

SOLVED EXAMPLES

Ex. 1 Prove that $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

Sol. $D = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$ (By interchanging rows & columns)

$$= - \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix} \quad (C_1 \leftrightarrow C_2)$$

$$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} \quad (R_1 \leftrightarrow R_2)$$

Ex. 2 If $D_r = \begin{vmatrix} r & r^3 & 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix}$, find $\sum_{r=0}^n D_r$.

Sol. $\sum_{r=0}^n D_r = \begin{vmatrix} \sum_{r=0}^n r & \sum_{r=0}^n r^3 & \sum_{r=0}^n 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = \begin{vmatrix} \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = 0$

Ex. 3 Simplify $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Sol. Let $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Apply $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$

$$\begin{aligned}
 &= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} \\
 &= (a+b+c) ((b-c)(a-b) - (c-a)^2) \\
 &= (a+b+c) (ab+bc-ca-b^2-c^2+2ca-a^2) \\
 &= (a+b+c) (ab+bc+ca-a^2-b^2-c^2) \equiv 3abc - a^3 - b^3 - c^3
 \end{aligned}$$

Ex. 4 Determinant $\begin{vmatrix} a+b+nc & (n-1)a & (n-1)b \\ (n-1)c & b+c+na & (n-1)b \\ (n-1)c & (n-1)a & c+a+nb \end{vmatrix}$ is equal to -

Sol. Applying $C_1 \rightarrow C_1 + (C_2 + C_3)$

$$\begin{aligned}
 D &= n(a+b+c) \begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 1 & b+c+na & (n-1)b \\ 1 & (n-1)a & c+a+nb \end{vmatrix} \\
 D &= n(a+b+c) \begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} \begin{cases} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{cases} \\
 &= n(a+b+c)^3
 \end{aligned}$$

Ex. 5 Prove that $\begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix} = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$

Sol. $D = \begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$

Using factor theorem, **Put** $x=y$

$$D = \begin{vmatrix} (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$$

R_1 and R_2 are identical which makes $D=0$. Therefore, $(x-y)$ is a factor of D .

Similarly $(y-z)$ & $(z-x)$ are factors of D

Now put $a=b$

$$D = \begin{vmatrix} (x-b)^2 & (x-b)^2 & (x-c)^2 \\ (y-b)^2 & (y-b)^2 & (y-c)^2 \\ (z-b)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$$

C_1 and C_2 become identical which makes $D = 0$. Therefore, $(a-b)$ is a factor of D .

Similarly $(b-c)$ and $(c-a)$ are factors of D .

Therefore, $D = \lambda(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$

To get the value of λ put $x = -1 = a, y = 0 = b$ and $z = 1 = c$

$$D = \begin{vmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{vmatrix} = \lambda(-1)(-1)(2)(-1)(-1)(2)$$

$$\Rightarrow 4\lambda = 8 \Rightarrow \lambda = 2$$

$$\therefore D = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$$

Ex. 6 Prove that $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix} = -(1-\alpha^3)^2$.

Sol. This is a cyclic determinant.

$$\begin{aligned} \Rightarrow \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix} &= -(1+\alpha+\alpha^2)(1+\alpha^2+\alpha^4-\alpha-\alpha^2-\alpha^3) \\ &= -(1+\alpha+\alpha^2)(-\alpha+1-\alpha^3+\alpha^4) = -(1+\alpha+\alpha^2)(1-\alpha)^2(1+\alpha+\alpha^2) \\ &= -(1-\alpha)^2(1+\alpha+\alpha^2)^2 = -(1-\alpha^3)^2 \end{aligned}$$

Ex. 7 If the system of equations $x + \lambda y + 1 = 0, \lambda x + y + 1 = 0$ & $x + y + \lambda = 0$ is consistent then find the value of λ .

Sol. For consistency of the given system of equations

$$\begin{aligned} D &= \begin{vmatrix} 1 & \lambda & 1 \\ \lambda & 1 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \\ \Rightarrow 3\lambda &= 1 + 1 + \lambda^3 \text{ or } \lambda^3 - 3\lambda + 2 = 0 \Rightarrow (\lambda-1)^2(\lambda+2) = 0 \\ \Rightarrow \lambda &= 1 \text{ or } \lambda = -2 \end{aligned}$$

Ex. 8 For a 3×3 skew-symmetric matrix A , show that $\text{adj } A$ is a symmetric matrix.

Sol. $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ $\text{cof } A = \begin{bmatrix} c^2 & -bc & ca \\ -bc & b^2 & -ab \\ ca & -ab & a^2 \end{bmatrix}$

$$\text{adj } A = (\text{cof } A)' = \begin{bmatrix} c^2 & -bc & ca \\ -bc & b^2 & -ab \\ ca & -ab & a^2 \end{bmatrix} \text{ which is symmetric.}$$

Ex. 9 If $a, b, c, x, y, z \in \mathbb{R}$, then prove that

$$\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$$

Sol. L.H.S. = $\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} a^2 - 2ax + x^2 & b^2 - 2bx + x^2 & c^2 - 2cx + x^2 \\ a^2 - 2ay + y^2 & b^2 - 2by + y^2 & c^2 - 2cy + y^2 \\ a^2 - 2az + z^2 & b^2 - 2bz + z^2 & c^2 - 2az + z^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times \begin{vmatrix} a^2 & -2a & 1 \\ b^2 & -2b & 1 \\ c^2 & -2c & 1 \end{vmatrix} \text{ (Row by Row)}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times (-1) \begin{vmatrix} a^2 & 2a & 1 \\ b^2 & 2b & 1 \\ c^2 & 2c & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times (-1)(-1) \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix} (C_1 \leftrightarrow C_3)$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix}$$

Multiplying row by row

$$= \begin{vmatrix} 1+2ax+a^2x^2 & 1+2bx+b^2x^2 & 1+2cx+c^2x^2 \\ 1+2ay+a^2y^2 & 1+2by+b^2y^2 & 1+2cy+c^2y^2 \\ 1+2az+a^2z^2 & 1+2bz+b^2z^2 & 1+2cz+c^2z^2 \end{vmatrix}$$

$$= \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix} = \text{R.H.S.}$$

Ex. 10 For two non-singular matrices A & B, show that $\text{adj} (AB) = (\text{adj} B) (\text{adj} A)$

