SOLVED EXAMPLES

Ex. 1 Evaluate : $\int_{-10}^{20} [\cot^{-1} x]dx$. Here [.] is the greatest integer function.

Sol. $I = \int_{-10}^{20} [\cot^{-1} x]dx$, we know $\cot^{-1} x \in (0, \pi)\ \forall\ x \in R$

Thus $[\cot^{-1} x] = \begin{cases} 3, & x \in (-\infty, \cot 3) \\ 2, & x \in (\cot 3, \cot 2) \\ 1, & x \in (\cot 2, \cot 1) \\ 0 & x \in (\cot 1, \infty) \end{cases}$

Hence $I = \int_{-10}^{\cot 3} 3\,dx + \int_{\cot 3}^{\cot 2} 2\,dx + \int_{\cot 2}^{\cot 1} 1\,dx + \int_{\cot 1}^{20} 0\,dx = 30 + \cot 1 + \cot 2 + \cot 3$

Ex. 2 Evaluate $\int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right) dx.$

Sol. Let $f(x) = \log_e\left(\frac{2-x}{2+x}\right)$

$\Rightarrow$ $f(-x) = \log_e\left(\frac{2+x}{2-x}\right) = -\log_e\left(\frac{2-x}{2+x}\right) = -f(x)$

i.e. f(x) is odd function

$\therefore$ $\int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right) dx = 0$

Ex. 3 Evaluate $\int_{-\pi}^{\pi} \frac{x \sin x}{e^x + 1} dx$

Sol. $I = \int_{-\pi}^{0} \frac{x \sin x}{e^x + 1} dx + \int_{0}^{\pi} \frac{x \sin x}{e^x + 1} dx = I_1 + I_2$

where $I_1 = \int_{-\pi}^{0} \frac{x \sin x}{e^x + 1} dx$

Put $x = -t$

$\Rightarrow$ $dx = -\,dt$

$\Rightarrow$ $I_1 = \int_{\pi}^{0} \frac{(-t)\sin(-t)(-dt)}{e^{-t} + 1} = \int_{0}^{\pi} \frac{t \sin t\,dt}{e^{-t} + 1} = \int_{0}^{\pi} \frac{e^t\, t \sin t\,dt}{e^t + 1} = \int_{0}^{\pi} \frac{e^x\, x \sin x\,dx}{e^x + 1}$

Hence $I = I_1 + I_2 = \int_{0}^{\pi} \frac{e^x\, x \sin x}{e^x + 1} dx + \int_{0}^{\pi} \frac{x \sin x}{e^x + 1} dx$

$I = \int_{0}^{\pi} x \sin x dx = \int_{0}^{\pi} (\pi - x) \sin(\pi - x)\, dx = \pi \int_{0}^{\pi} \sin x dx - I$

$\Rightarrow$ $2I = \pi \int_{0}^{\pi} \sin x dx = \pi \left|-\cos x\right|_0^{\pi} = 2\pi \Rightarrow I = \pi$

Ex. 4 Evaluate $\int_0^{\pi} \frac{dx}{1+2\sin^2 x}$ dx.

Sol. Let $f(x) = \frac{1}{1+2\sin^2 x}$

$\Rightarrow$ $f(\pi - x) = f(x)$

$\Rightarrow$ $\int_0^{\pi} \frac{dx}{1+2\sin^2 x} = 2\int_0^{\frac{\pi}{2}} \frac{dx}{1+2\sin^2 x} = 2\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{1+\tan^2 x + 2\tan^2 x}$

$$= 2\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{1+3\tan^2 x} = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\frac{\pi}{2}}$$

✈ $\tan\frac{\pi}{2}$ is undefined, we take limit

$$= \frac{2}{\sqrt{3}}\left[\underset{x\to\frac{\pi}{2}^-}{\text{Lt}} \tan^{-1}\left(\sqrt{3}\tan x\right) - \tan^{-1}\left(\sqrt{3}\tan 0\right)\right]$$

$$= \frac{2}{\sqrt{3}}\,\frac{\pi}{2} = \frac{\pi}{\sqrt{3}}$$

Ex. 5 If $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2 x/2 \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2} (x^2+1)(f(x)+f''(x))dx$

Sol. As, $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2 x/2 \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$

$\Rightarrow$ $f(-x) = -f(x)$ $\Rightarrow$ f(x) is odd

$\Rightarrow$ f'(x) is even $\Rightarrow$ f''(x) is odd

Thus, f(x) + f''(x) is odd function let,

$\phi(x) = (x^2+1).\{f(x)+f''(x)\}$

$\Rightarrow$ $\phi(-x) = -\phi(x)$

i.e. $\phi(x)$ is odd

$$\therefore \int_{-\pi/2}^{\pi/2} \phi(x)dx = 0$$

Ex. 6 For $x \in (0, 1)$ arrange $f_1(x) = \dfrac{1}{\sqrt{4-x^2}}$, $f_2(x) = \dfrac{1}{\sqrt{4-2x^2}}$ and $f_3(x) = \dfrac{1}{\sqrt{4-x^2-x^3}}$ in ascending order and hence prove that $\dfrac{\pi}{6} < \displaystyle\int_0^1 \dfrac{dx}{\sqrt{4-x^2-x^3}} < \dfrac{\pi}{4\sqrt{2}}$.

Sol. ∵ $0 < x^3 < x^2 \Rightarrow x^2 < x^2 + x^3 < 2x^2$

$\Rightarrow -2x^2 < -x^2 - x^3 < -x^2$

$\Rightarrow 4 - 2x^2 < 4 - x^2 - x^3 < 4 - x^2$

$\Rightarrow \sqrt{4-2x^2} < \sqrt{4-x^2-x^3} < \sqrt{4-x^2}$

$\Rightarrow f_1(x) < f_3(x) < f_2(x)$ for $x \in (0, 1)$

$$\Rightarrow \int_0^1 f_1(x)\,dx < \int_0^1 f_3(x)\,dx < \int_0^1 f_2(x)\,dx$$

$$\sin^{-1}\left(\frac{x}{2}\right)\Big]_0^1 < \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} < \frac{1}{\sqrt{2}}\sin^{-1}\frac{x}{\sqrt{2}}\Big]_0^1$$

$$\frac{\pi}{6} < \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} < \frac{\pi}{4\sqrt{2}}$$

Ex. 7 Evaluate $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

Sol. $$I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad .....(i)$$

then $$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}\,dx}{\sqrt{\cos(\pi/2 - x)}+\sqrt{\sin(\pi/2 - x)}} = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad .....(ii)$$

Adding **(i)** and **(ii)**, we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}.dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}.dx = \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$2I = \dfrac{\pi}{2} \Rightarrow I = \dfrac{\pi}{4}$

Ex. 8 Evaluate $\int_0^1 \frac{\tan^{-1}(ax)}{x\sqrt{1-x^2}}\,dx$, 'a' being parameter.

Sol. Let $I(a) = \int_0^1 \frac{\tan^{-1}(ax)}{x\sqrt{1-x^2}}\,dx$

$$\frac{dI(a)}{da} = \int_0^1 \frac{x}{(1+a^2x^2)}\,\frac{1}{x\sqrt{1-x^2}}\,dx = \int_0^1 \frac{dx}{(1+a^2x^2)\sqrt{1-x^2}}$$

Put $x = \sin t \Rightarrow dx = \cos t\,dt$

L.L. : $x = 0 \Rightarrow t = 0$

U.L. : $x = 1 \Rightarrow t = \frac{\pi}{2}$

$$\frac{dI(a)}{da} = \int_0^{\pi/2} \frac{1}{1+a^2\sin^2 t}\,\frac{1}{\cos t}\,\cos t\,dt = \int_0^{\pi/2} \frac{dt}{1+a^2\sin^2 t}$$

$$= \int_0^{\pi/2} \frac{\sec^2 t\,dt}{1+(1+a^2)\tan^2 t} = \frac{1}{\sqrt{1+a^2}}\tan^{-1}\left(\sqrt{1+a^2}\,\tan t\right)\Big]_0^{\pi/2} = \frac{1}{\sqrt{1+a^2}}\cdot\frac{\pi}{2}$$

$\Rightarrow$ $I(a) = \frac{\pi}{2}\ln\left(a+\sqrt{1+a^2}\right)+c$

But $I(0) = 0 \Rightarrow c = 0$

$\Rightarrow$ $I(a) = \frac{\pi}{2}\ln\left(a+\sqrt{1+a^2}\right)$

Ex. 9 Evaluate $\int_0^2 \frac{dx}{(17+8x-4x^2)[e^{6(1-x)}+1]}$

Sol. Let $I = \int_0^2 \frac{dx}{(17+8x-4x^2)[e^{6(1-x)}+1]}$

Also $I = \int_0^2 \frac{dx}{(17+8x-4x^2)[e^{-6(1-x)}+1]}$ $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

Adding, we get

$$2I = \int_0^2 \frac{1}{17+8x-4x^2}\left(\frac{1}{e^{6(1-x)}+1}+\frac{1}{e^{-6(1-x)}+1}\right)dx$$

$$= \int_0^2 \frac{1}{17+8x-4x^2}\,dx = -\frac{1}{4}\int_0^2 \frac{dx}{x^2-2x-17/4}$$

$$= -\frac{1}{4}\int_0^2 \frac{dx}{(x-1)^2 - 21/4} = -\frac{1}{4} \times \frac{1}{2 \times \frac{\sqrt{21}}{2}}\left[\log\left|\frac{x-1-\frac{\sqrt{21}}{2}}{x-1+\frac{\sqrt{21}}{2}}\right|\right]_0^2$$

$$= -\frac{1}{4\sqrt{21}}\left[\log\left|\frac{2x-2-\sqrt{21}}{2x-2+\sqrt{21}}\right|\right]_0^2$$

$$\Rightarrow \quad I = -\frac{1}{8\sqrt{21}}\left[\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right| - \log\left|\frac{2+\sqrt{21}}{\sqrt{21}-2}\right|\right]$$

$$= -\frac{1}{4\sqrt{21}}\left[\log\left|\frac{\sqrt{21}-2}{2+\sqrt{21}}\right|\right]$$

Ex. 10 Prove that $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2$

Sol. Let $I = \int_0^{\pi/2} \log(\sin x)dx$ .... **(i)**

then $I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2}-x\right)dx = \int_0^{\pi/2} \log(\cos x)dx$ .... **(ii)**

adding **(i)** and **(ii)**, we get

$$2I = \int_0^{\pi/2} \log\sin x\, dx + \int_0^{\pi/2} \log\cos x\, dx = \int_0^{\pi/2} (\log\sin x + \log\cos x)dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \log(\sin x\cos x)dx = \int_0^{\pi/2} \log\left(\frac{2\sin x\cos x}{2}\right)dx$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right)dx = \int_0^{\pi/2} \log(\sin 2x)dx - \int_0^{\pi/2} (\log 2)dx$$

$$= \int_0^{\pi/2} \log\sin 2x.dx - (\log 2)(x)_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \log(\sin 2x)dx - \frac{\pi}{2}\log 2 \quad \text{......... (iii)}$$

Let $I_1 = \int_0^{\pi/2} \log(\sin 2x)dx$, putting $2x = t$, we get

$$I_1 = \int_0^{\pi} \log(\sin t)\frac{dt}{2} = \frac{1}{2}\int_0^{\pi} \log(\sin t)dt = \frac{1}{2}.2\int_0^{\pi/2} \log(\sin t)dt$$

$$I_1 = \int_0^{\pi/2} \log(\sin x)dx$$

$\therefore$ **(iii)** becomes ; $2I = I - \frac{\pi}{2}\log 2$

Hence $\int_0^{\pi/2} \log\sin x\,dx = -\frac{\pi}{2}\log 2$

Ex. 11 Evaluate $\underset{n\to\infty}{\text{Lt}} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$.

Sol. Let $y = \underset{n\to\infty}{\text{Lt}} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$

$$\ln y = \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\ln\left(\frac{n!}{n^n}\right)$$

$$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\ln\left(\frac{1.2.3 ....n}{n^n}\right)$$

$$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\left[\ln\left(\frac{1}{n}\right)+\ln\left(\frac{2}{n}\right)+\ln\left(\frac{3}{n}\right)+.....+\ln\left(\frac{n}{n}\right)\right]$$

$$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right) = \int_0^1 \ln x\,dx = \Big[x\ln x - x\Big]_0^1$$

$$= (0-1) - \underset{x\to 0^+}{\text{Lt}} x\,\lambda n\,x + 0$$

$$= -1 - 0 = -1 \quad \Rightarrow \quad y = \frac{1}{e}$$

Ex. 12 $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx$

Sol. $I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx$ .....**(i)**

$I = \int_0^{\pi/2} \frac{a\sin(\pi/2 - x) + b\cos(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}dx = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\sin x + \cos x}dx$ .....**(ii)**

$\therefore$ $2I = \int_0^{\pi/2} \frac{(a+b)(\sin x + \cos x)}{\sin x + \cos x}dx = \int_0^{\pi/2}(a+b)dx = (a+b)\pi/2 \Rightarrow I = (a+b)\pi/4$

Ex. 13 Prove that $\left|\int_{10}^{19} \frac{\sin x}{1+x^8} dx\right| < 10^{-7}$

Sol. To find $I = \left|\int_{10}^{19} \frac{\sin x}{1+x^8} dx\right| \le \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx$ .... **(i)**

Since $|\sin x| \le 1$ for $x \ge 10$

The inequality $\left|\frac{\sin x}{1+x^8}\right| \le \frac{1}{|1+x^8|}$ .... **(ii)**

also, $10 \le x \le 19$

$\Rightarrow$ $1 + x^8 > 10^8$

$\Rightarrow$ $\frac{1}{1+x^8} < \frac{1}{10^8}$ or $\frac{1}{|1+x^8|} < 10^{-8}$ .... **(iii)**

from **(ii)** and **(iii)** ;

$$\left|\frac{\sin x}{1+x^8}\right| < 10^{-8}$$

$$\left|\int_{10}^{19} \frac{\sin x}{1+x^8} dx\right| < \int_{10}^{19} 10^{-8} dx$$

$\therefore$ $$\left|\int_{10}^{19} \frac{\sin x}{1+x^8} dx\right| < (19-10).10^{-8} < 10^{-7}$$

Ex. 14 Evaluate $\int_0^{n\pi+v} |\cos x| \, dx$, $\frac{\pi}{2} < v < \pi$ and $n \in z$.

Sol. $\int_0^{n\pi+v} |\cos x| \, dx = \int_0^{v} |\cos x| \, dx + \int_v^{n\pi+v} |\cos x| \, dx$

$= \int_0^{\frac{\pi}{2}} \cos x - \int_{\pi/2}^{v} \cos x \, dx + n \int_0^{\pi} |\cos x| \, dx$

$= (1-0) - (\sin v - 1) + 2n \int_0^{\frac{\pi}{2}} \cos x \, dx$

$= 2 - \sin v + 2n(1-0) = 2n + 2 - \sin v$

Ex. 15 Evaluate : $\int_0^{2n\pi} [\sin x + \cos x] dx$. Here [.] is the greatest integer function.

Sol. Let $I = \int_0^{2n\pi} [\sin x + \cos x] dx = n \int_0^{2\pi} [\sin x + \cos x] dx$

(✈ [sinx + cosx] is periodic function with period 2π]

$$[\sin x + \cos x] = \begin{cases} 1, & 0 \le x \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le x \le \frac{3\pi}{4} \\ -1, & \frac{3\pi}{4} < x \le \pi \\ -2, & \pi < x \le \frac{3\pi}{2} \\ -1, & \frac{3\pi}{2} < x \le \frac{7\pi}{4} \\ 0, & \frac{7\pi}{4} < x \le 2\pi \end{cases}$$

Hence $I = n\left[\int_0^{\pi/2} 1\,dx + \int_{\pi/2}^{3\pi/4} 0\,dx + \int_{3\pi/4}^{\pi} -1\,dx + \int_{\pi}^{3\pi/2} -2\,dx + \int_{3\pi/2}^{7\pi/4} -1\,dx + \int_{7\pi/4}^{2\pi} 0\,dx\right]$

$$I = n\left[\frac{\pi}{2} + 0 - \pi + \frac{3\pi}{4} - 3\pi + 2\pi - \frac{7\pi}{4} + \frac{3\pi}{2} + 0\right] = -n\pi$$

Ex. 16 Let f be an injective function such that $f(x)\,f(y) + 2 = f(x) + f(y) + f(xy)$ for all non negative real x and y with $f(0) = 1$ and $f'(1) = 2$ find f(x) and show that $3\int f(x)dx - x(f(x)+2)$ is a constant.

Sol. We have $f(x)f(y) + 2 = f(x) + f(y) + f(xy)$

Putting $x = 1$ & $y = 1$

then $f(1)f(1) + 2 = 3f(1)$

we get $f(1) = 1,2$

$f(1) \ne 1$ ($\because$ $f(0) = 1$ & function is injective)

then $f(1) = 2$

Replacing y by $\frac{1}{x}$ in (1) then

$$f(x)f\left(\frac{1}{x}\right) + 2 = f(x) + f\left(\frac{1}{x}\right) + f(1) \Rightarrow f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

Hence f(x) is of the type

$f(x) = 1 \pm x^n$

$\because$ $f(1) = 2$

$\therefore$ $f(x) = 1 + x^n$

and $f'(x) = nx^{n-1} \Rightarrow f'(1) = n = 2$

$f(x) = 1 + x^2$

$\therefore$ $3\int f(x)dx - x(f(x)+2) = 3\int (1 + x^2)dx - x(1 + x^2 + 2)$

$$= 3\left(x + \frac{x^3}{3}\right) - x(3 + x^2) + c = c = \text{constant}$$

Ex. 17 If $F(x) = \int_{e^{2x}}^{e^{3x}} \frac{t}{\log_e t}\,dt$, then find first and second derivative of F(x) with respect to $\ln x$ at $x = \ln 2$.

