HINTS & SOLUTIONS

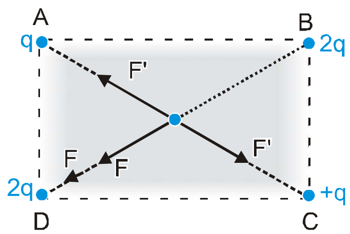
EXERCISE - 1

Single Choice

1. $\frac{K(9e)q}{r^2} = \frac{Keq}{(16-r)^2} \Rightarrow \frac{3}{r} = \frac{1}{16-r} \Rightarrow r = 12$ cm

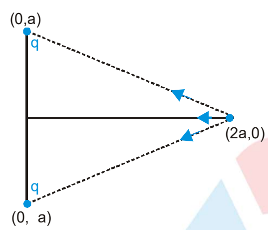
2. $[\epsilon_0] = \frac{[Q^2]}{[F][4\pi][\sigma^2]} = \frac{A^2T^2}{MLT^{-2} \times L^2} = M^{-1}L^{-3}T^4A^2$

3.



Hence net force is along BD

4. $F = QE = \frac{QKQx}{(R+x^2)^{3/2}}$



$F \propto (-x)$

(required equation for SHM)

5. Same charges repels each other.

6. $t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2x \times m}{qE}}$ $\therefore t \propto \sqrt{\frac{m}{q}} \propto \sqrt{m}$

[when x, q & E are same] $\therefore \frac{t_2}{t_1} = \sqrt{\frac{m_p}{m_e}}$

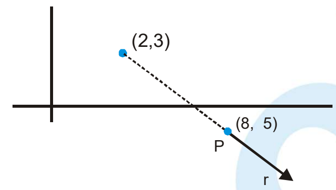
7. Force between two line charges On a unit length

$= \frac{2K\lambda}{r} \times \lambda = \frac{2 \times 9 \times 10^9 \times (5 \times 10^{-6})^2}{0.1} = 4.5$ N/m

9. If particle will loose KE (in that direction) against work done by electric field $\frac{K}{2} = qEd \Rightarrow E = \frac{K}{2qd}$

10. Charge moves $\perp^r$ to the field lines. So the work done will be zero.

11.



$E = \frac{KQ}{r^2}\hat{r} = \frac{9 \times 10^9 \times 50 \times 10^{-6}}{100} \times \left(\frac{6\hat{i} - 8\hat{j}}{10}\right)$

$= 4500$ V/m.

12. Work done by external force $= \Delta U$

[It is state function]

13. By mechanical energy conservation

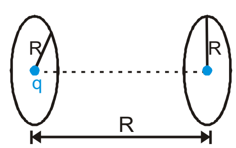
$(PE + KE)_i = (PE + KE)_f$

$0 + \frac{1}{2}mv^2 + 0 = \frac{KQ^2}{d} + \frac{1}{2}m\left(\frac{v}{2}\right)^2 x2$

(✈ from momentum conservation at closet approach, both particle will move with a common speed v/2)

$\therefore d = \frac{4KQ^2}{mv^2}$

14. $U_i = \frac{KQ_1q}{R} + \frac{KQ_2q}{\sqrt{R^2+R^2}}$



$U_f = \frac{KQ_1q}{\sqrt{R^2+R^2}} + \frac{KQ_2q}{R}$

Work done by external force : $\Delta U = U_f - U_i$

15. Let distance of closest approach be 'd' then

$\frac{1}{2}mv^2 = \frac{K(2e)(9)(2e)}{d^2} \Rightarrow d = 10^{-12}$ cm

16. $V = (n-1)\frac{KQ}{r}$, $E = \frac{KQ}{r^2} \Rightarrow \frac{V}{E} = (n-1)r$

17. $F = QE \Rightarrow 300 = 3 \times E \Rightarrow E = 100$ N/C

$E = \frac{dV}{dx} \Rightarrow \Delta V = 100 \times \frac{1}{10} = 10$ V

18. Equipotential lines are always $\perp^r$ to the electric field strength lines .

✈ slope of equipotential lines =2

$\therefore$ Slope of electric field must be = – 1/2

$\Rightarrow$ Electric field strength vector $= -8\hat{i} + 4\hat{j}$

19. Slope from x axis

$y = 3 + x;\ \tan\theta = 1 \Rightarrow \theta = 45°$

$\therefore$ Electric field in vector from $\vec{E} = 100\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$$\therefore V = -\int_{(3,1)}^{(1,3)} \vec{E}\cdot d\vec{r}$$

$$V = \frac{-100}{\sqrt{2}}\left[\int_3^1 dx + \int_1^3 dy\right] = 50\sqrt{2}\,[-2+2] = 0$$

Alternate solution : The direction of electric field and the slope of line A (3,1) & (1,3) is $\perp$ to each others so the dot product $\vec{E}.d\vec{r}$ becomes zero.

20. Slope of equipotential lines will be = 1/2

$\therefore$ Slope of electric lines must be = – 2

OR

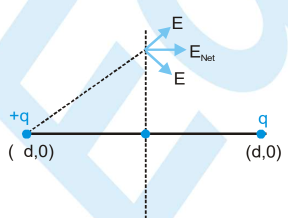
$$E_x = -\left(\frac{\Delta V}{\Delta x}\right)_{y=constant} = -\frac{(4-2)V}{(4-2)cm} = -100\ V/m$$

$$E_y = -\left(\frac{\Delta V}{\Delta y}\right)_{x=constant} = -\frac{(2-4)V}{(1-0)cm} = 200\ V/m$$

21. $$E = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -k\left[4x\hat{i} - 2y\hat{j} + 2z\hat{k}\right]$$

$$= -k\left[\sqrt{16+4+4}\right] = 2k\sqrt{6}$$

22. In figure (–d,0) to (d,0) on x–axis the direction of $\vec{E}$ in +ve x–axis and left side of (–d,0) the direction in –ve x–axis, but on y–axis, at any point the net electric field along the x–axis.



23. $-\int_{\infty}^{l=0} \vec{E}.\vec{dl}$ is the potential at the centre of the ring which is

$$V = \int dV = \int \frac{Kdq}{0.5} = \frac{Kq}{0.5}$$

$$V = \frac{9\times10^9 \times 1.11\times10^{-10}\times 2}{1} = 2\ \text{volt}$$

24. Electric field lines can't be closed

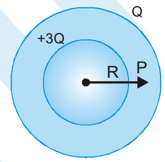
25. W=(–pEcosθ) – (– pE)=PE(1–cosθ)

26. From flux $= \int \vec{E}.\vec{ds} = \frac{Eq_{enclosed}}{\epsilon_0}$ $\quad\therefore$ flux $= \frac{Q}{\epsilon_0}$

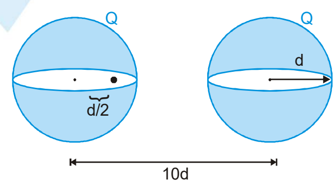
27. Area(100 m^2) in xy plane so area vector in $\hat{k}$

$$= \int \vec{E_z}\cdot d\vec{S} = \sqrt{3}\hat{k}.100\hat{k} = 173.2$$

28. $$q = \epsilon_0 \int \vec{E_x}.\vec{dx} = \epsilon_0 \times 600\int_{0.1}^{0.2}\frac{dx}{\sqrt{x}} = 7\times10^{-12}C$$

29. 

$$E_P = \frac{3Q}{4\pi\epsilon_0 R^2}$$

30. 

$$F = \frac{1}{4\pi\epsilon_0}\times\frac{Qq}{\left(\frac{19}{2}d\right)^2} + 0 = \frac{qQ}{361\pi\epsilon_0 d^2}$$

31. For any metallic surface, electric field lines are $\perp^r$ to the surface.

32. In hollow sphere potential remains constant inside the shell.

33. The potential difference (work done against field) between the shell & sphere is due to the field present inside that region which is only due to the sphere.

34. 1, 2, & 3 are wrong because in metallic solid sphere, there is no field inside the sphere. Option (4) is correct from given reason and also it has field lines perpendicular to the surface (as required in metallic surface)

EXERCISE - 2

Part # I : Multiple Choice

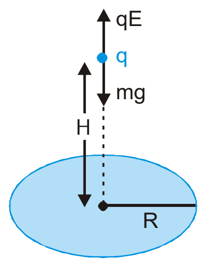
1. $mg = q \times \dfrac{KQH}{(R^2 + H^2)^{3/2}}$

The field due to ring on its axis will be maximum at

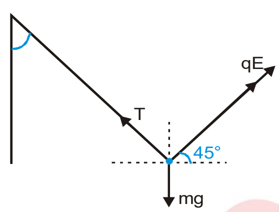
$H = \pm \dfrac{R}{\sqrt{2}}$

i.e. above that point qE force will decrease and resultant force becomes in downward direction (equilibrium position) and also in same way for below point.

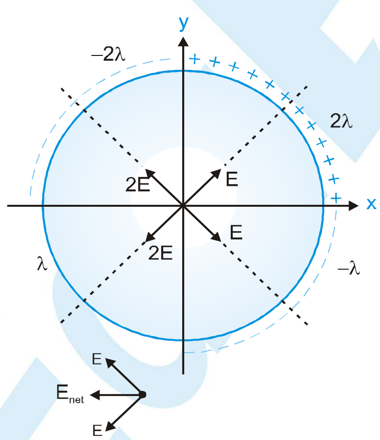
$mg = q \times \dfrac{KQH}{(R^2 + H^2)^{3/2}}$



2. T sinα = qE cos 45° and Tcosα+qE sin 45° = mg



3. $E_{net} = \sqrt{2}E = \sqrt{2}\left[\dfrac{\lambda\sqrt{2}}{4\pi \in_0 R}\right] \Rightarrow \overset{r}{E} = \dfrac{\lambda(-\hat{i})}{2\pi \in_0 R}$



4. $E = \dfrac{KQr}{(R^2 + r^2)^{3/2}}$.

It is maximum at $r = \pm \dfrac{R}{\sqrt{2}}$ and also E is not a linear function of r.

5. Potential at any point is a scalar quantity.

✈ Positive charge = negative charge i.e. the net charge is equal to zero. Hence potential is zero. Electric field is a vector quantity so it depends not only distance but also the way of distribution of charge.

6. $V = \dfrac{KQ}{r} = \dfrac{9 \times 10^9 \times 10^{-8}}{\sqrt{(1-4)^2 + (3-7)^2 + (2-2)^2}} = 18V$

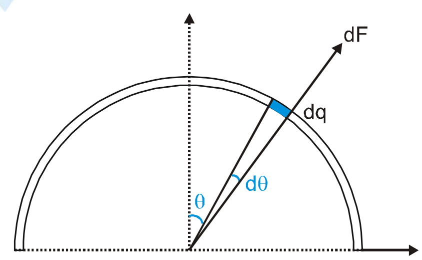
and electric field lines will be in all three direction.

$V = \dfrac{KQ}{r} = \dfrac{9 \times 10^9 \times 10^{-8}}{\sqrt{(1-4)^2 + (3-7)^2 + (2-2)^2}} = 18V$

7. At point A the field lines are much closer than B hence $E_A > E_B$.

Work done by external force in direction of field lines is negative, hence $V_B > V_A$.

8.