Sol. We have $(AB) (\text{adj} (AB)) = |AB| I_n$
 $= |A| |B| I_n$

$$A^{-1} (AB)(\text{adj} (AB)) = |A| |B| A^{-1}$$

$$\Rightarrow B \text{adj} (AB) = |B| \text{adj} A \quad (\rightarrow A^{-1} = \frac{1}{|A|} \text{adj} A)$$

$$\Rightarrow B^{-1} B \text{adj} (AB) = |B| B^{-1} \text{adj} A$$

$$\Rightarrow \text{adj} (AB) = (\text{adj} B) (\text{adj} A)$$

Ex. 11 If $D_1 = \begin{vmatrix} y^5 z^6 (z^3 - y^3) & x^4 z^6 (x^3 - z^3) & x^4 y^5 (y^3 - x^3) \\ y^2 z^3 (y^6 - z^6) & xz^3 (z^6 - x^6) & xy^2 (x^6 - y^6) \\ y^2 z^3 (z^3 - y^3) & xz^3 (x^3 - z^3) & xy^2 (y^3 - x^3) \end{vmatrix}$ and $D_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$. Then $D_1 D_2$ is equal to -

Sol. The given determinant D_1 is obtained by corresponding cofactors of determinant D_2 .

$$\text{Hence } D_1 = D_2^2$$

$$\Rightarrow D_1 D_2 = D_2^2 D_2 = D_2^3$$

Ex. 12 Obtain the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

Sol. Write $A = IA$, i.e., $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\text{or} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (\text{applying } R_1 \leftrightarrow R_2)$$

$$\text{or} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad (\text{applying } R_3 \rightarrow R_3 - 3R_1)$$

$$\text{or} \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad (\text{applying } R_1 \rightarrow R_1 - 2R_2)$$

$$\text{or} \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \quad (\text{applying } R_3 \rightarrow R_3 + 5R_2)$$

or
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A \quad (\text{applying } R_3 \rightarrow \frac{1}{2} R_3)$$

or
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A \quad (\text{Applying } R_1 \rightarrow R_1 + R_3)$$

or
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A \quad (\text{Applying } R_2 \rightarrow R_2 - 2R_3)$$

Hence
$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then $|3AB|$ is equal to
 (A) -9 (B) -81 (C) -27 (D) 81
2. If $\begin{vmatrix} \sin 2x & \cos^2 x & \cos 4x \\ \cos^2 x & \cos 2x & \sin^2 x \\ \cos^4 x & \sin^2 x & \sin 2x \end{vmatrix} = a_0 + a_1 (\sin x) + a_2 (\sin^2 x) + \dots + a_n (\sin^n x)$ then the value of a_0 is -
 (A) -1 (B) 1 (C) 0 (D) 2
3. Let $A = \begin{bmatrix} \cos^{-1} x & \cos^{-1} y & \cos^{-1} z \\ \cos^{-1} y & \cos^{-1} z & \cos^{-1} x \\ \cos^{-1} z & \cos^{-1} x & \cos^{-1} y \end{bmatrix}$ such that $|A| = 0$, then maximum value of $x + y + z$ is
 (A) 3 (B) 0 (C) 1 (D) 2
4. If $\begin{vmatrix} x^2 - 2x + 3 & 7x + 2 & x + 4 \\ 2x + 7 & x^2 - x + 2 & 3x \\ 3 & 2x - 1 & x^2 - 4x + 7 \end{vmatrix} = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ the value of g is
 (A) 2 (B) 1 (C) -2 (D) none of these
5. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5$ is equal to -
 (A) 0 (B) 25 (C) 625 (D) none of these
6. For any ΔABC , the value of determinant $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ is equal to -
 (A) 0 (B) 1 (C) $\sin A \sin B \sin C$ (D) $\sin A + \sin B + \sin C$
7. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in H.P. and $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, then the value of $[D]$ is
 (Where $[.]$ represents, the greatest integer fn.)
 (A) 0 (B) 1 (C) 2 (D) 3
8. Let $\Delta = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta \sin \phi & \sin \theta \cos \phi & 0 \end{vmatrix}$, then
 (A) Δ is independent of θ (B) Δ is independent of ϕ
 (C) Δ is a constant (D) none of these

9. If A is a square matrix of order 3 and A' denotes transpose of matrix A, $A' A = I$ and $\det A = 1$, then $\det (A - I)$ must be equal to

(A) 0 (B) -1 (C) 1 (D) none of these

10. The number of real values of x satisfying $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$ is -

(A) 3 (B) 0 (C) 1 (D) infinite

11. The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of :

(A) n (B) a (C) x (D) a, n and x

12. Three distinct points $P(3u^2, 2u^3)$; $Q(3v^2, 2v^3)$ and $R(3w^2, 2w^3)$ are collinear then

(A) $uv + vw + wu = 0$ (B) $uv + vw + wu = 3$
(C) $uv + vw + wu = 2$ (D) $uv + ww + wu = 1$

13. The determinant $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ is divisible by

(A) $1+x$ (B) $(1+x)^2$ (C) x^2 (D) x^2+1

14. $\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$ is equal to

(A) $(a-b)^2(b-c)^2(c-a)^2$ (B) $2(a-b)(b-c)(c-a)$
(C) $4(a-b)(b-c)(c-a)$ (D) $(a+b+c)^3$

15. If $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ then D =

(A) $1+a^2+b^2+c^2$ (B) $a^2+b^2+c^2$ (C) $(a+b+c)^2$ (D) none

16. Let $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix}$ then the maximum value of $f(x)$ is
- (A) 4 (B) 6 (C) 8 (D) 12
17. The determinant $\begin{vmatrix} b_1 + c_1 & c_1 + a_1 & a_1 + b_1 \\ b_2 + c_2 & c_2 + a_2 & a_2 + b_2 \\ b_3 + c_3 & c_3 + a_3 & a_3 + b_3 \end{vmatrix} =$
- (A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (B) $2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (C) $3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (D) none of these
18. If a, b, c are p th, q th and r th terms of a GP, then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to -
- (A) 0 (B) 1 (C) $\log abc$ (D) pqr
19. If $a_1, a_2, \dots, a_n, a_{n+1}, \dots$ are in GP and $a_i > 0 \forall i$, then $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to -
- (A) 0 (B) $n \log a_n$ (C) $n(n+1) \log a_n$ (D) none of these
20. The determinant $\begin{vmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) & \cos 2\phi \\ \sin \theta & \cos \theta & \sin \phi \\ -\cos \theta & \sin \theta & \cos \phi \end{vmatrix}$ is:
- (A) 0 (B) independent of θ (C) independent of ϕ (D) independent of θ & ϕ both
21. If A is a non singular matrix satisfying $AB - BA = A$, then which one of the following holds true
- (A) $\det. B = 0$ (B) $B = 0$ (C) $\det. A = 1$ (D) $\det. (B + I) = \det. (B - I)$
22. Let a determinant is given by $A = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ and suppose $\det. A = 6$. If $B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$ then
- (A) $\det. B = 6$ (B) $\det. B = -6$ (C) $\det. B = 12$ (D) $\det. B = -12$
23. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\frac{D_1}{D_2}$ where $b \neq 0$ and $ad \neq bc$, is
- (A) -2 (B) 0 (C) -2b (D) 2b

