different from zero. Find the path of the particle, if it passes through the point (3, 4).

Sol. $x^2 + y^2 = 25$

Q.8 Show that the curve for which the normal at every points passes through a fixed point is a circle.

Sol. Equation of normal at (x, y) is

$(Y - y) = -\frac{dx}{dy}(X - x)$

Let it is passes through a fixed point (h, k)

$\therefore (k - y) = -\frac{dx}{dy}(h - x)$

$\Rightarrow (x - h)\,dx = -(y - k)\,dy$

On integration

$\Rightarrow \frac{(x-h)^2}{2} = -\frac{(y-k)^2}{2} + C$

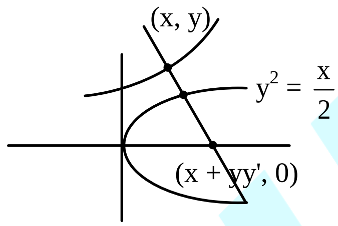
$\Rightarrow (x - h)^2 + (y - k)^2 = r^2$

Put $2C = r^2$

Which is a equation of circle.

Q.9 Find the equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$.

Sol.



$Y - y = -\frac{1}{Y}(X - x)$

$\left(\frac{2x + yy'}{2}, \frac{y}{2}\right)$ lies on $y^2 = \frac{x}{2}$

$\frac{y^2}{4} = \frac{2x + yy'}{4}$

$y \cdot \frac{dy}{dx} - y^2 + 2x = 0$

$y^2 = t$

$2y \cdot y' = \frac{dt}{dx}$

$\frac{dt}{dx} - 2t = -4x$

I.F. $= e^{-2x}$

$t \cdot e^{-2x} = -4\int x\,e^{-2x}dx$

$= -4\left[\frac{x\,e^{-2x}}{-2} - \frac{e^{-2x}}{4}\right] + c$

$t = 2x + 1 + e^{2x} \cdot c$

$y^2 = 2x + 1 + c \cdot e^{2x}$

$x = 0, y = 0$

$\Rightarrow c = -1$

$y^2 = 2x + 1 - e^{2x}$

Q.10 Find the equation of the curve for which the Cartesian sub-tangent varies as the reciprocal of the square of the abscissa.

Sol. Given that

$\frac{y}{\frac{dy}{dx}} = \frac{k}{x^2} \quad \Rightarrow \frac{x^2}{k}dx = \frac{dy}{y}$

$\Rightarrow \frac{x^3}{3k} = \lambda ny - \lambda nC \quad \Rightarrow \frac{y}{c} = e^{x^3/3k}$

$\Rightarrow\ y = c\ e^{x^3/3k}$

Q.11 Solve : $x\dfrac{dy}{dx} - y = x\sqrt{x^2+y^2}$

Sol. $\dfrac{dy}{dx} = \dfrac{\sqrt{x^2+y^2}+y}{x}$

Let $y = vx \Rightarrow \dfrac{dy}{dx} = v + \dfrac{xdv}{dx}$

$\Rightarrow\ v + \dfrac{xdv}{dx} = \sqrt{1+v^2} + v$

$\Rightarrow\ \dfrac{dv}{\sqrt{1+v^2}} = \dfrac{dx}{x}$

On integrating we get

$\Rightarrow\ \lambda n(v + \sqrt{1+v^2}) = \lambda nx + \lambda nC$

$\Rightarrow\ v + \sqrt{1+v^2} = cx$

$\Rightarrow y + \sqrt{x^2+y^2} = cx^2$

Q.12 A curve passes through the origin. Through any point (x, y) on the curve, lines are drawn parallel to the coordinate axes. If curve divides the area formed by these lines and coordinate axes in the ratio 2 : 1, show that the curve is a parabola.

Sol. $\int_0^x f(x)\,dx = \dfrac{xy}{3}$

diff w.r.t. x

$y\ f(x) = \dfrac{x}{3}\ y' + \dfrac{y}{3}$

$\dfrac{2y}{3} = \dfrac{x}{3}\ y'$

$2\int\dfrac{dx}{x} = \int\dfrac{dy}{y} \Rightarrow \lambda n\ x^2 = \lambda n\ yc$

$x^2 = cy$

parabola.

Q.13 The rate of disintegration of radium is proportional to the quantity of radium present. Radium disintegrate by one half in 1600yrs. The amount of radium that has disintegrated after 100 yrs is.....?

Sol. $\left[1-\left(\dfrac{1}{2}\right)^{1/16}\right]$ of initial quantity

Q.14 A wet porous substance in the open air loses its moisture at a rate proportional to the moisture containt. If a sheet hung in wind loses half its moisture during the first hour when will it have lost 99% of initial moisture containt. Assume that the weather conditions remain the same.

Sol. $\dfrac{\ln 100}{\ln 2}$ hrs

Q.15 A particle of mass m is projected vertically upwards with an initial velocity v_0 in the uniform gravitational field of earth. The force of resistance is k times the velocity, per unit mass. Find the duration of time when particle is moving vertically upwards.

Sol. $\dfrac{1}{k}\lambda n\left(\dfrac{g+kV_0}{g}\right)$

Q.16 Solve the following differential equation ;
(let p = dy/dx)
(i) $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$
(ii) $y = px + p - p^2$
(iii) $(y - px)(p - 1) = p$
(iv) $p^2(x^2 - a^2) - 2pxy + y^2 - b^2 = 0$

Q.17 Determine the function y = f(x) satisfying the equation $\dfrac{dy}{dx} = f(x) + \int_0^1 f(x)\,dx$ and f(0) = 1

Q.18 Determine the function y = f(x) satisfying the equation $x\int_0^x y(t)\,dt = (x + 1)\int_0^x t\,y(t)\,dt$; x > 0

Q.19 A cyclist moving on a level road at 4m/sec stops pedalling and lets the wheels comes to rest. The retardation of the cycle has two components; a constant 0.08 m/sec^2 due to friction and other is 0.02 v^2/m where v is speed in metres per second. What distance is traversed by the cycle before it comes to rest ? (consider λn5 = 1.61)

Sol. 161/4m

Q.20 A body at a temperature of 50F is placed outdoors where the temperature is 100 F. If the rate of change of the temperature of body is proportional to the temperature difference between the body and its surroundings. If after 5 min the temperature of body is 60 F, Find -

(A) How long will it take to reach a temp of 75 F
(B) the temp. of body after 20 min.

Sol. (A) 5 $\log_{5/4}2$ min. (B) 79.52 F

Q.21 A tank initially contain 40 gallons of fresh water. Brine contains 2 pounds per gallons of salt flows into the tank at the rate of 2 gallons per minutes and the mixture kept uniform by stirring, runs out at the same rate. It takes $k\lambda n\left(\frac{5}{2}\right)$ min. for the quantity of salt in the tank to increase from 30 to 60 pounds, where k is ......

Sol. 0020

Part-B Passage based Question

Passage I (Q. 22 to 24)

Let f(x) be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$, for ∀ x, y ∈ R, but y ≠ 0, f(y) ≠ 0. Slope of tangent to the curve y = f(x) at x = 1 is 2.

Q.22 The function f(x) is -

(A) $2x^2$ (B) x^2 (C) $\frac{x^4}{2}$ (D) $x^4 - 2$

Sol. **[B]**

$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

Put x = y = 1 we get f(1) = 1

y = f(x)

differential with respect to x we get

$\frac{dy}{dx} = f'(x)$

Θ at x = 1 ⇒ f ′(1) = 2

⇒ f ′(x) = 2x

integrating we get

$f(x) = x^2 + C$

Θ x = 1, f(x) = 1

⇒ C = 0

$f(x) = x^2$

⇒ $y = x^2$

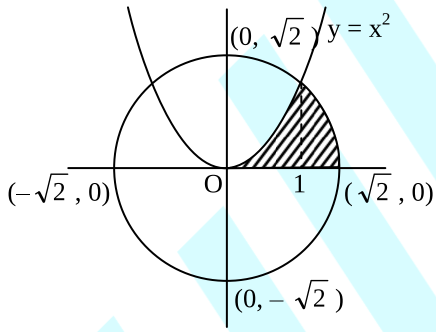
Q.23 Area bounded by the curve y = f(x), the x-axis, in the first quadrant and satisfying $x^2 + y^2 \le 2$ is

(A) $\frac{\pi}{2} - \frac{1}{3}$ (B) $\frac{\pi}{2} - \frac{1}{6}$

(C) $\frac{\pi}{4} - \frac{1}{6}$ (D) $\frac{\pi}{4} - \frac{1}{3}$

Sol. **[C]**

$y = x^2$, x-axis and satisfying $x^2 + y^2 \le 2$ is



Solving $x^2 + y^2 = 2$ and $x^2 = y$ we get

x = 1 So required area

$$= \int_0^1 x^2 dx + \int_1^{\sqrt{2}} \sqrt{2 - x^2}\, dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_1^{\sqrt{2}}$$

$$= \frac{1}{3} + \frac{\pi}{3} - \frac{1}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4} - \frac{1}{6}$$