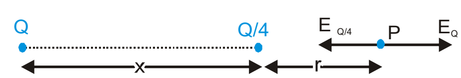


$dF = (dq)\left(\dfrac{\lambda}{2\pi \in_0 R}\right) = \left(\dfrac{qd\theta}{\pi}\right)\left(\dfrac{\lambda}{2\pi \in_0 R}\right)$

$F_{net} = \int_0^{\pi/2} 2\,dF\cos\theta = \dfrac{q\lambda}{\pi^2 \in_0 R}$

$\Rightarrow \int_0^{\pi/2} \cos\theta d\theta = \dfrac{q\lambda}{\pi^2 \in_0 R}$

9. No point exist in between the charges where field is zero.



$E = \dfrac{KQ}{(r+x)^2} - \dfrac{K\frac{Q}{4}}{r^2} = 0 \Rightarrow r = x$

10. Here potential decreases 2V as we moves unit distance. Hence for point (1, 1, 1) from (0, 0, 0). The total potential decrease is 2 +2 +2 = 6V. Hence the potential at point will (10 – 6) = 4V.

11. If Y is fixed i.e. another force is exist on Y additional to the mutual interaction between X & Y. So the net force on system (X &Y) is not zero so p is changed but total energy always remain conserved.

12. Equal electrostatic force (mutual interaction) acts on X and Y but in opposite direction which accelerate Y but retards X. After long time the velocity of X becomes zero while Y becomes u.

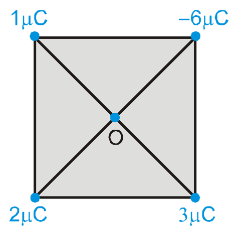
13. $V_A = 3$ volt, $V_B = 7$ volt

From energy conservation $(\text{Energy})_A = (\text{Energy})_B$

$\Rightarrow 1/2\, mv^2 - e \times 7V = -e \times 3V + 0$

$\Rightarrow 1/2\, mv^2 = 4eV$

14. Here total charge is zero.



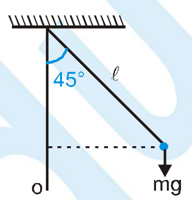
Any point on z–axis, having the same distance from the each vertex of a square. So the potential due to all is zero at that point.

15. If electric field and gravitational field will be in same line then the path may be straight line otherwise parabola.

16. Velocity due to acceleration $= \dfrac{10^{-6} \times 300}{10^{-3}} \times 10 = 3$ m/s

Then the resultant velocity may be between 1 m/s and 7 m/s

17. By work energy theorem $W_g + W_{elec} = \Delta KE$



$$mg\bullet\left(1 - \frac{1}{\sqrt{2}}\right) + qE \times \frac{1}{\sqrt{2}} = \frac{1}{2}mv^2 - 0$$

$$\left(Q\ E = \frac{mg}{q}\right)\quad u = \sqrt{2gl}\ \ ;\ W = \frac{u}{l} = \sqrt{\frac{2g}{l}}$$

18. As $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$

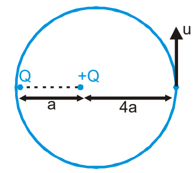
So $V = \dfrac{\lambda}{2\pi \epsilon_0} \ln r \Rightarrow W = \dfrac{\lambda}{2\pi \epsilon_0} \ln\left|\dfrac{r_2}{r_1}\right|$

$$W = \frac{\lambda}{2\pi \epsilon_0}\ln\left(\frac{\sqrt{2}}{\sqrt{1}}\right) + \frac{2\lambda}{2\pi \epsilon_0}\ln\left(\frac{\sqrt{2}}{\sqrt{2}}\right) + \frac{3\lambda}{2\pi \epsilon_0}\ln\left(\frac{\sqrt{2}}{\sqrt{1}}\right)$$

$$= \frac{\lambda}{2\pi \epsilon_0}\ln 2$$

19. Given V_A due to +Q is V, so $V_B = 4V$

By energy conservation



$$\frac{1}{2}mu^2 + qV = 4qV$$

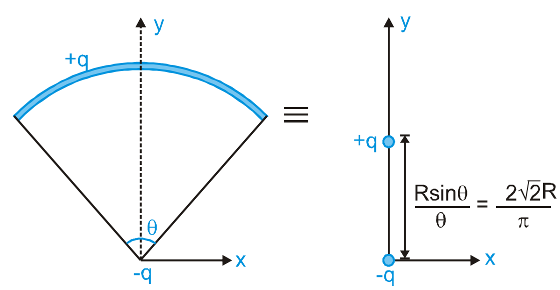
$$\Rightarrow u = \sqrt{\frac{6qV}{m}}$$

20. The mass of deuteron is twice,so momentum is different.

21. $E = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial x}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

Here $\dfrac{\partial V}{\partial x} = -\dfrac{V}{X_0}$ $\quad\therefore\quad E_{min} = \dfrac{V}{x_0}\hat{i}$.

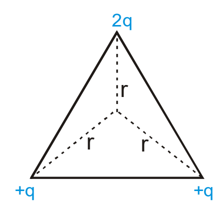
22.



Dipole moment $= (q)\left(\dfrac{2\sqrt{2}R}{\pi}\right) = \dfrac{2\sqrt{2}qR}{\pi}$

23. Potential at centroid of $\Delta = \dfrac{Kq}{r} + \dfrac{Kq}{r} - \dfrac{K2q}{r} = 0$

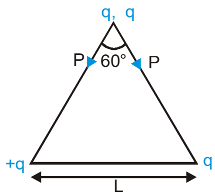
Dipole moment of system



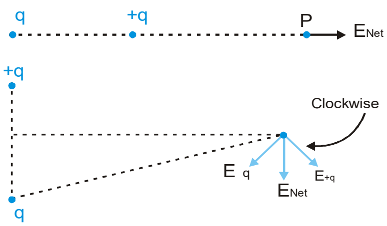
$= 2p \cos 30° = \sqrt{3}p$

$= \sqrt{3} \times q \times L$

$(\because p = q \times L)$



24.



25. Force on a dipole $F = p\dfrac{\partial E}{\partial x}$

At $x = \dfrac{R}{\sqrt{2}}$, $\dfrac{\partial E}{\partial x} = 0$, so $F = 0$

26. Interaction energy $U = -\vec{p}_1 \cdot \vec{E}$

where $\vec{E} = \dfrac{2Kp_2\cos\theta}{r^3}\hat{r} + \dfrac{Kp_2\sin\theta}{r^3}\hat{\theta}$

and $\vec{p}_1 = p_1\hat{r}$

Therefore $U = -\dfrac{2Kp_1p_2\cos\theta}{r^3}$

27. ✈ Electric flux depends only on the total charge enclosed by the Gaussian surface. Hence[A].

Potential at a distance r $= \dfrac{Kq}{r}$

Total charge of dipole $= 0$

28. Electric flux depends on the total charge enclosed by the closed surface S. Hence flux is related with charge Q_1. Electric field at a point is the vector sum of the all electric field intensities due to all charges.

29. Potential at 5 cm from surface $= \dfrac{KQ}{R+5} = 100$

Potential at 10 cm from surface

$= \dfrac{KQ}{R+10} = 75 \Rightarrow R = 10\text{ cm}$

$\therefore$ Potential at surface $= \dfrac{KQ}{R} = \dfrac{100 \times 15}{10} = 150\text{V}$

Electric field on surface $= \dfrac{KQ}{R^2} = \dfrac{100 \times 15\text{V} \times \text{cm}}{100\text{cm}^2}$

$= 1500$ V/m

30. ✈ Electric field $= \dfrac{\sigma}{\epsilon_0}$ due to infinitely uniform charged.

Electric flux $= \dfrac{\Sigma q_{enclosed}}{\epsilon_0} = \int \dfrac{\sigma}{\epsilon_0} \times [\pi R^2 - \pi x^2]$

31. $\dfrac{KQ}{R} + \dfrac{1}{2}mu^2 = \dfrac{KQ}{2R^3}\left(3R^2 - \dfrac{R^2}{4}\right) + 0$

$\dfrac{1}{4\pi\epsilon_0 R} \times \rho \times \dfrac{4}{3}\pi R^3 + \dfrac{1}{2}mu^2 = \dfrac{11 \times \rho \times \frac{4}{3}\pi R^3}{8 \times 2R^2}$

$\therefore \quad u = \left[\dfrac{\rho R^2}{4m\epsilon_0}\right]^{1/2}$

32. As the electric field converges at the origin so total charge contained in any spherical volume, irrespective of the location, is negative.

By Gauss theorem $\int \vec{E} \cdot d\vec{s} = \dfrac{q}{\epsilon_0}$

We have $-E(4\pi r^2) = \dfrac{q}{\epsilon_0} \Rightarrow q = -3 \times 10^{-13}\text{C}$

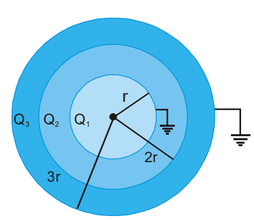
33. Energy at surface = Energy at centre

$\dfrac{1}{2}mu^2 + \dfrac{Kq \times q}{R} = \dfrac{3}{2}\dfrac{Kq}{R} \times q + 0$

$\therefore \quad U = \dfrac{q}{\sqrt{4\pi\epsilon_0 mR}}$

34. Due to the induction, the opposite nature of charge is induced at near by surface.

35.



Potential at outermost shell

$$=\frac{KQ_1}{3r}+\frac{KQ_2}{3r}+\frac{KQ_3}{3r}=0$$

$\Rightarrow Q_1+Q_2+Q_3=0$ ....(i)

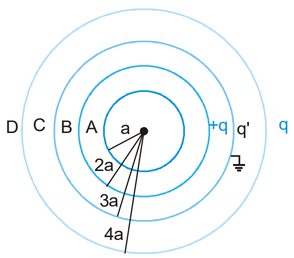
and potential at innermost surface

$$\frac{KQ_1}{r}+\frac{KQ_2}{2r}+\frac{KQ_3}{3r}=0$$

$$\Rightarrow \frac{K[6Q_1+3Q_2+2Q_3]}{6r}=0 \text{ ....(ii)}$$

From eq. (i) & (ii) $Q_1=-\frac{Q_2}{4}$ and also $\frac{Q_3}{Q_1}=3$.

36.

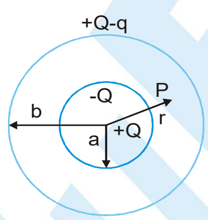


Potential at C $=\frac{Kq}{3a}-\frac{Kq}{4a}+\frac{Kq'}{3a}=0 \Rightarrow q'=-\frac{q}{4}$

Potential at A $=\frac{Kq}{2a}-\frac{Kq}{4a}-\frac{K\frac{q}{4}}{3a}=\frac{Kq}{6a}$.

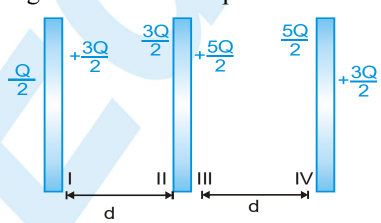
Hence $V_A-V_C=\frac{Kq}{6a}$

37.



from figure $V_P=\frac{K(Q-q)}{b}$

38. Final charge distribution on plate



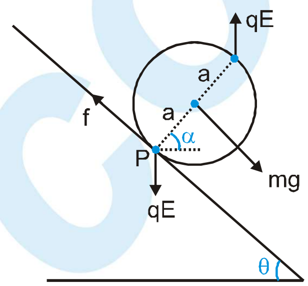
39. Surface charge density at inner surface of X is

$$\sigma=\frac{Q}{2A}$$

$\therefore$ Electric field at B due to this is $\frac{\sigma}{2\epsilon_0}=\frac{Q}{4\pi\epsilon_0}$

Towards right and in same direction of same value due to induced charged present inside surface of plate Y.

40.