Sol. $\frac{dF(x)}{d(\ln x)} = \frac{dF(x)}{dx}\,\frac{dx}{d(\ln x)} = \left[3 \,.\, e^{3x} \,.\, \frac{e^{3x}}{\ln e^{3x}} - 2 \,.\, e^{2x}\,\frac{e^{2x}}{\ln e^{2x}}\right] x = e^{6x} - e^{4x}.$

$$\frac{d^2F(x)}{d(\ln x)^2} = \frac{d}{d(\ln x)}(e^{6x} - e^{4x}) = \frac{d}{dx}(e^{6x} - e^{4x}) \times \frac{1}{d\ln x / dx} = (6\,e^{6x} - 4\,e^{4x})\,x$$

First derivative of F(x) at $x = \ln 2$ (i.e. $e^x = 2$) is $2^6 - 2^4 = 48$

Second derivative of F(x) at $x = \ln 2$ (i.e. $e^x = 2$) is $(6\,.\,2^6 - 4\,.\,2^4)\,.\ln 2 = 5\,.\,2^6\,.\ln 2.$

Ex. 18 Evaluate : $\int_{-1}^{1} [x[1 + \sin \pi x] + 1]dx$, [.] is the greatest integer function.

Sol. Let $I = \int_{-1}^{1} [x[1 + \sin \pi x] + 1]dx = \int_{-1}^{0} [x[1 + \sin \pi x] + 1]dx + \int_{0}^{1} [x[1 + \sin \pi x] + 1]dx$

Now $[1 + \sin\pi x] = 0$ if $-1 < x < 0$

$[1 + \sin\pi x] = 1$ if $0 < x < 1$

$\therefore \quad I = \int_{-1}^{0} 1\,.dx + \int_{0}^{1} [x + 1]dx = 1 + 1\int_{0}^{1} dx = 1 + 1 = 2.$

Ex. 19 Evaluate $\int_{0}^{1} \frac{x^b - 1}{\ln x}$, 'b' being parameter.

Sol. Let $I(b) = \int_{0}^{1} \frac{x^b - 1}{\ln x}\,dx$

$$\frac{dI(b)}{db} = \int_{0}^{1} \frac{x^b \ln x}{\ln x}\,dx + 0 - 0$$

(using modified Leibnitz Theorem)

$$= \int_{0}^{1} x^b\,dx = \left.\frac{x^{b+1}}{b+1}\right]_0^1 = \frac{1}{b+1}$$

$I(b) = \ln(b + 1) + c$

$b = 0 \quad \Rightarrow \quad I(0) = 0$

$\therefore \quad c = 0 \quad \therefore \quad I(b) = \ln(b+1)$

Ex. 20 Evaluate : $\int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{(\pi^2 - 3\pi x + 3x^2)} dx$

Sol. Let $I = \int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{(\pi^2 - 3\pi x + 3x^2)} dx$ .... (i)

$$= \int_0^{\pi} \frac{(\pi - x)^3 \cos^4(\pi - x)\sin^2(\pi - x)dx}{\pi^2 - 3\pi(\pi - x) + 3(\pi - x)^2} \quad \text{(By. Prop.)}$$

$$= \int_0^{\pi} \frac{(\pi^3 - x^3 - 3\pi^2 x + 3\pi x^2)\cos^4 x \sin^2 x}{(\pi^2 - 3\pi x + 3x^2)} dx \quad \text{......... (ii)}$$

Adding (i) and (ii) we have

$$2I = \int_0^{\pi} \frac{(\pi^3 - 3\pi^2 x + 3\pi x^2)\cos^4 x \sin^2 x}{(\pi^2 - 3\pi x + 3x^2)} dx$$

$\Rightarrow$ $2I = \pi \int_0^{\pi} \cos^4 x \sin^2 x dx$ $\Rightarrow$ $2I = 2\pi \int_0^{\pi/2} \cos^4 x \sin^2 x dx$

$\therefore$ $I = \pi \int_0^{\pi/2} \cos^4 x \sin^2 x\, dx$

Using walli's formula, we get $I = \pi \frac{(3.1)(1)}{6.4.2} \frac{\pi}{2} = \frac{\pi^2}{32}$

Ex. 21 If $f(x) = \begin{cases} 1 - |x|, & |x| \le 1 \\ |x| - 1, & |x| > 1 \end{cases}$, and $g(x) = f(x-1) + f(x+1)$. Find the value of $\int_{-3}^{5} g(x)dx$.

Sol. Given,

$$f(x) = \begin{cases} -x - 1, & x < -1 \\ 1 + x, & -1 \le x < 0 \\ 1 - x, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases} \; ; \; f(x-1) = \begin{cases} -x, & x - 1 < -1 & \Rightarrow x < 0 \\ x, & -1 \le x - 1 < 0 & \Rightarrow 0 \le x < 1 \\ 2 - x, & 0 \le x - 1 \le 1 & \Rightarrow 1 \le x \le 2 \\ x - 2, & x - 1 > 1 & \Rightarrow x > 2 \end{cases}$$

Similarly

$$f(x+1) = \begin{cases} -x - 2, & x + 1 < -1 & \Rightarrow x < -2 \\ x + 2, & -1 \le x + 1 < 0 & \Rightarrow -2 \le x < -1 \\ -x, & 0 \le x + 1 \le 1 & \Rightarrow -1 \le x \le 0 \\ x, & x + 1 > 1 & \Rightarrow x > 0 \end{cases}$$

$$\Rightarrow \quad g(x) = f(x-1) + f(x+1) = \begin{cases} -2x - 2 & x < -2 \\ 2, & -2 \le x < -1 \\ -2x, & -1 \le x \le 0 \\ 2x, & 0 < x < 1 \\ 2, & 1 < x \le 2 \\ 2x - 2, & 2 < x \end{cases}$$

Clearly g(x) is even,

Now $\int_{-3}^{5} g(x)dx = 2\int_0^3 g(x)dx + \int_3^5 g(x)dx = 2\left(\int_0^1 2x\, dx + \int_1^2 2\, dx + \int_2^3 (2x - 2)\, dx\right) + \int_3^5 (2x - 2)dx = 24$

Ex. 22 Evaluate $\int_0^{\pi} x\sin^5 x\cos^6 x\,dx$.

Sol. Let $I = \int_0^{\pi} x\sin^5 x\cos^6 x\,dx$

$$I = \int_0^{\pi} (\pi - x)\sin^5(\pi - x)\cos^6(\pi - x)\,dx = \pi\int_0^{\pi} \sin^5 .\cos^6 x\,dx - \int_0^{\pi} x\sin^5 x.\cos^6 x\,dx$$

$\Rightarrow$ $$2I = \pi.2\int_0^{\frac{\pi}{2}} \sin^5 x.\cos^6 x\,dx$$

$$I = \pi\frac{4.2.5.3.1}{11.9.7.5.3.1}$$

$$I = \frac{8\pi}{693}$$

Ex. 23 Find the limit, when $n \to \infty$ of

$$\frac{1}{\sqrt{(2n - 1^2)}} + \frac{1}{\sqrt{(4n - 2^2)}} + \frac{1}{\sqrt{(6n - 3^2)}} + ... + \frac{1}{n}$$

Sol. Let $$P = \lim_{n\to\infty}\left[\frac{1}{\sqrt{2n - 1^2}} + \frac{1}{\sqrt{4n - 2^2}} + \frac{1}{\sqrt{6n - 3^2}} + ... + \frac{1}{n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{\sqrt{1(2n) - 1^2}} + \frac{1}{\sqrt{2(2n) - 2^2}} + \frac{1}{\sqrt{3(2n) - 3^2}} + ... + \frac{1}{\sqrt{n(2n) - n^2}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{r(2n) - r^2}} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n.\sqrt{2\frac{r}{n} - \left(\frac{r}{n}\right)^2}} = \int_0^1 \frac{dx}{\sqrt{(2x - x^2)}}$$

Put $x = t^2 \Rightarrow dx = 2t\,dt$

$\therefore$ $$P = \int_0^1 \frac{2t\,dt}{t\sqrt{2 - t^2}} = \left[2\sin^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^1 = 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{\pi}{4}\right)$$

Hence $P = \pi/2$.

Exercise # 1

[Single Correct Choice Type Questions]

1. The value of the definite integral $\int_0^{\pi/2} \sin|2x - \alpha|\,dx$ where $\alpha \in [0, \pi]$

(A) 1 **(B)** $\cos\alpha$ **(C)** $\frac{1+\cos\alpha}{2}$ **(D)** $\frac{1-\cos\alpha}{2}$

2. Value of the definite integral $\int_{-1/2}^{1/2} (\sin^{-1}(3x - 4x^3) - \cos^{-1}(4x^3 - 3x))\,dx$

(A) 0 **(B)** $-\frac{\pi}{2}$ **(C)** $\frac{7\pi}{2}$ **(D)** $\frac{\pi}{2}$

3. If $\int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x}\,dx = k\log\left(\frac{3+2\sqrt{3}}{3}\right)$ then k is-

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{8}$

4. Suppose for every integer n, $\int_n^{n+1} f(x)\,dx = n^2$. The value of $\int_{-2}^{4} f(x)\,dx$ is :

(A) 16 **(B)** 14 **(C)** 19 **(D)** 21

5. If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t\,dt}{1+t^4}$ then $f'(2)$

(A) equals 2/17 **(B)** equals 0 **(C)** equals 1 **(D)** cannot be determined

6. $\int_{e^{e^e}}^{e^{e^{e^e}}} \frac{dx}{x\ln x \cdot \ln(\ln x) \cdot \ln(\ln(\ln x))}$ equals

(A) 1 **(B)** $\frac{1}{e}$ **(C)** $e - 1$ **(D)** $1 + e$

7. The value of the definite integral $\int_1^e ((x+1)e^x . \ln x)\,dx$ is -

(A) e **(B)** e^{e+1} **(C)** $e^e(e-1)$ **(D)** $e^e(e-1) + e$

8. $\int_0^{2n\pi} \left(|\sin x| - \left[\left|\frac{\sin x}{2}\right|\right]\right) dx$ (where [ ] denotes the greatest integer function and $n \in I$) is equal to :

(A) 0 **(B)** 2n **(C)** $2n\pi$ **(D)** 4n

9. $\int_2^4 \left[\log_x 2 - \frac{(\log_x 2)^2}{\ln 2}\right] dx =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

10. Suppose that F(x) is an antiderivative of $f(x)=\frac{\sin x}{x}$, $x>0$ then $\int_1^3 \frac{\sin 2x}{x}\,dx$ can be expressed as -

(A) $F(6)-F(2)$ **(B)** $\frac{1}{2}(F(6)-F(2))$ **(C)** $\frac{1}{2}(F(3)-F(1))$ **(D)** $2(F(6)-F(2))$

11. The value of the definite integral, $\int_0^{\sqrt{\ln\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right)\cdot 2x\,e^{x^2}\,dx$ is

(A) 1 **(B)** $1+(\sin 1)$ **(C)** $1-(\sin 1)$ **(D)** $(\sin 1)-1$

12. The value of the integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$, where $0<\alpha<\frac{\pi}{2}$, is equal to:

(A) $\sin\alpha$ **(B)** $\alpha\sin\alpha$ **(C)** $\frac{\alpha}{2\sin\alpha}$ **(D)** $\frac{\alpha}{2}\sin\alpha$

13. If x satisfies the equation $\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right)x^2-\left(\int_{-3}^{3}\frac{t^2\sin 2t}{t^2+1}dt\right)x-2=0$ $(0<\alpha<\pi)$, then the value x is

(A) $\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ **(B)** $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ **(C)** $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ **(D)** $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

14. The value of the definite integral $\int_0^\infty \frac{dx}{(1+x^a)(1+x^2)}$ $(a>0)$ is

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** π **(D)** some function of a.

15. The value of the definite integral $\int_1^\infty (e^{x+1}+e^{3-x})^{-1}\,dx$ is

(A) $\frac{\pi}{4e^2}$ **(B)** $\frac{\pi}{4e}$ **(C)** $\frac{1}{e^2}\left(\frac{\pi}{2}-\tan^{-1}\frac{1}{e}\right)$ **(D)** $\frac{\pi}{2e^2}$

16. Let f be a continuous functions satisfying $f'(\ln x)=\begin{cases}1 & \text{for } 0<x\le 1\\ x & \text{for } x>1\end{cases}$ and $f(0)=0$ then $f(x)$ can be defined as

(A) $f(x)=\begin{cases}1 & \text{if } x\le 0\\ 1-e^x & \text{if } x>0\end{cases}$ **(B)** $f(x)=\begin{cases}1 & \text{if } x\le 0\\ e^x-1 & \text{if } x>0\end{cases}$

(C) $f(x)=\begin{cases}x & \text{if } x<0\\ e^x & \text{if } x>0\end{cases}$ **(D)** $f(x)=\begin{cases}x & \text{if } x\le 0\\ e^x-1 & \text{if } x>0\end{cases}$

17. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x\, dx = 5$, then f(0) is equal to : (it is given that f(x) is continuous in $[0, \pi]$)

(A) 7 **(B)** 3 **(C)** 5 **(D)** 1

18. $\lim_{n\to\infty} \frac{\pi}{6n}\left[\sec^2\left(\frac{\pi}{6n}\right) + \sec^2\left(2\cdot\frac{\pi}{6n}\right) + ... + \sec^2(n-1)\frac{\pi}{6n} + \frac{4}{3}\right]$ has the value equal to

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $\frac{2}{\sqrt{3}}$

19. $\lim_{\lambda\to 0}\left(\int_0^1 (1+x)^{\lambda} dx\right)^{1/\lambda}$ is equal to

(A) $2\ln 2$ **(B)** $\frac{4}{e}$ **(C)** $\ln\frac{4}{e}$ **(D)** 4

20. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of g'(0) is

(A) 1 **(B)** 17 **(C)** $\sqrt{17}$ **(D)** none of these

21. $\int_{\ln\pi - \ln 2}^{\ln\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)} dx$ is equal to

(A) $\sqrt{3}$ **(B)** $-\sqrt{3}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** $-\frac{1}{\sqrt{3}}$

22. The value of the definite integral $\int_0^{3\pi/4} \left((1+x)\sin x + (1-x)\cos x\right) dx$, is

(A) $2\tan\frac{3\pi}{8}$ **(B)** $2\tan\frac{\pi}{4}$ **(C)** $2\tan\frac{\pi}{8}$ **(D)** 0

23. Let a, b, c be non-zero real numbers such that ; $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$, then the quadratic equation $ax^2 + bx + c = 0$ has -

(A) no root in (0,2)
(B) atleast one root in (0,2)
(C) a double root in (0,2)
(D) none

24. Let f be a one-to-one continuous function such that $f(2) = 3$ and $f(5) = 7$. Given $\int_2^5 f(x)dx = 17$, then the value of the definite integral $\int_3^7 f^{-1}(x)dx$ equals

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13

25. If $\int_a^y \cos t^2 dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{2\sin^2 x}{x\cos^2 y}$ **(B)** $\frac{2\sin x^2}{x\cos y^2}$ **(C)** $\frac{2\sin x^2}{x\left(1-2\sin\frac{y^2}{2}\right)}$ **(D)** none of these

26. If $g(x)=\int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(A) $g(x)+g(\pi)$ **(B)** $g(x)-g(\pi)$ **(C)** $g(x)\,g(\pi)$ **(D)** $[g(x)/g(\pi)]$

27. Let $S(x)=\int_{x^2}^{x^3} \ln t\,dt\ (x>0)$ and $H(x)=\dfrac{S'(x)}{x}$. Then H(x) is :

(A) continuous but not derivable in its domain
(B) derivable and continuous in its domain
(C) neither derivable nor continuous in its domain
(D) derivable but not continuous in its domain.