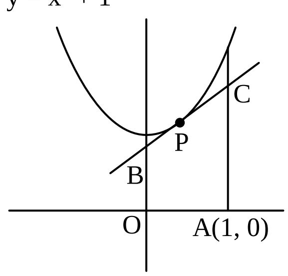
Q.24 A tangent to y = f(x) + 1, cuts off a trapezium of greatest area with x = 0, y = 0 and x = 1. The point of tangency is -

(A) $\left(\frac{1}{4}, \frac{5}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{5}{4}\right)$

(C) $\left(\frac{1}{4}, \frac{17}{4}\right)$ (D) $\left(\frac{1}{4}, \frac{5}{4}\right)$

Sol. **[B]**

$y = x^2 + 1$



Let Point of tangency is P (h, k)

Then equation of tangent

$= (y - k) = 2h (x - h)$

at $x = 0$ $y = k - 2h^2$

$\Rightarrow$ B $(0, k - 2h^2)$

and at $x = 1$ $y = k + 2h - 2h^2$

$\Rightarrow$ c $(1, k + 2h - 2h^2)$

Area $= \frac{1}{2}(2k + 2h - 4h^2) = k + h - 2h^2$

$= k - h (2h - 1)$

it is maximum when $2h - 1 = 0$

$\Rightarrow h = \frac{1}{2}$

and $k = h^2 + 1 \quad \Rightarrow \quad k = \frac{5}{4}$

$$P\left(\frac{1}{2}, \frac{5}{4}\right)$$

Passage II (Q. 25 to 27)

A drop of liquid evaporates at a rate proportional to its surface area, if the radius initially is 5mm and 5 minute later the radius is reduced to 2mm.

On the basis of above information, answer the following questions-

Q.25 The rate with which the surface area changes after 5 minutes is-

(A) 8 π sq. mm/min (B) 4.8 π sq. mm/min

(C) 9.6 π sq. mm/min (D) 16 π sq. mm/min

Sol,[C]

Q.26 The rate of change of radius is –

(A) linear function of time

(B) circular function of time

(C) parabolic function of time

(D) independent of time

Sol. [D]

Q.27 The rate of evaporation after, 5 minutes is –

(A) 8 π (B) 4.8 π (C) 9.6 π (D) 16 π

Sol. [C]

Passage III (Q. 28 to 31)

Newton Law of Cooling. The rate at which a body undergoes a change in temperature is proportional to the difference between its temperature and temperature of the surrounding medium. If y = f(t) is the temperature of the body at time t and if M(t) denotes the temperature of the surrounding medium, Newton's law leads to the differential equation

$y' = -k[y - M(t)]$ or $y' + ky = kM(t)$

Where k is a positive constant. This first-order linear equation is the mathematical model we use for cooling problems. The unique solution of the equation satisfying the initial condition f(a) = b is given by the formula

$$f(t) = be^{-kt} + e^{-kt}\int_a^t kM(z)e^{kz}\,dz$$

Q.28 A body cools from 200° to 100° in 40 minutes while immersed in a medium whose temperature is kept constant. Let M(t) = 10°. If we measure t in minutes and f(t) in degree then f(t) must be -

(A) $10 + 180\,e^{-kt}$ (B) $10 + 140e^{-kt}$

(C) $10 + 100e^{-kt}$ (D) $10 + 190\,e^{-kt}$

Sol. **[D]**

$$\frac{dy}{dt} = -k(y - 10)$$

$$\int_{200}^{T}\frac{dy}{y-10} = -k\int_0^t dt$$

$$\lambda n\left(\frac{T-10}{190}\right) = -kt$$

$$T = 10 + 190\,e^{-kt}$$

Q.29 The value of k must be -

(A) (log 19 – log9)/100

(B) (log 19 – log 9)/100

(C) $\frac{\log 19 - \log 9}{40}$

(D) none of these

Sol. **[C]**

t = 40

T = 100

$90 = 190\ e^{-k(40)}$

$\lambda n\frac{19}{9} = 40k$

$k = \frac{\lambda n19 - \lambda n9}{40}$

Q.30 Suppose in the same system a body cools from 400º to 200º with M(t) = 10º then time taken for cooling must be equal to -

(A) 40 log 19 (B) 40 log 9

(C) $40\frac{\log 19-\log 9}{\log 39-\log 19}$ (D) $40\frac{\log 39-\log 19}{\log 19-\log 9}$

Sol. **[D]**

$$\int_{400}^{200}\frac{dy}{y-10}=-k\int_0^t dt$$

$$\lambda n\left(\frac{190}{390}\right)=\frac{-(\lambda n19-\lambda n9)}{40}t$$

$$t = 40\left(\frac{\lambda n39-\lambda n19}{\lambda n19-\lambda n9}\right)$$

Q.31 Suppose the temperature drop from 100º to 10º takes t_1 minutes while from 200º to 100º t_2 minutes then t_1 is nearly equal to (Take M(t) = 5º)

(A) $4t_2$ (B) $8t_2$ (C) $2t_2$ (D) t_2

Sol. **[A]**

$$\int_{100}^{10}\frac{dy}{y-5}=-kt_1$$

$$\lambda n\left(\frac{5}{95}\right)=-kt_1$$

$$\int_{200}^{100}\frac{dy}{y-5}=-kt_2$$

$$\lambda n\left(\frac{95}{195}\right)=-kt_2$$

$$\frac{t_1}{t_2}=\frac{\lambda n19}{\lambda n39-\lambda n19}\approx 4$$

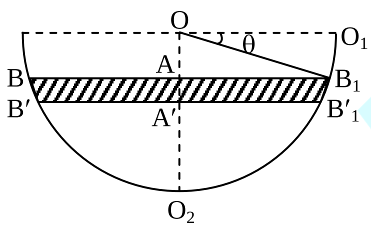
$t_1 = 4t_2$

EXERCISE # 4

Old IIT JEE Questions

Q.1 A hemispherical tank of radius 2 meters is initially full of water and has an outlet of 12 cm^2 cross-sectional area of the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = 0.6\sqrt{2gh(t)}$, where v(t) and h(t) are respectively the rate of flow at outlet and water level in tank at time t, and g is the acceleration due to gravity. Find the time takes to empty the tank. **[IIT-2001]**

Sol. Let 'O' be the centre of hemispherical tank. Let at any instant 't', water level be BAB_1 and at t + dt, water level is $B'A'B_1$. Let $\angle O_1OB_1 = \theta$.



$\Rightarrow$ $AB_1 = r\cos\theta$ and $OA = r\sin\theta$

decrease in the water volume in time dt,

$= \pi AB_1^2 \cdot d(OA)$

(πr^2 is surface area of water level and d(OA) is depth of water level).

$= \pi r^2 . \cos^2\theta . r\cos\theta\, d\theta$

$= \pi r^3 . \cos^3\theta d\theta$

Also $h(t) = O_2A = r - r\sin\theta = r(1-\sin\theta)$

Now outflow rate Q = A . v (t)

$= A.(0.6)\sqrt{2gr(1-\sin\theta)}$

Where A is the area of the outlet. Thus volume flowing out in time dt.

$\Rightarrow$ $Q\,dt = A.(0.6).\sqrt{2gr}\,.\sqrt{1-\sin\theta}\ dt$

We have, $\pi r^3\cos^3\theta\, d\theta = A(0.6).\sqrt{2gr}\,.\sqrt{1-\sin\theta}\ dt$

$$\Rightarrow \frac{\pi r^3}{A(0.6)\sqrt{2gr}}\ \frac{\cos^3\theta}{\sqrt{(1-\sin\theta)}}\ d\theta = dt$$

Let the time taken to empty the tank be T.

$$\text{Then } T = \int_0^{\pi/2}\frac{\pi r^3}{A(0.6).\sqrt{2gr}}\cdot\frac{\cos^3\theta}{\sqrt{1-\sin\theta}}\ d\theta$$

$$= \frac{-\pi r^3}{A(0.6)\sqrt{2gr}}\int_0^{\pi/2}\frac{\cos^2\theta(-\cos\theta)}{\sqrt{1-\sin\theta}}d\theta$$