For rotational equilibrium, $\sum \tau_P=0$

$\Rightarrow mga\cos\alpha-qE(2a\cos\alpha)=0 \Rightarrow E=\frac{mg}{2q}$

Part # II : Assertion & Reason

1. C	**2.** D	**3.** C	**4.** A	**5.** B	**6.** C
7. B	**8.** C	**9.** D	**10.** B	**11.** B	**12.** A
13. A	**14.** D	**15.** D	**16.** D		

EXERCISE - 3

Part # I : Matrix Match Type

1. Electric field $\vec{E}=-\frac{\partial V}{\partial x}\hat{i}-\frac{\partial V}{\partial y}\hat{j}$

For (A) : $\vec{E}=-\hat{i}+\frac{1}{\sqrt{3}}\hat{j}$

For (B) : $\vec{E}=+\hat{i}$

For (C) : $\vec{E}=-\hat{i}+\sqrt{3}\hat{j}$

For (D) : $\vec{E}=+\frac{1}{\sqrt{3}}\hat{i}+\sqrt{3}\hat{j}$

2. Electric field due to metallic plates remains same and constant at near by points.

[A] For $\sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_1 = -\sigma_2$

$\therefore$ Electric field at a point is equal & opposite in direction.

$\sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_1 = -\sigma_2$

[B] $\sigma_1 + \sigma_2 > 0 \Rightarrow \sigma_1 \& \sigma_2$ [densities]

either both positive or opposite but positive has a greater magnitude. So the net electric field will be away from the plates in region I & III.

[C] Same explanation according to [B]

3. $\because$ Electric field due to an electric dipole at a point on equatorial line of dipole makes either 0° or 180° with the dipole moment of another dipole.

$\therefore$ Torque on dipole $\vec{\tau} = \vec{P} \times \vec{E}$ becomes zero

($\because \theta = 0$ or $\theta = 180°$) Hence in column II,

[P] option is suit for every queries for column I.

Electrostatic potential energy (U) =– PE cosθ

θ = Angle between moment & electric field.

[A] Here θ = 180° $\therefore$ U = – PEcos 180° = PE (+ve)

[B] Here θ=0° $\therefore$ U = –PE cos θ = –PE (–ve)

[C] & [D] : θ < 90° $\therefore$ U = –ve

4. **(A)** Initially, the potential difference exist between both shells, so positive charge is flow from high to low potential.

Every system wants to acquire minimum potential energy if possible for stability. So charge flown to achieve it.

(B) As explained in [A], charge flow does not depend on the size of sphere.

(C) Charge flow through wire until the potential becomes same for both shells.

(D) Potential is same everywhere inside a conducting shell. So no charge is flow through connecting wire, so no heat is produced.

5. **(A)** Electric field at a point is the vector sum of all individual fields at that point

(B) Electric flux $\oint \vec{E} \cdot d\vec{S} = \dfrac{q_{enc}}{\epsilon_0}$

(C) Electric flux $\oint \vec{E} \cdot d\vec{S} = \dfrac{q_{enclosed}}{\epsilon_0}$

6. Electrostatic potential energy

= self energy + interaction energy

(A) Self energy of uniformly charged thin shell

$$= \frac{KQ^2}{2a} \text{ (radius a)}$$

(B) Self energy $= \dfrac{KQ^2}{2 \times \dfrac{5a}{2}}$ &

Interaction energy $= \dfrac{KQ^2}{\dfrac{5a}{2}}$

(C) Self energy of solid sphere of radius 'a'= $\dfrac{3KQ^2}{5a}$

Part # II : Comprehension

Comprehension# 1

1. $\because$ Gravity is absent and path of the particle is parabola i.e. a downward (qE) force is necessary.

2. Acceleration of the particle is given by

$$a = \frac{qE}{m} \Rightarrow a \propto \frac{q}{m}$$

3. $$h = \frac{u^2 \sin^2\theta}{2a} = \frac{100 \times 100 \times \left(\frac{1}{2}\right)^2}{2 \times \frac{1 \times 10}{1}} = 125\text{ m}$$

4. $$a = \frac{qE}{m} \Rightarrow m \uparrow \Rightarrow a \downarrow \Rightarrow h = \frac{u^2 \sin^2\theta}{2 \times \frac{qE}{m}}$$

$$\Rightarrow h \propto \frac{m}{q} \text{ [tan u, } \theta \text{ \& E is constant]}$$

Comprehension# 2

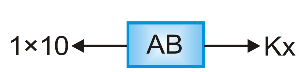
1. Velocity of B, when it strikes 'A' is

$$v_B = \sqrt{2 \times \frac{1 \times 10}{1} \times 1.8} = 6 \text{ m/s}$$

From COLM between 'A' & 'B' is

0 + 1 × 6 = (1+1) × v = 3m/s [left]

2. Equilibrium position $\Rightarrow \Sigma \vec{F}_{ext} = 0$



$$Kx = 10 \Rightarrow x = \frac{10}{18} = \frac{5}{9} \text{ (Left from x = 0)}$$

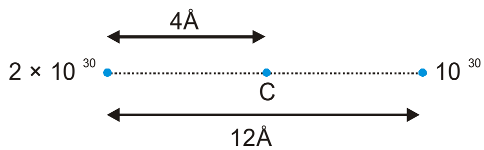
3. At equilibrium position the spring is compressed by x = 5/9. Let the amplitude of oscillation be 'A'

$$\therefore \frac{1}{2}KA^2 = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$$

$$A = \sqrt{x^2 + \frac{m}{K}v^2} = \sqrt{\frac{25}{81} + \frac{2}{18} \times 9} = \frac{\sqrt{106}}{9}$$

Comprehension# 3

1.



For first particle

$$a = \frac{v^2}{r} = \frac{(10^3)^2}{4 \times 10^{-10}} = 2.5 \times 10^{15} \text{ m/s}^2$$

2. Acceleration of second particle

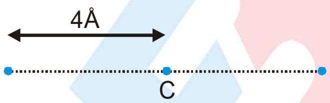
$$= \frac{v^2}{r} = \frac{(2 \times 10^3)^2}{8 \times 10^{-10}} = \frac{4}{8} \times 10^{16} = 5 \times 10^{15} \text{m/s}^2$$

3. Velocity of centre of mass

$$= \frac{10^{-30} \times 2 \times 10^3}{2 \times 10^{-30} + 10^{-30}} = \frac{2}{3} \times 10^3 \text{ m/s}$$

4. Both particles move in a circular orbit about their centre of mass.

5. Angular velocity

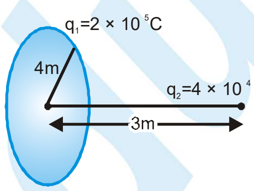


$$\omega = \frac{v}{r}$$

$$\omega = \frac{10^3}{4 \times 10^{-10}} = 2.5 \times 10^{12} \text{rad/s}$$

Comprehension# 4

1.



From energy conservation

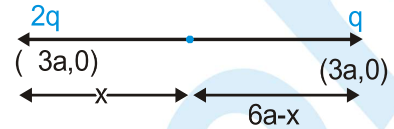
$$\frac{Kq_1q_2}{\sqrt{4^2+3^2}} + 0 + 0 = 2 \times \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{\frac{9 \times 10^9 \times 2 \times 10^{-5} \times 4 \times 10^{-4}}{5}} = \frac{3 \times 2\sqrt{2}}{\sqrt{5}} = 3.1 \text{ m/s}$$

2. $\frac{Kq_1q_2}{5} = \frac{1}{2}mv^2 \Rightarrow v = \frac{2}{\sqrt{5}}$

Comprehension# 5

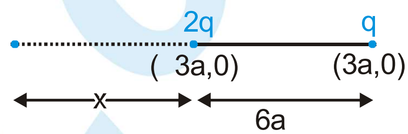
1. Let the potential will be zero at x distance from –2q.



$$\therefore \frac{Kx - 2q}{x} + \frac{Kq}{(6a - x)} = 0 \quad \therefore x = 4a$$

$$\frac{Kx - 2q}{x} + \frac{Kq}{6a + x} = 0 \Rightarrow x = -12a$$

OR



2. At x = – 3a & x = 3a, the potential becomes – ∞ & + ∞ respectively and from the above question potential becomes zero at (a, 0) and (9a, 0)

Comprehension# 6

1. P.E. = $\frac{1}{4\pi \epsilon_0} \frac{Q_e}{r}$ and KE = $\frac{1}{2}mv^2$

Hence total energy is greater than $\frac{eQ}{4\pi \epsilon_0 r}$

2. After long time sphere gets positive charge so the trajectory of the proton is (4) due to repulsion.

3. From the angular momentum conservation

$$mv_0 \times \frac{R}{2} = mv \times R \Rightarrow v = \frac{v_0}{2}$$

4. Limiting electric potential = change in ΔKE

$$(KE)_i = \frac{1}{2}mv_0^2 \, ; (KE)_f = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2$$

$$\therefore \text{Electric potential} = \frac{3mv_0^2}{8e}$$

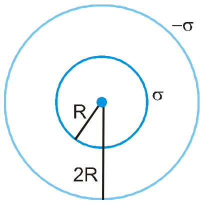
5. From previous questions we can see that the final potential energy of the sphere is equal to the 3/4 of initial kinetic energy

$$\Rightarrow \frac{3}{4} \times 2.56 = 1.92 \text{ kV}$$

Comprehension# 7

1. $E = \frac{KQ}{r^2} \Rightarrow Q = \frac{Er^2}{K} = 13\mu C$

Comprehension# 8



1. $Q_1 = \sigma \times 4\pi R^2$

$Q_2 = -\sigma \times 4\pi (2R)^2 = -4Q_1$

Potential at outer shell

$$= \frac{K[Q_1 - Q_2]}{2R} = \frac{K[Q_1 - 4Q_1]}{2R}$$

$$= \frac{-1 \times 3 \times \sigma \times 4\pi R}{4\pi \epsilon_0 \times 2R} = -\frac{3}{2}\frac{\sigma}{\epsilon_0} \times R$$

2. Electric field (r > 2R) before connecting the shells

$$E_1 = \frac{1 \times \sigma \times 4\pi R^2}{4\pi \epsilon_0 \times r^2} + \frac{1(-\sigma) \times 4\pi R^2 \times 4}{4\pi \epsilon_0 \times r^2}$$

$$= \frac{\sigma}{\epsilon_0}\left[\frac{R^2}{r^2} - \frac{4R^2}{r^2}\right]$$

After connecting :

$$E_2 = \frac{1}{4\pi \epsilon_0}\frac{[Q_1 - Q_2]}{r^2} = \frac{\sigma}{\epsilon_0}\left[\frac{R^2}{r^2} - \frac{4R^2}{r^2}\right]$$

$\therefore E_1 / E_2 = 1$

3. Electric field at $r = \frac{3R}{2}$ [Before connecting]

$$E_1 = \frac{1}{4\pi \epsilon_0} \times \frac{\sigma 4\pi R^2}{\left(\frac{3R}{2}\right)^2} + 0$$

After connecting $E_2 = 0$.

Hence $\left|\frac{E_2}{E_1}\right| = 0$

Comprehension# 9

1. As given in paragraph, it is treated as +q and –q point charge at a distance 2a

$$F = \frac{1}{4\pi \epsilon_0}\frac{(-q)(q)}{(2d)^2}$$

2. Potential energy $= -\int \vec{F} \cdot d\vec{r}$

$$= +\int_{\infty}^{d} \frac{q^2}{4\pi \epsilon_0 (2r)^2} dr = -\frac{1}{4\pi \epsilon_0}\frac{q^2}{4d}$$

Note : Here potential energy $\neq -\frac{1}{4\pi \epsilon_0}\frac{q^2}{2d}$ because it is not an electrostatic field.