28. The expression $\dfrac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$, where [x] and {x} are integral and fractional parts of x and $n \in N$, is equal to :

(A) $\dfrac{1}{n-1}$ **(B)** $\dfrac{1}{n}$ **(C)** n **(D)** $n-1$

29. The true set of values of 'a' for which the inequality $\int_a^0 (3^{-2x} - 2.\,3^{-x})\,dx \ge 0$ is true is:

(A) $[0,1]$ **(B)** $(-\infty,-1]$ **(C)** $[0,\infty)$ **(D)** $(-\infty,-1]\cup[0,\infty)$

30. The value of $\int_0^1 \left(\prod_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\dfrac{1}{x+k}\right)dx$ equals

(A) n **(B)** $n\,!$ **(C)** $(n+1)\,!$ **(D)** $n\cdot n\,!$

31. The value of the definite integral $\int_0^{\pi/2} \sin x \sin 2x \sin 3x\,dx$ is equal to :

(A) $\dfrac{1}{3}$ **(B)** $-\dfrac{2}{3}$ **(C)** $-\dfrac{1}{3}$ **(D)** $\dfrac{1}{6}$

32. If $g(x)=\int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals -

(A) $g(x)+g(\pi)$ **(B)** $g(x)-g(\pi)$ **(C)** $g(x)g(\pi)$ **(D)** $\dfrac{g(x)}{g(\pi)}$

33. $\lim_{n\to\infty} \sum_{r=1}^{n}\left(\dfrac{r^3}{r^4+n^4}\right)$ equals to :

(A) ●n 2 **(B)** $\dfrac{1}{2}$ ●n 2 **(C)** $\dfrac{1}{3}$ ●n 2 **(D)** $\dfrac{1}{4}$ ●n 2

34. Let $a_n=\int_0^{\pi/2}(1-\sin t)^n \sin 2t\,dt$ then $\lim_{n\to\infty}\sum_{n=1}^{n}\dfrac{a_n}{n}$ is equal to

(A) 1/2 **(B)** 1 **(C)** 4/3 **(D)** 3/2

35. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\lim_{n\to\infty} n(I_n + I_{n-2}) =$

(A) 1 (B) 1/2 (C) ∞ (D) 0

36. $\int_2^3 \frac{(x+2)^2}{2x^2-10x+53} dx$ is equal to -

(A) 2 (B) 1 (C) 1/2 (D) 5/2

37. The slope of the tangent to the curve $y = \int_x^{x^2} \cos^{-1} t^2$ at $x = \frac{1}{\sqrt[4]{2}}$ is

(A) $\left(\frac{\sqrt[4]{8}}{2} - \frac{3}{4}\right)\pi$ (B) $\left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4}\right)\pi$ (C) $\left(\frac{\sqrt[5]{8}}{4} - \frac{1}{3}\right)\pi$ (D) None of these

38. A function $f(x)$ satisfies $f(x) = \sin x + \int_0^x f'(t)(2\sin t - \sin^2 t)\,dt$ then $f(x)$ is

(A) $\frac{x}{1-\sin x}$ (B) $\frac{\sin x}{1-\sin x}$ (C) $\frac{1-\cos x}{\cos x}$ (D) $\frac{\tan x}{1-\sin x}$

39. The value of $\sqrt{\pi\left(\int_0^{2008} x|\sin \pi x|dx\right)}$ is equal to

(A) $\sqrt{2008}$ (B) $\pi\sqrt{2008}$ (C) 1004 (D) 2008

40. For any integer n the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1)x dx$ has the value

(A) π (B) 1 (C) 0 (D) none of these

41. The value of the definite integral $\int_0^{\pi/2} \sqrt{\tan x}\, dx$, is

(A) $\sqrt{2}\pi$ (B) $\frac{\pi}{\sqrt{2}}$ (C) $2\sqrt{2}\pi$ (D) $\frac{\pi}{2\sqrt{2}}$

42. Let $f: R \to R$, $g: R \to R$ be continuous functions. Then the value of integeral $\int_{\ln\lambda}^{\ln 1/\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x)+g(-x)]} dx$ is :

(A) depend on λ (B) a non-zero constant (C) zero (D) none of these

43. If g (x) is the inverse of f (x) and f (x) has domain $x \in [1, 5]$, where f (1) = 2 and f (5) = 10 then the values of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$ equals

(A) 48 (B) 64 (C) 71 (D) 52

44. Let $f : R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim\limits_{x\to 1}\int_{4}^{f(x)} \frac{2t}{x-1}\,dt$ is -

(A) $8f'(1)$ **(B)** $4f'(1)$ **(C)** $2f'(1)$ **(D)** $f'(1)$

45. The value of $\int_{-1}^{1} \frac{dx}{(2-x)\sqrt{1-x^2}}$ is

(A) 0 **(B)** $\frac{\pi}{\sqrt{3}}$ **(C)** $\frac{2\pi}{\sqrt{3}}$ **(D)** cannot be evaluated

46. $\lim\limits_{n\to\infty} \frac{\pi}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots.. + \sin\frac{(n-1)\pi}{n}\right]$ is equals to :

(A) 0 **(B)** π **(C)** 2 **(D)** none of these

47. Let $C_n = \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{\tan^{-1}(nx)}{\sin^{-1}(nx)}\,dx$ then $\lim\limits_{n\to\infty} n^2\cdot C_n$ equals

(A) 1 **(B)** 0 **(C)** -1 **(D)** $\frac{1}{2}$

48. $\int_{2}^{3} \frac{\sqrt{x}}{\sqrt{(5-x)}+\sqrt{x}}\,dx =$

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{5}$ **(D)** none

49. Positive value of 'a' so that the definite integral $\int_{a}^{a^2} \frac{dx}{x+\sqrt{x}}$ achieves the smallest value is

(A) $\tan^2\left(\frac{\pi}{8}\right)$ **(B)** $\tan^2\left(\frac{3\pi}{8}\right)$ **(C)** $\tan^2\left(\frac{\pi}{12}\right)$ **(D)** 0

50. $\int_{2-\lambda n 3}^{3+\lambda n 3} \frac{\ln(4+x)}{\ln(4+x)+\ln(9-x)}\,dx$ is equal to :

(A) cannot be evaluated **(B)** is equal to $\frac{5}{2}$

(C) is equal to $1 + 2$ ●n 3 **(D)** is equal to $\frac{1}{2} +$ ●n 3

51. Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n$ $(n \in N)$ satisfies the equation $\int_{-1}^{1}(px+q)\,g_n(x)\,dx = 0$ for all linear functions $(px + q)$ then

(A) $a_n = b_n = 0$ **(B)** $b_n = 0;\ a_n = -\frac{3}{2n+3}$

(C) $a_n = 0;\ b_n = -\frac{3}{2n+3}$ **(D)** $a_n = \frac{3}{2n+3};\ b_n = -\frac{3}{2n+3}$

52. For $n \in N$, the value of the definite integral $\int_0^{n\pi+V} \sqrt{\frac{1+\cos 2x}{2}}\,dx$ where $\frac{\pi}{2} < V < \pi$ is -

(A) $2n+1-\cos V$ **(B)** $2n-\sin V$ **(C)** $2n+2-\sin V$ **(D)** $2n+1-\sin V$

53. Let $f(x)=\int_{-1}^{x} e^{t^2}dt$ and $h(x)=f(1+g(x))$, where g (x) is defined for all x, g'(x) exists for all x, and $g(x)<0$ for $x>0$.

If $h'(1)=e$ and $g'(1)=1$, then the possible values which g(1) can take

(A) 0 **(B)** – 1 **(C)** – 2 **(D)** – 4

54. $\int_{\pi/2}^{2\pi/3} e^{-x/2}\frac{\sqrt{1-\sin x}}{1+\cos x}\,dx$ is

(A) $\left[e^{-\pi/2}\frac{2}{\sqrt{3}}-e^{-\pi/4}\sqrt{2}\right]$ **(B)** $2e^{-\pi/3}\left[\frac{e^{\pi/6}}{\sqrt{3}}-1\right]$

(C) $2e^{-\pi/2}\left(\frac{e^{\pi/3}}{\sqrt{3}}-\sqrt{2}\,e^{\pi/4}+e^{\pi/6}\right)$ **(D)** $\left[2e^{-\pi/3}-\sqrt{2}\,e^{-\pi/4}\right]$

55. $\underset{h\to 0}{\text{Limit}}\ \frac{\int_a^{x+h} \ln^2 t\,dt-\int_a^{x} \ln^2 t\,dt}{h}$ equals to :

(A) 0 **(B)** $\ln^2 x$ **(C)** $\frac{2\ln x}{x}$ **(D)** does not exist

56. $\lim_{x\to\infty}\left(x^3\int_{-1/x}^{1/x}\frac{\ln(1+t^2)}{1+e^t}dt\right)$ equals

(A) 1/3 **(B)** 2/3 **(C)** 1 **(D)** 0

57. Let $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^2}}$ where $g(x)=\int_0^{\cos x}(1+\sin t^2)\,dt$. Also $h(x)=e^{-|x|}$ and $f(x)=x^2\sin\frac{1}{x}$ if $x\neq 0$ and $f(0)=0$ then

$f'\left(\frac{\pi}{2}\right)$ equals

(A) $h'(0)$ **(B)** $h'(0^-)$ **(C)** $h'(0^+)$ **(D)** $\lim_{x\to 0}\frac{1-\cos x}{x\sin x}$

58. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B$, $f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)dx=\frac{2A}{\pi}$, then the constant A and B are-

(A) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ **(B)** $\frac{2}{\pi}$ and 3π **(C)** 0 and $\frac{-4}{\pi}$ **(D)** $\frac{4}{\pi}$ and 0

59. The value of the definite integral $\int_{19}^{37}\left(\{x\}^2+3(\sin 2\pi x)\right)dx$ where { x } denotes the fractional part function.

(A) 0 **(B)** 6 **(C)** 9 **(D)** can not be determined

60. The true solution set of the inequality, $\sqrt{5x-6-x^2}+\left(\frac{\pi}{2}\int_0^x dz\right) > x\int_0^{\pi}\sin^2 x\,dx$ is :

(A) R **(B)** (1, 6) **(C)** (–6, 1) **(D)** (2, 3)

61. The value of $\lim_{n\to\infty}\sum_{r=1}^{r=4n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to

(A) $\frac{1}{35}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{5}$

62. If $\int_{-1/\sqrt{3}}^{1/\sqrt{3}}\frac{x^4}{1-x^4}\cos^{-1}\frac{2x}{1+x^2}dx = k\int_0^{1/\sqrt{3}}\frac{x^4}{1-x^4}dx$ then 'k' equals

(A) π **(B)** 2π **(C)** 2 **(D)** 1

63. For $U_n=\int_0^1 x^n(2-x)^n\,dx$; $V_n=\int_0^1 x^n(1-x)^n\,dx$ $n\in N$, which of the following statement(s) is/are ture ?

(A) $U_n=2^n V_n$ **(B)** $U_n=2^{-n}V_n$ **(C)** $U_n=2^{2n}V_n$ **(D)** $U_n=2^{-2n}V_n$

64. $\int_0^{(\pi/2)^{1/3}} x^5 . \sin x^3\,dx$ equals to :

(A) 1 **(B)** 1/2 **(C)** 2 **(D)** 1/3

65. Suppose that the quadratic function $f(x)=ax^2+bx+c$ is non-negative on the interval [–1, 1]. Then the area under the graph of f over the interval [–1, 1] and the x-axis is given by the formula

(A) $A=f(-1)+f(1)$ **(B)** $A=f\left(-\frac{1}{2}\right)+f\left(\frac{1}{2}\right)$

(C) $A=\frac{1}{2}[f(-1)+2f(0)+f(1)]$ **(D)** $A=\frac{1}{3}[f(-1)+4f(0)+f(1)]$

66. The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx$ is -

(A) 0 **(B)** $\pi-\frac{\pi^3}{3}$ **(C)** $2\pi-\pi^3$ **(D)** $\frac{7}{2}-2\pi^3$

67. The interval [0, 4] is divided into n equal sub-intervals by the points x_0, x_1, x_2, ..., x_{n-1}, x_n where $0=x_0<x_1<x_2<x_3\ldots\ldots<x_n=4$. If $\delta x=x_i-x_{i-1}$ for i = 1, 2, 3, ... n then $\lim_{\delta x\to 0}\sum_{i=1}^{n}x_i\delta x$ is equal to

(A) 4 **(B)** 8 **(C)** $\frac{32}{3}$ **(D)** 16

68. $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$ is equal to :

(A) $\frac{e^{\pi/2}}{2e^2}$ **(B)** $2e^2e^{\pi/2}$ **(C)** $\frac{2}{e^2}e^{\pi/2}$ **(D)** none of these

69. $\int_0^1 x\ln\left(1+\frac{x}{2}\right)dx =$

(A) $\frac{3}{4}\left(1-2\ln\frac{3}{2}\right)$ **(B)** $\frac{3}{2}-\frac{7}{2}\ln\frac{3}{2}$ **(C)** $\frac{3}{4}+\frac{1}{2}\ln\frac{1}{54}$ **(D)** $\frac{1}{2}\ln\frac{27}{2}-\frac{3}{4}$

70. Let $I(a)=\int_0^\pi\left(\frac{x}{a}+a\sin x\right)^2 dx$ where 'a' is positive real. The value of 'a' for which I (a) attains its minimum value is

(A) $\sqrt{\pi\sqrt{\frac{2}{3}}}$ **(B)** $\sqrt{\pi\sqrt{\frac{3}{2}}}$ **(C)** $\sqrt{\frac{\pi}{16}}$ **(D)** $\sqrt{\frac{\pi}{13}}$

71. The absolute value of $\int_{10}^{19}\frac{(\sin x)dx}{(1+x^8)}$ is less than

(A) 10^{-10} **(B)** 10^{-11} **(C)** 10^{-7} **(D)** 10^{-9}

72. If $\int_0^{11}\frac{11^x}{11^{[x]}}dx=\frac{k}{\log 11}$, (where [ ] denotes greatest integer function) then value of k is

(A) 11 **(B)** 101 **(C)** 110 **(D)** none of these

73. Let $a>0$ and let $f(x)$ is monotonic increasing such that $f(0)=0$ and $f(a)=b$ then $\int_0^a f(x)dx+\int_0^b f^{-1}(x)dx$ equals

(A) $a+b$ **(B)** $ab+b$ **(C)** $ab+a$ **(D)** ab

74. $\int_{-\pi}^{\pi}(\cos ax-\sin bx)^2 dx$ where a and b are integer is equal to -

(A) $-\pi$ **(B)** 0 **(C)** π **(D)** 2π

75. $\lim_{n\to\infty}\frac{n}{(n!)^{1/n}}$ is equal to

(A) e **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** $\int_0^1 \ln x\,dx$

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. The equation $10x^4 - 3x^2 - 1 = 0$ has

(A) at least one root in (–1, 0)
(B) at least one root in (0, 1)
(C) at least two roots in (–1, 1)
(D) no root in (–1, 1)

2. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ is-

(A) $\pi/4$
(B) $\pi/2$
(C) $\int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$
(D) $\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x}$

3. The function f is continuous and has the property

$f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $J = \int_0^1 f(x)\,dx$ then

(A) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$
(B) the value of J equal to $\frac{1}{2}$
(C) $f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$
(D) $\int_0^{\pi/2} \frac{\sin x\,dx}{(\sin x + \cos x)^3}$ has the same value as J

4. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ & $v = \int_0^{\infty} \frac{x^2\,dx}{x^4 + 7x^2 + 1}$ then -

(A) $v > u$
(B) $6v = \pi$
(C) $3u + 2v = 5\pi/6$
(D) $u + v = \pi/3$

5. If $f(x) = \int_0^x (\cos^4 t + \sin^4 t)\,dt$, then $f(x + \pi)$ is equal to :

(A) $f(x) + f(\pi)$
(B) $f(x) + 2\,f(\pi)$
(C) $f(x) + f\left(\frac{\pi}{2}\right)$
(D) $f(x) + 2f\left(\frac{\pi}{2}\right)$

6. Let f (x) and g (x) are differentiable function such that $f(x) + \int_0^x g(t)\,dt = \sin x\,(\cos x - \sin x)$, and $(f'(x))^2 + (g(x))^2$ = 1 then f (x) and g (x) respectively, can be

(A) $\frac{1}{2}\sin 2x, \sin 2x$
(B) $\frac{\cos 2x}{2}, \cos 2x$
(C) $\frac{1}{2}\sin 2x, -\sin 2x$
(D) $-\sin^2 x, \cos 2x$

7. $\int_0^{\infty} \frac{x}{(1 + x)(1 + x^2)} dx$

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) is same as $\int_0^{\infty} \frac{dx}{(1 + x)(1 + x^2)}$
(D) cannot be evaluated

8. Which of the following definite integral(s) vanishes

(A) $\int_{0}^{\pi/2} \ln(\cot x)\,dx$ (B) $\int_{0}^{2\pi} \sin^3 x\,dx$ (C) $\int_{1/e}^{e} \frac{dx}{x(\ln x)^{1/3}}$ (D) $\int_{0}^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\,dx$

9. The value of $\int_{0}^{1} \frac{2x^2+3x+3}{(x+1)\left(x^2+2x+2\right)}\,dx$ is :

(A) $\frac{\pi}{4}+2\ln 2-\tan^{-1}2$ (B) $\frac{\pi}{4}+2\ln 2-\tan^{-1}\frac{1}{3}$

(C) $2\ln 2-\cot^{-1}3$ (D) $-\frac{\pi}{4}+\ln 4+\cot^{-1}2$

10. Let $f(x)=\int_{-1}^{1}(1-|t|)\cos(xt)\,dt$ then which of the following hold true?

(A) $f(0)$ is not defined (B) $\lim_{x\to 0} f(x)$ exists and equals 2

(C) $\lim_{x\to 0} f(x)$ exists and is equal to 1 (D) $f(x)$ is continuous at $x=0$

11. Which of the following are true ?

(A) $\int_{a}^{\pi-a} x.f(\sin x)dx = \frac{\pi}{2}.\int_{a}^{\pi-a} f(\sin x)dx$ (B) $\int_{-a}^{a} f(x^2)dx = 2.\int_{0}^{a} f(x^2)dx$

(C) $\int_{0}^{n\pi} f(\cos^2 x)dx = n.\int_{0}^{\pi} f(\cos^2 x)dx$ (D) $\int_{0}^{b-c} f(x+c)dx = \int_{c}^{b} f(x)dx$

12. Let f(x) is a real valued function defined by :

$$f(x)=x^2+x^2\int_{-1}^{1} t\cdot f(t)dt + x^3\int_{-1}^{1} f(t)dt$$

then which of the following hold(s) good ?