Let $t_1 = \sqrt{1-\sin\theta}$

$$\Rightarrow dt_1 = \frac{-\cos\theta}{\sqrt{1-\sin\theta}}\ d\theta$$

$$\Rightarrow T = \frac{-2\pi r^3}{A(0.6)\sqrt{2gr}}\int_1^0(1-\sin^2\theta).dt_1$$

$$\Rightarrow T = \frac{-2\pi r^3}{A(0.6)\sqrt{2gr}}\int_1^0[1-(\sin\theta)^2]dt_1$$

$$\Rightarrow T = \frac{-2\pi r^3}{A(0.6)\sqrt{2gr}}\int_1^0[1-(1-t_1^2)^2]dt_1$$

$$\Rightarrow T = \frac{-2\pi r^3}{A(0.6)\sqrt{2gr}}\int_1^0[1-(1+t_1^4-2t_1^2)]dt_1$$

$$\Rightarrow T = \frac{2\pi r^3}{A(0.6)\sqrt{2gr}}\int_1^0(t_1^4-2t_1^2)dt_1$$

$$\Rightarrow T = \frac{2\pi r^3}{A(0.6)\sqrt{2gr}}\left[\frac{t_1^5}{5}-\frac{2t_1^3}{3}\right]_1^0$$

$$\Rightarrow T = \frac{2\pi r^{5/2}}{A\left(\frac{6}{10}\right)\sqrt{2gr}}.\left[0-\frac{1}{5}-0+\frac{2}{3}\right]$$

$$\Rightarrow T = \frac{2\pi.2^{5/2}(10^2)^{5/2}}{12.\frac{3}{5}.\sqrt{2}.\sqrt{g}}\left[\frac{2}{3}-\frac{1}{5}\right]$$

$$= \frac{2\pi.2^{5/2}(10^2)^{5/2}}{12.\frac{3}{5}.\sqrt{2}.\sqrt{g}}\left[\frac{2}{3}-\frac{1}{5}\right]$$

$$= \frac{2\pi\times10^5.4.5}{(12\times3)\sqrt{g}}\left[\frac{10-3}{15}\right]$$

$$= \frac{2\pi\times10^5\times7}{3.3.\sqrt{g}.3} = \frac{14\pi\times10^5}{27\sqrt{g}}\ \text{units.}$$

Q.2 Let P(x) be a polynomial such that

$\frac{d}{dx}$ P (x) > P (x) for x > 1, and P(1) = 0.

Prove that P(x) > 0 for x > 1

[IIT-2003]

Sol. Given P(1) = 0 and

$\frac{d}{dx}$ P(X) > P(x) $\forall$ x > 1

$\Rightarrow \frac{dP(x)}{dx} - P(x) > 0$

Multiplying by e^{-x} (e^{-x} > 0)

$\Rightarrow e^{-x} \frac{d}{dx} P(x) + P(x) \frac{d}{dx} e^{-x} > 0$

$\Rightarrow \frac{d}{dx} (P(x) e^{-x}) > 0$

$\Rightarrow$ P(x) e^{-x} is a increasing function

$\Rightarrow P(x) e^{-x} > P(1) e^{-1} \quad \forall x > 1$

$\Rightarrow P(x) > 0 \quad \forall \ x > 1$

Q.3 A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = K > 0). Find the time after which the cone is empty.

[IIT-2003]

Sol. Given : liquid evaporates at a rate proportional to its surface area

$\Rightarrow \frac{dv}{dt} \propto - S$ ....(1)

We know, volume of cone

$= \frac{1}{3} \pi r^2 h$ and surface area

$= \pi r^2$ (of liquid in contact with air)

or $V = \frac{1}{3} \pi r^2 h$ and $S = \pi r^2$ ....(2)

Where $\tan \theta = \frac{R}{H}$ and $\frac{r}{h} = \tan \theta$ ....(3)

from (2) and (3)

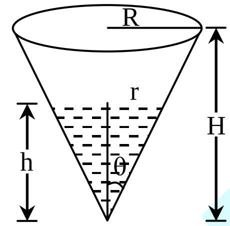
$V = \frac{1}{3} \pi r^3 \cot\theta$ and $S = \pi r^2$ ....(4)

Substituting (4) in (1), we get

$\frac{1}{3} \cot\theta \,.\, 3r^2 \frac{dr}{dt} = - k\pi r^2$

$\Rightarrow \cot \theta \int_R^0 dr = - k \int_0^T dt$

$\Rightarrow \cot\theta (0 - R) = - k (T - 0)$



$\Rightarrow R \cot\theta = kT$

$\Rightarrow H = kT$ (using (3))

$\Rightarrow T = \frac{H}{k}$

$\therefore$ required time after which the cone is empty

$= T = \frac{H}{k}$

Q.4 If y(t) is a solution of $(1+ t)\frac{dy}{dt} - ty = 1$ and y (0) = – 1, then y (1) is equal to- **[IIT-2003S]**

(A) –1/2 (B) e + 1/2

(C) e – 1/2 (D) 1/2

Sol. [A] $\frac{dy}{dt} - \frac{t}{1+t} y = \frac{1}{1+t}$

IF $= e^{-\int \frac{t}{1+t} dt} = e^{-\int \left(1 - \frac{1}{1+t}\right) dt}$

$= e^{-t + \ln(1+t)} = e^{-t} (1 + t)$

$= e^{-t} (1 + t) = \int e^{-t} dt$

$\Rightarrow e^{-t} y (1 + t) = - e^{-t} + C$

y (0) = – 1

$\Rightarrow$ y (1 + t) = – 1

$\Rightarrow y = - \frac{1}{1+t}$

$y(1) = - \frac{1}{2}$

Q.5 If y = y(x) and $\frac{2+\sin x}{y+1} \left(\frac{dy}{dx}\right) = -\cos x$, y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ equals - **[IIT-2004S]**

(A) 1/3 (B) 2/3 (C) – 1/3 (D) 1

Sol. [A] $\frac{dy}{dx} + \frac{\cos x}{2+\sin x} y = - \frac{\cos x}{2+\sin x}$

IF $= e^{\int \frac{\cos x}{2+\sin x} dx} = (2 + \sin x)$

$\Rightarrow y (2 + \sin x) = - \int \cos x \, dx$

$\Rightarrow$ y (2 + sin x) = – sinx + C

$\because$ y(0) = 1 $\Rightarrow$ c = 2

$\Rightarrow$ y (2 + sin x) = 2 – sin x

$\Rightarrow$ $y = \dfrac{2 - \sin x}{2 + \sin x}$

at $y(\pi/2) = \dfrac{1}{3}$

Q.6 Find the curve passing through (2, 0) and having slope of tangent at any point P(x, y) as $\dfrac{(x+1)^2 + y - 3}{x+1}$. Find equation of the curve, find also the area enclosed by the curve and x-axis in the IV quadrant. **[IIT-2004]**

Sol. Hence, slope of tangent

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(x+1)^2 + y - 3}{(x+1)}$

$\Rightarrow \dfrac{dy}{dx} = (x + 1) + \dfrac{(y-3)}{(x+1)}$; put x + 1 = X and y – 3 = Y

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dY}{dX}$, $\therefore \dfrac{dY}{dX} = X + \dfrac{Y}{X}$

or $\dfrac{dY}{dX} - \dfrac{1}{X} Y = X$

where integration factor = $\dfrac{1}{X} = e^{-\log X} = \dfrac{1}{X}$

$\therefore$ Solution is, $Y = \dfrac{1}{X} = \int X \cdot \dfrac{1}{X} dX + c$

or $\dfrac{Y}{X} = X + c$

or $y - 3 = (x + 1)^2 + c\,(x + 1)$, which passes through (2, 0)

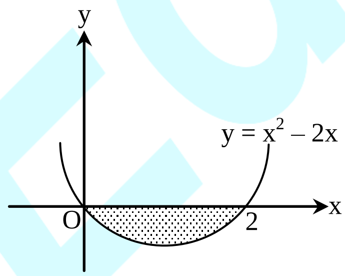
$-3 = 1 + c$

$\Rightarrow$ $c = -4$

$\therefore$ required curve is,

$y = (x + 1)^2 - 4(x + 1) + 3$ or $y = x^2 - 2x$

As shown in the figure



Thus, required area

$$= \left| \int_0^2 (x^2 - 2x)\,dx \right| = \left| \left(\frac{x^3}{3} - x^2 \right)_0^2 \right|$$

$= \dfrac{4}{3}$ square units

Q.7 $(x^2 + y^2)dy = xydx$ (initial value problem), $y > 0$, $x > 0$, $y(1) = 1$, $y(x_0) = e$ then find x_0 =? **[IIT-2005]**

(A) $\sqrt{\dfrac{e^2 - 1}{2}}$ (B) $\sqrt{2e^2 - 1}$

(C) $\sqrt{e^2 - 2}$ (D) $\sqrt{3}\,e$

Sol. [D] $\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$

Let $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Put the value and solving we get

$x\dfrac{dv}{dx} = -\dfrac{v^3}{1 + v^2}$

$\Rightarrow -\int \dfrac{1+v^2}{v^3} dv = \int \dfrac{dx}{x}$

$\Rightarrow \dfrac{1}{2v^2} - \lambda nv = \lambda nx + C$

$\Rightarrow \dfrac{x^2}{2y^2} - \lambda ny = C$

$\because$ $y(1) = 1 \Rightarrow c = \dfrac{1}{2}$

$\Rightarrow \dfrac{x^2}{2y^2} - \lambda ny = \dfrac{1}{2}$

$\because$ $y(x_0) = e$

$\Rightarrow \dfrac{x_0^2}{2e^2} = \dfrac{3}{2}$

$\Rightarrow x_0 = \sqrt{3}\,e$

Q.8 $xdy - ydx = y^2dy$, $y > 0$ & $y(1) = 1$ then find $y(-3)$ = ? **[IIT- 2005]**