24. If $\Delta_1 = \begin{vmatrix} 2a & b & e \\ 2d & e & f \\ 4x & 2y & 2z \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} f & 2d & e \\ 2z & 4x & 2y \\ e & 2a & b \end{vmatrix}$, then the value of $\Delta_1 - \Delta_2$ is

- (A) $x + \frac{y}{2} + z$ (B) 2 (C) 0 (D) 3

25. If a, b, c are non zeros, then the system of equations

$$(\alpha + a)x + \alpha y + \alpha z = 0$$

$$\alpha x + (\alpha + b)y + \alpha z = 0$$

$$\alpha x + \alpha y + (\alpha + c)z = 0$$

has a non-trivial solution if

- (A) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$ (B) $\alpha^{-1} = a + b + c$
(C) $\alpha + a + b + c = 1$ (D) none of these

26. If A, B are two $n \times n$ non-singular matrices, then

- (A) AB is non-singular (B) AB is singular
(C) $(AB)^{-1} = A^{-1} B^{-1}$ (D) $(AB)^{-1}$ does not exist

27. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

- (A) has no real solution (B) has 4 real solutions
(C) has two real and two non-real solutions (D) has infinite number of solutions, real or non-real

28. The system of linear equations $x + y - z = 6$, $x + 2y - 3z = 14$ and $2x + 5y - \lambda z = 9$ ($\lambda \in \mathbb{R}$) has a unique solution if

- (A) $\lambda = 8$ (B) $\lambda \neq 8$ (C) $\lambda = 7$ (D) $\lambda \neq 7$

29. The value of 'k' for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$, $2x + 3y - 4z = 0$ has a non-trivial solution over the set of rational is:

- (A) $33/2$ (B) $31/2$ (C) 16 (D) 15

30. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$, then $\sum_{n=1}^N U_n$ is equal to

- (A) $2 \sum_{n=1}^N n$ (B) $2 \sum_{n=1}^N n^2$ (C) $\frac{1}{2} \sum_{n=1}^N n^2$ (D) 0

31. If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $x + 4y + \mu z = 3$ has an infinite number of solutions, then:

- (A) $p = 2, \mu = 3$ (B) $p = 2, \mu = 4$ (C) $3p = 2\mu$ (D) $p = 4, \mu = 2$

32. If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = 0$ then -
- (A) $a+b+c=0$ (B) $ab+bc+ca=0$
 (C) $a^2+b^2+c^2=ab+bc+ca$ (D) $abc=0$
33. If $a, b, \& c$ are nonzero real numbers, then $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to -
- (A) $a^2b^2c^2(a+b+c)$ (B) $abc(a+b+c)^2$ (C) zero (D) none of these
34. An equilateral triangle has each of its sides of length 6 cm. If $(x_1, y_1); (x_2, y_2) \& (x_3, y_3)$ are its vertices then the value of the determinant, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ is equal to -
- (A) 192 (B) 243 (C) 486 (D) 972
35. If the system of equations $x+2y+3z=4, x+py+2z=3, \mu x+4y+z=3$ has an infinite number of solutions, then -
- (A) $p=2, \mu=3$ (B) $p=2, \mu=4$ (C) $3p=2\mu$ (D) none of these
36. The values of θ, λ for which the following equations $\sin\theta x - \cos\theta y + (\lambda+1)z=0; \cos\theta x + \sin\theta y - \lambda z=0; \lambda x + (\lambda+1)y + \cos\theta z=0$ have non trivial solution, is
- (A) $\theta=n\pi, \lambda \in \mathbb{R} - \{0\}$ (B) $\theta=2n\pi, \lambda$ is any rational number
 (C) $\theta=(2n+1)\pi, \lambda \in \mathbb{R}^+, n \in \mathbb{I}$ (D) $\theta=(2n+1)\frac{\pi}{2}, \lambda \in \mathbb{R}, n \in \mathbb{I}$
37. The set of equations
- $$\begin{aligned} \lambda x - y + (\cos\theta)z &= 0 \\ 3x + y + 2z &= 0 \\ (\cos\theta)x + y + 2z &= 0 \end{aligned}$$
- $0 \leq \theta < 2\pi$, has non-trivial solution(s)
- (A) for no value of λ and θ (B) for all values of λ and θ
 (C) for all values of λ and only two values of θ (D) for only one value of λ and all values of θ
38. If A, B and C are $n \times n$ matrices and $\det(A)=2, \det(B)=3$ and $\det(C)=5$, then the value of the $\det(A^2BC^{-1})$ is equal to
- (A) $\frac{6}{5}$ (B) $\frac{12}{5}$ (C) $\frac{18}{5}$ (D) $\frac{24}{5}$
39. If a, b, c are real then the value of determinant $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1$ if
- (A) $a+b+c=0$ (B) $a+b+c=1$ (C) $a+b+c=-1$ (D) $a=b=c=0$

40. The system of equations $(\sin\theta)x + 2z = 0$, $(\cos\theta)x + (\sin\theta)y = 0$, $(\cos\theta)y + 2z = a$ has
 (A) no unique solution
 (B) a unique solution which is a function of a and θ
 (C) a unique solution which is independent of a and θ
 (D) a unique solution which is independent of θ only
41. If the system of equations

$$\begin{aligned} x - 2y + z &= a \\ 2x + y - 2z &= b \end{aligned}$$
 and $x + 3y - 3z = c$
 have atleast one solution, then the relationship between a , b and c is
 (A) $a + b + c = 0$ (B) $a - b + c = 0$ (C) $-a + b + c = 0$ (D) $a + b - c = 0$
42. The equation
$$\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$$

 (A) has no real solution (B) has 4 real solutions
 (C) has two real and two non-real solutions (D) has infinite number of solutions, real or non-real
43. Three digit numbers $x17$, $3y6$ and $12z$ where x, y, z are integers from 0 to 9, are divisible by a fixed constant k . Then the determinant
$$\begin{vmatrix} x & 3 & 1 \\ 7 & 6 & z \\ 1 & y & 2 \end{vmatrix}$$
 must be divisible by
 (A) k (B) k^2 (C) k^3 (D) None

