EXERCISE # 1

Q.1 If $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\tan\beta = 1$, find the value of $\sin(\alpha + \beta)$, where α and β both are acute.

Q.2 If $\cos\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, find the value of $\sin(\alpha + \beta)$, where α and β both are acute.

Q.3 Without using trigonometric tables evaluate the following :

(i) $\frac{\sin 20^\circ}{\cos 70^\circ}$ (ii) $\frac{\cos 19^\circ}{\sin 71^\circ}$ (iii) $\frac{\sin 21^\circ}{\cos 69^\circ}$

(iv) $\frac{\tan 10^\circ}{\cot 80^\circ}$ (v) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$ (vi) $\frac{\sin 20^\circ 30'}{\cos 69^\circ 30'}$

Q.4 Without using trigonometric tables evaluate the following :

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

(ii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cot 35^\circ}{\sin 55^\circ}\right)$

Q.5 Without using trigonometric tables evaluate the following :

(i) $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$

(ii) $\operatorname{cosec}^2 67^\circ - \tan^2 23^\circ$

Q.6 Without using trigonometric tables evaluate the following :

(i) $\sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ$

(ii) $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Q.7 Without using trigonometric tables prove the following :

(i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

(ii) $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2$

(iii) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

(iv) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - \cos 70^\circ \operatorname{cosec} 20^\circ = 1$

(v) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$

Q.8 Prove the following :

(i) $\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta) = 0$

(ii) $\frac{\sin\theta \cos(90^\circ - \theta) \cos\theta}{\sin(90^\circ - \theta)} + \frac{\cos\theta \sin(90^\circ - \theta) \sin\theta}{\sin(90^\circ - \theta)} = 1$

(iii) $\frac{\sin\theta}{\sin(90^\circ - \theta)} + \frac{\cos\theta}{\cos(90^\circ - \theta)} = \sec\theta \operatorname{cosec}\theta$

(iv) $\sin(90^\circ - \theta) \cos(90^\circ - \theta) = \frac{\tan\theta}{1 + \cot^2(90^\circ - \theta)}$

(v) $\frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2\operatorname{cosec}\theta$

(vi) $\frac{1}{1 + \cos(90^\circ - \theta)} + \frac{1}{1 - \cos(90^\circ - \theta)} = 2 \operatorname{cosec}^2(90^\circ - \theta)$

(vii) $\sin^2(90^\circ - \theta)(1 + \cot^2(90^\circ - \theta)) = 1$

(viii) $\frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan\theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} = 2$

(ix) $\frac{\tan(90^\circ - A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

(x) $\frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

Q.9 Without using trigonometric tables, evaluate each of the following :

(i) $\sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos\theta \sin(90^\circ - \theta) + \sin\theta \cos(90^\circ - \theta)}$

(ii) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$

(iii) $\cot\theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec}\theta + \sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}\,(\tan 5^\circ \tan 45^\circ \tan 85^\circ)$

(iv) $\cot\theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec}\theta + \sqrt{3}\,(\tan 5^\circ \tan 30^\circ \tan 85^\circ) + \sin^2 25^\circ + \sin^2 65^\circ$

(v) $\dfrac{-\tan\theta\cot(90^\circ-\theta)+\sec\theta\,\mathrm{cosec}(90^\circ-\theta)}{\tan10^\circ\tan20^\circ\tan45^\circ\tan70^\circ\tan80^\circ}$

$+\dfrac{\sin^2 35^\circ+\sin^2 55^\circ}{\tan10^\circ\tan20^\circ\tan45^\circ\tan70^\circ\tan80^\circ}$

Q.10 The round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is r $\sin\beta$ cosec $\alpha/2$.

Q.11 If $\tan\theta = 8/15$ and $0^\circ < \theta < 90^\circ$, find $\sin\theta$.

Q.12 If $\sin\theta = 8/17$ and $0^\circ < \theta < 90^\circ$, find $\tan\theta$.

Q.13 If $\sin A = \dfrac{24}{25}$, find the value of tanA + secA, where $0^\circ < A < 90^\circ$.

Q.14 If $5\tan\theta = 12$, find the value of $\dfrac{2\cos\theta+\sin\theta}{\sin\theta-\cos\theta}$.

Q.15 If $\tan\theta = \dfrac{3}{4}$, find the value of $\dfrac{1-\cos\theta}{1+\cos\theta}$.

Q.16 If $\tan\theta = \dfrac{12}{5}$, find the value of $\dfrac{1+\sin\theta}{1-\sin\theta}$.

Q.17 If $\tan A = \dfrac{1}{2}$ and $\tan B = \dfrac{1}{3}$, by using $\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$, prove that $A + B = 45^\circ$

Q.18 If $4\tan\theta = 3$, find the value of $\dfrac{4\sin\theta-2\cos\theta}{4\sin\theta+3\cos\theta}$.