(A) 3 (B) 2 (C) 4 (D) 5

Sol. [A] $-\left(\dfrac{ydx - xdy}{y^2}\right) = dy$

$\Rightarrow -d\left(\dfrac{x}{y}\right) = dy$

On integrating we get

$-\dfrac{x}{y} = y + C$

Θ $y(1) = 1 \Rightarrow C = -2$

$\Rightarrow y^2 - 2y + x = 0$

at $x = -3$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow (y-3)(y+1) = 0$

$y = 3, -1$

Q.9 If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x- axis is of length 1. Find the equation of curve. **[IIT- 2005]**

Sol. $\sqrt{1-y^2} + \ln\left|\dfrac{1-\sqrt{1-y^2}}{y}\right| = \pm x + c$

Q.10 A tangent at P(x, y) point on the curve $y = f(x)$ intersects to the axes at A and B points respectively such that AP : BP ≡ 1 : 3, given that $f(1) = 1$, then- **[IIT- 2006]**

(A) normal at (1, 1) is $x + 3y = 4$

(B) equation of curve is $3y + xy' = 0$

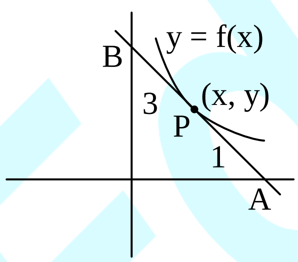
(C) curve passes through (2, 1/8)

(D) equation of curve is $xy' - 3y = 0$

Sol. **[B, C]**

Equation of tangent to the curve $y = f(x)$ at (x, y) is

$$(Y - y) = \frac{dy}{dx}(X - x)$$



Θ $A\left(\dfrac{x\frac{dy}{dx} - y}{\frac{dy}{dx}}, 0\right)$

and $B\left(0, -x\dfrac{dy}{dx} + y\right)$

Θ BP : PA = 3 : 1

$\Rightarrow x = \dfrac{3\left(\dfrac{x\frac{dy}{dx} - y}{\frac{dy}{dx}}\right) + 1 \times 0}{4}$

$\Rightarrow x\dfrac{dy}{dx} + 3y = 0 \Rightarrow xy' + 3y = 0$

$\Rightarrow \displaystyle\int \frac{dy}{y} = -3\int \frac{dx}{x}$

λny = – 3λnx + λnC

$\Rightarrow x^3y = C$

Curve passes through (1, 1) $\Rightarrow C = 1$

curve is $x^3y = 1$

Which also passes through $\left(2, \dfrac{1}{8}\right)$

$\Rightarrow$ Option B, C are correct.

Q.11 Match the following - **[IIT-2006]**

Column I	**Column II**
(A) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$	(P) 0
(B) cosine of the angle of intersection of curves $y = x^{x-1}$ and $y = 3^{x-1}\log x$	(Q) 6λn 2
(C) $\displaystyle\int_0^{\pi/2} (\sin x)^{\cos x}(\cos x \cdot \cot x - \log(\sin x)^{\sin x})dx$	(R) 4/3,
(D) Let $\dfrac{dy}{dx} = \dfrac{6}{x+y}$ where $y(0) = 0$ then value of y when $x + y = 6$ is	(S) 1

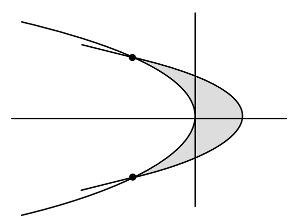
Sol. **A → R ; B → S ; C → S; D → Q**

(A) $4y^2 + 1 = +5y^2$

$y^2 = 1$

$y = \pm 1$

$x = -4$



$$A = \int_{-1}^{1} (-4y^2 - 1 + 5y^2)\, dy$$

$$= \int_{-1}^{1}(y^2-1)\,dy$$

$$= \left|\frac{2}{3}-2\right| = \frac{4}{3}$$

(B) $y' = x^x \lambda n\, x + x.x^{x-1}$

$= x^x(\lambda nx + 1)$ ...(1)

$y' = \frac{3^{x-1}}{x} + \lambda nx\,.\,3^{x-1}\,\lambda n3$ ....(2)

(1) $y' = 1$

(2) $y' = 1$

$\angle$ of intersection = 0

$\cos 0 = -1$

(C) $(\sin x)^{\cos x} = t$

$(\sin x)^{\cos x}\lambda n(\sin x)(-\sin x) + \cos^2 x(\sin x)^{\cos x - 1}.\,dx = dt$

$I = \int_0^1 dt = [t]_0^1 = 1$

(D) $\frac{dy}{dx} = \frac{6}{x+y}$

$x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$1 + \frac{6}{t} = \frac{dt}{dx}$

$\int dx = \int \frac{t+6-6}{t+6}dt$

$c + x = t - \lambda n\,(t + 6)$

$c + x = x + y - \lambda n(x + y + 6)$

$\lambda n(x + y + 6) = y - c$

$x = 0, y = 0$

$c = -\lambda n\, 6$

$\lambda n\, 12 = y + \lambda n\, 6$

$y = \lambda n\, 2$

Q.12 The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with- **[IIT-2007]**

(A) variable radii and a fixed centre at (0, 1)

(B) variable radii and a fixed centre at (0, –1)

(C) fixed radius 1 and variable centres along the x–axis

(D) fixed radius 1 and variable centres along the y–axis

Sol. **[C]**

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y} \Rightarrow \int \frac{y}{\sqrt{1-y^2}}dy = \int dx$

$\Rightarrow \sqrt{1-y^2} = x + C$

$\Rightarrow 1 - y^2 = x^2 + 2Cx + C^2$

$\Rightarrow x^2 + y^2 + 2Cx + C^2 - 1$

Center is $(-C, 0)$

radius = $\sqrt{C^2 - C^2 + 1} = 1$

$\Rightarrow$ circle has fixed radius 1 and variable center along the x-axis.

Q.13 Let f(x) be differentiable on the interval $(0,\infty)$ such that f(1) = 1 , and $\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$ for each x > 0. Then f(x) is- **[IIT 2007]**

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $\frac{-1}{3x} + \frac{4x^2}{3}$

(C) $\frac{-1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

Sol. **[A]**

Q.14 Let a solution y = y(x) of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$

STATEMENT-1 : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$ and

STATEMENT-2

y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

[IIT-2008]

(A) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

(C) Statement–1 is true, Statement–2 is false

(D) Statement–1 is false, Statement–2 is true

Sol. **[C]**

$x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$

$\frac{dy}{dx} = \frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}}$

$$\int \frac{dy}{y\sqrt{y^2-1}} = \int \frac{dx}{x\sqrt{x^2-1}}$$

$\sec^{-1} y = \sec^{-1} x + C$

at $x = 2$, $y = \frac{2}{\sqrt{3}}$

$\sec^{-1} \frac{2}{\sqrt{3}} = \sec^{-1} 2 + C$

$\frac{\pi}{6} = \frac{\pi}{3} + C$

$C = -\frac{\pi}{6}$

$\sec^{-1} y = \sec^{-1} x - \frac{\pi}{6}$

$y = \sec(\sec^{-1} x - \frac{\pi}{6})$

Again $\sec^{-1} y = \sec^{-1} x - \frac{\pi}{6}$

$\cos^{-1} \frac{1}{x} - \cos^{-1} \frac{1}{y} = \frac{\pi}{6}$

$$\cos^{-1}\left(\frac{1}{xy} + \sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{y^2}}\right) = \frac{\pi}{6}$$

$$\frac{1}{xy} + \sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{y^2}} = \frac{\sqrt{3}}{2}$$

Q.15 Match the statements/ expressions in Column I with the open intervals in Column- II **[IIT-2009]**

Column I	**Column II**
(A) Interval contained in the domain of definition of non- zero solutions of the differential equations $(x-3)^2 y' + y = 0$	(P) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
	(Q) $\left(0, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(R) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(S) $\left(0, \frac{\pi}{8}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(T) $(-\pi, \pi)$

Sol. **A → P,Q,R,S,T ; B → P,T ; C → P,Q,R,T ; D → S**

$A \to (x-3)^2 \frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{(x-3)^2}$

or $\frac{dy}{y} = -\frac{1}{(x-3)^2}dx$

$\Rightarrow \ln y = \frac{1}{(x-3)}$ $(x \neq 3)$

$B \to I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)$

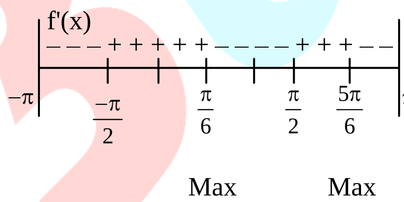
$\Rightarrow I = -I$

$\Rightarrow I = 0$

$C \to f(x) = \cos^2 x + \sin x$

$\Rightarrow f'(x) = -2\cos x \sin x + \cos x$

$= \cos x(-2\sin x + 1)$



$\sin x + \cos x \to (\cos x - \sin x)$, $\cos x > \sin x$

Q.16 Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\frac{d f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is. **[IIT-2011]**