3. Work done by external force is change in potential energy

Comprehension# 10

1. $E = \frac{-dV}{dr}$; $\frac{\sigma}{\epsilon_0} = \frac{-30}{0.3}$

$\sigma = 8.85 \times 10^{-12} \times 10^2 = 8.85 \times 10^{-10}\ C/m^2$

2. Positive charge has a tendency to move from higher potential to lower potential hence it will move from B (–20V) to A (–30V).

3. Here $V_{DE} = V_D - V_E = 20 - (-20) = 40V$

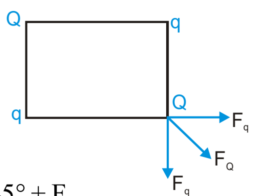
Work done $= qV_{DE}$

$= -1 \times 10^{-6} \times 40$

$= -4 \times 10^{-5}\ J$

EXERCISE - 4
Subjective Type

1. $(F_Q)_{total} = \left|\vec{F}_{Qq} + \vec{F}_{Qq} + \vec{F}_{qQ}\right|$

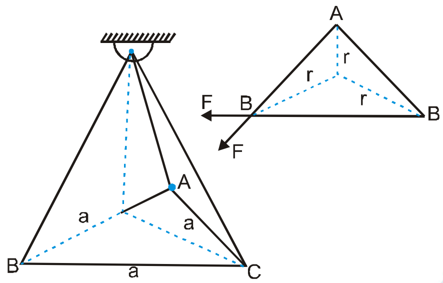


$O = 2F_q \cos 45° + F_Q$

$O = 2 \times \frac{1}{\sqrt{2}} \times \frac{KQq}{a^2} + \frac{KQ^2}{2a^2}$

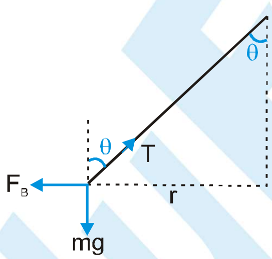
$q = \frac{-Q}{2\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}} = -\frac{1}{2}\mu C$

2.



$F_B = 2F \cos 30° = \frac{2Kq^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}Kq^2}{a^2}$

For Charge at B : 2r cos 30° = a



$r = \frac{a}{\sqrt{3}}$

$T \sin\theta = F_B$

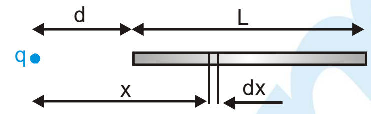
$T \cos\theta = mg$

$\tan\theta = \frac{F_B}{mg}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{99 \times 10^{-2}}{\sqrt{3} \times 10^{-2}} = \frac{\sqrt{3} \times 9 \times 10^9 \times q^2}{(3 \times 10^{-2})2} \times \frac{1}{10^{-3} \times 10}$

$\Rightarrow q = 3.17 \times 10^{-9} C$

3. We consider an element of thickness dx at a distance x from q



Force on q = $\int \frac{1}{4\pi \epsilon_0} \times \frac{dQ.q}{x^2}$

Where $dQ = \frac{Q}{L}dx$ then ,$F = \frac{KQq}{L}\int_d^{d+L}\frac{dx}{x^2} = \frac{KQq}{d(d+L)}$

4.

$q_1 = +10\ \mu C = q_3 = q_5$

$q_2 = q_4 = q_6 = -10\ \mu C$

$r_1 = 1m$

$r_2 = 3m$

$r_3 = 9\ m = 3^2 m$

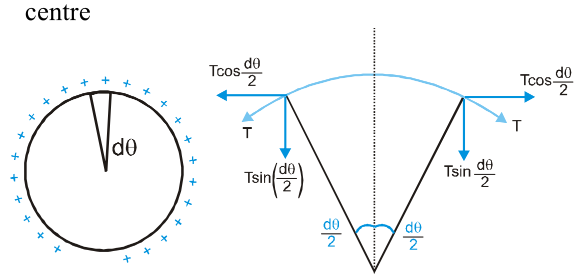
$E_{at\,0} = \frac{Kq_1}{r_1^2} - \frac{Kq_2}{r_2^2} + \frac{Kq_3}{r_3^2} - \frac{Kq_4}{r_4^2} + ...$

$= Kq\left[\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{3^4} - \frac{1}{3^6} + ...\right]$

$= \frac{Kq}{1 - \frac{1}{3^2}} = \frac{9}{8}Kq = \frac{9}{8} \times 9 \times 10^9 \times 1 \times 10^{-6}$

$= \frac{8.1}{8} \times 10^4 N/C$

5. We consider an element subtending angle dθ at the centre



F = electrostatic force on element

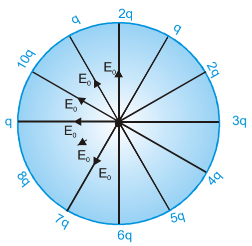
$= \frac{KdQ.q}{R^2} = \frac{\left(\frac{KQ}{2\pi R} \times Rd\theta\right)}{R^2}q = \frac{KQq}{2\pi R^2}d\theta$

$2T\sin\frac{d\theta}{2}$; $2T \times \frac{\theta}{2} = \frac{KQqd\theta}{2\pi R^2}$ $\therefore T = \frac{Qq}{8\pi^2 \epsilon_0 r^2}$

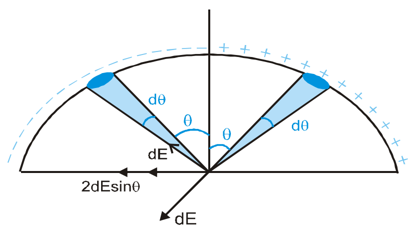
6. $E_x = E_0 + 2E_0\cos 30° + 2E_0\cos 60° = \left(2+\sqrt{3}\right)E_0$

$E_y = E_0;\ \tan\theta = \dfrac{E_y}{E_x} = \dfrac{1}{2+\sqrt{3}} \times 2-\sqrt{3}\ ;\ \theta = 15°$

The hour hand will point towards 9 : 30



7.



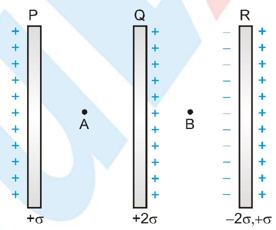
$$E_{net} = 2\int_0^{\pi/2} dE\sin\theta$$

$$\Rightarrow 2\int_0^{\pi/2}\frac{\dfrac{Kq}{(\pi R/2)}\times R d\theta}{R^2}\times\sin\theta = \frac{4Kq}{\pi R^2}$$

$$\Rightarrow \vec{E} = \left(\frac{4Kq}{\pi R^2}\right)(-\hat{i})\,N/C$$

8. Charge on outer plates $= \dfrac{\sigma A + 2\sigma A - \sigma A}{2} = \sigma A$

Charge distribution



Field at A : $E_A = 0$

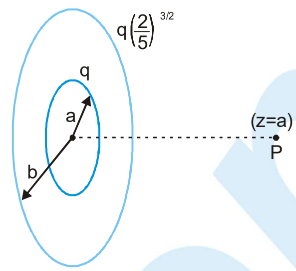
Field at B :

$E_B = \dfrac{2\sigma}{\varepsilon_0}\ \therefore\ \dfrac{E_A}{E_B} = 0$

9. The charge at 'P' is in equilibrium.

Hence $E_P = 0;\ \vec{E}_P = \vec{E}_{Pa} + \vec{E}_{Pb} = \vec{0}$

$$\frac{Kqa}{\left(a^2+a^2\right)^{3/2}} - \frac{Kq\left(\dfrac{2}{5}\right)^{-3/2} a}{\left(a^2+b^2\right)^{3/2}} = 0 \Rightarrow \frac{b}{a} = 2$$



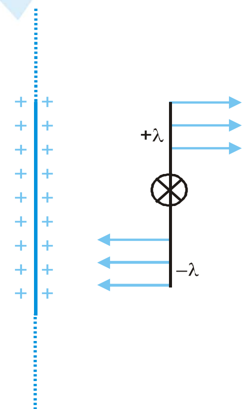
10. $U_{total} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$U_{initial} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{a}$$

$$U_{final} = \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{2a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{2a} + \frac{Kq^2}{\sqrt{2}a}$$

$$\Delta U = U_f - U_i = -\left(3-\sqrt{2}\right)\frac{Kq^2}{a}$$

11.



Net force on upper half $= \left(\dfrac{\lambda l}{2}\right)\left(\dfrac{\sigma}{2\,\epsilon_0}\right)$

Torque on upper half $= \left(\dfrac{\lambda l}{2}\right) \times \dfrac{\sigma}{2\,\epsilon_0} \times \dfrac{l}{4}$

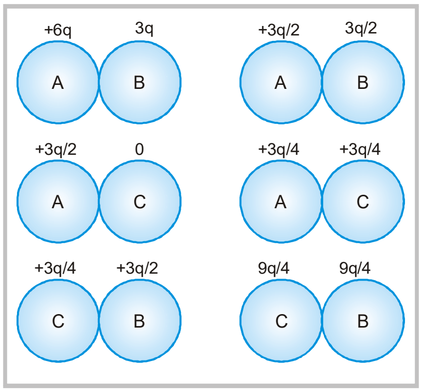
[$\dfrac{l}{4}$ = distance from hinge point for centre of mass]

Total torque on rod

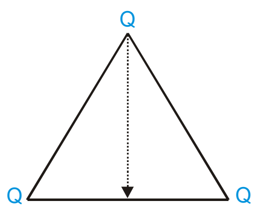
$$= \frac{\lambda l}{2} \times \frac{\sigma}{2\,\epsilon_0} \times \frac{l}{4} \times 2 = I\alpha = \frac{ml^2}{12}\propto$$

$$\therefore \alpha = \frac{3\lambda\sigma}{2m\,\epsilon_0}$$

12.



13. $U_{initial} = \frac{KQ^2}{a} \times 2 = \frac{2KQ^2}{a}$

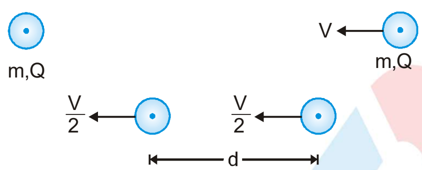


$U_{final} = \frac{KQ^2}{\left(\frac{a}{2}\right)} \times 2 = \frac{4KQ^2}{a}$

$\Delta U = \frac{2KQ^2}{a} = 1.8 \times 10^8 \text{ J} = P \propto t$

$t = \frac{1.8 \times 10^8}{10^3} \text{ sec} = 1.8 \times 10^5 \text{ sec}$

14.