Exercise # 2

Part # I

[Multiple Correct Choice Type Questions]

1. Which of the following determinant(s) vanish(es) ?

$$(A) \begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$$

$$(B) \begin{vmatrix} 1 & ab & \frac{1}{a} + \frac{1}{b} \\ 1 & bc & \frac{1}{b} + \frac{1}{c} \\ 1 & ca & \frac{1}{c} + \frac{1}{a} \end{vmatrix}$$

$$(C) \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$$

$$(D) \begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$$

2. If $D(x) = \begin{vmatrix} \sin 2x & e^x \sin x + x \cos x & \sin x + x^2 \cos x \\ \cos x + \sin x & e^x + x & 1 + x^2 \\ e^x \cos x & e^{2x} & e^x \end{vmatrix}$, then the value of $|D(x)|$ will be -

(A) independent of x

(B) dependent on x

(C) 0

(D) non-existent

3. The determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if -

(A) a, b, c are in AP

(B) a, b, c are in GP

(C) α is a root of the equation $ax^2 + bx + c = 0$ (D) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

4. If the system of linear equations $x + ay + az = 0$, $x + by + bz = 0$, $x + cy + cz = 0$ has a non-zero solution then

(A) System has always non-trivial solutions.

(B) System is consistent only when $a = b = c$ (C) If $a \neq b \neq c$ then $x = 0$, $y = t$, $z = -t \forall t \in \mathbb{R}$ (D) If $a = b = c$ then $y = t_1$, $z = t_2$, $x = -a(t_1 + t_2) \forall t_1, t_2 \in \mathbb{R}$

5. The value of θ lying between $\theta = 0$ & $\theta = \pi/2$ & satisfying the equation :

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0 \text{ are :}$$

(A) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$ (C) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$ 

6. Let $a, b > 0$ and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then
- (A) $a + b - x$ is a factor of Δ (B) $x^2 + (a + b)x + a^2 + b^2 - ab$ is a factor of Δ
 (C) $\Delta = 0$ has three real roots if $a = b$ (D) none of these
7. The determinant $\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix}$ is equal to zero if
- (A) b, c, d are in A.P. (B) b, c, d are in G.P.
 (C) b, c, d are in H.P. (D) α is a root of $ax^3 - bx^2 - 3cx - d = 0$
8. Let $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a + b & (a + b)^2 \\ 0 & 1 & 2a + 3b \end{vmatrix}$, then
- (A) $a + b$ is a factor of Δ (B) $a + 2b$ is a factor of Δ
 (C) $2a + 3b$ is a factor of Δ (D) a^2 is a factor of Δ
9. The value of θ lying between $-\frac{\pi}{4}$ & $\frac{\pi}{2}$ and $0 \leq A \leq \frac{\pi}{2}$ and satisfying the equation
- $$\begin{vmatrix} 1 + \sin^2 A & \cos^2 A & 2 \sin 4\theta \\ \sin^2 A & 1 + \cos^2 A & 2 \sin 4\theta \\ \sin^2 A & \cos^2 A & 1 + 2 \sin 4\theta \end{vmatrix} = 0$$
- are -
- (A) $A = \frac{\pi}{4}, \theta = -\frac{\pi}{8}$ (B) $A = \frac{3\pi}{8}, \theta = \frac{\pi}{8}$ (C) $A = \frac{\pi}{5}, \theta = -\frac{\pi}{8}$ (D) $A = \frac{\pi}{6}, \theta = \frac{3\pi}{8}$
10. If $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = 0$, then x may be equal to -
- (A) a (B) b (C) $a + b$ (D) m
11. The value of the determinant $\begin{vmatrix} \alpha & \beta & 1 \\ \alpha & x & n \\ \alpha & \beta & x \end{vmatrix}$ is
- (A) independent of α (B) independent of n (C) $\alpha(x - \beta)(x - \beta)$ (D) $\alpha\beta(x - \beta)(x - n)$
12. The determinant $\begin{vmatrix} a^2 & a^2 - (b - c)^2 & bc \\ b^2 & b^2 - (c - a)^2 & ca \\ c^2 & c^2 - (a - b)^2 & ab \end{vmatrix}$ is divisible by -
- (A) $a + b + c$ (B) $(a + b)(b + c)(c + a)$ (C) $a^2 + b^2 + c^2$ (D) $(a - b)(b - c)(c - a)$

13. Let A and B be two 2×2 matrix with real entries. If $AB = O$ and $\text{tr}(A) = \text{tr}(B) = 0$ then
 (A) A and B are comutative w.r.t. operation of multiplication.
 (B) A and B are not commutative w.r.t. operation of multiplication.
 (C) A and B are both null matrices.
 (D) $BA = 0$
14. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then
 (A) $|A| = 2$ (B) A is non-singular
 (C) $\text{Adj. } A = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$ (D) A is skew symmetric matrix
15. If the system of equations $x + y - 3 = 0$, $(1 + K)x + (2 + K)y - 8 = 0$ & $x - (1 + K)y + (2 + K) = 0$ is consistent then the value of K may be -
 (A) 1 (B) $\frac{3}{5}$ (C) $-\frac{5}{3}$ (D) 2
16. The set of equations $x - y + 3z = 2$, $2x - y + z = 4$, $x - 2y + \alpha z = 3$ has -
 (A) unique solution only for $\alpha = 0$
 (B) unique solution for $\alpha \neq 8$
 (C) infinite number of solutions of $\alpha = 8$
 (D) no solution for $\alpha = 8$
17. The determinant

$$\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$$
 is divisible by
 (A) x (B) x^2 (C) x^3 (D) x^4
18. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then
 (A) $a + d = 0$ (B) $k = -|A|$ (C) $k = |A|$ (D) none of these
19. If the system of equations, $a^2x - by = a^2 - b$ & $bx - b^2y = 2 + 4b$ possess an infinite number of solutions then the possible values of 'a' and 'b' are
 (A) $a = 1$, $b = -1$ (B) $a = 1$, $b = -2$
 (C) $a = -1$, $b = -1$ (D) $a = -1$, $b = -2$

20. If p, q, r, s are in A.P. and $f(x) = \begin{vmatrix} p + \sin x & q + \sin x & p - r + \sin x \\ q + \sin x & r + \sin x & -1 + \sin x \\ r + \sin x & s + \sin x & s - q + \sin x \end{vmatrix}$

such that $\int_0^2 f(x) dx = -4$ then the common difference of the A.P. can be :

- (A) -1 (B) $\frac{1}{2}$ (C) 1 (D) 2

Part # II

[Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

- (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
 (B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
 (C) Statement-I is true, Statement-II is false.
 (D) Statement-I is false, Statement-II is true.