Sol. **[0]**

$\frac{dy}{dg} + y = g$

I. F. $= \int 1.dg = g$

$y.e^g = \int ge^g.dg = ge^g - \int e^g.dg$

$ye^g = ge^g - e^g + c$

$y = g - 1 + ce^{-g}$

$\because y(0) = 0$ & $g(0) = 0$

at $x = 0$

$0 = 0 - 1 + Ce^{-0}$

$C = 1$

$y = g - 1 + e^{-g}$

at $x = 2$

$y(2) = 0 - 1 + e^{-0} = 0$

Q.17 If y (x) satisfies the differential equations $y' - y \tan x = 2x \sec x$ and $y(0) = 0$, then **[IIT-2012]**

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Sol. [A,D] $\frac{dy}{dx} - y \tan x = 2x \sec x$

$P = \tan x$

$\text{I.F.} = e^{-\int \tan x dx} = \cos x$

$y.\cos x = \int 2x.dx$

$y \cos x = x^2 + C$

$\Theta\ y(0) = 0 \Rightarrow C = 0$

$y = x^2 \sec x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ $y\left(\frac{\pi}{3}\right) = \frac{2\pi^2}{9}$

$y'(\pi) = 2x \sec x + x^2 \sec x \tan x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}} + \frac{\pi^2}{16}\sqrt{2} = \frac{\pi}{\sqrt{2}} + \frac{\pi^2}{8\sqrt{2}} = \frac{9\pi^2}{8\sqrt{2}}$

$y'\left(\frac{\pi}{3}\right) = \frac{2\pi}{3} \times 2 + \frac{\pi^2}{9} 2.\sqrt{3}$

EXERCISE # 5

Q.1 The differential equation

$(x + y)\,dx + x\,dy = 0$, is- **[REE- 1995]**

(A) homogeneous but not linear

(B) linear but not homogeneous

(C) both homogeneous and linear

(D) neither homogeneous nor linear

Sol. [C] Given equation can be written as

$$\frac{dy}{dx} = -\frac{x+y}{x}$$

or $\frac{dy}{dx} + \frac{y}{x} = -1$

$\Rightarrow$ Given equation is linear and homogeneous.

Q.2 The order of the differential equation whose general solution is given by **[IIT-1998]**

$y = (c_1 + c_2)\cos(x + c_3) - c_4\, e^{x + c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constant is-

(A) 5 (B) 4

(C) 3 (D) 2

Sol. [C] $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^x \cdot e^{c_5}$

$\Rightarrow$ $y = A\cos(x + c_3) - Be^x$

Hence three arbitrary constant

$\Rightarrow$ order = 3

Q.3 The differential equation

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \sin y + x^2 = 0$ is of which of the following types- **[REE-1998]**

(A) linear (B) homogeneous

(C) order two (D) degree one

Sol. [C, D] $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \sin y + x^2 = 0$

Clearly it is of order two and degree one

$\Rightarrow$ option C, D are correct.

Q.4 The differential equation representing the family of curve $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of- **[IIT-1999]**

(A) order 1 (B) order 2

(C) degree 3 (D) degree 4

Sol.[A, C] $y^2 = 2c(x + \sqrt{c})$

$2y\frac{dy}{dx} = 2c \quad \Rightarrow \quad c = y\frac{dy}{dx}$

$\Rightarrow \quad y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{\frac{ydy}{dx}}\right)$

$\Rightarrow \quad y - 2x\frac{dy}{dx} = 2y^{1/2}\left(\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow \quad \left(y - 2x\frac{dy}{dx}\right)^2 = 4y\left(\frac{dy}{dx}\right)^3$

$\Rightarrow$ order = 1 and degree = 3.

Q.5 The solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is- **[IIT-1999]**

(A) $y = 2$ (B) $y = 2x$

(C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

Sol. [C] $p^2 - xp + y = 0$

$\Rightarrow$ $y = xp - p^2$

Solution is

$y = cx - c^2$

Put c = 2 we get

$y = 2x - 4$

Q.6 If $x^2 + y^2 = 1$, then- **[IIT-2000S]**

(A) $yy'' - 2(y')^2 + 1 = 0$

(B) $yy'' + (y')^2 + 1 = 0$

(C) $yy'' + (y')^2 - 1 = 0$

(D) $yy'' + 2(y')^2 + 1 = 0$

Sol.[B] $x^2 + y^2 = 1$

$x + yy' = 0$

$1 + yy'' + (y')^2 = 0$

$\Rightarrow$ $yy'' + (y')^2 + 1 = 0$

Q.7 A normal is drawn at a point P(x, y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves, is

$y\dfrac{dy}{dx} = \pm\sqrt{k^2 - y^2}$ **[IIT 1994]**

Sol. Length of normal

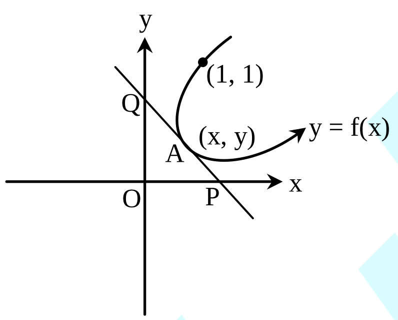
$y\sqrt{1+(y')^2} = k$

$y^2 + y^2 (y')^2 = k^2$

$y\,y' = \pm\sqrt{k^2 - y^2}$

Q.8 Let y = *f* (x) be a curve passing through (1, 1) such that the triangle formed by the coordinate axes and tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves. **[IIT- 1995]**

Sol. Equation of tangent to the curve y = f(x) at point A (x, y) is $Y - y = f'(x)(X - x)$



whose, x-intercept $\left(x - y.\dfrac{dx}{dy}, 0\right)$

y-intercept $\left(0, y - x\dfrac{dy}{dx}\right)$; ΔOPQ = 2

$\Rightarrow \dfrac{1}{2}.\left(x - y\dfrac{dx}{dy}\right)\left(y - x\dfrac{dy}{dx}\right) = 2$

$\Rightarrow \left(x - y\dfrac{1}{p}\right)(y - xp) = 4$, where $p = \dfrac{dy}{dx}$

$\Rightarrow p^2x^2 - 2pxy + 4p + y^2 = 0$

$\Rightarrow (y - px)^2 + 4p = 0$

$\therefore\ y - px = 2\sqrt{-p}$

$\Rightarrow y = px + 2\sqrt{-p}$ ....(1)

differentiating w.r.t x, we get

$p = p + \dfrac{dp}{dr}.x + 2.\left(\dfrac{1}{2}\right)(-p)^{-1/2}.(-1)\dfrac{dp}{dx}$

$\Rightarrow \dfrac{dp}{dx}.x - (p)^{-1/2}.\dfrac{dp}{dx} = 0$

$\Rightarrow \dfrac{dp}{dx}\{x - (-p)^{-1/2}\} = 0$

$\Rightarrow \dfrac{dp}{dx} = 0$ or $x = (-p)^{-1/2}$

If $\dfrac{dp}{dx} = 0 \Rightarrow p = c$

putting this value in (i), we get

$y = cx + 2\sqrt{-c}$

This curve passes through (1, 1)

$\Rightarrow 1 = c + 2\sqrt{-c}$

$\Rightarrow c = -1$

$\therefore\ y = -x + 2$

or $x + y = 2$

again if $x = (-p)^{1/2}$

$\Rightarrow -p = x^2$ putting in (1)

$y = \dfrac{-x}{x^2} + 2.\dfrac{1}{x} = \dfrac{1}{x}$

$\Rightarrow xy = 1$

Thus, the two curves are xy = 1 and x + y = 2.

Q.9 If $y + \dfrac{d}{dx}(xy) = x(\sin x + \log x)$, find y (x). **[REE -1995]**

Sol. $y + \dfrac{xdy}{dx} + y = x(\sin x + \log x)$

$\dfrac{dy}{dx} + \dfrac{2}{x}y = (\sin x + \log x)$

$IF = e^{\int \frac{2}{x}dx} = x^2$

$\Rightarrow yx^2 = \int x^2(\sin x + \log x)\,dx$

$\Rightarrow yx^2 = -x^2\cos x + \int 2x\cos dx$

$+\dfrac{x^3}{3}\log x - \int \dfrac{x^2}{3}dx$

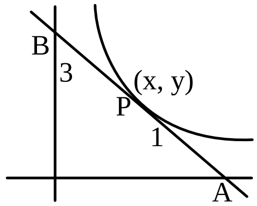
$= -x^2\cos x + \dfrac{x^3}{3}\log x + 2x\sin x + 2\cos x - \dfrac{x^3}{9} + C$

$\Rightarrow y = \dfrac{x}{3}\log x - \dfrac{x}{9} - \cos x + \dfrac{2}{x}\sin x + \dfrac{2}{x^2}\cos x + \dfrac{C}{x^2}$

Q.10 A curve y = f (x) passes through the point P(1, 1). The normal to the curve at P is;

a (y –1) + (x –1) = 0. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point determine the equation of the curve. Also obtain the area bounded by the y-axis, by the curve and the normal to the curve at P. **[IIT-1996]**

Sol.