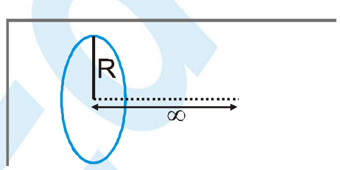
Conservation of momentum (COM)

$0 + mV = (m+m)\,V_f,\ V_f = \frac{V}{2}$

$\text{COME}: \frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{KQ^2}{d}$

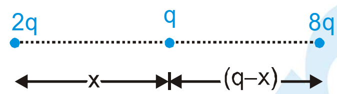
$d = \frac{Q^2}{\pi m \epsilon_0 v^2}$

15. COME : $K_i + U_i = K_f + U_f$



$0 + 0 = \frac{1}{2}mV^2 - \frac{KQ^2}{R} \Rightarrow V = \sqrt{\frac{2KQ^2}{mR}}$

16. For minimum potential energy of the system, q should be placed in the middle



$U_{system} = \frac{K2q \times q}{x} \times 100 + \frac{Kq8q}{(9-x)} \times 100$

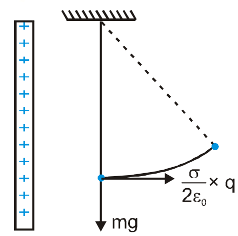
$+ \left(\frac{K \times 2q \times 8q}{9}\right) \times 100$

$\frac{dU}{dx} = Kq^2 \times 100 \left[\frac{-2}{x^2} + \frac{8}{(9-x)^2} + 0\right] \Rightarrow x = 3 \text{ cm}$

Electric field at the position of

$q = \frac{K \times 2q}{(0.03)^2} - \frac{K \times 8q}{(0.06)^2} = 0$

17. From work energy theorem (WET)



$W_{EF} + W_{mg} = \Delta KE$

$q_0 \times \frac{\sigma}{2\epsilon_0} \times L\sin\theta - mgL(1-\cos\theta) = 0$

$= q_0 \times \frac{\sigma}{2\epsilon_0} \times L \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= mgL \times 2\sin^2\frac{2\theta}{2};\ \tan\frac{\theta}{2} = \frac{\sigma}{2\epsilon_0} \times \frac{q_0}{mg}$

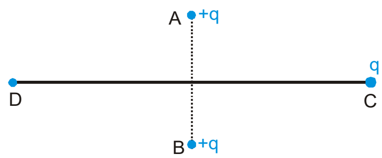
18. $V_A = \frac{KQ}{2} - \frac{KQ}{3}$

$V_B = \frac{KQ}{3} - \frac{KQ}{2}$

$V_A - V_B = 2\left(\frac{KQ}{2} - \frac{KQ}{3}\right)$

$= 9 \times 10^9 \times 10^{-6}\left(1 - \frac{2}{3}\right) = 3000 \text{ volt}$

19.



COME : $K_C + U_C = K_D + U_D$

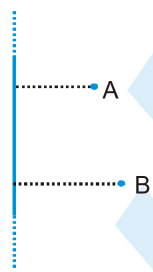
$$4 + 2\left[\frac{Kq\times(-q)}{5}\right] = 0 + \left(\frac{Kq\times -2}{r}\right)$$

$$4 - \frac{2\times 9\times 10^9\times 25\times 10^{-10}}{5}$$

$$= \frac{-2\times 9\times 10^9\times 25\times 10^{-10}}{r}\ ; r = 9\text{ m}$$

$$OD = \sqrt{AD^2 - OA^2} = \sqrt{9^2 - 3^2} = 8.48\text{m}$$

20. Force at A $\dfrac{\lambda q}{2\pi\in_0 r} = 100$



$$\frac{\lambda q}{2\pi\in_0\times 0.2} = 100 \qquad \int_A^B dv = -\int_A^B \vec{E}.d\vec{r}$$

$$V_B - V_A = \frac{-\lambda}{2\pi\in_0}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{-\lambda}{2\pi\in_0}\ln\frac{r_2}{r_1}$$

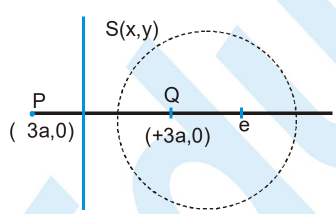
$$V_A - V_B = \frac{\lambda}{2\pi\in_0}\ln\frac{r_2}{r_1} = \frac{\lambda}{2\pi\in_0}\ln 2$$

COME : $K_A + U_A = K_B + U_B$

$$0 + \frac{\lambda Q}{2\pi\in_0}\ln 2 = \frac{1}{2}mv^2\ ;20\ \ln 2 = \frac{1}{2}\times 0.1\times V^2$$

$$V = 20\sqrt{\ln 2}$$

21.



[A] $V_S = 0 = \dfrac{KQ}{\sqrt{(x-3a)^2 + y^2}} - \dfrac{2KQ}{\sqrt{(x+3a)^2 + y^2}}$

$\Rightarrow (x-5a)^2 + y^2 = (4a)^2$

$\therefore$ C = $(5a, 0)$ and radius = 4a

[B] $V_{(x)} = \dfrac{KQ}{x-3a} - \dfrac{2KQ}{x+3a}$ for $x > 3a$

$= \dfrac{KQ}{(3a-x)} - \dfrac{2KQ}{3a+x}$ for $x < 3a$

[C] A positive charge when released will move from high potential to low potential

COME : $K_1 + U_1 = K_2 + U_2$

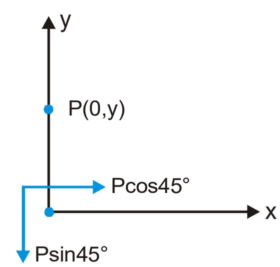
$$0 + q\left[\frac{KQ}{2a} - \frac{2KQ}{8a}\right] = \frac{1}{2}mv^2 + 0 \Rightarrow V = \sqrt{\frac{Qq}{8\pi\in_0 ma}}$$

22. Electric field due to dipole $\vec{P} = -\dfrac{KP}{1^3}\times\hat{k}$

Electric field due to dipole $\dfrac{\vec{P}}{2} = +\dfrac{2K\times\frac{P}{2}}{2^3}\hat{k}$

Resultant electric field $= -\dfrac{7}{8}KP\hat{k}$

23. $\vec{E}_x = \dfrac{-K(P\cos 45°)}{y^3}\hat{i}$



$\vec{E}_y = \dfrac{-2x(P\sin 45°)}{y^3}\hat{j}$

$\vec{E}_P = \dfrac{KP}{\sqrt{2}y^3}\left[-\hat{i} - 2\hat{j}\right]$

24. Eight cubes encloses a charge. For a cube, the three surfaces meeting the charge contribute to zero flux. Hence the rest three surfaces including the shaded surface have equal flux passing through them and are equal to $\dfrac{q}{24\in_0}$.

25. $|V_P| = |E_P|$

$$\frac{KP\cos\theta}{r^2} = \frac{KP}{r^3}\sqrt{1 + 3\cos^2\theta}\ ;\ \cos\theta = \frac{1}{\sqrt{r^2 - 3}}$$

$r^2 - 3 \geq 1;\ r \geq 2$

Now $\cos\theta = \dfrac{1}{\sqrt{r^2 - 3}}\left(r = \sqrt{5}\right) = \dfrac{1}{\sqrt{2}}$

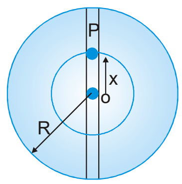
then $\theta = 45°, 135°, 225°, 315°$

26. Flux $= \frac{q}{4\in_0} = \frac{q}{\in_0}\frac{(1-\cos\theta)}{2}$

where θ is the semi–vertex angle

$\Rightarrow \cos\theta = \frac{1}{2} = \frac{a}{\sqrt{a^2+R^2}}$; $a = \frac{R}{\sqrt{3}}$

27.

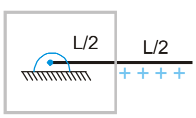


Force on charge at position P

$= -\left(\frac{1}{4\pi\in_0}\times\frac{Q}{R^3}x\right)q = -m\omega^2 x$

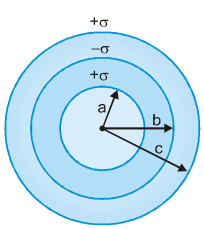
$T = 2\pi\sqrt{\frac{qQ}{4\pi\in_0 \times mR^3}}$

28.



Minimum flux through cube $= \frac{Q}{2\in_0}$

29.



(i) $V_A = \frac{1}{4\pi\in_0}\left[\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$

$= \frac{\sigma}{\in_0}(a-b+c)$

$V_B = \frac{1}{4\pi\in_0}\left[\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$

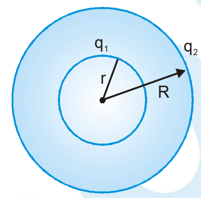
$V_C = \frac{1}{4\pi\in_0}\left[\frac{\sigma 4\pi a^2}{c} - \frac{\sigma 4\pi b^2}{c} + \frac{\sigma 4\pi c^2}{c}\right]$

$= \frac{\sigma}{\in_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$

(ii) $V_A = V_C$; $\frac{\sigma}{\in_0}(a-b+c) - \frac{\sigma}{\in_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$; $c = (a+b)$

30. $q_1 + q_2 = Q$

$\sigma 4\pi r^2 + \sigma 4\pi R^2 = Q$



$\sigma = \frac{Q}{4\pi(r^2+R^2)}$

$V_{centre} = \frac{1}{4\pi\in_0}\left[\frac{\sigma 4\pi r^2}{r} + \frac{\sigma 4\pi R^2}{R}\right]$

$= \frac{Q(r+R)}{4\pi\in_0(r^2+R^2)}$

31. $(U_{self})_{initial} = \frac{1}{4\pi\in_0}\left(\frac{Q^2}{2R}\right)$

$(U_{self})_{final} = \frac{1}{4\pi\in_0}\left(\frac{Q^2}{4R}\right)$

Workdone against electric forces = ΔU

$= -\frac{1}{4\pi\in_0}\frac{Q^2}{4R} = -\frac{\pi\sigma^2R^3}{\in_0}$

32. From mass conservation

$27\times\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$; $R = 3r$

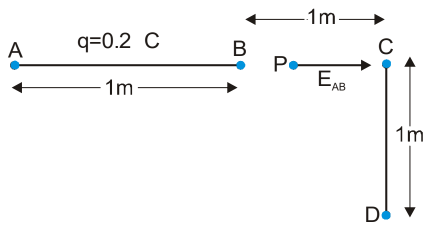
Given that $V_0 = \frac{Kq}{r}$ then $V_{bigdrop}$

$= \frac{K\times(27q)}{R} = \frac{K(27q)}{3r} = \frac{9Kq}{r} = 9V_0$

33. After earthing $V_{inner\ shell} = 0$; $\frac{Kq}{R} + \frac{KQ}{3R} = 0$

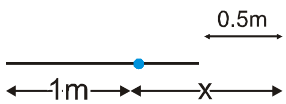
$q = \frac{-Q}{3}$ (charge on inner shell)

34.



$$\vec{E}_{AB} = \int_{0.5}^{1.5} \frac{K\lambda dx}{x^2} = -K\lambda\left[\frac{2}{3} - 2\right] = \frac{\lambda}{3\pi \epsilon_0}$$

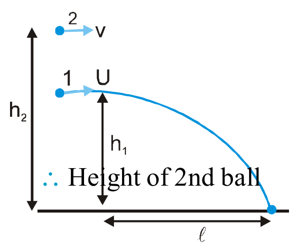
From the wire CD



$E_{II} = \frac{K\lambda}{r}(\cos\theta_2 - \cos\theta_1)$ and

$\{\theta_2 = 0, \theta_1 = \tan^{-12}$ & $E_{\perp} = \frac{K\lambda}{r}[\sin\theta_2 + \sin\theta_1]$

35. Time to fall first ball



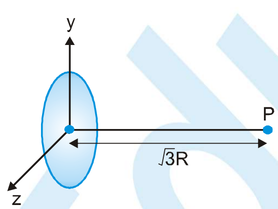
∴ Height of 2nd ball

$t = \frac{1}{v}$ ∴ $h_1 = \frac{1}{2} \times g \times \frac{1^2}{v^2}$

$= h_2 - \frac{1}{2} g \frac{1^2}{v^2}$

$pr = h_2 + \frac{1}{2} g \frac{1^2}{v^2} - g\frac{1^2}{v^2}$; $h_2 + h_1 - g\left(\frac{1^2}{v^2}\right)$

36.