1. **Statement - I :** Consider the system of equations,

$$2x + 3y + 4z = 5 \quad x + y + z = 1 \quad x + 2y + 3z = 4$$

This system of equations has infinite solutions.

Statement - II : If the system of equations is

$$e_1 : a_1x + b_1y + c_1z - d_1 = 0$$

$$e_2 : a_2x + b_2y + c_2z - d_2 = 0$$

$$e_3 : e_1 + \lambda e_2 = 0, \text{ where } \lambda \in \mathbb{R} \text{ \& } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Then such system of equations has infinite solutions.

2. **Statement - I :** Consider $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Let B_1, B_2, B_3 be the co-factors of $b_1, b_2,$ and b_3 respectively then $a_1B_1 + a_2B_2 + a_3B_3 = 0$

Statement - II : If any two rows (or columns) in a determinant are identical then value of determinant is zero.

3. **Statement - I :** If $a, b, c \in \mathbb{R}$ and $a \neq b \neq c$ and x, y, z are non zero. Then the system of equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0 \text{ has infinite solutions.}$$

Statement - II : If the homogeneous system of equations has non trivial solution, then it has infinitely many solutions.

4. **Statement-I :** If $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$ then $\det(A)$ is real.

Statement-II : If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers then $\det(A)$ is always real.



Exercise # 3

Part # I

[Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. Column-I

Column-II

- | | |
|---|--------|
| (A) If A is a skew-symmetric matrix of odd order then $\det(A)$ is | (p) 7 |
| (B) If A is a square matrix such that $A^2 = A$ and $(I + A)^3 = I + kA$, then k is equal to | (q) 8 |
| (C) If A and B are two invertible matrices such that $AB = C$ and $ A = 2, C = -2$, then $\det(B)$ is | (r) 0 |
| (D) If $A = [a_{ij}]_{3 \times 3}$ is a scalar matrix with $a_{11} = a_{22} = a_{33} = 2$ and $A(\text{adj}A) = kI_3$ then k is | (s) -1 |

2. Column-I

Column-II

- | | |
|---|-------|
| (A) If the determinant $\begin{vmatrix} a+p & 1+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the values of K is | (p) 3 |
| (B) The values of λ for which the system of equations
$\begin{aligned} x + y + z &= 6, \\ x + 2y + 3z &= 10 \\ \& \quad x + 2y + \lambda z &= 12 \end{aligned}$ is inconsistent | (q) 8 |
| (C) If x, y, z are in A.P. then the value of the determinant
$\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}$ is | (r) 5 |
| (D) Let p be the sum of all possible determinants of order 2 having 0, 1, 2 & 3 as their four elements (without repetition of digits). The value of 'p' is | (s) 0 |

Comprehension # 1

Consider the system of linear equations

$$\alpha x + y + z = m$$

$$x + \alpha y + z = n$$

and $x + y + \alpha z = p$

On the basis of above information, answer the following questions :

- If $\alpha \neq 1, -2$ then the system has -
 (A) no solution (B) infinite solutions
 (C) unique solution (D) trivial solution if $m \neq n \neq p$
- If $\alpha = -2$ & $m + n + p \neq 0$ then system of linear equations has -
 (A) no solution (B) infinite solutions (C) unique solution (D) finitely many solution
- If $\alpha = 1$ & $m \neq p$ then the system of linear equations has -
 (A) no solution (B) infinite solutions (C) unique solution (D) unique solution if $p = n$

Comprehension # 2

Let $x, y, z \in \mathbb{R}^+$ & $D = \begin{vmatrix} x & x^3 & x^4 - 1 \\ y & y^3 & y^4 - 1 \\ z & z^3 & z^4 - 1 \end{vmatrix}$

On the basis of above information, answer the following questions

- If $x \neq y \neq z$ & x, y, z are in GP and $D = 0$, then y is equal to -
 (A) 1 (B) 2 (C) 4 (D) none of these
- If x, y, z are the roots of $t^3 - 21t^2 + bt - 343 = 0$, $b \in \mathbb{R}$, then D is equal to-
 (A) 1 (B) 0 (C) dependent on x, y, z (D) data inadequate
- If $x \neq y \neq z$ & x, y, z are in A.P. and $D = 0$, then $2xy^2z + x^2z^2$ is equal to-
 (A) 1 (B) 2 (C) 3 (D) none of these

Exercise # 4

[Subjective Type Questions]

1. Without expanding the determinant prove that :

$$(A) \begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$$

$$(B) \begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$$

2. Prove that :

$$(A) \begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$$

$$(B) \begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$$

3. If $a^2 + b^2 + c^2 = 1$, then prove that $\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1 - \cos\phi) & ac(1 - \cos\phi) \\ ba(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1 - \cos\phi) \\ ca(1 - \cos\phi) & cb(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$

is independent of a, b, c

4. Prove that $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3$.

5. If $a, b, c, x, y, z \in \mathbb{R}$, then prove that,

$$\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}.$$

6. Using properties of determinants, prove that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$.

7. Prove that : $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

8. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

9. For a fixed positive integer n , if $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3} - 4 \right]$ is divisible by n .

10. If $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$ then prove that $\sum_{r=1}^n D_r = 0$.

11. Using the properties of determinants, evaluate the following : $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$.

12. Using properties of determinants prove the following : $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

13. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.

14. Solve the following sets of equations using Cramer's rule and remark about their consistency.

$x + y + z - 6 = 0$

(A) $2x + y - z - 1 = 0$

$x + y - 2z + 3 = 0$

$x - 3y + z = 2$

(C) $3x + y + z = 6$

$5x + y + 3z = 3$

$x + 2y + z = 1$

(B) $3x + y + z = 6$

$x + 2y = 0$

$7x - 7y + 5z = 3$

(D) $3x + y + 5z = 7$

$2x + 3y + 5z = 5$

15. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations

$\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$.