Q.19 If $\mathrm{cosec}\,\theta = \dfrac{13}{12}$, find the value of $\dfrac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta}$.

ANSWER KEY

1. 1

2. 1

3. (i) 1 , (ii) 1, (iii) 1, (iv) 1, (v) 1, (vi) 1

4. (i) 2, (ii) $\frac{1}{2}$

5. (i) 1, (ii) 1

6. (i) 0, (ii) 2

9. (i) 2, (ii) 1, (iii) $\sqrt{3}$, (iv) 1, (v) 2

11. $\frac{8}{17}$

12. $\frac{8}{15}$

13. 7

14. $\frac{22}{7}$

15. $\frac{1}{9}$

16. 25

18. $\frac{1}{6}$

19. 3

EXERCISE # 2

Q.1 Find the value of $\left(\frac{3\pi}{5}\right)$ radians in degrees.

Q.2 Find the value of 150º in radians.

Q.3 If $\sin\theta = \frac{5}{13}$, then find the values of $\tan\theta$ and $\sec\theta$.

Q.4 If $\tan\theta = \frac{x}{y}$, then find the value of $\left(\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta}\right)$.

Q.5 If $5\tan\theta = 4$, find the value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}\right)$.

Q.6 If $16\cot x = 12$, then find the value of $\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$.

Q.7 If $\tan\theta = (3/4)$ and $0 < \theta < 90^\circ$, then find the value of $(\sin\theta\cos\theta)$.

Q.8 If $8\tan x = 15$, then find the value of $(\sin x - \cos x)$.

Q.9 If $\tan\theta = \frac{1}{\sqrt{7}}$, then find the value of $\left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right)$.

Q.10 If $\cot\theta = \frac{1}{\sqrt{3}}$, then find the value of $\left(\frac{1-\cos^2\theta}{2-\sin^2\theta}\right)$.

Q.11 If $\tan\theta = \frac{4}{3}$, then find the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$.

Q.12 If $3\cos\theta = 5\sin\theta$, then find the value of $\left(\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}\right)$.

Q.13 If $\tan\theta = (3/4)$, then find the value of $(\cos^2\theta - \sin^2\theta)$.

Q.14 Find the value of $\tan 75^\circ$.

Q.15 If $\tan\theta = \frac{a}{x}$, then find the value of $\frac{x}{\sqrt{a^2 + x^2}}$.

Q.16 If $3\sin x + 5\cos x = 5$, then the value of $(3\cos x - 5\sin x)^2$.

Q.17 Find the value of $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$.

Q.18 Find the value of $\sqrt{\frac{1+\sin A}{1-\sin A}}$.

Q.19 Find the value of $\sqrt{\frac{1-\sin A}{1+\sin A}}$.

Q.20 Find the value of $\sqrt{\frac{1-\cos x}{1+\cos x}}$.

Q.21 Find the value of $\sqrt{\frac{1+\cos x}{1-\cos x}}$.

Q.22 Find the value of $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$.

Q.23 Find the value of

$$\left(\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}\right).$$

Q.24 Find the value of

$$\left(\frac{\sin A + \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A - \sin B}\right).$$

Q.25 Find the value of $\sin 15^\circ$.

Q.26 Find the value of $(\sin 40^\circ - \cos 50^\circ)$.

Q.27 If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, then which is correct ?

(A) $x^2 + y^2 + z^2 = r^2$ (B) $x^2 - y^2 + z^2 = r^2$

(C) $x^2 + y^2 - z^2 = r^2$ (D) $-x^2 + y^2 + z^2 = r^2$

Q.28 Find the value of

$(\cot 15^\circ \cot 16^\circ \cot 17^\circ .... \cot 73^\circ \cot 74^\circ \cot 75^\circ)$.

ANSWER KEY

1. $108°$ **2.** $\left(\frac{5\pi}{6}\right)^c$ **3.** $\frac{5}{12}$ and $\frac{13}{12}$

4. $\frac{x^2+y^2}{x^2-y^2}$ **5.** $\frac{1}{6}$ **6.** $\frac{1}{7}$

7. $\frac{12}{25}$ **8.** $\frac{7}{17}$ **9.** $\frac{3}{4}$

10. $\frac{3}{5}$ **11.** $\frac{1}{3}$ **12.** $\frac{271}{979}$

13. $\frac{7}{25}$ **14.** $2-\sqrt{3}$ **15.** $\cos\theta$

16. 9 **17.** 2

18. sec A + tan A **19.** sec A – tan A **20.** cosec x – cot x

21. cosec x + cot x **22.** sec x – tan x **23.** 1

24. 0 **25.** $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **26.** 0

27. (A) **28.** 1