$(\sqrt{3}R, 0, 0)$

K.E. at 'P' must be sufficient to reach the charge particle at the centre of the ring.

$(ME)_P = (ME)_{\text{centre of ring}}$

$$\frac{1}{2}mv^2 + \frac{K\pi\lambda \times 2\pi Rq}{\sqrt{(R^2 + \sqrt{3}R)^2}} = O + \frac{K \times \lambda 2\pi Rq}{R}$$

$$v = \sqrt{\frac{\lambda q}{2 \epsilon_0 m}} \left(Q\ K = \frac{1}{4\pi \epsilon_0}\right)$$

37. Momentum of 1st ball

$P_i = mvi$; $P_f = m\frac{v}{2}(\cos 60°\hat{i} + \sin 60\hat{j})$

change in momentum

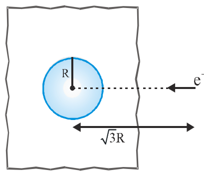
$\Delta P = P_f - P_i = -\frac{3v}{4}\hat{i} + \frac{\sqrt{3}v}{4}\hat{j}$

Momentum of IInd ball :

$P_i = mv\hat{i}$ $\quad P_f = mv\hat{j}$

Change in momentum $= mv[\hat{j} - \hat{i}]$

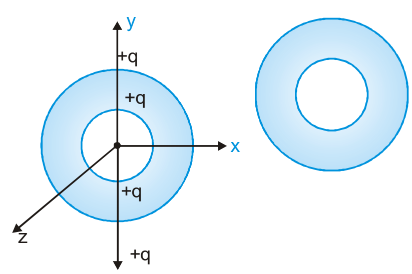
38. From the energy conservation



$\left[\frac{\sigma R}{2 \epsilon_0} - \frac{\sigma}{2 \epsilon_0} \times 2R\right]e = \frac{1}{2}mv^2 -$

$-\frac{\sigma Re}{2 \epsilon_0} + \frac{\sigma Re}{\epsilon_0} = \frac{1}{2}mv^2$ $\sqrt{\frac{\sigma e R}{m \epsilon_0}}$

39.



P.E. of outer ring charges does not change

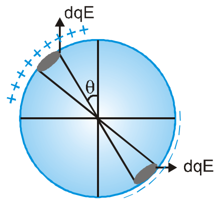
$= \frac{Kq^2}{R} \times 2 + \frac{Kq^2}{3R} \times 2 = \frac{8Kq^2}{3R}$; $U_f = \frac{Kq^2}{\sqrt{5}R} \times 4$.

Hence (∇r%) $W_{\text{electron}} = -\Delta U$

$$W_{\text{electron}} = -(V_f - V_i) = -\frac{Kq^2}{R}\left[\frac{4}{\sqrt{5}} - \frac{8}{3}\right]$$

$$= \frac{Kq^2}{R}\left[\frac{8}{3} - \frac{4}{\sqrt{5}}\right]$$

40. $\int_0^{\frac{\pi}{2}} 2(\lambda R d\theta) \times E_0 R\cos\theta = F \times R$



$2\lambda R^2 E_0 \int_0^{\frac{\pi}{2}} \cos\theta d\theta = 2\lambda R^2 E_0 = FR$

$\Rightarrow F = 2\lambda RE_0$

42. **(i)** $r < a \Rightarrow$ Point inside $\Rightarrow E=0$

(ii) $a < r < b \Rightarrow$ Point outside the inner and inside the outer so field due to only inner cylinder

hence $E = \dfrac{2 \times \lambda}{r}$

(iii) $r > b \Rightarrow$ point outside from both cylinders so the charge per unit length (λ) is zero for point $(r > b)$

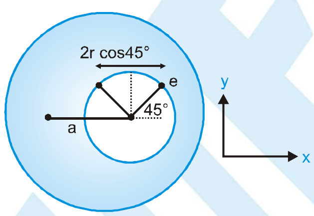
$\therefore E=0$

43. $q = \epsilon_0 \int \vec{E} \cdot d\vec{S} = \epsilon_0 \int \dfrac{E_0 x}{l} \hat{i}$

$\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right) = \epsilon_0 \times \left[\dfrac{E_0 x^2}{2l}\right]_0^a$

Put given values $\Rightarrow 2.2 \times 10^{-12}$ C

44. The direction of electric field inside the cavity in –ve x direction and of constant magnitude $\dfrac{\rho a}{3\epsilon_0}$.



For touch the sphere again, electron must move 2r cos45° (as shown) distance

$\therefore \dfrac{1}{2}\left[\dfrac{\rho a}{3\epsilon_0} \times \dfrac{e}{m}\right] \times t^2 = \sqrt{2}r$

$\therefore t = \sqrt{\dfrac{6\sqrt{2}mr\epsilon_0}{e\rho a}}$

45. **(i)** $q = \int dq = \int_0^R \rho_0 \dfrac{r}{R} \times 4\pi r^2 dr = \dfrac{\rho_0 \times 4\pi R^4}{R} = \pi\rho_0 R^3$

(ii) $E = \dfrac{KQ'}{r^2} = \dfrac{K}{r^2} \times \dfrac{\rho_0 \pi r^4}{R} = \dfrac{KQr^2}{R^4}$ where

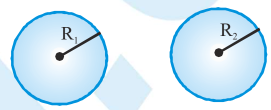
$Q' = \int_0^r \rho_0 \dfrac{x}{R} \times 4\pi x^2 dx = \dfrac{\rho_0}{4R} \times 4\pi \times r^4 = \dfrac{\rho_0 \pi r^4}{R}$

46. From the energy conservation

$\dfrac{KQq}{r} + \dfrac{1}{2}mv^2 = \dfrac{11}{8}\dfrac{KQq}{R} + 0$

$v = \sqrt{\dfrac{2KQq}{mR}\left[\dfrac{11}{8} - \dfrac{R}{r}\right]}$

47.



Electrostatic energy of system = Interaction Energy + Self Energy

(Let the total charge of balls be Q)

$U = 0 + \dfrac{KQ_1^2}{2R_1} + \dfrac{K(Q-Q_1)^2}{2R_2}$

Here for it's minimum value $\dfrac{dU}{dQ_1} = 0$

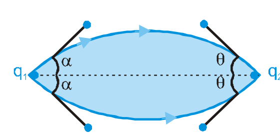
$= K\left[\dfrac{2Q_1}{2R_1} + \dfrac{2(Q-Q_1)}{2R_2}(0-1)\right] = 0$

$\Rightarrow \dfrac{Q_1}{R_1} - \dfrac{Q_2}{R_2} = 0 \quad [\because Q - Q_1 = Q_2] \Rightarrow \dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

48. Solid angle $= 2\pi(1-\cos\alpha)$

$\dfrac{q_1}{2\epsilon_0}(1-\cos\alpha) = \dfrac{q_2}{2\epsilon_0}(1-\cos\theta)$

$q_1 \dfrac{\sin^2\alpha}{2} = q_2 \dfrac{\sin^2\theta}{2}$



$$\sin\frac{\theta}{2} = \sqrt{\frac{q_1}{q_2}}\,\sin\frac{\alpha}{2}$$

$$\theta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\,\sin\frac{\alpha}{2}\right]$$

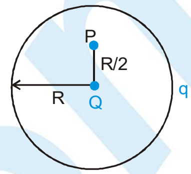
49. Let n^{th} number last drop that can entre

$$\frac{KQ^2}{R}\times n = mg\,(h-R) \quad \therefore\ n = \frac{4\pi\epsilon_0\, mg(h-R)R}{Q^2}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. At P, potential due to shell :



$$V_1 = \frac{q}{4\pi\varepsilon_o R}$$

At P , potential due to Q :

$$V_2 = \frac{Q}{4\pi\varepsilon_o \frac{R}{2}} \qquad \therefore \text{ Net potential at P}$$

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_o R} + \frac{2Q}{4\pi\varepsilon_o R}$$

2. $$\vec{F}_{12(x)} = \frac{q_1 q_2}{4\pi_0 b^2}(-\hat{i}) \Rightarrow \vec{F}_{13\,(x)} = \frac{q_1 q_3}{4\pi\varepsilon_0 a^2}(-\sin\theta\hat{i})$$

$$\therefore \text{ Net } \overset{r}{F_x} = \frac{Kq_1q_2}{b^2}(-\hat{i}) + \frac{kq_1q_3}{a^2}\sin\theta(-\hat{i})$$

or $$F_x = -Kq_1\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

$$\therefore\ F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$

3. $$\Delta\phi = \frac{q}{\varepsilon_0} \qquad \Rightarrow \phi_2 - \phi_1 = \frac{q}{\varepsilon_0}$$

So, the electric charge inside the surface will be

$$\Rightarrow\ q = (\phi_2 - \phi_1)\varepsilon_0$$

4. Initially : $\dfrac{kQ^2}{r^2} = F \quad \Rightarrow$ Finally : Charge on B = Q/2

and Charge on C = $\dfrac{3Q}{4}$ (By conduction)

$$\therefore\ F' = \frac{k(Q/2)(3Q/4)}{r^2} = \frac{3kQ^2}{8r^2} = \frac{3F}{8}$$

5. By energy conservation :

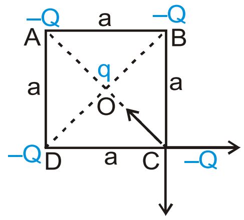
$$\text{Initially} : 0 + \frac{1}{2}mv^2 = \frac{kQq}{r}$$

$$\text{Finally} : \frac{1}{2}\,m\,(2v)^2 = \frac{kQq}{r'}$$

So , $$\frac{4kQq}{r} = \frac{kQq}{r'} \quad \text{or} \quad r' = \frac{r}{4}$$

6. Charge q at O is in equilibrium.

For –Q to be in equilibrium, we see charge at C.



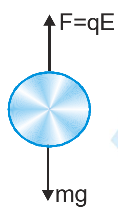
$\therefore$ F_{net} on – Q (at C) = 0

or $\dfrac{kQ^2}{(\sqrt{2}a)^2} + \dfrac{\sqrt{2}\,kQ^2}{a^2} - \dfrac{kQq}{(a/\sqrt{2})^2} = 0$

$\therefore$ $q = \dfrac{Q}{4}(1 + 2\sqrt{2})$

7. In steady state,

electric force on drop = weight of drop



$\therefore$ $qE = mg$

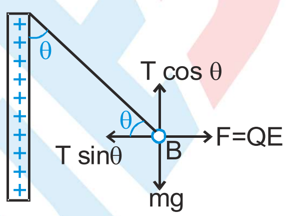
$\Rightarrow$ $q = \dfrac{mg}{E} = \dfrac{9.9 \times 10^{-15} \times 10}{3 \times 10^4} = 3.3 \times 10^{-18}$C

8. Electric field due to a charged conducting sheet of surface charge density σ is given by $E = \dfrac{\sigma}{2\varepsilon_0 \varepsilon_r}$

Where, ε_0 = permittivity in vacuum and ε_r = relative permittivity of medium.