(A) $\int_{-1}^{1} t\cdot f(t)dt=\frac{10}{11}$ (B) $f(1)+f(-1)=\frac{30}{11}$

(C) $\int_{-1}^{1} t\cdot f(t)dt > \int_{-1}^{1} f(t)dt$ (D) $f(1)-f(-1)=\frac{20}{11}$

13. Given f is an odd function defined everywhere, periodic with period 2 and integrable on every interval. Let $g(x)=\int_{0}^{x} f(t)\,dt$. Then :

(A) $g(2n)=0$ for every integer n (B) g(x) is an even function

(C) g(x) and f(x) have the same period (D) none of these

14. If a, b, c ∈ R and satisfy $3a + 5b + 15c = 0$, the equation $ax^4 + bx^2 + c = 0$ has -

(A) atleast one root in $(-1, 0)$ **(B)** atleast one root in $(0, 1)$

(C) atleast two roots in $(-1, 1)$ **(D)** no root in $(-1, 1)$

15. Which of the following are true ?

(A) $\int_a^{\pi-a} x \, . \, f(\sin x)\, dx = \frac{\pi}{2} . \int_a^{\pi-a} f(\sin x)\, dx$ **(B)** $\int_{-a}^{a} f(x)^2\, dx = 2. \int_0^a f(x)^2\, dx$

(C) $\int_0^{n\pi} f\left(\cos^2 x\right) dx = n. \int_0^{\pi} f\left(\cos^2 x\right) dx$ **(D)** $\int_0^{b-c} f(x+c)\, dx = \int_c^b f(x)\, dx$

16. If $f(x) = \int_1^x \frac{\ln t}{1+t} dt$ where $x > 0$ then the value(s) of x satisfying the equation, $f(x) + f(1/x) = 2$ is :

(A) 2 **(B)** e **(C)** e^{-2} **(D)** e^2

17. Let $S_n = \frac{n}{(n+1)(n+2)} + \frac{n}{(n+2)(n+4)} + \frac{n}{(n+3)(n+6)} + ... + \frac{1}{6n}$, then $\lim_{n\to\infty} S_n$ is -

(A) $\ln \frac{3}{2}$ **(B)** $\ln \frac{9}{2}$ **(C)** greater than one **(D)** less than two

18. Suppose $I_1 = \int_0^{\pi/2} \cos(\pi \sin^2 x)dx$; $I_2 = \int_0^{\pi/2} \cos(2\pi \sin^2 x)dx$ and $I_3 = \int_0^{\pi/2} \cos(\pi \sin x)dx$, then

(A) $I_1 = 0$ **(B)** $I_2 + I_3 = 0$ **(C)** $I_1 + I_2 + I_3 = 0$ **(D)** $I_2 = I_3$

29. The value of definite integral $\int_{\infty}^{0} \frac{ze^{-z}}{\sqrt{1-e^{-2z}}} dz$

(A) $-\frac{\pi}{2} \ln 2$ **(B)** $\frac{\pi}{2} \ln 2$ **(C)** $-\pi$●n2 **(D)** π●n $\frac{1}{\sqrt{2}}$

20. If $I = \int_0^{2\pi} \sin^2 x dx$, then

(A) $I = 2\int_0^{\pi} \sin^2 x dx$ **(B)** $I = 4\int_0^{\pi/2} \sin^2 x dx$ **(C)** $I = \int_0^{2\pi} \cos^2 x dx$ **(D)** $I = 8\int_0^{\pi/4} \sin^2 x dx$

21. If $I_n = \int_0^1 \frac{dx}{\left(1+x^2\right)^n}$; n ∈ N, then which of the following statements hold good ?

(A) $2n\, I_{n+1} = 2^{-n} + (2n-1)\, I_n$ **(B)** $I_2 = \frac{\pi}{8} + \frac{1}{4}$

(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ **(D)** $I_3 = \frac{\pi}{16} - \frac{5}{48}$

22. The value of integral $\int_0^{\pi} x f(\sin x)dx =$

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ **(B)** $\pi \int_0^{\pi/2} f(\sin x)dx$ **(C)** $\pi \int_0^{\pi/2} f(\cos x)dx$ **(D)** $\frac{\pi}{2}\int_0^{\pi} f(\cos x)dx$

23. Let $f(x) = \int_{-x}^{x} (t\sin at + bt + c)\,dt$ where a,b, c are non zero real numbers, then $\lim_{x\to 0} \frac{f(x)}{x}$ is

(A) independent of a
(B) independent of a and b and has the value equals to c.
(C) independent a, b and c.
(D) dependent only on c.

24. Let $L = \lim_{n\to\infty} \int_{a}^{\infty} \frac{n\,dx}{1+n^2x^2}$ where $a \in R$ then L can be

(A) π
(B) $\frac{\pi}{2}$
(C) 0
(D) 1

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** If $f(x) = \int_{1}^{x} \frac{\ln t\,dt}{1+t+t^2}$ $(x > 0)$, then $f(x) = -f\left(\frac{1}{x}\right)$

Statement-II : If $f(x) = \int_{1}^{x} \frac{\ln t\,dt}{t+1}$, then $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{2}(\ln x)^2$.

2. Consider $I = \int_{-\pi/4}^{\pi/4} \frac{dx}{1-\sin x}$

Statement-I : $I = 0$

Statement-II : $\int_{-a}^{a} f(x)\,dx = 0$, wherever f (x) is an odd function

3. **Statement-I :** If {.} represents fractional part function, then $\int_{0}^{5.5} \{x\}\,dx = \frac{21}{8}$

Statement-II : If [.] and {.} represent greatest integer and frational part functions respectively, then $\int_{0}^{t} \{x\}\,dx = \frac{[t]}{2} + \frac{\{t\}^2}{2}$

4. **Statement-I :** The equation $4x^3 - 9x^2 + 2x + 1 = 0$ has atleast one real root in (0, 1).

Statement-II : If 'f' is a continuous function such that $\int_{a}^{b} f(x) = 0$, then the equation f(x) = 0 has atleast one real root in (a, b).

5. **Statement-I :** $\int_{1/3}^{3} \frac{1}{x}\text{cosec}^{99}\left(x - \frac{1}{x}\right)dx = 0$.

Statement-II : $\int_{-a}^{a} f(x)dx = 0$ if $f(-x) = -f(x)$.

6. **Statement-I :** If $f(x)=\int_0^1 (x f(t)+1)\,dt$, then $\int_0^3 f(x)\,dx = 12$

Statement-II : $f(x)=3x+1$

7. Let $f(x)=x-x^2+1$.

Statement-I : $g(x)=\max\{f(t): 0\le t\le x\}$, then $\int_0^1 g(x)dx = \frac{29}{24}$

Statement-II : f(x) is increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$.

8. **Statement-I :** $\int_0^{\pi} x\tan x\cos^3 x\,dx = \frac{\pi}{2}\int_0^{\pi}\tan x\cos^3 x\,dx$.

Statement-II : $\int_a^b x f(x)dx = \frac{a+b}{2}\int_a^b f(x)dx$.

9. **Statement-I :** The function $f(x)=\int_0^x \sqrt{1+t^2}\,dt$ is an odd function and $g(x)=f'(x)$ is an even function.

Statement-II: For a differentiable function $f(x)$ if $f'(x)$ is an even function then $f(x)$ is an odd function.

10. **Statement-I :** $\int_0^{10\pi} |\cos x|\,dx = 20$

Statement-II : $\int_a^b f(x)\,dx \ge 0$, then $f(x)\ge 0, \forall x \in (a, b)$

11. **Statement-I :** $\sum_{r=0}^{n-1}\frac{1}{n}\left(\sqrt{\frac{r}{n}}+1\right) < \int_0^1 (\sqrt{x}+1)dx < \sum_{r=1}^{n}\frac{1}{n}\left(\sqrt{\frac{r}{n}}+1\right)$, $n\in N$.

Statement-II : If $f(x)$ is continuous and increasing in [0, 1], then $\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right) < \int_0^1 f(x)dx < \sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$, where $n \in N$

12. Given $f(x)=\sin^3 x$ and $P(x)$ is a quadratic polynomial with leading coefficient unity.

Statement-I : $\int_0^{2\pi} P(x)\cdot f''(x)\,dx$ vanishes.

Statement-II : $\int_0^{2\pi} f(x)\,dx$ vanishes

13. **Statement-I :** Let m & n be positive integers. $a=\cos\left\{\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx\right\}$, if $m\ne n$ &

$b=\cos\left\{\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx\right\}$ if $m=n$, then $a+b=2$.

Statement-II : $\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx = \begin{cases}0, & m\ne n\\ \pi, & m=n\end{cases}$, where m & n are positive integers.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. **Column-I** / **Column-II**

(A) The function $f(x) = \dfrac{e^{x\cos x} - 1 - x}{\sin x^2}$ is not defined at x = 0. The value of $f(0)$ so that f is continuous at x = 0 is — **(p)** -1

(B) The value of the definite integral $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ equals a + b ln 2 where a and b are integers then (a + b) equals — **(q)** 0

(C) Given $e^n \displaystyle\int_0^n \frac{\sec^2\theta - \tan\theta}{e^\theta} d\theta = 1$ then the value of tan (n) is equal to — **(r)** 1/2

(D) Let $a_n = \displaystyle\int_{\frac{1}{n+1}}^{\frac{1}{n}} \tan^{-1}(nx)\,dx$ and $b_n = \displaystyle\int_{\frac{1}{n+1}}^{\frac{1}{n}} \sin^{-1}(nx)\,dx$ then — **(s)** 1

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n}$ has the value equal to

2. **Column-I** / **Column-II**

(A) $\displaystyle\int_0^1 \left(1+(2008)x^{2008}\right)e^{x^{2008}} dx$ equals — **(p)** e^{-1}

(B) The value of the definite integral $\displaystyle\int_0^1 e^{-x^2}dx + \int_1^{1/e} \sqrt{-\ln x}\,dx$ is equal to — **(q)** $e^{-1/4}$

(C) $\displaystyle\lim_{n\to\infty} \left(\frac{1^1 \cdot 2^2 \cdot 3^3 \ldots\ldots\ldots (n-1)^{n-1} \cdot n^n}{n^{1+2+3+\ldots\ldots+n}}\right)^{\frac{1}{n^2}}$ equals — **(r)** $e^{1/2}$

(s) e

3. **Column-I** / **Column-II**

(A) If [ ] denotes the greatest integer function and — **(p)** 1

$f(x) = \begin{cases} 3[x] - \dfrac{5|x|}{x}; & x \neq 0 \\ 2 & ; x = 0 \end{cases}$, then is equal to $\displaystyle\int_{-3/2}^{2} f(x)dx$

(B) The value of $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ of is — **(q)** $-\dfrac{11}{2}$

(C) If $I_1 = \int_1^{\sin\theta} \frac{x}{1+x^2}dx$ and $I_2 = \int_1^{\operatorname{cosec}\theta} \frac{1}{x(x^2+1)}dx$ then the value of $\begin{vmatrix} I_1 & I_1^2 & I_2 \\ e^{I_1+I_2} & I_2^2 & -1 \\ 1 & I_1^2+I_2^2 & -1 \end{vmatrix}$, is — (r) $\frac{3}{2}$

(D) If f(x) and g(x) are two continuous functions defined on R, then the value of $\int_{-a}^{a} \{f(x)+f(-x)\}\{g(x)-g(-x)\}dx$, is — (s) 0

4. Let $f(\theta) = \int_0^1 (x+\sin\theta)^2\,dx$ and $g(\theta) = \int_0^1 (x+\cos\theta)^2\,dx$ where $\theta \in [0, 2\pi]$.

The quantity $f(\theta) - g(\theta)\ \forall\ \theta$ in the interval given in column-I, is

	Column-I		Column-II
(A)	$\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$	(p)	negative
(B)	$\left(\frac{3\pi}{4}, \pi\right]$	(q)	positive
(C)	$\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right)$	(r)	non negative
(D)	$\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$	(s)	non positive

5.

	Column-I		Column-II
(A)	Suppose, $f(n) = \log_2(3)\cdot\log_3(4)\cdot\log_4(5)\,.........\,\log_{n-1}(n)$ then the sum $\sum_{k=2}^{100} f(2^k)$ equals	(p)	5010
(B)	Let $f(x) = \sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)(x+4)}}}$ then $\int_0^{100} f(x)dx$ is	(q)	5050
		(r)	5100
(C)	In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is	(s)	5049
(D)	$\lim_{x\to 0} \frac{\prod_{r=1}^{100}(1+rx)-1}{x}$ equals		

6. **Column-I** | **Column-II**

(A) $\int_0^{\pi/2} \ln(\tan x + \cot x)\, dx =$ **(p)** $\frac{\pi^2}{4}$

(B) $\int_0^{\pi/2} \frac{\sin x - \cos x}{(\sin x + \cos x)^2}\, dx =$ **(q)** $\pi \ln 2$

(C) $\int_0^{2\pi} x(\sin^2 x \cos^2 x)\, dx =$ **(r)** 0

(D) $\int_0^{\pi/2} (2\ln \sin x - \ln \sin 2x)\, dx =$ **(s)** $-\frac{\pi}{2} \ln 2$

7. **Column-I** | **Column-II**

(A) $\int_{-1}^{1} \frac{3x^2}{1+4^{\tan x}} dx =$ **(p)** 7

(B) $\int_6^8 \frac{\sin x^2 dx}{\sin x^2 + \sin(x-14)^2} =$ **(q)** $\frac{1}{2}$

(C) $\frac{1}{156}\int_1^{13} [x]dx =$ **(r)** 1

{where [.] denotes greatest integer function}

(D) $\frac{1}{\pi \ln 2}\int_{\pi/2}^{0} \ln \sin 2x dx =$ **(s)** 2

8. **Column-I** | **Column-II**

(A) If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$ where $g(x) = \int_0^{\cos x} (1+\sin t^2)dt$ then the value of $f'(\pi/2)$ **(p)** 3

(B) If $f(x)$ is a non zero differentiable function such that $\int_0^x f(t)dt = (f(x))^2$ for all x, then $f(2)$ equals **(q)** 2 **(r)** 1

(C) If $\int_a^b (2+x-x^2)dx$ is maximum then $(a+b)$ is equal to **(s)** -1

(D) If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0$ then $(3a+b)$ has the the value equal to

9. **Column-I** | **Column-II**

(A) $\int_0^{\pi} x\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx$ — **(p)** π^2

(B) $\int_0^{\pi} \frac{x\,dx}{1+\sin^2 x}$ — **(q)** $\frac{\pi^2}{2}$

(C) $\int_0^{\pi^2/4} \left(2\sin\sqrt{x}+\sqrt{x}\cos\sqrt{x}\right)dx$ equals — **(r)** $\frac{\pi^2}{4}$

(s) $\frac{\pi^2}{2\sqrt{2}}$

10. **Column-I** | **Column-II**

(A) Let $f(x)=\int x^{\sin x}\left(1+x\cos x\cdot\ln x+\sin x\right)dx$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$ then the value of $f(\pi)$ is — **(p)** rational

(B) Let $g(x)=\int\frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0)=0$ — **(q)** irrational

then the value of $g\left(\frac{\pi}{2}\right)$ is — **(r)** integral

(C) If real numbers x and y satisfy $(x+5)^2+(y-12)^2=(14)^2$ then the minimum value of $\sqrt{(x^2+y^2)}$ is — **(s)** prime

(D) Let $k(x)=\int\frac{(x^2+1)\,dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1)=\frac{1}{\sqrt[3]{2}}$ then the value of $k(-2)$ is

11. **Column-I** | **Column-II**

(A) $\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}=$ — **(p)** $\frac{1}{100}$

{where [.] denotes greatest integer function}

(B) $\int_{-1}^{2}\frac{|x|}{x}dx=$ — **(q)** 3

(C) $\lim_{n\to\infty}\frac{1^{99}+2^{99}+.....+n^{99}}{n^{100}}=$ — **(r)** $\frac{1}{3}$

(D) $5050\int_{-1}^{1}\sqrt{x^{200}}\,dx=\frac{1}{\alpha}$, then $\alpha=$ — **(s)** 1

12. Let $\lim_{T\to\infty} \frac{1}{T}\int_0^T (\sin x + \sin ax)^2\,dx = L$ then

	Column I		Column II
(A)	for a = 0, the value of L is	(p)	0
(B)	for a = 1 the value of L is	(q)	1/2
(C)	for a = – 1 the value of L is	(r)	1
(D)	$\forall\ a \in R - \{-1, 0, 1\}$ the value of L is	(s)	2

13.