Exercise # 5

Part # I

[Previous Year Questions] [AIEEE/JEE-MAIN]

1. If a, b, c are p th, q th and r th terms of a GP, and all are positive then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to- [AIEEE-2002]
- (1) 0 (2) 1 (3) $\log abc$ (4) pqr
2. If $1, \omega, \omega^2$ are cube roots of unity and $n \neq 3p, p \in \mathbb{Z}$, then $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is equal to- [AIEEE-2003]
- (1) 0 (2) ω (3) ω^2 (4) 1
3. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2), (1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals- [AIEEE-2003]
- (1) 1 (2) 0 (3) 2 (4) -1
4. If $a_1, a_2, \dots, a_n, a_{n+1}, \dots$ are in GP and $a_i > 0 \forall i$, then $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to- [AIEEE-2004,05]
- (1) 0 (2) $n \log a_n$ (3) $n(n+1) \log a_n$ (4) none of these
5. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then $f(x)$ is a polynomial of degree- [AIEEE 2005]
- (1) 2 (2) 3 (3) 0 (4) 1
6. The system of equations $\alpha x + y + z = \alpha - 1; x + \alpha y + z = \alpha - 1; x + y + \alpha z = \alpha - 1$ has no solution, If α is [AIEEE 2005]
- (1) 1 (2) not -2 (3) either -2 or 1 (4) -2
7. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is- [AIEEE - 2007]
- (1) Divisible by both x and y (2) Divisible by x but not y
(3) Divisible by y but not x (4) Divisible by neither x nor y
8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$, if $|A^2| = 25$ then $|\alpha|$ equals- [AIEEE - 2007]
- (1) 5 (2) 5^2 (3) 1 (4) $1/5$
9. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that

$x = cy + bz$, $y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc$ is equal to

- (1) 2 (2) -1 (3) 0 (4) 1

[AIEEE - 2008]

10. Let a, b, c be such that $b(a + c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$,

then the value of n is :-

- (1) Any odd integer (2) Any integer (3) Zero (4) Any even integer

[AIEEE - 2009]

11. Consider the system of linear equations :

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ 3x_1 + 5x_2 + 2x_3 &= 1 \end{aligned}$$

The system has

- (1) Infinite number of solutions (2) Exactly 3 solutions
(3) A unique solution (4) No solution

[AIEEE - 2010]

12. The number of values of k for which the linear equations

$$\begin{aligned} 4x + ky + 2z &= 0 \\ kx + 4y + z &= 0 \\ 2x + 2y + z &= 0 \end{aligned}$$

possess a non-zero solution is :-

- (1) 1 (2) zero (3) 3 (4) 2

[AIEEE - 2011]

13. If the trivial solution is the only solution of the system of equations

$$\begin{aligned} x - ky + z &= 0 \\ kx + 3y - kz &= 0 \\ 3x + y - z &= 0 \end{aligned}$$

Then the set of all values of k is:

- (1) $\{2, -3\}$ (2) $\mathbb{R} - \{2, -3\}$ (3) $\mathbb{R} - \{2\}$ (4) $\mathbb{R} - \{-3\}$

[AIEEE - 2011]

14. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to :

- (1) $\alpha\beta$ (2) $\frac{1}{\alpha\beta}$ (3) 1 (4) -1

[Main 2014]

15. The set of all values of λ for which the system of linear equations :

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$

has a non-trivial solution,

- (1) contains two elements (2) contains more than two elements..
(3) is an empty set. (4) is a singleton

[Main 2015]

1. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$ [REE 2001, (Mains)]

2. Test the consistency and solve them when consistent, the following system of equations for all values of λ :

$$x + y + z = 1$$

$$x + 3y - 2z = \lambda$$

$$3x + (\lambda + 2)y - 3z = 2\lambda + 1$$

[JEE 2001, (Mains)]

3. Let a, b, c , be real numbers with $a^2 + b^2 + c^2 = 1$, Show that the equation

[JEE 2001, (Mains)]

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0 \text{ represents a straight line.}$$

4. The number of values of k for which the system of equations

$$(k+1)x + 8y = 4k$$

$$kx + (k+3)y = 3k - 1$$

has infinitely many solutions is

[JEE 2002, (Screening)]

- (A) 0 (B) 1 (C) 2 (D) infinite

5. The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is

[JEE 2004 (Screening)]

- (A) 3 (B) -3 (C) 2 (D) -2

6. (a) Consider three point $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and

$$R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta)), \text{ where } 0 < \alpha, \beta, \theta < \pi/4$$

- (A) P lies on the line segment RQ (B) Q lies on the line segment PR
(C) R lies on the line segment QP (D) P, Q, R are non collinear

- (b) Consider the system of equations $x - 2y + 3z = -1$; $-x + y - 2z = k$; $x - 3y + 4z = 1$.

Statement-I : The system of equations has no solution for $k \neq 3$.

Statement-II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

[JEE 2008]

- (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.



7. The number of all possible values of θ , where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

[JEE 2010]

8. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

[JEE Ad. 2014]

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3×3 matrix U , if $(M^2 + MN^2)U$ equal the zero matrix then U is the zero matrix

9. Which of the following values of α satisfy the equation

[JEE Ad. 2015]

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = 648\alpha$$

(A) -4

(B) 9

(C) -9

(D) 4

10. The total number of distinct $x \in \mathbb{R}$ for which
- $$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$
- is

[JEE Ad. 2016]

11. Let $a, \lambda, \mu \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statements (s) is (are) correct ?

[JEE Ad. 2016]

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If A is a square matrix of order 3 such that $|A| = 2$ then $|\text{adj } A^{-1}|^{-1}$ is
 (A) 1 (B) 2 (C) 4 (D) 8
2. If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$, then:
 (A) $k = -3$ (B) $k = -1$ (C) $k = 1$ (D) $k = 3$
3. If $p + q + r = 0 = a + b + c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is
 (A) 0 (B) $pa + qb + rc$ (C) 1 (D) none of these
4. If the determinant $\begin{vmatrix} a + p & 1 + x & u + f \\ b + q & m + y & v + g \\ c + r & n + z & w + h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K =
 (A) 6 (B) 7 (C) 8 (D) 9
5. $|A_{3 \times 3}| = 3$, $|B_{3 \times 3}| = -1$ and $|C_{2 \times 2}| = +2$ then $|2ABC| =$
 (A) $2^3(6)$ (B) $2^3(-6)$ (C) $2(-6)$ (D) none of these
6. If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$, then:
 (A) $abc(ab + bc + ca) = a + b + c$ (B) $(a + b + c)(ab + bc + ca) = abc$
 (C) $abc(a + b + c) = ab + bc + ca$ (D) none of these
7. If A is a non-diagonal involutory matrix, then
 (A) $A - I = O$ (B) $A + I = O$
 (C) $A - I$ is non zero singular (D) none of these
8. If $f(x) = \log_{10} x$ and $g(x) = e^{i\pi x}$ and $h(x) = \begin{vmatrix} f(x)g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2)g(x^2) & [f(x^2)]^{g(x^2)} & 0 \\ f(x^3)g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$, then the value of $h(10)$ is
 (A) 0 (B) 2 (C) 1 (D) 4

9. If a, b, c , are real numbers, and $D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$ then D is

- (A) purely real (B) purely imaginary (C) non real (D) integer

10. S_1 : The value of the determinant $D = \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln 2x & \ln 2y & \ln 2z \\ \ln 3x & \ln 3y & \ln 3z \end{vmatrix}$ is $\bullet n 216 xyz$.