Here, electrostatic force on B

$QE = \dfrac{Q\sigma}{2\varepsilon_0 \varepsilon_r}$



where, $T \cos\theta = mg$

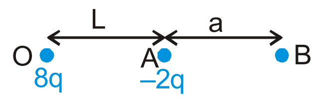
and $T \sin\theta = \dfrac{Q\sigma}{2\varepsilon_0 \varepsilon_r}$

Thus, $\tan\theta = \dfrac{Q\sigma}{2\varepsilon_0 \varepsilon_r mg}$

$\therefore$ $\tan\theta \propto \sigma$ or $\sigma \propto \tan\theta$

9. Suppose that at point B, where net electric field is zero due to charges 8q and 2q.

$E_{BO} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{8q}{(L+a)^2}$ $\Rightarrow$ $E_{BA} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{-2q}{(a)^2}$



According to condition $E_{BO} + E_{BA} = 0$

$\therefore$ $\dfrac{1}{4\pi\varepsilon_0} \dfrac{8q}{(a+L)^2} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2q}{(a)^2} \Rightarrow 2 = \dfrac{a+L}{a}$

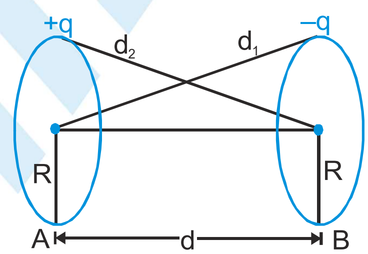
So, $a = L$

Thus, at distance 2L from origin, net electric field will be zero.

10. V_A = (potential due to charge +q on ring A) + (potential due to charge – q on ring B)

$= \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{R} - \dfrac{q}{d_1}\right)$; $d_1 = \sqrt{R^2 + d^2}$

$= \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{R} - \dfrac{q}{\sqrt{R^2 + d^2}}\right)$ .... **(i)**



$V_B = \dfrac{1}{4\pi\varepsilon_0}\left(-\dfrac{q}{R} + \dfrac{q}{\sqrt{R^2 + d^2}}\right)$

Potential difference $V_A - V_B$

$= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R} + \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\sqrt{R^2+d^2}} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\sqrt{R^2+d^2}}$

$= \dfrac{1}{2\pi\varepsilon_0}\left(\dfrac{q}{R} - \dfrac{q}{\sqrt{R^2 + d^2}}\right)$

11. The dipole will have some distance along the electric field, so, option (1) is correct.

12. By work energy theorem:

$W_{\text{all forces}} = \Delta KE$

So $q(\Delta V) = KE_{final} - KE_{Initial}$

or $1.6 \times 10^{-19} \times (20) = \dfrac{1}{2} \times 9.11 \times 10^{-31} \times v^2$

$\therefore$ $v = 2.65 \times 10^6$ m/s

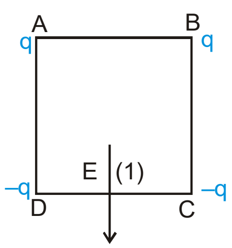
13. After connecting with conducting wire, let the charges on both spheres are q_1 & q_2.

$$\therefore \frac{q_1}{q_2} = \frac{r_1}{r_2} = \frac{1}{2} \quad \therefore \frac{E_1}{E_2} = \frac{q_1}{r_1^2} \cdot \frac{r_2^2}{q_2} = 2:1$$

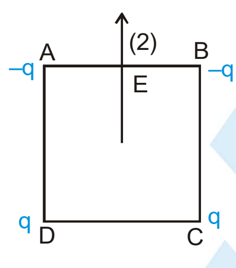
14. Since $V_A = 9\times10^9 \dfrac{10^{-9}}{\{2+2\}^{\frac{1}{2}}} = 4.5$ volt,

$$V_B = 9\times10^9 \frac{10^{-9}}{\{4+0\}^{\frac{1}{2}}} \text{ volt}, V_A - V_B = 0$$

15. In initial case, E is along (1) whereas in final case E is along (2). Potential at centre remains same.



Initial



Final

16. $v(x) = \dfrac{20}{x^2-4}$,

$$E = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2-4}\right) = \frac{20}{(x^2-4)^2}(2x)$$

$$E \text{ at } x = 4\ \mu m, \frac{(20)(2\times4)}{144} = \frac{10}{9} \text{ volt}/\mu m$$

Also as x increases, V decreases.

So, E is along + ve x -axis.

17. Since, the electric field inside the shell is zero and outside, the electric field is given as $\dfrac{kQ}{r^2}$,

where r = distance from centre.

So, graph is as shown in option (4).

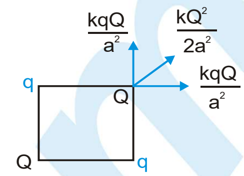
18. $W_{P\to Q}|_{ext} = q(V_Q - V_P)$

$= -1.6\times10^{-19}\times100(-4-10)$

$= 2.24\times10^{-16}$ J

19. Since, F_{net} on Q is zero, so :

$$\frac{kqQ}{a^2}[\sqrt{2}] + \frac{kQ^2}{2a^2} = 0$$



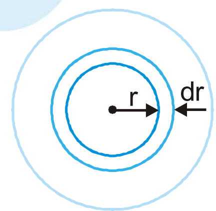
$$\frac{Q}{q} = -2\sqrt{2}$$

20. Statement–1 : Correct as the field is conservative

statement –2 : Correct Explanation

21. Consider a spherical shell having radius r and thickness dr

$$dq = \frac{Q}{\pi R^4} r \times 4\pi r^2 dr$$



$$\text{or } q = \frac{4Q}{R^4}\int_0^{r_1} r^3 dr$$

$$\text{so } q = \frac{Q.r_1^4}{R^4}$$

Electric field at a distance r_1 from the center (inside)

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_1^2} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\times\frac{Q.r_1^2}{R^4}$$

22. $\vec{E} = \left(\dfrac{2k\lambda}{r}\right)(-\hat{j}) \Rightarrow \vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}(-\hat{j})$

$\Rightarrow \lambda = \dfrac{q}{\pi r} \quad \Rightarrow \vec{E} = \dfrac{q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$

23. Consider a spherical shell of radius x and thickness dx.

Charge on it dq

$dq = \rho\times4\pi x^2 . dx$

$$dq = \rho_0\left(\frac{5}{4}-\frac{x}{R}\right) x\, 4\pi x^2 dx$$

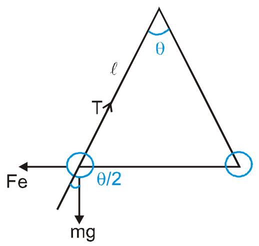
$$\Rightarrow q = 4\pi\rho_0\int_0^r\left(\frac{5x^2}{4}-\frac{x^3}{R}\right)dx$$

$$q = 4\rho_0\left(\frac{5r^3}{3\times4}-\frac{r^4}{4R}\right) \Rightarrow E = \frac{kq}{r^2} = \frac{1}{4\pi r^2}\times4\pi\rho_0$$

$$\left(\frac{5r^3}{3\times4}-\frac{r^4}{4R}\right) \Rightarrow E = \frac{\rho_0 r}{4\rho_0}\left(\frac{5}{3}-\frac{r}{R}\right)$$

24. At equilibrium

$$\tan\theta/2 = \frac{F_e}{mg} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[l\sin(\theta/2)]^2}\cdot\frac{1}{mg}$$



When suspended in liquid

$$\tan\frac{\theta}{2} = \frac{1}{4\pi\epsilon_0 K}\frac{q^2}{[l\sin(\theta/2)]^2}\frac{1}{(mg - F_B)}$$

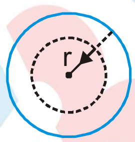
$$= \frac{1}{4\pi\epsilon_0 K}\frac{q^2}{[l\sin(\theta/2)]^2}\cdot\frac{1}{\left(mg - \frac{m}{1.6}\times 0.8\,g\right)}$$

on comparing the two equation we get

$$K\left(1 - \frac{0.8}{1.6}\right) = 1 \Rightarrow K = 2.$$

25. $\phi = ar^2 + b \quad \Rightarrow E = -\frac{d\phi}{dt} = -2ar$

$$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0} \Rightarrow -2ar \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$



$$q = -8\,\varepsilon_0 a\pi r^3 \quad \Rightarrow \rho = \frac{q}{\frac{4}{3}\pi r^3}$$

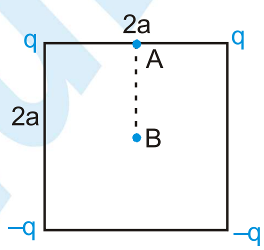
$\Rightarrow \rho = -6a\varepsilon_0$ **Ans.**

26. Potential at point A,

$$V_A = \frac{2Kq}{a} - \frac{2Kq}{a\sqrt{5}}$$

Potential at point B,

$V_B = 0$

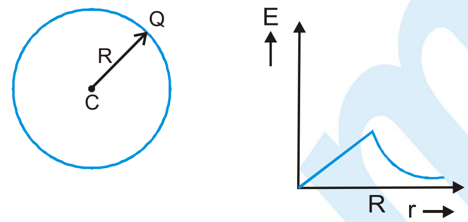


$\therefore$ Using work energy theroem,

$W_{AB})_{electric} = Q(V_A - V_B)$

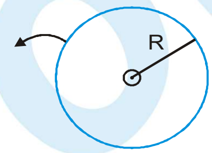
$$= \frac{2KqQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right] = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{2Qq}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

27.



28. $U_c = \frac{3}{2}\frac{KQ}{R}q$

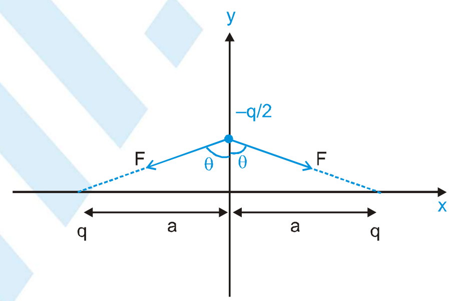
$U_S = \frac{KQ}{R}q$



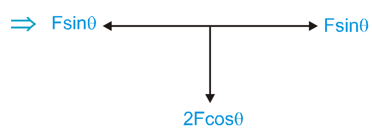
$\therefore \quad \Delta U = \frac{KQ}{2R}q$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{2R}\rho\frac{4\pi R^3}{3}q = \frac{\rho R^2 q}{6\varepsilon_0}$$

29.



$\Rightarrow$



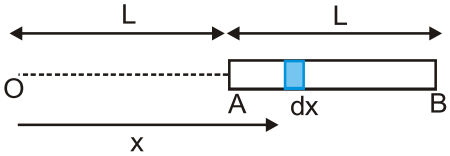
$\Rightarrow F_{net} = 2F\cos\theta$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}$$

$$\Rightarrow \frac{kq^2y}{a^3} \propto y$$

30.