$Y - y = y'(X - x)$

$A \equiv \left(x - \frac{y}{y}, 0\right)$

$B \equiv (0, y - xy')$

$3x - \frac{3y}{y'} = 4x$

$-x = \frac{3y}{y'}$

$3y + xy' = 0$

$\int \frac{dy}{y} = -3\int \frac{dx}{x}$

$\ln y = -3 \ln x + \ln c$

$y = \frac{c}{x^3}$

$c = 1$

$y = \frac{1}{x^3}$

$y' = -\frac{3x^2}{x^6 x^4} = -3$

$-\frac{1}{y'} = \frac{1}{3}$

Q.11 Solve the differential equation

$\cos^2 x \frac{dy}{dx} - (\tan 2x)\, y = \cos^4 x,\ |x| < \pi/4.$

When $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$. **[REE -1996]**

Sol. $\frac{dy}{dx} - \frac{\tan 2x}{\cos^2 x} y = \cos^2 x$

I.F. $= e^{-\int \frac{\tan 2x}{\cos^2 x}dx}$

$= e^{-\int \frac{2\sin x \cos x}{(2\cos^2 x - 1)\cos^2 x}dx}$

$= e^{\int \frac{2}{(2t^2-1)t}dt}$ where t = cos x

$= e^{2\int\left(\frac{2t}{2t^2-1} - \frac{1}{t}\right)dt}$

$= e^{2\left(\frac{1}{2}\ln(2t^2-1) - \ln t\right)} = e^{\ln\frac{2t^2-1}{t^2}}$

$= \frac{2t^2-1}{t^2} = \frac{2\cos^2 x - 1}{\cos^2 x} = \frac{\cos 2x}{\cos^2 x}$

$\Rightarrow \quad y\frac{\cos 2x}{\cos^2 x} = \int \cos 2x\, dx$

$\Rightarrow \quad y\frac{\cos 2x}{\cos^2 x} = \frac{\sin 2x}{2} + C$

$y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8} \Rightarrow c = 0$

$\Rightarrow \quad y = \frac{1}{2}\tan 2x \cos^2 x$

Q.12 Determine the equation of the curve passing through the origin, in the form y = f (x), which satisfies the differential equation.

$\frac{dy}{dx} = \sin(10x + 6y)$ **[IIT -1996]**

Sol. $\frac{dy}{dx} = \sin(10x + 6y)$

Let 10x + 6y = t (given) ....(1)

$\Rightarrow \quad 10 + 6\frac{dy}{dx} = \left(\frac{dt}{dx}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$

Now, the given differential equation becomes

$\sin t = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$

$\Rightarrow \quad 6\sin t = \frac{dt}{dx} - 10$

$\Rightarrow \quad \frac{dt}{dx} = 6\sin t + 10$

$\Rightarrow \quad \frac{dt}{dx} = 6\sin t + 10$

$\Rightarrow \quad \frac{dt}{6\sin t + 10} = dx$ apply variable separable

Integrating both the sides, we get

$$\int \frac{dt}{6\sin t + 10} = \int dx$$

$$\Rightarrow \frac{1}{2}\int \frac{dt}{3\sin t + 5} = x + c \quad \text{....(2)}$$

Let $I_1 = \int \frac{dt}{3\sin t + 5}$

Put tan t / 2 = u

$$\Rightarrow \frac{1}{2}\sec^2 t/2\, dt = du$$

$$\Rightarrow dt = \frac{2du}{\sec^2 t/2}$$

$$\Rightarrow dt = \frac{2du}{1+u^2}$$

Also, $I_1 = \int \frac{dt}{3\sin t + 5}$

$$= \int \frac{dt}{3\left(\frac{2\tan t/2}{1+\tan^2 t/2}\right)+5}$$

$$= \int \frac{(1+\tan^2 t/2)\,dt}{\left(6\tan\frac{t}{2}+5+5\tan^2\frac{t}{2}\right)}$$

$$= \int \frac{2(1+u^2)\,du}{(1+u^2)(5u^2+6u+5)}$$

$$= \frac{2}{5}\int \frac{du}{u^2+(6/5)u+1}$$

$$= \frac{2}{5}\int \frac{du}{u^2+\frac{6}{5}u+1}$$

$$= \frac{2}{5}\int \frac{du}{u^2+\frac{6}{5}u+\frac{9}{25}+1}$$

$$= \frac{2}{5}\int \frac{du}{\left(u+\frac{3}{5}\right)^2+\frac{16}{25}}$$

$$= \frac{2}{5}\int \frac{du}{\left(u+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$$

$$= \frac{2}{5}\cdot\frac{5}{4}\tan^{-1}\left(\frac{u+3/5}{4/5}\right)$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5u+3}{4}\right]$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5\tan t/2+3}{4}\right]$$

Putting this in (2)

Now $\frac{1}{2}I_1 = x + c.$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left[\frac{5\tan\frac{t}{2}+3}{4}\right] = x + c$$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left[\frac{5\tan\frac{t}{2}+3}{4}\right] = 4x + 4c$$

$$\Rightarrow \frac{1}{4}[5\tan(5x+3y)+3] = \tan(4x+4c)$$

$\Rightarrow$ 5 tan (5x + 3y) + 3 = 4 tan (4x + 4c)

When x = 0, y = 0, we get

5 tan 0 + 3 = 4 tan (4c)

$$\Rightarrow \frac{3}{4} = \tan 4c$$

$$\Rightarrow 4c = \tan^{-1}\frac{3}{4}$$

Then, 5 tan (5x + 3y) + 3 =4 tan (4x + $\tan^{-1}$ 3/4)

$$\Rightarrow \tan(5x+3y) = \frac{4}{5}\tan(4x+\tan^{-1}3/4) - \frac{3}{5}$$

$$\Rightarrow 5x+3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4)\}-\frac{3}{5}\right]$$

$$\Rightarrow 3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4)\}-\frac{3}{5}\right]-5x$$

$$\Rightarrow y = \frac{1}{3}\tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4)-\frac{3}{5}\right]-\frac{5x}{3}$$

Q.13 Solve the differential equation

$$y\cos\frac{y}{x}(xdy - ydx) + x\sin\frac{y}{x}(xdy + ydx) = 0.$$

When y (1) = π/2 **[REE -1997]**

Sol. Given equation can be written as

$$\frac{y}{x}\cos x\frac{y}{x}\left(\frac{dy}{dx}-\frac{y}{x}\right)+\sin\frac{y}{x}\left(\frac{dy}{dx}+\frac{y}{x}\right)=0$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2\cos\frac{y}{x}-\frac{y}{x}\sin\frac{y}{x}}{\frac{y}{x}\cos\frac{y}{x}+\sin\frac{y}{x}}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v}{v\cos v + \sin v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = -\frac{2v\sin v}{v\cos v + \sin v}$$

$$\Rightarrow \quad \int\left(\frac{v\cos v + \sin v}{v\sin v}\right)dv = -\int\frac{2}{x}dx$$

$$\Rightarrow \quad \lambda n \sin v + \lambda n\, v = -\lambda n x^2 + \lambda n c$$

$$y(1) = \frac{\pi}{2} \quad \Rightarrow \quad c = \frac{\pi}{2}$$

$$\Rightarrow \quad (v \sin v)\, x^2 = \frac{\pi}{2}$$

$$\Rightarrow \quad xy \sin\frac{y}{x} = \frac{\pi}{2}$$

Q.14 A and B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity in reservoirs A is 1½ times the quantity of water is reservoirs B. After how many hours do both the reservoirs have the same quantity of water? **[IIT -1997]**

Sol. $\frac{dv}{dt} \propto v$ for each reservoir

$$\frac{dv}{dx} \propto -v_A$$

$$\Rightarrow \quad \frac{dv_A}{dt} = -K_1 v_A$$

(K_1 is the proportional constant).

$$\Rightarrow \quad \int_{v_A}^{v'_A}\frac{dv_A}{v_A} = -K_1\int_0^1 dt$$

$$\Rightarrow \quad \log\frac{v'_A}{v_A} = -K_1 t \Rightarrow v'_A = v_A \, . \, e^{-K_v} \quad ....(i)$$

Similarly for B,

$$v_B' = v_B \, . \, e^{-k_2 t} \quad ....(ii)$$

Dividing (i) by (ii), we get

$$\frac{v'_A}{v'_B} = \frac{v_A}{v_B} \, . \, e^{-(K_1-K_2)t}$$

It is given that at t = 0, $v_A = 2v_B$ and at

$$t = 3/2,\ v'_A = \frac{3}{2}\, v'_B.$$

Thus, $\frac{3}{2} = 2 \, . \, e^{-(K_1-K_2)t}$

$$\Rightarrow \quad e^{-(K_1-K_2)} = 3/4$$

Now, let at t = t_0 both the reservoirs have some quantity of water. Then

$$v'_A = v'_B.$$

Hence, $2e^{-(K_1-K_2)} = 1$

$$\Rightarrow \quad 2 \, . \left(\frac{3}{4}\right)^{t_0} = 1 \quad \Rightarrow \quad t_0 = \log_{3/4}(1/2)$$

Q.15 Let u(x) and v(x) satisfy the differential equations $\frac{du}{dx} + P(x)u = f(x)$ & $\frac{dv}{dx} + P(x)v = g(x)$, where P(x), f(x) and g(x) are continuous functions. If $u(x_1) > v(x_1)$ for some x_1 and f(x) > g (x) for all x > x_1. Prove that any point (x, y) where x > x_1 does not satisfy the equation y = u(x) and y = v(x) **[IIT -1997]**

Sol. Let w (x) = u (x) – v (x)

and h (x) = f (x) (x) – g (x)

differentiating with respect to x

$$\frac{dw}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

= {f(x) – p(x) . u (x)} – {g(x) – p(x) v (x)} (given)

= {f(x) – g (x)} – p (x) [u (x) – v(x)]

$$\Rightarrow \quad \frac{dw}{dx} = h(x) - p(x)\, .\, w(x) \quad ....(1)$$

$\Rightarrow \quad \frac{dw}{dx} + p(x)\, w(x) = h(x)$ which is linear differential equation.