	Column-I		Column-II
(A)	$\lim_{x\to 0} \frac{1}{\sin x}\int_0^{\ln(1+x)} (1-\tan 2y)^{1/y}\,dy$ equals	(p)	1
(B)	$\lim_{x\to\infty} (e^{2x} + e^x + x)^{1/x}$ equals	(q)	e
(C)	Let $f(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2x^2}$ then $\lim_{x\to 0} f(x)$ equals	(r)	e^2
		(s)	e^{-2}

Part # II [Comprehension Type Questions]

Comprehension # 1

Suppose $\lim_{x\to 0} \dfrac{\int_0^x \frac{t^2\,dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l$ where $p \in N, p \geq 2, a > 0, r > 0$ and $b \neq 0$.

1. If l exists and is non zero then

(A) $b > 1$ (B) $0 < b < 1$ (C) $b < 0$ (D) $b = 1$

2. If $p = 3$ and $l = 1$ then the value of 'a' is equal to

(A) 8 (B) 3 (C) 6 (D) 3/2

3. If $p = 2$ and $a = 9$ and l exists then the value of l is equal to

(A) 3/2 (B) 2/3 (C) 1/3 (D) 7/9

Comprehension # 2

Consider $g(x) = \begin{cases} \left\{\dfrac{\max.(f(t)) + \min.(f(t))}{2},\ 0 \leq t \leq x\right\} & 0 \leq x \leq 4 \\ |x-5| + |x-4| & 4 < x < 5 \\ \tan\left(\sin^{-1}\left(\dfrac{6-x}{\sqrt{x^2 - 12x + 37}}\right)\right) & x \geq 5 \end{cases}$

where $f(x) = x^2 - 4x + 3$.

On the basis of above information, answer the following questions :

1. $\int_{2}^{5} g(x)dx$ is equal to

(A) 5/3 **(B)** 3 **(C)** 13/3 **(D)** 3/2

2. If $h(x) = \int_{0}^{x^2} g(t)dt$, then complete set of values of x in the interval [0, 7] for which h(x) is decreasing, is -

(A) (6, 7] **(B)** (5, 7] **(C)** $\left(\sqrt{6}, \sqrt{7}\right]$ **(D)** $\left(\sqrt{6}, 7\right]$

3. $\lim\limits_{x \to 4} \frac{g(x) - g(2)}{\ln(\cos(4 - x))}$ is equal to -

(A) 0 **(B)** 1 **(C)** 2 **(D)** does not exist

Comprehension # 3

Let $g(t) = \int_{x_1}^{x_2} f(t, x)\,dx$. Then $g'(t) = \int_{x_1}^{x_2} \frac{\partial}{\partial t}(f(t, x))\,dx$. Consider $f(x) = \int_{0}^{\pi} \frac{\ln(1 + x\cos\theta)}{\cos\theta}\,d\theta$.

1. Range of f(x) is

(A) $(0, \pi)$ **(B)** $(0, \pi^2)$ **(C)** $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ **(D)** $\left(\frac{-\pi^2}{2}, \frac{\pi^2}{2}\right)$

2. The number of critical points of f(x), in the interior of its domain, is

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinitely many

3. f(x) is

(A) discontinuous at x = 0 **(B)** continuous but not differentiable at x = 1

(C) continuous at x = 0 **(D)** differentiable at x = 1

Comprehension # 4

Consider the function defined on $[0, 1] \to R$

$$f(x) = \frac{\sin x - x\cos x}{x^2} \text{ if } x \neq 0 \text{ and } f(0) = 0$$

1. $\int_{0}^{1} f(x)\,dx$ equals

(A) 1 – sin (1) **(B)** sin (1) – 1 **(C)** sin (1) **(D)** – sin (1)

2. $\lim\limits_{t \to 0} \frac{1}{t^2}\int_{0}^{t} f(x)\,dx$ equals

(A) 1/3 **(B)** 1/6 **(C)** 1/12 **(D)** 1/24

Comprehension # 5

Suppose a and b are positive real numbers such that $ab = 1$. Let for any real parameter t, the distance from the origin to the line $(ae^t)x + (be^{-t})y = 1$ be denoted by D(t) then

1. The value of the definite integral $I = \int_0^1 \frac{dt}{(D(t))^2}$ is equal to

(A) $\frac{e^2-1}{2}\left(b^2+\frac{a^2}{e^2}\right)$
(B) $\frac{e^2+1}{2}\left(a^2+\frac{b^2}{e^2}\right)$
(C) $\frac{e^2-1}{2}\left(a^2+\frac{b^2}{e^2}\right)$
(D) $\frac{e^2+1}{2}\left(b^2+\frac{a^2}{e^2}\right)$

2. The value of 'b' at which I is minimum, is

(A) e (B) $\frac{1}{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) $\sqrt{e}$

3. Minimum value of I is

(A) e – 1 (B) $e - \frac{1}{e}$ (C) e (D) $e + \frac{1}{e}$

Comprehension # 6

If $y = \int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x)\,f^2(v(x)) - u'(x)\,f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{(a,b)}(x-a)$

1. If $y = \int_x^{x^2} t^2\,dt$, then equation of tangent at x = 1 is

(A) y = x + 1 (B) x + y = 1 (C) y = x – 1 (D) y = x

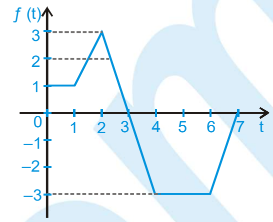
2. If $F(x) = \int_1^x e^{t^2/2}(1-t^2)\,dt$, then $\frac{d}{dx}F(x)$ at x = 1 is

(A) 0 (B) 1 (C) 2 (D) – 1

3. If $y = \int_{x^3}^{x^4} \ln t\,dt$, then $\lim_{x\to 0^+}\frac{dy}{dx}$ is

(A) 0 (B) 1 (C) 2 (D) – 1

Comprehension # 7



Let $g(x)=\int_0^x f(t)\,dt$, where f is a function whose graph is show adjacently.

On the basis of above information, answer the following questions :

1. Maximum value of g(x) in $x \in [0, 7]$ is -

(A) 3 **(B)** 9/2 **(C)** 3/2 **(D)** 6

2. Value of x at which g(x) becomes zero, is -

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

3. Set of values of x in [0, 7] for which g(x) is negative is -

(A) (2, 7) **(B)** (3, 7) **(C)** (4, 6) **(D)** (5, 7)

Comprehension # 8

Let the function f satisfies

$$f(x)\cdot f'(-x) = f(-x)\cdot f'(x) \text{ for all x and } f(0) = 3.$$

1. The value of $f(x)\cdot f(-x)$ for all x, is

(A) 4 **(B)** 9 **(C)** 12 **(D)** 16

2. $\int_{-51}^{51}\frac{dx}{3+f(x)}$ has the value equal to

(A) 17 **(B)** 34 **(C)** 102 **(D)** 0

3. Number of roots of $f(x) = 0$ in [–2, 2] is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

Comprehension # 9

Let f(x) be a differentiable function, satisfying $f(0) = 2$, $f'(0) = 3$ and $f''(x) = f(x)$

1. Graph of y = f(x) cuts x -axis at

(A) $x = -\frac{1}{2}\ln 5$ **(B)** $x = \frac{1}{2}\ln 5$ **(C)** $x = -$●n5 **(D)** $x =$ ●n5

2. Area enclosed by y = f(x) in the second quadrant is

(A) $3+\frac{1}{2}\ln\sqrt{5}$ **(B)** $2+\frac{1}{2}\ln 5$ **(C)** $3-\sqrt{5}$ **(D)** 3

3. Area enclosed by $y = f(x)$, $y = f^{-1}(x)$, $x + y = 2$ and $x + y = -\frac{1}{2}\ln 5$ is

(A) $8+\frac{1}{8}(\ln 5)^2$ **(B)** $8-2\sqrt{5}+\frac{1}{8}(\ln 5)^2$ **(C)** $2\sqrt{5}-\frac{1}{8}(\ln 5)^2$ **(D)** $8+2\sqrt{5}-\frac{1}{8}(\ln 5)^2$

Comprehension # 10

The average value of a function f(x) over the interval, [a, b] is the number $\mu = \frac{1}{b-a}\int_a^b f(x)dx$

The square root $\left\{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx\right\}^{1/2}$ is called the root mean square of f on [a, b]. The average value of μ is attained if f is continuous on [a, b].

On the basis of above information, answer the following questions :

1. The average ordinate of y = sin x over the interval [0, π] is -

(A) $1/\pi$ **(B)** $2/\pi$ **(C)** $4/\pi^2$ **(D)** $2/\pi^2$

2. The average value of the pressure varying from 2 to 10 atm if the pressure p and the volume v are related by $pv^{3/2} = 160$ is -

(A) $\frac{20}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$ **(B)** $\frac{10}{\sqrt[3]{10}+\sqrt[3]{2}}$ **(C)** $\frac{40}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$ **(D)** $\frac{160}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$

3. The average value of $f(x) = \frac{\cos^2 x}{\sin^2 x + 4\cos^2 x}$ on [0, π/2] is -

(A) $\pi/6$ **(B)** $4/\pi$ **(C)** $6/\pi$ **(D)** $1/6$

Comprehension # 11

Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \le x \le 1$.

Given $f(x) = \int_0^1 e^{x+t}\cdot f(t)\,dt$ and $g(x) = \int_0^1 e^{x+t}\cdot g(t)\,dt + x$.

1. The value of $f(1)$ equals

(A) 0 **(B)** 1 **(C)** e^{-1} **(D)** e

2. The value of $g(0) - f(0)$ equals

(A) $\frac{2}{3-e^2}$ **(B)** $\frac{3}{e^2-2}$ **(C)** $\frac{1}{e^2-1}$ **(D)** 0

3. The value of $\frac{g(0)}{g(2)}$ equals

(A) 0 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{e^2}$ **(D)** $\frac{2}{e^2}$

Exercise # 4 [Subjective Type Questions]

1. Compute the integrals :

(I) $\int_{2}^{-13} \frac{dx}{\sqrt[5]{(3-x)^4}}$ (II) $\int_{0}^{1} (e^x - 1)^4 e^x dx$ (III) $\int_{\pi/4}^{\pi/3} \frac{x dx}{\sin^2 x}$

(IV) $\int_{0}^{1} \frac{\sqrt{x} dx}{1+x}$ (V) $\int_{1}^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx$ (VI) $\int_{\sqrt{2}}^{2} \frac{dx}{x^5\sqrt{x^2-1}}$

(VII) $\int_{0}^{\frac{1}{\sqrt{3}}} \frac{dx}{(2x^2+1)\sqrt{x^2+1}}$ (VIII) $\int_{0}^{3} |(x-1)(x-2)| dx$ (IX) $\int_{0}^{\pi} |\cos x| dx$

(X) $\int_{0}^{2} [x^2] dx$ (XI) $\int_{-1}^{1} [\cos^{-1} x] dx$, where [.] represents the greatest integer function

(XII) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$ (XIII) $\int_{\sqrt{2}}^{\infty} \frac{dx}{x\sqrt{x^2-1}}$ (XIV) $\int_{0}^{4} \frac{x^2}{1+x} dx$

(XV) $\int_{0}^{\pi/2} \sqrt{\cos\theta} \sin^3\theta \, d\theta$ (XVI) $\int_{0}^{1} \sin^{-1} x \, dx$ (XVII) $\int_{1}^{2} \frac{\ln x}{x^2} dx$

(XVIII) $\int_{0}^{1} x e^x dx$ (XIX) $\int_{0}^{1} x^2 \sin^{-1} x \, dx$.

(XX) $\int_{0}^{\pi} e^{\cos^2 x} \cos^3 (2n+1) x \, dx, n \in I$ (XXI) Evaluate: $\int_{1/2}^{2} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$.

2. Evaluate :

(I) $\int_{0}^{\pi} \log(1+\cos x) dx$ (II) $\int_{0}^{2t} \frac{f(x)}{f(x)+f(2t-x)} dx$

(III) $\int_{0}^{\frac{\pi}{4}} \log(1+\tan x) dx$ (IV) $\int_{0}^{\frac{\pi}{2}} \frac{x \sin 2x dx}{\cos^4 x + \sin^4 x}$

(V) $\int_{0}^{1} \frac{x^4(1-x)^4}{1+x^2} dx$ (VI) $\int_{0}^{\pi/2} \frac{a \sin x + b \cos x}{\sin\left(\frac{\pi}{4}+x\right)} dx$

(VII) $\int_0^{2\pi} \frac{dx}{2+\sin 2x}$

(VIII) $\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7+3x^6-10x^5-7x^3-12x^2+x+1}{x^2+2}dx$

(IX) $\int_0^1 \frac{1-x}{1+x} \cdot \frac{dx}{\sqrt{x+x^2+x^3}}$

(X) $\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} \ln\left(1+x-\frac{1}{x}\right)dx$

(XI) $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

(XII) $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx$

(XIII) $\int_a^b \sqrt{(x-a)(b-x)}\ dx, a>b$

(XIV) $\int_0^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

(XV) $\int_0^{\infty} \frac{dx}{e^x+e^{-x}}$

(XVI) $\int_0^1 \frac{x}{1+\sqrt{x}}\ dx$

(XVII) $\int_0^{\pi/2} \frac{\sin x\cos x}{\cos^2 x+3\cos x+2}\ dx$

(XVIII) $\int_0^{\pi/2} \frac{\sin 2\theta\ d\theta}{\sin^4\theta+\cos^4\theta}$

(XIX) $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}\ dx$

(XX) $\int_{-1}^{1} \sin^5 x\cos^4 x\ dx$

(XXI) $\int_{-\pi/2}^{\pi/2} \frac{g(x)-g(-x)}{f(-x)+f(x)}\ dx$

(XXII) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\ dx$

(XXIII) $\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}+e^{\cos x}}\ dx$

(XXIV) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\ dx$

(XXV) $\int_0^{\pi/2} \frac{a\sin x+b\cos x}{\sin x+\cos x}\ dx$

3. Integrate following

(I) $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}\ dx$

(II) $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$

(III) $\int_0^{\pi/4} \frac{\cos x-\sin x}{10+\sin 2x}dx$

(IV) $\int_0^{\pi} \frac{x\sin 2x\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}\ dx$

(V) $\int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

(VI) $\int_0^{\pi/4} \frac{x^2(\sin 2x-\cos 2x)}{(1+\sin 2x)\cos^2 x}\ dx$

4. Prove that : **(I)** $\int_{\alpha}^{\beta}\sqrt{(x-\alpha)(\beta-x)}\,dx=\frac{(\beta-\alpha)^2\pi}{8}$

(II) $\int_{\alpha}^{\beta}\sqrt{\frac{x-\alpha}{\beta-x}}\,dx=(\beta-\alpha)\frac{\pi}{2}$

(III) $\int_{\alpha}^{\beta}\frac{dx}{x\sqrt{(x-\alpha)(\beta-x)}}=\frac{\pi}{\sqrt{\alpha\beta}}$ where $\alpha,\beta>0$

(IV) $\int_{\alpha}^{\beta}\frac{x\,.\,dx}{\sqrt{(x-\alpha)(\beta-x)}}=(\alpha+\beta)\frac{\pi}{2}$ where $\alpha<\beta$

(V) $\int_{a}^{b}\frac{x^{n-1}\left((n-2)x^2+(n-1)(a+b)x+nab\right)}{(x+a)^2(x+b)^2}dx=\frac{b^{n-1}-a^{n-1}}{2(a+b)}$

(VI) $\int_{0}^{x}\left(\int_{0}^{u}f(t)dt\right)du=\int_{0}^{x}f(u).(x-u)du$

(VII) $I_{m,n}=\int_{0}^{1}x^m.(1-x)^n dx=\frac{m!\,n!}{(m+n+1)!}$ $m,n\in N.$

(VIII) $I_{m,n}=\int_{0}^{1}x^m.(\ell n\,x)^n dx=(-1)^n\frac{n!}{(m+1)^{n+1}}$ $m,n\in N.$

(IX) If $I_n=\int_{0}^{\pi/4}\tan^n x\,dx$, then show that $I_n+I_{n-2}=\frac{1}{n-1}$

(X) $\int_{0}^{x}\left(\int_{0}^{u}f(t)\,dt\right)du=\int_{0}^{x}f(u).(x-u)\,du.$

5. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x}=2[\cos x+\cos 3x+......+\cos(2k-1)x]$

Hence prove that $\int_{0}^{\pi/2}\sin 2kx\cot x\,dx=\frac{\pi}{2}$

6. Given a function f(x) such that

(A) it is integrable over every interval on the real line and

(B) $f(T+x)=f(x)$, for every x and a real T, then show that the integral $\int_{a}^{a+T}f(x)dx$ is independent of a.

7. If a_1, a_2 and a_3 are the three values of a which satisfy the equation $\int_{0}^{\pi/2}(\sin x+a\cos x)^3dx-\frac{4a}{\pi-2}\int_{0}^{\pi/2}x\cos x\,dx=2$ then find the value of $1000\left(a_1^2+a_2^2+a_3^2\right)$.