S_2 : The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = 0$ are independent of λ, μ, v, a, b

S_3 : If a, b, c , are sides of a scalene triangle, then value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

S_4 : Let $f(x) = \begin{vmatrix} \frac{1}{x} & \ln x & x^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, if $f^{(n)}(x)$ is the n^{th} derivative of $f(x)$ then $f^{(n)}(1)$ is independent of a .

- (A) FFFT (B) FTTT (C) FFFT (D) TTTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If A and B are invertible square matrices of the same order, then which of the following is correct ?

- (A) $\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$
 (B) $(\text{adj}A)' = (\text{adj}A')$
 (C) $|\text{adj}A| = |A|^{n-1}$, where n is the order of matrix A
 (D) $\text{adj}(\text{adj}B) = |B|^{n-2} B$, where n is the order of matrix B

12. Let $f(x) = \begin{vmatrix} 2\sin x & \sin^2 x & 0 \\ 1 & 2\sin x & \sin^2 x \\ 0 & 1 & 2\sin x \end{vmatrix}$, then

- (A) $f(x)$ is independent of x (B) $f'(\pi/2) = 0$
 (C) $\int_{-\pi/2}^{\pi/2} f(x)dx = 0$ (D) tangent to the curve $y = f(x)$ at $x = 0$ is $y = 0$

13. System of equation $\begin{matrix} x + 3y + 2z = 6 & x + \lambda y + 2z = 7 \\ x + 3y + 2z = \mu & \text{has} \end{matrix}$

- (A) unique solution if $\lambda = 2, \mu \neq 6$ (B) infinitely many solution if $\lambda = 4, \mu = 6$
 (C) no solution if $\lambda = 5, \mu = 7$ (D) no solution if $\lambda = 3, \mu = 5$



14. Let $f(x) = \begin{vmatrix} 1/x & \log x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, then
- (A) $f^n(1)$ is independent of a
 (B) $f^n(1)$ is independent of n
 (C) $f^n(1)$ depends on a and n
 (D) $y = a(x - f^n(1))$ represents a straight line through the origin
15. Which of the following statement is always true
- (A) Adjoint of a symmetric matrix is a symmetric matrix
 (B) Adjoint of a unit matrix is unit matrix
 (C) $A(\text{adj } A) = (\text{adj } A)A$
 (D) Adjoint of a diagonal matrix is diagonal matrix

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** The system of equations possess a non trivial solution over the set of rationals $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$, then the value of k is $31/2$.
Statement-II : For non trivial solution $\Delta = 0$.
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
17. **Statement-I :** The determinants of a matrix $A = [a_{ij}]_{5 \times 5}$ where $a_{ij} + a_{ji} = 0$ for i and j is zero
Statement-II : The determinant of a skew symmetric matrix of odd order is zero.
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
18. **Statement-I :** For a singular square matrix A , if $AB = AC \Rightarrow B = C$
Statement-II : If $|A| = 0$ then A^{-1} does not exist
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
19. **Statement-I :** $(a_{11}, a_{22}, \dots, a_{nn})$ is a diagonal matrix then $A^{-1} = \text{dia}(a_{11}^{-1}, a_{22}^{-1}, \dots, a_{nn}^{-1})$
Statement-II : If $A = \text{dia}(2, 1, -3)$ and $B = \text{dia}(1, 1, 2)$ then $\det(AB^{-1}) = 3$
 (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True

20. **Statement-I :** If a, b, c are distinct and x, y, z are not all zero and $ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0$, then $a + b + c = 0$.

Statement-II : $a^2 + b^2 + c^2 > ab + bc + ca$, if a, b, c are distinct.

- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. Match the following

Column - I

(A) If $\Delta(x) = \begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 + 6x + 9 & 14x - 6 & 21 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then

(B) If $\Delta(x) = \begin{vmatrix} x - 1 & 5x & 7 \\ x^2 - 1 & x - 1 & 8 \\ 2x & 3x & 0 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then

(C) If $\Delta(x) = \begin{vmatrix} 2x^3 - 3x^2 & 5x + 7 & 2 \\ 4x^3 - 7x & 3x + 2 & 1 \\ 7x^3 - 8x^2 & x - 1 & 3 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then

Column - II

(p) $3a + 4b + 5c + d = 141$

(q) $a + 2b + 3c + 5d = 156$

(r) $c - d = 19$

(s) $b - c = 25$

(t) $3a + 2b + 5c + 5d = 187$

22. Match the following

Column - I

(A) Let $|A| = |a_{ij}|_{3 \times 3} \neq 0$. Each element a_{ij} is multiplied by k_i^{-j} . Let $|B|$ the resulting determinant, where $k_1|A| + k_2|B| = 0$. Then $k_1 + k_2 =$

(B) The maximum value of a third order determinant each of its entries are ± 1 equals

(C) $\begin{vmatrix} 1 & \cos \alpha & \cos \beta \\ \cos \alpha & 1 & \cos \gamma \\ \cos \beta & \cos \gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos \alpha & \cos \beta \\ \cos \alpha & 0 & \cos \gamma \\ \cos \beta & \cos \gamma & 0 \end{vmatrix}$

if $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma =$

(D) $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = Ax + B$ where A and B

are determinants of order 3. Then $A + 2B =$

Column - II

(p) 0

(q) 4

(r) 1

(s) 2

(t) $\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Consider the determinant $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$

M_{ij} = Minor of the element of i^{th} row and j^{th} column

C_{ij} = Cofactor of the element of i^{th} row and j^{th} column

- Value of $b_1 \cdot C_{31} + b_2 \cdot C_{32} + b_3 \cdot C_{33}$ is
(A) 0 (B) Δ (C) 2Δ (D) Δ^2
- If all the elements of the determinant are multiplied by 2, then the value of new determinant is
(A) 0 (B) 8Δ (C) 2Δ (D) $2^9 \cdot \Delta$
- $a_3 M_{13} - b_3 \cdot M_{23} + d_3 \cdot M_{33}$ is equal to
(A) 0 (B) 4Δ (C) 2Δ (D) Δ

24. Read the following comprehension carefully and answer the questions.

Let $\Delta \neq 0$ and Δ^c denotes the determinant of cofactors, then $\Delta = \Delta^{n-1}$, where $n(> 0)$ is the order of Δ .