The integrating factor is given by

$$\text{I.F.} = e^{\int p(x)dx} = r(x) \quad \text{(let)}$$

Multiplying both sides of (1) of r(x), we get

$$r(x)\, .\, \frac{dw}{dx} + p(x)\,\{r(x)\}\, w(x) = r(x)\, .\, h(x)$$

$$\Rightarrow \quad \frac{d}{dx}[r(x)\, w(x)] = r(x)\, .\, h(x)$$

$$\left[\text{since } \frac{dr}{dx} = p(x).r(x)\right]$$

Now $\quad r(x) = e^{\int p(x)dx} > 0 \ \forall\ x$

and $\quad h(x) = f(x) - g(x) > 0$ for $x > x_1$

Thus, $\frac{d}{dx}[r(x)\,w(x)] > 0 \ \forall\ x > x_1$

r(x) w (x) increases on the interval $[x, \infty)$

Therefore for all $x > x_1$

$r(x)\,w(x) > r(x_1)\,w(x_1) > 0$

$[\Theta\ r(x_1) > 0$ and $u(x_1) > v(x_1)]$

$\Rightarrow\ w(x) > 0\ \forall\ x > x_1$

$\Rightarrow\ u(x) > v(x)\ \forall x > x_1 \qquad [\Theta\ r(x) > 0]$

Hence, there exist a point (x, y) such that $x > x_1$ and $y = u(x)$ and $y = v(x)$.

Q.16 Solve the differential equation

$(1+\tan y)(dx - dy) + 2x\,dy = 0$ **[REE-1998]**

Sol. $\frac{dx}{dy} - 1 + \frac{2x}{1+\tan y} = 0$

$\Rightarrow \frac{dx}{dy} + \frac{2x}{1+\tan y} = 1$

IF = $e^{\int \frac{2}{1+\tan y}dy}$

$\Theta \int \frac{2}{1+\tan y}dy$

Let $\tan y = t \Rightarrow dy = \frac{dt}{1+t^2}$

$\Rightarrow \int \frac{2}{(1+t)(1+t^2)}dt$

Solving we get

$= \lambda n(\sin y + \cos y) + y$

$\Rightarrow e^{\lambda n(\sin y + \cos y)+y} = (\sin y + \cos y)\,e^y$

$\Rightarrow x(\sin y + \cos y)\,e^y = \int (\sin y + \cos y)\,e^y\,dy$

$\Rightarrow x(\sin y + \cos y)\,e^y = e^y \sin y + C$

$\Rightarrow x(\sin y + \cos y) = \sin y + ce^{-y}$

Solving we get

$\Rightarrow x(\sin y + \cos y) = -\cos y + \sqrt{2}\cos\left(y - \frac{\pi}{4}\right) + Ce^{-y}$

Q.17 Solve the following differential equation :

$(x^2 + 4y^2 + 4xy)\,dy = (2x + 4y + 1)\,dx$

[REE- 1999]

Sol. $\frac{dy}{dx} = \frac{2(x+2y)+1}{(x+2y)^2}$

Let $x + 2y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{dt}{dx} - 1\right)$

$\Rightarrow \frac{1}{2}\left(\frac{dt}{dv} - 1\right) = \frac{2t+1}{t^2}$

$\Rightarrow \frac{dt}{dx} = \frac{t^2+4t+2}{t^2}$

$\Rightarrow \int \frac{t^2}{(t+2)^2 - 2}dt = \int dx$

Put $t + 2 = P \Rightarrow dt = dp$ and

Solving we get

$P - 2\lambda n(p^2 - 2) - \frac{3}{\sqrt{2}}\lambda n\frac{p-\sqrt{2}}{p+\sqrt{2}} = x + C$

$\Rightarrow t - 2\lambda n(t^2 + 4t + 2) - \frac{3}{\sqrt{2}}\lambda n\frac{t+2-\sqrt{2}}{t+2+\sqrt{2}} = x + C$

Where $t = x + 2y$

Q.18 A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve. **[IIT -1999]**

Sol. Equation of normal at point (x, y) is

$Y - y = -\frac{dx}{dy}(X - x)$

Distance of perpendicular from the origin to (1)

$= \frac{\left|y + \frac{dx}{dy}.x\right|}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}$

Also distance between P and x-axis is | y |

$\therefore \frac{\left|y + \frac{dx}{dy}.x\right|}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}} = |y|$

$\Rightarrow y^2 + \frac{dx}{dy}.x^2 + 2xy\frac{dx}{dy} = y^2\left[1+\left(\frac{dx}{dy}\right)^2\right]$

$\Rightarrow \left(\frac{dx}{dy}\right)^2(x^2 - y^2) + 2xy\frac{dx}{dy} = 0$

$\Rightarrow \frac{dx}{dy}\left[\left(\frac{dx}{dy}\right)(x^2 - y^2) + 2xy\right] = 0$

$\Rightarrow \frac{dx}{dy} = 0$ or $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

But $\frac{dx}{dy} = 0 \Rightarrow x = c$, where c is a constant.

As curve passes through (1, 1), we get the equation of the curve as x = 1.

The equation $\frac{dx}{dy} = \frac{y^2 - x^2}{2xy}$ is a homogeneous equation.

Put y = vx, so that

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x^2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v} = -\frac{v^2+1}{2v}$$

$$\Rightarrow \frac{-2v}{v^2+1}dv = \frac{dx}{x}$$

$\Rightarrow c_1 - \log(v^2 + 1) = \log|x|$

$\Rightarrow \log|x|(v^2 + 1) = c_1$

$$\Rightarrow |x|\left(\frac{y^2}{x^2}+1\right) = e^{c_1}$$

$x^2 + y^2 = \pm e^{c_1}x$ or $x^2 + y^2 = \pm e^c x$ is passing through (1, 1) so $1 + 1 = \pm e^c . 1 \Rightarrow \pm e^c = 2$

Therefore, required curve is $x^2 + y^2 = 2x$

Q.19 A country has a food deficit of 10%. Its population grows continuously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant. Prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{\lambda n10 - \lambda n9}{\lambda n(1.04) - 0.03}$. **[IIT- 2000]**

Sol. Let X_0 be initial population of the country and Y_0 be its initial food production.

Let the average consumption be a units. Therefore, food required initially aX_0. It is given

$$Y_p = aX_0\left(\frac{90}{100}\right) = 0.9\, aX_0 \qquad ....(1)$$

Let X be the population of the country in year t.

Then $\frac{dX}{dt}$ = rate of change of population

$$= \frac{3}{100}X = 0.03X$$

$$\frac{dX}{X} = 0.03\,dt$$

integrating, $\int\frac{dX}{X} = \int 0.03\,dt$

$\Rightarrow \log X = 0.03t + c$

$\Rightarrow X = A . e^{0.03t}$ when $A = e^c$

At t = 0, $X = X_0$, thus $X_0 = A$

$\therefore X = X_0 e^{0.03t}$

Let Y be the food production in year t.