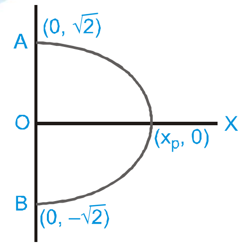
8. If f, g, h be continuous functions on [0, a] such that $f(a-x)=f(x)$, $g(a-x)=-g(x)$ and $3h(x)-4h(a-x)=5$, then prove that, $\int_{0}^{a}f(x)g(x)h(x)=0.$

9. Evaluate $\int_0^1 (tx+1-x)^n\,dx$, where n is a positive integer and t is a parameter independent of x. Hence show that $\int_0^1 x^k(1-x)^{n-k}\,dx = \frac{1}{[{}^nC_k(n+1)]}$ for k = 0, 1, ... n.

10. Given that $U_n = \{x(1-x)\}^n$ & $n \geq 2$ prove that $\frac{d^2U_n}{dx^2} = n(n-1)U_{n-2} - 2n(2n-1)U_{n-1}$, further if $V_n = \int_0^1 e^x . U_n\,dx$, prove that when $n \geq 2$, $V_n + 2n(2n-1).V_{n-1} - n(n-1)V_{n-2} = 0$

11. Let $f(x) = \begin{cases} 1-x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } 1 < x \leq 2 \\ (2-x)^2 & \text{if } 2 < x \leq 3 \end{cases}$. Define the function $F(x) = \int_0^x f(t)\,dt$ and show that F is continuous in [0, 3] and differentiable in (0, 3).

12. If 'f' is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x| \to \infty$, then show that every line y = mx intersects the curve $y^2 + \int_0^x f(t)dt = 2!$



13. If $f(x) = \frac{\sin x}{x}$ $\forall\, x \in (0, \pi]$, prove that $\frac{\pi}{2}\int_0^{\pi/2} f(x)\,f\left(\frac{\pi}{2} - x\right)dx = \int_0^{\pi} f(x)\,dx$

14. Prove that the sum to (n +1) terms of $\frac{C_0}{n(n+1)} - \frac{C_1}{(n+1)(n+2)} + \frac{C_2}{(n+2)(n+3)} - \ldots\ldots$ equals $\int_0^1 x^{n-1}.(1-x)^{n+1}\,dx$ & evaluate the integral.

15. $\int_1^2 \frac{(x^2-1)dx}{x^3.\sqrt{2x^4 - 2x^2 + 1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$

16. The tangent to the graph of the function y = f(x) at the point with abscissa x = a forms with the x-axis an angle of π/3 and at the point with abscissa x = b at an angle of π/4, then find the value of the integral,

$\int_a^b f'(x).\, f''(x)\,dx$ [assume $f''(x)$ to be continuous]

17. Evaluate : $\int_0^{2\pi} \frac{dx}{2 + \sin 2x}$

18. Show that the sum of the two integrals $\int_{-4}^{-5} e^{(x+5)^2}\,dx + 3\int_{1/3}^{2/3} e^{9(x-2/3)^2}\,dx$ is zero.

19. Comment upon the nature of roots of the quadratic equation $x^2 + 2x = k + \int_0^1 |t+k|\,dt$ depending on the value of $k \in R$.

20. **(i)** If $f(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$, then prove that $f'(x) = 0\ \forall\ x \in R$.

(ii) If $f(x) = 2x^3 - 15x^2 + 24x$ and $g(x) = \int_0^x f(t)\,dt + \int_0^{5-x} f(t)\,dt$ $(0 < x < 5)$. Find the interval in which g(x) is increasing.

(iii) Find the value of x for which function $f(x) = \int_{-1}^{x} t\,(e^t - 1)\,(t - 1)\,(t - 2)^3\,(t - 3)^5\,dt$ has a local minimum

21. Prove the inequalities :

(I) $\frac{\pi}{6} < \int_0^1 \frac{dx}{\sqrt{4 - x^2 - x^3}} < \frac{\pi\sqrt{2}}{8}$ **(II)** $2\,e^{-1/4} < \int_0^2 e^{x^2 - x}\,dx < 2e^2$ **(III)** $\frac{1}{2} \le \int_0^2 \frac{dx}{2 + x^2} \le \frac{5}{6}$

(IV) $\frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}\,dx < \frac{\sqrt{2}}{6}$ **(V)** $4 \le \int_1^3 \sqrt{(3 + x^3)}\,dx \le 2\sqrt{30}$

22. Let α, β be the distinct positive roots of the equation tan x = 2x then evaluate $\int_0^1 (\sin\alpha x.\sin\beta x)dx$ independent of α and β.

23. Let $f(x) = \ell n\left(\frac{1 - \sin x}{1 + \sin x}\right)$, then show that $\int_a^b f(x)\,dx = \int_b^a \ell n\left(\frac{1 + \sin x}{1 - \sin x}\right) dx$.

24. Show that $\int_0^\infty \frac{dx}{x^2 + 2x\cos\theta + 1} = 2\int_0^1 \frac{dx}{x^2 + 2x\cos\theta + 1} = \begin{cases} \frac{\theta}{\sin\theta} & \text{if } \theta \in (0, \pi) \\ \frac{\theta - 2\pi}{\sin\theta} & \text{if } \theta \in (\pi, 2\pi) \end{cases}$

25. Let h(x) = (fog)(x) + K where K is any constant. If $\frac{d}{dx}(h(x)) = \frac{\sin x}{\cos^2(\cos x)}$ then compute the value of j(0) where $j(x) = \int_{g(x)}^{f(x)} \frac{f(t)}{g(t)}\,dt$, where f and g are trigonometric functions.

26. Show that $\int_0^\infty f\left(\frac{a}{x} + \frac{x}{a}\right).\frac{\ell n\,x}{x}\,dx = \ell n\,a.\int_0^\infty f\left(\frac{a}{x} + \frac{x}{a}\right).\frac{dx}{x}$

27. Determine a positive integer $n \le 5$, such that $\int_0^1 e^x (x - 1)^n\,dx = 16 - 6e$

28. **(A)** $f(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt,\quad x \in \left[0, \frac{\pi}{2}\right]$ determine f(x)

(B) $f(x) = \int_{e^x}^{e^{3x}} \frac{t\,dt}{\ell n\,t}$, $x > 0$ find differential coefficient of f(x) w.r.t. $\ell n x$ when $x = \ell n 2$

29. **(i)** If $f(x) = 5^{g(x)}$ and $g(x) = \int_2^{x^2} \frac{t}{\ell n\,(1 + t^2)}\,dt$, then find the value of $f'(\sqrt{2})$.

(ii) The value of $\lim_{x \to 0} \frac{\frac{d}{dx}\int_0^{x^3} \sqrt{\cos t}\,dt}{1 - \sqrt{\cos x}}$ is 12

30. If the derivative of f(x) wrt x is $\frac{\cos x}{f(x)}$ then show that f(x) is a periodic function.

31. Prove that if $J_m = \int_1^e \ln^m x\,dx$, then $J_m = e - mJ_{m-1}$ (m a positive integer).

32. If f(x) is an odd function defined on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T, then prove that $\phi(x) = \int_a^x f(t)\,dt$ is also periodic with period T.

33. **(A)** If $|x| < 1$ prove that $\frac{1-2x}{1-x+x^2} + \frac{2x-4x^3}{1-x^2+x^4} + \frac{4x^3-8x^7}{1-x^4+x^8} + \ldots\ldots\infty = \frac{1+2x}{1+x+x^2}$.

(B) Prove the identity $f(x) = \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \ldots\ldots + \frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}} = \frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}} - 2\cot 2x$

34. Suppose g(x) is the inverse of f(x) and f(x) has a domain $x \in [a, b]$. Given $f(a) = \alpha$ and $f(b) = \beta$, then find the value of $\int_a^b f(x)dx + \int_\alpha^\beta g(y)dy$ in terms of a, b, α and β.

35. Find the limits

(I) $\lim_{n\to\infty} \left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\ldots\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

(II) For potitive integers n, let $A_n = \frac{1}{n}\{(n+1)+(n+2)+\ldots\ldots(n+n)\}$, $B_n = \{(n+1)(n+2)\ldots\ldots(n+n)\}^{1/n}$.

If $\lim_{n\to\infty}\frac{A_n}{B_n} = \frac{ae}{b}$ where $a, b \in N$ and relatively prime find the value of (a + b).

(III) $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots+\frac{1}{6n}\right)$

(IV) $\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{n+1}+\frac{2}{n+2}+\ldots\ldots+\frac{3n}{4n}\right]$

$\lim_{n\to\infty}$

(V) $\lim_{x\to 0^+}\frac{\int_0^{x^2}\sin\sqrt{x}\,dx}{x^3}$

(VI) $\lim_{x\to+\infty}\frac{\left(\int_0^x e^{x^2}dx\right)^2}{\int_0^x e^{2x^2}dx}$

(VII) $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$

(VIII) $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$

(IX) $\lim_{n\to\infty}\frac{3}{n}\left[1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+\ldots\ldots\ldots+\sqrt{\frac{n}{n+3(n-1)}}\right]$

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $I_n = \int_0^{\pi/4} \tan^n x\,dx$ then the value of $n(I_{n-1} + I_{n+1})$ is- **[AIEEE-2002]**

(1) 1 **(2)** $\pi/2$ **(3)** $\pi/4$ **(4)** n

2. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} =$ **[AIEEE-2002]**

(1) π^2 **(2)** $\pi^2/4$ **(3)** $\pi/8$ **(4)** $\pi^2/8$

3. $\int_{\pi}^{10\pi} |\sin x|\,dx =$ **[AIEEE-2002]**

(1) 9 **(2)** 10 **(3)** 18 **(4)** 20

4. $\int_0^{\sqrt{2}} [x^2]\,dx$ is equal to (where [.] denotes greatest integer function) **[AIEEE-2002]**

(1) $\sqrt{2} - 1$ **(2)** $2(\sqrt{2} - 1)$ **(3)** $\sqrt{2}$ **(4)** none of these

5. $\lim_{n\to\infty} \frac{1^P + 2^P + 3^P + \ldots + n^P}{n^{P+1}}$ equals - **[AIEEE-2002]**

(1) 1 **(2)** $\frac{1}{P+1}$ **(3)** $\frac{1}{P+2}$ **(4)** P^2

6. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3}\,dx = F(k) - F(1)$, then one of the possible values of k, is- **[AIEEE-2003]**

(1) 64 **(2)** 15 **(3)** 16 **(4)** 63

7. If $f(a + b - x) = f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to- **[AIEEE-2003]**

(1) $\frac{a+b}{2}\int_a^b f(a+b-x)\,dx$ **(2)** $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ **(3)** $\frac{a+b}{2}\int_a^b f(x)\,dx$ **(4)** $\frac{b-a}{2}\int_a^b f(x)\,dx$

8. The value of the integral $I = \int_0^1 x(1-x)^n\,dx$ is- **[AIEEE-2003]**

(1) $\frac{1}{n+1} + \frac{1}{n+2}$ **(2)** $\frac{1}{n+1}$ **(3)** $\frac{1}{n+2}$ **(4)** $\frac{1}{n+1} - \frac{1}{n+2}$

9. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x \sin x}$ is - **[AIEEE-2003]**

(1) 0 **(2)** 3 **(3)** 2 **(4)** 1

10. $\lim\limits_{n\to\infty}\dfrac{(1)^4+2^4+3^4+\ldots+n^4}{n^5}-\lim\limits_{n\to\infty}\dfrac{(1)^3+2^3+3^3+\ldots+n^3}{n^5}$ is equal to - **[AIEEE-2003]**

(1) 1/5 **(2)** 1/30 **(3)** zero **(4)** 1/4

11. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)g(y)\,dy$, then- **[AIEEE-2003]**

(1) $F(t) = te^{-t}$ **(2)** $F(t) = 1 - e^{-t}(1 + t)$
(3) $F(t) = e^t - (1 + t)$ **(4)** $F(t) = te^t$

12. Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g(x) be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)\,dx$ is - **[AIEEE-2003]**

(1) $e+\dfrac{e^2}{2}+\dfrac{5}{2}$ **(2)** $e-\dfrac{e^2}{2}-\dfrac{5}{2}$ **(3)** $e+\dfrac{e^2}{2}-\dfrac{3}{2}$ **(4)** $e-\dfrac{e^2}{2}-\dfrac{3}{2}$

13. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{n}e^{r/n}$ is- **[AIEEE-2004]**

(1) e **(2)** e – 1 **(3)** 1 – e **(4)** e + 1

14. The value of $\int_{-2}^{3}|1-x^2|\,dx$ is- **[AIEEE-2004]**

(1) 28/3 **(2)** 14/3 **(3)** 7/3 **(4)** 1/3

15. The value of $I=\int_0^{\pi/2}\dfrac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}\,dx$ is- **[AIEEE-2004]**

(1) 0 **(2)** 1 **(3)** 2 **(4)** 3

16. If $\int_0^{\pi} x\, f(\sin x)\,dx = A\int_0^{\pi/2} f(\sin x)\,dx$, then A is - **[AIEEE-2004]**

(1) 0 **(2)** π **(3)** $\pi/4$ **(4)** 2π

17. If $f(x)=\dfrac{e^x}{1+e^x}$, $I_1=\int_{f(-a)}^{f(a)} xg\{x(1-x)\}\,dx$ and $I_2=\int_{f(-a)}^{f(a)} g\{x(1-x)\}\,dx$, then the value of $\dfrac{I_2}{I_1}$ is- **[AIEEE-2004]**

(1) 2 **(2)** –3 **(3)** –1 **(4)** 1

18. $\lim\limits_{n\to\infty}\left[\dfrac{1}{n^2}\sec^2\dfrac{1}{n^2}+\dfrac{2}{n^2}\sec^2\dfrac{4}{n^2}+\ldots+\dfrac{1}{n}\sec^2 1\right]$ equals- **[AIEEE-2005]**

(1) $\dfrac{1}{2}\sec 1$ **(2)** $\dfrac{1}{2}\operatorname{cosec} 1$ **(3)** $\tan 1$ **(4)** $\dfrac{1}{2}\tan 1$

19. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then- **[AIEEE-2005]**

(1) $I_2 > I_1$ **(2)** $I_1 > I_2$ **(3)** $I_3 = I_4$ **(4)** $I_3 > I_4$

20. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2)=\left(\dfrac{1}{48}\right)$. Then $\lim\limits_{x\to 2}\int_6^{f(x)}\dfrac{4t^3}{x-2}\,dt$ equals -

(1) 24 **(2)** 36 **(3)** 12 **(4)** 18 **[AIEEE-2005]**

21. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ is- [AIEEE-2005]

(1) $a\pi$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{a}$ (4) 2π

22. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is - [AIEEE-2006]

(1) $\frac{3}{2}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

23. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$ is equal to- [AIEEE-2006]

(1) $(\pi^4/32)+(\pi/2)$ (2) $\pi/2$ (3) $(\pi/4)-1$ (4) $\pi^4/32$

24. $\int_0^{\pi} x f(\sin x)dx$ is equal to- [AIEEE-2006]

(1) $\pi\int_0^{\pi} f(\sin x)dx$ (2) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (3) $\pi\int_0^{\pi/2} f(\cos x)dx$ (4) $\pi\int_0^{\pi} f(\cos x)dx$

25. The value of $\int_1^a [x]f'(x)dx$, a > 1, where [x] denotes the greatest integer not exceeding x is- [AIEEE-2006]

(1) $[a] f(a) - \{f(1) + f(2) + ... + f([a])\}$ (2) $[a] f([a]) - \{f(1) + f(2) + ... + f(a)\}$
(3) $a f([a]) - \{f(1) + f(2) + ... + f(a)\}$ (4) $a f(a) - \{f(1) + f(2) + ... + f([a])\}$

26. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then F(e) equals- [AIEEE-2007]

(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) 2

27. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is- [AIEEE-2007]

(1) 2 (2) π (3) $\sqrt{3}/2$ (4) $2\sqrt{2}$

28. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true ? [AIEEE-2008]

(1) $I > \frac{2}{3}$ and $J > 2$ (2) $I < \frac{2}{3}$ and $J < 2$ (3) $I < \frac{2}{3}$ and $J > 2$ (4) $I > \frac{2}{3}$ and $J < 2$

29. $\int_0^{\pi} [\cot x]dx$, where [.] denotes the greatest integer function, is equal to - [AIEEE-2009]

(1) –1 (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) 1

30. Let p(x) be a function defined on R such that p'(x) = p'(1 – x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41.

Then $\int_0^1 p(x)\,dx$ equals :- **[AIEEE-2010]**

(1) $\sqrt{41}$ **(2)** 21 **(3)** 41 **(4)** 42

31. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is :- **[AIEEE-2011]**

(1) $\frac{\pi}{2}\log 2$ **(2)** log 2 **(3)** π log 2 **(4)** $\frac{\pi}{8}\log 2$

32. Let [.] denote the greatest integet function then the value of $\int_0^{1.5} x[x^2]dx$ is :- **[AIEEE-2011]**

(1) $\frac{5}{4}$ **(2)** 0 **(3)** $\frac{3}{2}$ **(4)** $\frac{3}{4}$

33. If $g(x) = \int_0^x \cos 4t\,dt$, then g(x + π) equals : **[AIEEE-2012]**

(1) g(x) . g(π) **(2)** $\frac{g(x)}{g(\pi)}$ **(3)** g(x) + g(π) **(4)** g(x) – g(π)

34. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. **[JEE-MAIN-2013]**

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(1) Statement-I is true, Statement-II is true; Statement-II is a correct explanation for Statement-I.
(2) Statement-I is true, Statement-II is true; Statement-II is not a correct explanation for Statement-I.
(3) Statement-I is true, Statement-II is false.
(4) Statement-I is false, Statement-II is true.

35. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals : **[JEE-MAIN-2014]**

(1) $\pi - 4$ **(2)** $\frac{2\pi}{3}-4-4\sqrt{3}$ **(3)** $4\sqrt{3}-4$ **(4)** $4\sqrt{3}-4-\frac{\pi}{3}$

36. The integral $I = \int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$ is equal to : **[JEE-MAIN-2014]**

(1) 1 **(2)** 6 **(3)** 2 **(4)** 4

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. **(A)** The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is -

(A) 3/2 **(B)** 5/2 **(C)** 3 **(D)** 5

(B) Let $g(x) = \int_0^x f(t)\,dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in (0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in (1, 2]$. Then g(2) satisfies the inequality -

(A) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ **(B)** $0 \le g(2) < 2$ **(C)** $\frac{3}{2} < g(2) \le \frac{5}{2}$ **(D)** $2 < g(2) < 4$

(C) If $f(x) = \begin{cases} e^{\cos x} . \sin x & \text{for } |x| \le 2 \\ 2 & \text{otherwise} \end{cases}$. Then $\int_{-2}^{3} f(x)dx$ -

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **[JEE 2000]**

(D) For $x > 0$, let $f(x) = \int_1^x \frac{\ln t}{1+t} dt$. Find the function $f(x) + f(1/x)$ and show that, $f(e) + f(1/e) = 1/2$.

2. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ is - **[JEE 2001]**

(A) π **(B)** $a\pi$ **(C)** $\frac{\pi}{2}$ **(D)** 2π

3. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1 + x)$, then f(4) equals - **[JEE 2001]**

(A) $\frac{5}{4}$ **(B)** 7 **(C)** 4 **(D)** 2

4. **(A)** Let $f(x) = \int_1^x \sqrt{2-t^2}\, dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are -

(A) ± 1 **(B)** $\pm\frac{1}{\sqrt{2}}$ **(C)** $\pm\frac{1}{2}$ **(D)** 0 and 1

(B) Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$ $f(x + T) = f(x)$.

If $I = \int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is -

(A) $\frac{3}{2}I$ **(B)** 2I **(C)** 3I **(D)** 6I

(C) The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equals -

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** 1 **(D)** $2\ln\left(\frac{1}{2}\right)$ **[JEE 2002]**

5. **(A)** If $\bullet(m,n)=\int_0^1 t^m(1+t)^n\,dt$, then the expression for $\bullet(m,n)$ in terms of $\bullet(m+1,n-1)$ is -

(A) $\frac{m}{n+1}\bullet(m+1,n-1)$ **(B)** $\frac{n}{m+1}\bullet(m+1,n-1)$

(C) $\frac{2^n}{m+1}+\frac{n}{m+1}\bullet(m+1,n-1)$ **(D)** $\frac{2^n}{m+1}-\frac{n}{m+1}\bullet(m+1,n-1)$

(B) If function f defined by $f(x)=\int_{x^2}^{x^2+1} e^{-t^2}\,dt$ increases in the interval -

(A) nowhere **(B)** $x\le 0$ **(C)** $x\in[-2,2]$ **(D)** $x\ge 0$ **[JEE 2003]**

6. If f(x) is an even function, then prove that $\int_0^{\pi/2} f(\cos 2x)\cos x\,dx=\sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x\,dx$ **[JEE 2003]**

7. **(A)** The value of the integral $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx$ is - **[JEE 2004]**

(A) $\frac{\pi}{2}+1$ **(B)** $\frac{\pi}{2}-1$ **(C)** -1 **(D)** 1

(B) If f(x) is differentiable and $\int_0^{t^2} x f(x)\,dx=\frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals -

(A) $\frac{2}{5}$ **(B)** $-\frac{5}{2}$ **(C)** 1 **(D)** $\frac{5}{2}$ **[JEE 2004]**

(C) If $y(x)=\int_{\pi^2/16}^{x^2}\frac{\cos x.\ \cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}\,d\theta$, then find $\frac{dy}{dx}$ at $x=\pi$.

(D) Evaluate : $\int_{-\pi/3}^{\pi/3}\frac{\pi+4x^3}{2-\cos\left(|x|+\frac{\pi}{3}\right)}\,dx$ **[JEE 2004]**

8. **(A)** If $\int_{\sin x}^{1} t^2(f(t))\,dt=(1-\sin x)$ then $f\left(\frac{1}{\sqrt{3}}\right)$ is - **[JEE 2005]**

(A) 1/3 **(B)** $1/\sqrt{3}$ **(C)** 3 **(D)** $\sqrt{3}$

(B) $\int_{-2}^{0}\left(x^3+3x^2+3x+3+(x+1)\cos(x+1)\right)dx$ is equal to -

(A) -4 **(B)** 0 **(C)** 4 **(D)** 6

9. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\,dx$ **[JEE 2005]**

10 to 12 are based on the following Comprehension

Suppose we define the definite integral using the following formula $\int_a^b f(x)\,dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b)$ $F(c) = \frac{c-a}{2}(f(a)+f(c)) + \frac{b-c}{2}(f(b)+f(c))$.

When $c = \frac{a+b}{2}$, $\int_a^b f(x)\,dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$

10. $\int_0^{\pi/2} \sin x\,dx$ is equal to -

(A) $\frac{\pi}{8}(1+\sqrt{2})$ **(B)** $\frac{\pi}{4}(1+\sqrt{2})$ **(C)** $\frac{\pi}{8\sqrt{2}}$ **(D)** $\frac{\pi}{4\sqrt{2}}$ **[JEE 2006]**

11. If $f''(x) < 0$, $\forall\ x \in (a, b)$ and c is a point such that $a < c < b$ and (c, f(c)) is the point lying on the curve for which F(c) is maximum then f'(c) is equal to -

(A) $\frac{f(b)-f(a)}{b-a}$ **(B)** $\frac{2(f(b)-f(a))}{b-a}$ **(C)** $\frac{2(f(b)-f(a))}{2b-a}$ **(D)** 0 **[JEE 2006]**

12. If f(x) is a polynomial and if $\lim_{t\to a} \frac{\int_a^t f(x)\,dx - \left(\frac{t-a}{2}\right)(f(t)+f(a))}{(t-a)^3} = 0$ for all a, then the degree of f(x) can atmost be -

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **[JEE 2006]**

13. The value of $\frac{5050\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is. **[JEE 2006]**

14. Match the following : **[JEE 2006]**

Column-I	Column-II
(A) $\int_0^{\pi/2} (\sin x)^{\cos x}\left(\cos x \cot x - \sin x . \ln(\sin x)\right)dx$	**(p)** 4/3
(B) $\left\|\int_0^1 (1-y^2)\,dy\right\| + \left\|\int_1^0 (y^2-1)\,dy\right\|$	**(q)** 1
(r) $\left\|\int_0^1 \sqrt{1-x}\,dx\right\| + \left\|\int_{-1}^0 \sqrt{1+x}\,dx\right\|$	

15. $\lim_{x\to\frac{\pi}{4}} \dfrac{\int_{2}^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ equals - **[JEE 2007]**

(A) $\frac{8}{\pi} f(2)$ **(B)** $\frac{2}{\pi} f(2)$ **(C)** $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ **(D)** $4f(2)$

16. Match the integrals in Column-I with the values in Column-II **[JEE 2007]**

Column-I	**Column-II**
(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$	**(p)** $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	**(q)** $2\log\left(\frac{2}{3}\right)$
(C) $\int_{2}^{3} \frac{dx}{1-x^2}$	**(r)** $\frac{\pi}{3}$
(D) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	**(s)** $\frac{\pi}{2}$

17. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ **[JEE 2008]**
for n = 1, 2, 3, ... Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ **(B)** $S_n > \frac{\pi}{3\sqrt{3}}$ **(C)** $T_n < \frac{\pi}{3\sqrt{3}}$ **(D)** $T_n > \frac{\pi}{3\sqrt{3}}$

18. Let f be a non-negative function defined on the interval [0, 1]. If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt$, $0 \le x \le 1$, and $f(0) = 0$, then - **[JEE 2009]**

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ **(B)** $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ **(D)** $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

19. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, n = 0, 1, 2, ..., then - **[JEE 2009]**

(A) $I_n = I_{n+2}$ **(B)** $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ **(C)** $\sum_{m=1}^{10} I_{2m} = 0$ **(D)** $I_n = I_{n+1}$

20. Let f: R $\to$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of f(ln 5) is **[JEE 2009]**

21. The value of $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}dt$ is **[JEE 2010]**

(A) 0 **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{24}$ **(D)** $\frac{1}{64}$

22. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are) **[JEE 2010]**

(A) $\frac{22}{7}-\pi$ **(B)** $\frac{2}{105}$ **(C)** 0 **(D)** $\frac{71}{15}-\frac{3\pi}{2}$

23. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x-[x] & \text{if } [x] \text{ is odd,} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is **[JEE 2010]**

24. Let f be a real-valued function defined on the interval (–1,1) such that $e^{-x}f(x) = 2+\int_0^x \sqrt{t^4+1}\,dt$, for all x $\in$ (1,1), and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to - **[JEE 2010]**

(A) 1 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{e}$

25. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$ is **[JEE 2011]**

(A) $\frac{1}{4}\ln\frac{3}{2}$ **(B)** $\frac{1}{2}\ln\frac{3}{2}$ **(C)** $\ln\frac{3}{2}$ **(D)** $\frac{1}{6}\ln\frac{3}{2}$

26. Let S be the area of the region enclosed by $y = e^{-x^2}$, y = 0, x = 0, and x = 1. Then - **[JEE 2012]**

(A) $S \ge \frac{1}{e}$ **(B)** $S \ge 1-\frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ **(D)** $S \le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

27. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is **[JEE 2012]**

(A) 0 **(B)** $\frac{\pi^2}{2}-4$ **(C)** $\frac{\pi^2}{2}+4$ **(D)** $\frac{\pi^2}{2}$

28. For $a \in R$ (the set of all real numbers), $a \neq -1$. $\lim_{n\to\infty} \frac{(1^a + 2^a + \ldots\ldots + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots\ldots+(na+n)]} = \frac{1}{60}$

Then a = **[JEE Ad. 2013]**

(A) 5 **(B)** 7 **(C)** $\frac{-15}{2}$ **(D)** $\frac{-17}{2}$

29. The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}\left(1-x^2\right)\right\} dx$ is **[JEE Ad. 2014]**

30. The following integral $\int_{\pi/4}^{\pi/3} (2\operatorname{cosec} x)^{17}\, dx$ is equal to **[JEE Ad. 2014]**

(A) $\int_0^{\log(1+\sqrt{2})} 2\left(e^u + e^{-u}\right)^{16} du$ **(B)** $\int_0^{\log(1+\sqrt{2})} \left(e^u + e^{-u}\right)^{17} du$

(C) $\int_0^{\log(1+\sqrt{2})} \left(e^u - e^{-u}\right)^{17} du$ **(D)** $\int_0^{\log(1+\sqrt{2})} 2\left(e^u - e^{-u}\right)^{16} du$

31. Let $f : [0, 2] \to R$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with $f(0) = 1$.

Let $F(x) = \int_0^{x^2} f\left(\sqrt{t}\right) dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then F(2) equals **[JEE Ad. 2014]**

(A) $e^2 - 1$ **(B)** $e^4 - 1$ **(C)** $e - 1$ **(D)** e^4

Comprehension

Given that for each $a \in (0, 1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\, dt$ exists. Let this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0, 1).

32. The value of $g\left(\frac{1}{2}\right)$ is **[JEE Ad. 2014]**

(A) π **(B)** 2π **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

33. The value of $g'\left(\frac{1}{2}\right)$ is **[JEE Ad. 2014]**

(A) $\frac{\pi}{2}$ **(B)** π **(C)** $-\frac{\pi}{2}$ **(D)** 0

34. Match the following **[JEE Ad. 2014]**

	List - I	List - II	
(p)	The number of polynomials f(x) with non-negative integer coefficient of degree $\leq$ 2, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is	**(1)**	8
(q)	The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	**(2)**	2
(r)	$\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equal	**(3)**	4
(s)	$\frac{\left(\int_{-1/2}^{1/2} \cos 2x . \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{1/2} \cos 2x . \log\left(\frac{1+x}{1-x}\right) dx\right)}$	**(4)**	0

Codes :

	p	q	r	s
(A)	3	2	4	1
(B)	2	3	4	2
(C)	3	2	1	4
(D)	2	3	1	4

35. Let $f : R \rightarrow R$ be a function defined by $f(x) = \begin{cases} [x], & x \leq 2 \\ 0, & x > 2 \end{cases}$, where [x] is the greatest integer less than or equal to x, If $I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)}$ dx, then the value of (4I – 1) is . **[JEE Ad. 2015]**

36. Let $F(x) = \int_x^{x^2 \frac{\pi}{6}} 2\cos^2 t \, dt$ for all $x \in R$ and $f : \left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by x = 0, y = 0, y = f(x) and x = a, then f(0) is. **[JEE Ad. 2015]**

37. If $\alpha = \int_0^1 \left(e^{9x + 3\tan^{-1} x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right) dx$ where tan–1 x takes only principal values, then the value of $\left(\log_e |1 + \alpha| - \frac{3\pi}{4}\right)$ is **[JEE Ad. 2015]**

38. Let $f : R \rightarrow R$ be a continuous odd function, which vanishes exactly at one pointand $f(1) = \frac{1}{2}$. Suppose that F(x) = $\int_{-1}^{x} f(t) \, dt$ for all $x \in [-1, 2]$ and G(x) = $\int_{-1}^{x} t|f(f(t))| \, dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is **[JEE Ad. 2015]**

39. The option(s) with the values of a and L that satisfy the following equation is (are) **[JEE Ad. 2015]**

$$\frac{\int_0^{4\pi} e^t\left(\sin^6 at+\cos^4 at\right)dt}{\int_0^{\pi} e^t\left(\sin^6 at+\cos^4 at\right)dt}=L$$

(A) $a=2, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ **(B)** $a=2, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

(C) $a=4, L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ **(D)** $a=4, L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

40. Let $f(x) = 7\tan^8x + 7\tan^6x - 3\tan^4x - 3\tan^2x$ for all $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the correct expression(s) is (are) **[JEE Ad. 2015]**

(A) $\int_0^{\pi/4} xf(x)dx=\frac{1}{12}$ **(B)** $\int_0^{\pi/4} f(x)dx=0$

(C) $\int_0^{\pi/4} xf(x)dx=\frac{1}{6}$ **(D)** $\int_0^{\pi/4} f(x)dx=1$

41. Let $f'(x)=\frac{192x^2}{2+\sin^4 \pi x}$ for all $x \hat{I} R$ with $f\left(\frac{1}{2}\right)=0$. If $m \le \int_{1/2}^{1} f(x)dx \le M$, then the possible values of m and M are **[JEE Ad. 2015]**

(A) $m=13, M=24$ **(B)** $m=\frac{1}{4}, M=\frac{1}{2}$

(C) $m=-11, M=0$ **(D)** $m=1, M=12$

42. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$ is **[JEE Ad. 2016]**

43. The value of $\int_{-\pi/2}^{\pi/2} \frac{x^2\cos x}{1+e^x}dx$ is equal to **[JEE Ad. 2016]**

(A) $\frac{\pi^2}{4}-2$ **(B)** $\frac{\pi^2}{4}+2$ **(C)** $\pi^2 - e^{\frac{\pi}{2}}$ **(D)** $\pi^2 + e^{\frac{\pi}{2}}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The value of the integral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

(A) π (B) 2π (C) 4π (D) none of these

2. Let $\lambda=\int_0^1\frac{dx}{1+x^3}$, $p=\lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}(n^3+r^3)}{n^{3n}}\right]^{1/n}$, then ●np is equal to

(A) $●n\,2-1+\lambda$ (B) $●n\,2-3+3\lambda$ (C) $2\,●n\,2-\lambda$ (D) $●n\,4-3+3\lambda$

3. The value of the definite integral $\int_2^3\left[\sqrt{2x-\sqrt{5(4x-5)}}+\sqrt{2x+\sqrt{5(4x-5)}}\right]dx$ is equal to

(A) $4\sqrt{3}-\frac{2\sqrt{2}}{3}$ (B) $4\sqrt{2}$ (C) $4\sqrt{3}-\frac{4}{3}$ (D) $\frac{\sqrt{10}}{2}+\frac{7\sqrt{7}-5\sqrt{5}}{3\sqrt{2}}$

4. Consider the integrals

$$I_1=\int_0^1 e^{-x}\cos^2 x\,dx,\ I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx,\ I_3=\int_0^1 e^{-\frac{x^2}{2}}\cos^2 x\,dx,\ I_4=\int_0^1 e^{-\frac{x^2}{2}}\,dx$$

Then

(A) $I_2>I_4>I_1>I_3$ (B) $I_2<I_4<I_1<I_3$ (C) $I_1<I_2<I_3<I_4$ (D) $I_1>I_2>I_3>I_4$

5. The tangent to the graph of the function $y=f(x)$ at the point with abscissa $x=1$ form an angle of $\pi/6$ and at the point $x=2$, an angle of $\pi/3$ and at the point $x=3$, an angle of $\pi/4$. The value of

$\int_1^3 f'(x)f''(x)dx+\int_2^3 f''(x)dx$ ($f''(x)$ is supposed to be continuous) is :

(A) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ (B) $\frac{3\sqrt{3}-1}{2}$ (C) $\frac{4-\sqrt{3}}{3}$ (D) None of these

6. If $S_n=\frac{1}{2n}+\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-4}}+..........+\frac{1}{\sqrt{3n^2+2n-1}}$, $n\in N$, then $\lim_{n\to\infty}S_n$ is equal to

(A) $\frac{\pi}{2}$ (B) 2 (C) 1 (D) $\frac{\pi}{6}$

7. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2}$ is equal to

(A) $\frac{1}{7}$ (B) $\frac{1}{10}$ (C) $\frac{1}{14}$ (D) none of these

8. The value of $\int_a^b (x-a)^3 (b-x)^4 \, dx$ is $\frac{(b-a)^m}{n}$. Then (m, n) is

(A) (6, 260) (B) (8, 280) (C) (4, 240) (D) none of these

9. Let $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$, ... are in :

(A) A.P. (B) G.P. (C) H.P. (D) none

10. S_1 : In $\int_{-1}^{1} f(\cot^{-1} x) dx$ putting $\cot^{-1} x = t$ may change the limits to $\int_{3\pi/4}^{\pi/4}$

S_2 : If f(x) has removable discontinuities at finite number of points in (a, b) then if $\int f(x)dx = F(x)$, $\int_a^b f(x)dx = F(b) - F(a)$.