- If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of $\begin{vmatrix} bc - a^2 & ca - b^2 & c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix}$ is
(A) p^2 (B) p^4 (C) p^6 (D) p^9

- If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are real quantities satisfying the six relations :

$$l_1^2 + m_1^2 + n_1^2 = l_2^2 + m_2^2 + n_2^2 = l_3^2 + m_3^2 + n_3^2 = 1$$

$$l_2 l_3 + m_2 m_3 + n_2 n_3 = l_3 l_1 + m_3 m_1 + n_3 n_1 = l_1 l_2 + m_1 m_2 + n_1 n_2 = 0, \text{ then the value of } \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \text{ is}$$

- (A) 0 (B) ± 1 (C) ± 2 (D) ± 3
- If a, b, c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$ is
(A) 9 (B) 27 (C) 81 (D) 0

25. Read the following comprehension carefully and answer the questions.

Let A be a $m \times n$ matrix. If there exists a matrix L of type $n \times m$ such that $LA = I_n$, then L is called left inverse of A . Similarly, if there exists a matrix R of type $n \times m$ such that $AR = I_m$, then R is called right inverse of A .

For example to find right inverse of matrix



$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \text{ we take } R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} \text{ and solve } AR = I_3 \text{ i.e.}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{lll} x - u = 1 & y - v = 0 & z - w = 0 \\ x + u = 0 & y + v = 1 & z + w = 0 \\ 2x + 3u = 0 & 2y + 3v = 0 & 2z + 3w = 1 \end{array}$$

As this system of equations is inconsistent, we say there is no right inverse for matrix A.

1. Which of the following matrices is NOT left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$

(A) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

2. The number of right inverses for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

(A) 0

(B) 1

(C) 2

(D) infinite

3. For which of the following matrices number of left inverses is greater than the number of right inverses

(A) $\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix}$

SECTION - VI : INTEGER TYPE

26. If $f(x)$ satisfies the equation $\begin{vmatrix} f(x+1) & f(x+8) & f(x+1) \\ 1 & 2 & -5 \\ 2 & 3 & \lambda \end{vmatrix} = 0$ for all real x . If f is periodic with period 7, then find the value of $|\lambda|$.

27. Let $f(x) = \begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If $f(x)$ be an odd function and its odd values is equal $g(x)$, then find the value of λ .

If $f(1)g(1) = -4\lambda$



28. If $f(x) = \begin{vmatrix} \cos(x + \alpha) & \cos(x + \beta) & \cos(x + \gamma) \\ \sin(x + \alpha) & \sin(x + \beta) & \sin(x + \gamma) \\ \sin(\beta - \gamma) & \sin(\gamma - \alpha) & \sin(\alpha - \beta) \end{vmatrix}$ and $f(2) = 6$, then find $\frac{1}{5} \sum_{r=1}^{25} f(r)$,

29. Find the coefficient of x in the determinant $\begin{vmatrix} (1+x)^{a_1 b_1} & (1+x)^{a_1 b_2} & (1+x)^{a_1 b_3} \\ (1+x)^{a_2 b_1} & (1+x)^{a_2 b_2} & (1+x)^{a_2 b_3} \\ (1+x)^{a_3 b_1} & (1+x)^{a_3 b_2} & (1+x)^{a_3 b_3} \end{vmatrix}$, where $a_i, b_j \in \mathbb{N}$

30. If $p, q \in \mathbb{R}$ and $p^2 + q^2 - pq - p - q + 1 \leq 0$, $\alpha + \beta + \gamma = 0$, then $\begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & p & \cos \alpha \\ \cos \beta & \cos \alpha & q \end{vmatrix} = 0$.

ANSWER KEY

EXERCISE - 1

1. B 2. A 3. A 4. D 5. D 6. A 7. C 8. B 9. A 10. D 11. A 12. A 13. C
14. A 15. A 16. B 17. B 18. A 19. A 20. B 21. D 22. C 23. A 24. C 25. A 26. A
27. D 28. B 29. A 30. B 31. D 32. A 33. C 34. D 35. D 36. D 37. A 38. B 39. D
40. B 41. B 42. D 43. A

EXERCISE - 2 : PART # I

1. ABCD 2. AC 3. BD 4. ACD 5. AC 6. ABC 7. BD 8. AB 9. ABCD
10. AB 11. BC 12. ACD 13. AD 14. BC 15. AC 16. BD 17. ABCD 18. AC
19. ABCD 20. AC

PART - II

1. A 2. A 3. A 4. C

EXERCISE - 3 : PART # I

1. $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$ 2. $A \rightarrow q$ $B \rightarrow p$ $C \rightarrow s$ $D \rightarrow s$

PART - II

Comprehension #1 : 1. C 2. A 3. A Comprehension #2 : 1. A 2. D 3. C

Comprehension #3 : 1. A 2. B 3. C

EXERCISE - 5 : PART # I

1. 1 2. 1 3. 4 4. 1 5. 1 6. 4 7. 1 8. 4 9. 4 10. 1 11. 4 12. 4 13. 2
14. 3 15. 2

PART - II

1. $x = n\pi, n \in \mathbb{I}$
2. If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,

$$y = \frac{1}{2}(3K + 4) \text{ and } x = -\frac{1}{2}(5K + 2) \text{ where } K \in \mathbb{R}$$

If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.

4. B 5. D 6. A.D B.A 7. 3 8. AB 9. BC 10. 2 11. BCD



MOCK TEST

1. C 2. B 3. A 4. C 5. D 6. A 7. C 8. A 9. A
10. A 11. ABCD 12. BCD 13. BCD 14. ABD 15. ABCD 16. D 17. A 18. D
19. C 20. A
21. $A \rightarrow p$ $B \rightarrow q, s$ $C \rightarrow r, t$ 22. $A \rightarrow p, t$ $B \rightarrow q$ $C \rightarrow r$ $D \rightarrow p, t$
23. 1. A 2. B 3. D 24. 1. C 2. B 3. D 25. 1. C 2. D 3. C
26. 4 27. 1 28. 3 29. 0 30. 0