Then $Y = Y_0\left(1+\frac{4}{100}\right)^t = 0.9\, aX_0 (1.04)^t$

$\because y_0 = 0.9a X_0$

Food consumption in the year t is $aX_0 e^{0.03t}$

again $Y - X \geq 0$ (given)

$\Rightarrow 0.9 X_0 a (1.04)^t > aX_0 e^{0.03t}$

$$\Rightarrow \frac{(1.04)^t}{e^{0.03t}} > \frac{1}{0.9} = \frac{10}{9},$$

Taking log on both sides,

$t[\lambda n(1.04) - 0.03] \geq \lambda n 10 - \lambda n 9$

$$\Rightarrow t \geq \frac{\lambda n10 - \lambda n9}{\lambda n(1.04) - 0.03}$$

Thus, the least integral values of the year n, when the country becomes self-sufficient, is the smallest integer greater than or equal to

$$\frac{\lambda n10 - \lambda n9}{\lambda n(1.04) - 0.03}$$

Q.20 A curve y = f(x) passes through O(0, 0) and slope of tangent line at any point P(x, y) of the curve is $\frac{x^4 + 2xy - 1}{1 + x^2}$, then the value of least integer which is greater than or equal to f(–1) is

Sol.
$$\frac{dy}{dx} = x^2 - 1 + \frac{2xy}{1+x^2}$$

$$\frac{dy}{dx} - \frac{2x}{1+x^2}y = x^2 - 1$$

$$\text{I.F.} = e^{-\int\frac{2x}{1+x^2}dx} = \frac{1}{1+x^2}$$

$$\frac{y}{1+x^2} = \int\frac{x^2-1}{x^2+1}dx$$

$$\frac{y}{1+x^2} = x - 2\tan^{-1}x + c$$

$x = 0, y = 0 \Rightarrow c = 0$

$y = (x^2 + 1)x - 2(x^2 + 1)\tan^{-1}x$

$$f(-1) = -2 + 4\ \frac{\pi}{4} = \pi - 2$$

Q.21 Solve the differential equations

$(x^3 + y^2 + 2)\,dx + 2y\,dy = 0$

Sol. $y^2 = t$

$$(x^3 + t + 2) + \frac{dt}{dx} = 0$$

$$\frac{dt}{dx} + t = -x^3 - 2$$

I.F. $= e^x$

Its solution

$$t\,.\,e^x = -\int (x^3 + 2)e^x\,dx$$

$t\,.\,e^x = [(x^3 + 2)e^x - 3x^2e^x + 6x\,e^x - 6e^x + c]$

$t = -(x^3 + 2) + 3x^2 - 6x + 6 + ce^{-x}$

Q.22 Solve the differential equations

$yy'\sin x = \cos x\,(\sin x - y^2)$

Sol. $y^2 = t$

$2yy' = t'$

$$\frac{t'}{2}\ \sin x = \frac{\cos x}{\sin x}\ (\sin x - t)$$

$$\frac{t'}{2} = \cos x - t\cot x$$

$$\frac{dt}{dx} + 2\cot x\,.\,t = 2\cos x$$

I.F. $= e^{2\int \cot x\,dx} = \sin^2 x$

Its solution is

$$t\,.\,\sin^2 x = 2\int \cos x \sin^2 x\,dx$$

Put $\sin x = u$

$$= 2\int u^2\,du$$

$$y^2\sin^2 x = \frac{2}{3}(\sin x)^3 + c$$

Q.23 Solve :

$$\frac{dy}{dx} - y\,\lambda n\,2 = 2^{\sin x}.\,(\cos x - 1)\,\lambda n2,$$

y being bounded when $x \to +\infty$.

Sol. I.F. $= e^{-\lambda n 2\int dx} = \dfrac{1}{2x}$

Its solutions

$$\frac{y}{2x} = \int \frac{2^{\sin x}(\cos x - 1)\lambda n2}{2x}\,dx$$

$2^{\sin x - x} = t$

$2^{\sin x - x}\,\lambda n2\,.\,(\cos x - 1)\,dx = dt$

$$\frac{y}{2x} = 2^{\sin x - x} + c$$

$y = 2^{\sin x} + c\,.\,2x$

$c = 0$

$y = 2^{\sin x}$

Q.24 Solve :

$$(1 - x^2)^2 dy + \left(y\sqrt{1-x^2} - x - \sqrt{1-x^2}\right)dx = 0.$$

Sol. $$\frac{dy}{dx} + \frac{y}{(1-x^2)^{3/2}} = \frac{x + \sqrt{1-x^2}}{(1-x^2)^2}$$

I.F. $= e^{\int \frac{dx}{(1-x^2)^{3/2}}}$

$x = \sin\theta$

$$e^{\int \frac{\cos\theta\,d\theta}{\cos^3\theta}} = e^{\int \sec^2\theta\,d\theta}$$

I.F. $= e^{\tan\theta}$

Its solution

$$y\,.\,e^{\tan\theta} = \int e^{\tan\theta}.\ \frac{x + \sqrt{1-x^2}}{(1-x^2)^2}\ dx$$

$$= \int e^{\frac{x}{\sqrt{1-x^2}}}\left(\frac{x}{(1-x^2)^2} + \frac{1/dx}{(1-x^2)^3}\right)$$

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	B	A	D	A	B	A	C	A	D	B	D	B	A	A	B	D	C	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**								
Ans.	B	B	A	D	A	B	C	A	D	A	B	B								

33. True **34.** False **35.** True **36.** False **37.** $y^2 = x^2(\lambda nx - 1) + c$ **38.** ellipse

39. $6y - 3x = \lambda n\left|\frac{3x+3y+2}{2}\right|$

40. $y^3(x+1)^2 = \frac{x^6}{6} + \frac{2x^5}{5} + \frac{x^4}{4} + C$

EXERCISE # 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	C	A	A	A	C	A	B	A	B	A	B	A	C
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**						
Ans.	B	B	A	B	C	D	B	C	C						

PART-B

Q.No.	25	26	27	28	29	30	31	32	33	34	35	36	37
Ans.	A,B	A,B	C,D	A,B,C,D	A,C	A,B	A,B,C	C,D	A,B,D	C	A	A	A

PART-C

38. (B) **39.** (C) **40.** (D) **41.** (A) **42.** (B)

PART-D

43. A $\rightarrow$ P, S ; B $\rightarrow$ Q ; C $\rightarrow$ Q, S; D $\rightarrow$ R

44. A $\rightarrow$ R ; B $\rightarrow$ P ; C $\rightarrow$ S ; D $\rightarrow$ Q

EXERCISE # 3

1. $y\,\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{2}{3}x^{3/2} + c$ **2.** $y^2 = 5 \pm 2\sqrt{5}\,x$ **4.** $x\sqrt{\cot y} = c + \sqrt{\tan y}$ **5.** $2x^2 + 3y^2 = c^2$

6. $y = \frac{1}{9}[26e^{3x-3} - 6x - 2]$ **7.** $x^2 + y^2 = 25$ **9.** $y^2 = 2x + 1 - e^{2x}$ **10.** $y = c\,e^{x^3/3k}$

11. $y = x \sinh(x + c)$

13. $\left[1-\left(\frac{1}{2}\right)^{1/16}\right]$ of initial quantity

14. $\frac{\ln 100}{\ln 2}$ hrs

15. $\frac{1}{k}\lambda n\left(\frac{g+kV_0}{g}\right)$

19. 161/4m

20. (A) 5 $\log_{5/4}2$ min. (B) 79.52 F

21. 0020

22. (B) **23.** (C) **24.** (B) **25.** (C) **26.** (D)

27. (C) **28.** (D) **29.** (C) **30.** (D) **31.** (A)

EXERCISE # 4

1. $T = \frac{14\pi\ 10^5}{27\sqrt{g}}$ units, where g is in cm/time^2

3. $T = \frac{H}{K}$

4. (A) **5.** (A)

6. (i) $y = x(x-2)$; $x \neq -1$ (ii) 4/3 sq. units

7. (D) **8.** (A)

9. $\sqrt{1-y^2} + \ln\left|\frac{1-\sqrt{1-y^2}}{y}\right| = \pm x + c$

10. (B, C)

11. $A \to R$; $B \to S$; $C \to S$; $D \to Q$

12. (C) **13.** (A) **14.** (C)

15. $A \to P,Q,R,S,T$; $B \to P,T$; $C \to P,Q,R,T$; $D \to S$

16. 0

17. (A, D)

EXERCISE # 5

1. (C) **2.** (C) **3.** (C, D) **4.** (A, C) **5.** (C) **6.** (B)

8. $\left(x\frac{dy}{dx} - y\right)^2 + 4\frac{dy}{dx} = 0$, $\sqrt{1-xy} = 1 - x$, $\sqrt{1-xy} = x - 1$, $xy = 1$

9. $y = \frac{x}{3}\log x - \frac{x}{3} - \cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{c}{x^2}$, $c \in R$.

10. $e^{a(x-1)}$, $\frac{1}{a}$ $[a - \frac{1}{2} + e^{-a}]$ sq.unit

11. $y = \frac{1}{2}\tan 2x \cdot \cos^2 x$

12. $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$

13. $xy \sin\frac{y}{x} = \frac{\pi}{2}$

14. $\frac{\ln(1/2)}{\ln(3/4)}$

16. $x(\sin y + \cos y) = -\cos y + \sqrt{2}\cos\left(y - \frac{\pi}{4}\right) + c\,e^{-y}$

17. $t - 2\lambda n(t^2 + 4t + 2) + \frac{3}{\sqrt{2}}\lambda n\frac{t+2-\sqrt{2}}{t+2+\sqrt{2}} = x + c$; where $t = x + 2y$

18. $x^2 + y^2 - 2x = 0,\ x - 1 = 0$

20. 2

21. $y^2 = 3x^2 - 6x - x^3 + ce^{-x} + 4$

22. $y^2 = \frac{2}{3}\sin x + \frac{c}{\sin^2 x}$

23. $y = 2^{\sin x}$

24. $y = \frac{x}{\sqrt{1-x^2}} + ce^{-\frac{x}{\sqrt{1-x^2}}}$