S_3 : If f(x) has an infinite discontinuity in (a, b), then we can always write $\int_a^b f(x)dx = F(b) - F(a)$ where $\int f(x)dx = F(x)$

S_4 : If f(x) : [0, 1] $\to$ R has single point continuity in (0, 1) then $\int_0^1 f(x)dx$ can be evaluated.

(A) FTTF (B) TFFT (C) FFFF (D) TTFF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If f(x) is integrable over [1, 2], then $\int_1^2 f(x) \, dx$ is equal to

(A) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (B) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$ (C) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

12. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$, n $\in$ N, then which of the following statements hold good?

(A) $2n \, I_{n+1} = 2^{-n} + (2n-1) \, I_n$ (B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$

(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

13. If $f(x) = 2^{\{x\}}$, where {x} denotes the fractional part of x. Then which of the following is true ?

(A) f is periodic (B) $\int_0^1 2^{\{x\}} dx = \frac{1}{\ln 2}$ (C) $\int_0^1 2^{\{x\}} \, dx = \log_2 e$ (D) $\int_0^{100} 2^{\{x\}} \, dx = 100 \log_2 e$

14. If f(2 – x) = f(2 + x) and f(4 – x) = f(4 + x) and f(x) is a function for which $\int_0^2 f(x)dx = 5$, then $\int_0^{50} f(x)dx$ is equal to

(A) 125 (B) $\int_{-4}^{46} f(x)dx$ (C) $\int_1^{51} f(x)dx$ (D) $\int_2^{52} f(x)dx$

15. If $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, then $F'(4)$ equals –

(A) $\frac{32}{9}$ (B) $\frac{64}{9}$ (C) $\frac{F(8)}{28}$ (D) $\frac{11F(8)}{28}$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** $\int_0^{\pi/4} \sec x \sqrt{\frac{1-\sin x}{1+\sin x}}\,dx = 2-\sqrt{2}$

Statement-II : $\int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}\,dx = \frac{1}{2}\log(\sqrt{2}-1) + \frac{\pi}{6\sqrt{2}}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$ and $\int_1^4 \frac{3e^{\sin x^3}}{x}\,dx = F(k) - F(1)$ then one possible value of K is 64.

Statement-II : If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0\ \forall\ x \in R_0$ then $\int_{\sin\theta}^{\text{cosec}\theta} f(x)\,dx = \sin\theta - \text{cosec}\theta$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** $\int_0^n \{x\}\,dx = \frac{n}{2}$, where {.} represents fractional part function and $n \in N$.

Statement-II : $\int_0^n [x]\,dx = \frac{n(n-1)}{2}$, where [.] represents greatest integer function and $n \in N$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement-I :** $\int_0^{\pi} x \sin x \cos^2 x \, dx = \frac{\pi}{2}\int_0^{\pi} \sin x \cos^2 x \, dx$

Statement-II : $\int_a^b x\, f(x)\, dx = \frac{a+b}{2}\int_a^b f(x)\, dx$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** Let f be real valued function such that $f(2) = 2$ and $f'(2) = 1$, then $\lim_{x\to 2} \int_2^{f(x)} \frac{4t^3}{x-2} dt = 12$

Statement-II : Let $f(x) = \int_{u(x)}^{v(x)} g(t)\, dt$, then $f'(x) = g(v(x))\, v'(x) - g(u(x))\, u'(x)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I	Column – II
(A) $\int_4^{10} \frac{[x^2]dx}{[x^2-28x+196]+[x^2]} =$ {where [.] denotes greatest integer function}	**(p)** $\frac{1}{101}$
(B) $\int_{-1}^{2} \frac{\|x\|}{x} dx =$	**(q)** 3
(C) $\lim_{n\to\infty} \frac{1^{99}+2^{99}+\ldots+n^{99}}{n^{100}} =$	**(r)** $\frac{1}{3}$
(D) $5050\int_{-1}^{1}\sqrt{x^{200}}\, dx = \frac{1}{\alpha}$, then $\alpha =$	**(s)** 1
	(t) $\frac{1}{100}$

22.

Column – I		Column – II	
(A)	$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n-1}\frac{\sqrt{n^2-r^2}}{n^2} =$	**(p)**	$\frac{\pi}{4}$
(B)	$\int_0^{\frac{\pi}{4}}\left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx =$	**(q)**	$\frac{\pi}{\sqrt{2}}$
(C)	$\int_{-1}^{1}\sin^3 x\cos^2 x\, dx =$	**(r)**	$-\frac{\pi}{4}$
(D)	$\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin^3 x}dx}{\sqrt{\sin^3 x}+\sqrt{\cos^3 x}} =$	**(s)**	$\frac{\pi}{2}$
		(t)	0

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Definite integral of any discontinuous or non-differentiable function is normally solved by the property

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $c \in (a, b)$ is the point of discontinuity or non-differentiability.

1. The value of $A = \int_1^{\infty}[\operatorname{cosec}^{-1}x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) $\operatorname{cosec}1 - 1$ **(B)** 1 **(C)** $1 - \sin 1$ **(D)** none of these

2. The value of $B = \int_1^{100}[\sec^{-1}x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) sec 1 **(B)** 100 – sec 1 **(C)** 99 – sec 1 **(D)** none of these

3. The value of integral $\int_A^B[\tan^{-1}x]\,dx$, {where [.] denotes greatest integer function} ,is equal to

(A) tan 1 **(B)** 100 – tan 1 – sec1 **(C)** 99 – sec1 **(D)** none of these

24. **Read the following comprehension carefully and answer the questions.**

Using integral $\int_0^{\pi/2}\ln(\sin x)\,dx = -\int_0^{\pi/2}\ln(\sec x)\,dx = -\frac{\pi}{2}\ell n\,2$,

$\int_0^{\pi/2}\ln(\tan x)\,dx = 0$ and $\int_0^{\pi/4}\ln(1+\tan x)\,dx = \frac{\pi}{8}\ell n2$.

1. Evaluate $\int_{-\pi/4}^{\pi/4}\ln\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)dx =$

(A) $\pi\ell n2$ **(B)** $\frac{\pi\ln 2}{2}$ **(C)** 0 **(D)** $-\pi\ell n2$

2. Evalute $\int_{-\pi/4}^{\pi/4} \lambda n\,(\sin x + \cos x)dx =$

(A) $\frac{\pi \ln 2}{2}$ **(B)** $\frac{-\pi \ln 2}{4}$ **(C)** π●n2 **(D)** 0

3. Evalute $\int_0^{\pi/4} \ln(\sin 2x)\,dx =$

(A) $\frac{-\pi \ln 2}{2}$ **(B)** π●n2 **(C)** $\frac{\pi \ln 2}{4}$ **(D)** $-\frac{\pi \ln 2}{4}$

25. Read the following comprehension carefully and answer the questions.

Integral $\int_a^b f(x)\,dx$ can be represented as a limit of a sum of infinite series $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{r=na+c}^{nb+c} \frac{1}{n} f\left(\frac{r}{n}\right)$ where na +c $\le$ r $\le$ nb + c, r, n $\in$ N, c $\in$ R and any limit of sum of series of same form can be changed to definite integral by replacing

(1) $\lim_{n\to\infty} \sum \to \int$ **(2)** $\frac{1}{n} \to dx$ **(3)** $\frac{r}{n} \to x$

(4) Lower limit $= \lim_{n\to\infty} \left(\frac{r}{n}\right)_{min} = \lim_{n\to\infty} \left(\frac{na+c}{n}\right) = a$

(5) Upper limit $= \lim_{n\to\infty} \left(\frac{r}{n}\right)_{max} = \lim_{n\to\infty} \left(\frac{nb+c}{n}\right) = b$

1. Find the value of $\lim_{n\to\infty} \left(\frac{n}{(n+1)\sqrt{2n+1}} + \frac{n}{(n+2)\sqrt{2(2n+2)}} + \frac{n}{(n+3)\sqrt{3(2n+3)}} + ... + \frac{1}{2n\sqrt{3}}\right)$

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{4}$ **(D)** none of these

2. The n^{th} term of the corresponding series of $\int_0^1 \tan^{-1}x\,dx$ is

(A) $\frac{\pi}{4n}$ **(B)** $\frac{1}{n}\tan^{-1}(n-1)$ **(C)** $\frac{\pi}{2n}$ **(D)** $\tan^{-1}n$

3. $\lim_{n\to\infty} \sum_{r=0}^{2n-1} \frac{1}{n} \sec^2\left(\frac{r}{n}\right)$ is

(A) sec 2 **(B)** tan 2 **(C)** $\sec^2$ **(D)** not defined

SECTION - VI : INTEGER TYPE

26. $\underset{n\to\infty}{\text{Limit}}\ \frac{1}{n}\left[\cos^{2p}\frac{\pi}{2n}+\cos^{2p}\frac{2\pi}{2n}+\cos^{2p}\frac{3\pi}{2n}+......+\cos^{2p}\frac{\pi}{2}\right]=\frac{\lambda}{\pi}\left[\frac{2p-1}{2p}\cdot\frac{2p-3}{2p-2}\cdot............\cdot\frac{1}{2},\frac{\pi}{2}\right]$, $p\in N$, then find λ

27. Evaluate : $\lim\limits_{m\to\infty}\int\limits_{-\infty}^{\infty}\frac{dx}{1+x^2+x^4+....+x^{2m}}$; $m\in N$

28. $\int\limits_0^{2\pi}\frac{dx}{a+b\cos x+c\sin x}=\frac{\lambda\pi}{\sqrt{a^2-b^2-c^2}}$ where $a>\sqrt{b^2+c^2}>0$, then find λ

29. Given that $\lim\limits_{x\to\infty}\sum\limits_{r=1}^{n}\frac{\log_e(n^2+r^2)-2\log_e n}{n}=\log_e{}^2+\frac{\pi}{2}-2$, then

Evaluate : $\lim\limits_{n\to\infty}\frac{1}{n^{2m}}\left[(n^2+1^2)^m(n^2+2^2)^m.......(2n^2)^m\right]^{1/n}$.

30. A function f: $R\to R$ satisfies the equation $f(x+y)=f(x)+f(y)\ \forall\ x,y\in R$ and is continuous throughout the domain.

If $I_1+I_2+I_3+I_4+I_5=450$ where $I_n=n\int\limits_0^n f(x)\,dx$ and $f(x)=\lambda x$, then find λ

ANSWER KEY

EXERCISE - 1

1. A	2. B	3. C	4. C	5. A	6. A	7. D	8. D	9. A	10. A	11. C	12. C	13. D
14. A	15. A	16. D	17. B	18. A	19. B	20. C	21. A	22. A	23. B	24. C	25. B	26. A
27. B	28. D	29. D	30. D	31. D	32. A	33. D	34. A	35. A	36. C	37. B	38. B	39. D
40. C	41. B	42. C	43. A	44. A	45. B	46. C	47. D	48. A	49. A	50. D	51. B	52. C
53. C	54. D	55. B	56. A	57. C	58. D	59. B	60. D	61. C	62. A	63. C	64. D	65. D
66. A	67. B	68. C	69. A	70. A	71. C	72. C	73. D	74. D	75. A			

EXERCISE - 2 : PART # I

1. ABC	2. AD	3. ABD	4. BCD	5. AD	6. CD	7. AC	8. ABC	9. ACD
10. CD	11. ABCD	12. BD	13. ABC	14. ABC	15. ABCD	16. CD	17. AD	18. ABC
19. AD	20. ABC	21. AB	22. ABC	23. AD	24. ABC			

PART - II

1. D	2. D	3. A	4. A	5. A	6. C	7. A	8. C	9. C	10. C	11. A	12. A	13. D

EXERCISE - 3 : PART # I

1. $A \to r$ $B \to p$ $C \to s$ $D \to r$
2. $A \to s$ $B \to p$ $C \to q$
3. $A \to q$ $B \to p$ $C \to s$ $D \to s$
4. $A \to q$ $B \to r$ $C \to s$ $D \to p$
5. $A \to s$ $B \to r$ $C \to s$ $D \to q$
6. $A \to q$ $B \to r$ $C \to p$ $D \to s$
7. $A \to r$ $B \to r$ $C \to q$ $D \to q$
8. $A \to s$ $B \to r$ $C \to r$ $D \to q$
9. $A \to q$ $B \to s$ $C \to q$
10. $A \to q$ $B \to p$ $C \to p,r$ $D \to p,r,s$
11. $A \to q$ $B \to s$ $C \to p$ $D \to p$
12. $A \to q$ $B \to s$ $C \to p$ $D \to r$
13. $A \to s$ $B \to r$ $C \to p$ $D \to q,r$

PART - II

Comprehension # 1 : 1. D 2. A 3. B

Comprehension # 2 : 1. B 2. D 3. A

Comprehension # 3 : 1. D 2. A 3. C

Comprehension # 4 : 1. A 2. B

Comprehension # 5 : 1. C 2. D 3. B

Comprehension # 6 : 1. C 2. **A** 3. A

Comprehension # 7 : 1. B 2. C 3. D

Comprehension # 8 : 1. B 2. A 3. A

Comprehension # 9 : 1. A 2. C 3. B

Comprehension # 10 : 1. B 2. C 3. D

Comprehension # 11 : 1. A 2. A 3. B

EXERCISE - 5 : PART # I

1. 1	2. 1	3. 3	4. 1	5. 2	6. 1	7. 3	8. 4	9. 4	10. 1	11. 3	12. 4	13. 2
14. 1	15. 3	16. 2	17. 1	18. 4	19. 2	20. 4	21. 2	22. 1	23. 2	24. 3	25. 1	26. 1
27. 1	28. 2	29. 2	30. 2	31. 3	32. 4	33. 3,4	34. 4	35. 4	36. 1			

PART - II

1. **(A)** B **(B)** B **(C)** C **(D)** $\frac{1}{2}\ln^2 x$ **2.** C **3.** C **4.** **(A)** A **(B)** C **(C)** B

5. **(A)** D **(B)** B **7.** **(A)** B **(B)** A **(C)** 2π **(D)** $\frac{4\pi}{\sqrt{3}}\tan^{-1}\frac{1}{2}$ **8.** **(A)** C **(B)** C

9. $\frac{24}{5}\left[e\cos\left(\frac{1}{2}\right)+\frac{e}{2}\sin\left(\frac{1}{2}\right)-1\right]$ **10.** A **11.** A

12. A **13.** 5051 **14.** **A** $\rightarrow$ q **B** $\rightarrow$ p,r **15.** A **16.** **A** $\rightarrow$ s **B** $\rightarrow$ s **C** $\rightarrow$ p **D** $\rightarrow$ r

17. AD **18.** C **19.** ABC **20.** 0 **21.** B **22.** A **23.** 4 **24.** B **25.** A

26. ABD **27.** B **28.** B **29.** 2 **30.** A **31.** B **32.** A **33.** D **34.** D

35. 0 **36.** 3 **37.** 9 **38.** 7 **39.** AC **40.** AB **41.** D **42.** 1 **43.** A

MOCK TEST

1. A **2.** B **3.** D **4.** C **5.** D **6.** D **7.** C **8.** B **9.** A

10. D **11.** BC **12.** AB **13.** ABCD **14.** ABD **15.** AD **16.** C **17.** C **18.** B

19. C **20.** D **21.** A $\rightarrow$ q B $\rightarrow$ s C $\rightarrow$ t D $\rightarrow$ t **22.** A $\rightarrow$ p B $\rightarrow$ q C $\rightarrow$ t D $\rightarrow$ p

23. **1.** A **2.** B **3.** B **24.** **1.** C **2.** B **3.** D **25.** **1.** A **2.** A **3.** B

26. 2 **27.** $\frac{4}{3}$ **28.** 2 **29.** $\left(\frac{2\sqrt{e^{\pi}}}{e^2}\right)^m$ **30.** 4