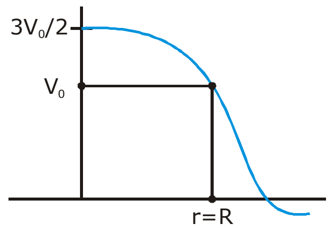


$$V=\int_L^{2L}\frac{kdq}{x}=\int_L^{2L}\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{L}\right)dx}{x}$$

$$=\frac{q}{4\pi\varepsilon_0 L}\ln(2)$$

31. None

32.



$$R_1=\frac{3V_0}{2}\ ;\ R_2=\frac{5V_0}{4}\ ;\ R_3\ \frac{3V_0}{4}\ ;\ R_4=\frac{V_0}{4}$$

$$\therefore\ r<R \qquad V=\frac{KQ}{2R^3}\left(3R^2-r^2\right)$$

$$v=\frac{3V_0}{2},\ R_1=0 \ \Rightarrow\ \frac{5V_0}{4}=\frac{KQ}{2R^3}\left(3R^2-R_2^2\right)$$

$$\therefore\ R_2=\frac{R}{\sqrt{2}} \ \Rightarrow\ r>R$$

$$\frac{3V_0}{4}=\frac{KQ}{R_3} \ \Rightarrow\ R_3=\frac{4KQ}{3V_0}=\frac{KQ\times R}{3\times KQ}=\frac{R}{3}$$

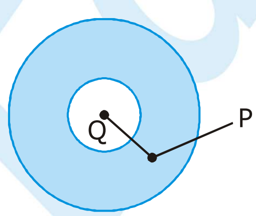
$$\frac{V_0}{4}=\frac{KQ}{R_4} \quad \therefore\ R_4=\frac{4KQ}{V_0}=\frac{4KQ}{KQ}\times R=4R$$

On comparing we get

(1) or (2)

33. Tangent to the electrical field lines will give us the direction at a given point.

34.



$$E(r^2)=\frac{1}{\epsilon_0}\int_a^r \rho r^2 dr\ ;E(r^2)=\frac{A}{\epsilon_0}\int_a^r r dr=\frac{A}{\epsilon_0}\left[\frac{r^2}{2}\right]_a^r$$

$$E(r^2)=\frac{A}{2\epsilon_0}[r^2-a^2]\ ;$$

$$E=\frac{A}{2\epsilon_0 r^2}(r^2-a^2)+\frac{kQ}{r^2}$$

$$E=\frac{A}{2\epsilon_0}\left[1-\frac{a^2}{r^2}\right]+\frac{kQ}{r^2}$$

$$\frac{dE}{dr}=0\ ;\ \frac{a^2A}{2\epsilon_0}=kQ$$

$$A=\frac{2kQ\epsilon_0}{a^2}\ ;=\frac{2}{4\pi\varepsilon_0}\ \frac{Q\varepsilon_0}{a^2}\ =\frac{Q}{2\pi a^2}$$

Part # II : IIT-JEE ADVANCED

1. Electric field is zero everywhere inside a metal (conductor). i.e., field lines do not enter into metal Also, these are perpendicular to a metal surface (equipotential surface).

2. For potential energy of the system of charges, total number of charge pairs will be $^8C_2$ or 28.

Of these 28 pairs, 12 unlike charges are at a separation 'a', 12 likes charges are at separation $\sqrt{2}$ a and 4 unlike charges are at separation $\sqrt{3}a$. Therefore, the potential energy of the system :

$$U=\frac{1}{4\pi\varepsilon_0}\left[\frac{(12)(q)(-q)}{a}+\frac{(12)(q)(q)}{\sqrt{2}a}+\frac{(4)(q)(-q)}{\sqrt{3}a}\right]$$

$$=\frac{4Kq^2}{a}\cdot\left[-3+\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right]$$

The binding energy of this system is therefore

$$|U|=\frac{4Kq^2}{a}\left[3+\frac{1}{\sqrt{3}}-\frac{3}{\sqrt{2}}\right]$$

So, work done by external forces in disassembling, this system of charges is :

$$W=|U|=\frac{4Kq^2}{a}\left[3+\frac{1}{\sqrt{3}}-\frac{3}{\sqrt{2}}\right]\text{Joules.}$$

3. **(a)** Applying energy conservation principle, Increase in kinetic energy of the dipole = Decrease in electrostatic potential energy of the dipole.

$\therefore$ kinetic energy of dipole at distance d from origin

$$= U_i - U_f$$

or $$K.E. = 0 - \left(-Q\frac{kp}{d^2}\right) = \frac{kpQ}{d^2}$$ **Ans.**

(b) Electric field at origin due to the dipole.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i} \quad (\vec{E}_{axis} \uparrow\uparrow \vec{p})$$

$\therefore$ Force on charge Q :

$$\vec{F} = Q\vec{E} = \frac{pQ}{2\pi\varepsilon_0 d^3}\hat{i} = \frac{2kpQ}{d^3}\hat{i}$$ **Ans.**

4. According to option (A) the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence the correct option is (A).

5. The electric field at the surface will be due to all charges. However, net flux coming out of the surface will be =

$$\varphi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{q_1 - q_1}{\varepsilon_0} = 0.$$

6. Electric field near a large metallic plate is given by $E = \sigma/\varepsilon_0$. In between the plates the two fields will be in opposite direction. Hence,

$$E_{net} = \frac{\sigma_1 - \sigma_2}{2\varepsilon_0} = E_0(\text{say})$$

Now, W = (Q) (potential difference)

$$= Q(E_0\, a\cos 45^\circ) = (Q)\left(\frac{\sigma_1 - \sigma_2}{2\varepsilon_0}\right)\left(\frac{a}{\sqrt{2}}\right)$$

$$= \frac{(\sigma_1 - \sigma_2)Qa}{2\sqrt{2}\varepsilon_0}.$$

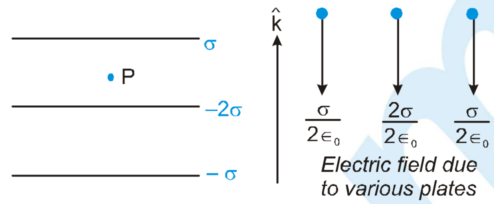
7. Electric dipole moment $= q \times 2\bullet = L^1T^1A^1$

$$\text{Electric flux} = \int E \cdot ds = (F/q)\,ds = \frac{M^1L^1T^{-2}}{AT} \times L^2$$

$$= M^1L^3T^{-3}A^{-1}$$

$$\text{Electric field} = \frac{F}{q} = M^1L^1T^{-3}A^{-1}$$

8.



$$\text{Resultant electric field} = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$= \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0}\downarrow = \frac{2\sigma}{\epsilon_0}(-\hat{k})$$

9. Let q be the charge on the bubble, then

$$V = \frac{Kq}{a}\left(\text{Here } K = \frac{1}{4\pi\varepsilon_0}\right) \qquad \therefore q = \frac{Va}{K}$$

Let, after collapsing the radius of droplet becomes R, then equating the volume, we have

$$(4\pi a^2)\,t = \frac{4}{3}\pi R^3 \qquad \therefore R = (3a^2t)^{1/3}$$

Now, potential of droplet will be $V' = \frac{Kq}{R}$

Substituting the values, we have

$$V' = \frac{(K)\left(\frac{Va}{K}\right)}{(3a^2t)^{1/3}} \quad \text{or} \quad V' = V\left(\frac{a}{3t}\right)^{1/3}$$

10. This is the variation of potential of a hollow sphere with the distance from the centre and the total charge within the sphere will be q.

As no electric field is developed within it, so electrostatic energy for $r \le R_0$ is zero.

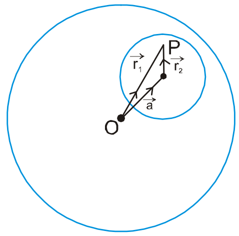
and the total charge resides over its surface and also at $r = R_0$ electric field is discontinuous, since it changes abruptly.

11. When inner cylinder is charged (outer cylinder may or maynot be charged) an electric field will be present in the gap between the cylinders which will produce a potential difference.

12. Sphere is electrically neutral and net charge never changes due to induction. Therefore, net charge will be zero.

13. Electric field at P = $\vec{E}$ due to full sphere – $\vec{E}$ due to charge that would be present in cavity

$$= \frac{\rho \vec{r}_1}{3\varepsilon_0} - \frac{\rho \vec{r}_2}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}(\vec{r}_1 - \vec{r}_2)$$



$$= \frac{\rho}{3\varepsilon_0}\vec{a}$$

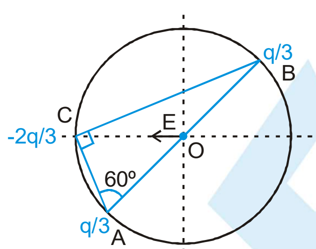
It is uniform.

14. Both the points are at equitorial position of given dipole. So, Potential is zero at both the points.

15. Net electric field due to both q/3 charges, will get cancelled. Electric field due to $\left(\frac{-2q}{3}\right)$ will be directed in –ve axis

$$E = \frac{k\left(\frac{2q}{3}\right)}{R^2} \Rightarrow E = \frac{q}{6\pi\varepsilon_0 R^2}$$

$$\text{P.E. of system} = \frac{K\left(\frac{q}{3}\right)^2}{2R} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60°} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60°}$$



P.E. of system ≠ 0

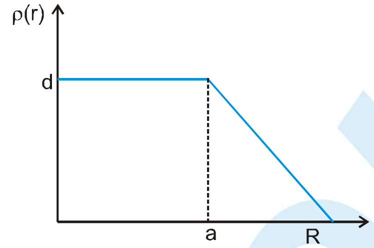
Force between B and C

$$F = \frac{K\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60°)^2} = \frac{4\times 2Kq^2}{9\times 4\times 3R^2} = \frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2}$$

$$\text{(attractive)} = \frac{1}{54}\frac{q^2}{\pi\epsilon_0 R^2}$$

$$\text{Potential at O } V = \frac{K\left(\frac{q}{3}+\frac{q}{3}-\frac{2q}{3}\right)}{R} = 0$$

16. Electric field at r = R



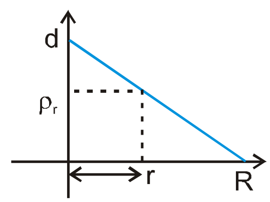
$$E = \frac{KQ}{R^2}$$

where Q = Total charge within the nucleus = Ze

So $\quad E = \frac{KZe}{R^2}$

So electric field is independent of a

17. $Q = \int \rho_r 4\pi r^2 dr$



for a = 0, $\frac{d}{R} = \frac{\rho_r}{R-r}$

$\therefore \quad \rho_r = \frac{d}{R}(R-r)$

or, $\quad Q = \int_0^R \frac{d}{R}(R-r)\,4\pi r^2\,dr$

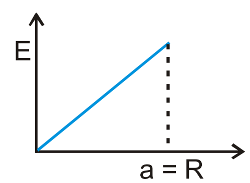
$$= \frac{4\pi d}{R}\left[R\int_0^R r^2\,dr - \int_0^R r^3\,dr\right]$$

$$= \frac{4\pi d}{R}\left[\frac{R^4}{3} - \frac{R^4}{4}\right] = \frac{\pi d R^3}{3}$$

$\therefore \quad Q = Ze = \frac{\pi d R^3}{3}$ or $\quad d = \frac{3Ze}{\pi R^3}$

18. From the formula of uniformly (volume) charged solid sphere

$$E = \frac{\rho r}{3\epsilon_0}$$



For E ∝ r, ρ should be constant throughout the volume of nucleus

This will be possible only when a = R.

19. Statement-1 is true by information

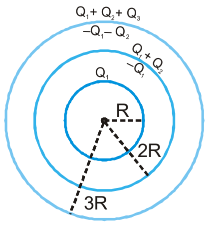
Statement-2 is true by formula. But statement-2 is not the explanation of 1 correct explanation is the very large size of earth.

20. From Gauss's law

$$\frac{\Sigma Q_{in}}{\varepsilon_0} = \frac{(8C/4) - 7C + (6C/2)}{\varepsilon_0} = -\frac{2C}{\varepsilon_0}.$$

21. The charge distribution on the surfaces of the shells are given.

As per the given condition.



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$$\Rightarrow \frac{Q_1}{1} = \frac{Q_2}{3} = \frac{Q_3}{5}.$$

22. Torque about Q of charge –q is zero, so angular momentum of charge –q is constant, but distance between charges is changing, so force is changing, so speed and velocity are changing.

23. Total charge

$$Q = \int \rho dV = \int_{r=0}^{r=R} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}(R^{a+3}) \text{ and}$$

$$Q' = \int \rho dV = \int_{r=0}^{r=R/2} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$$

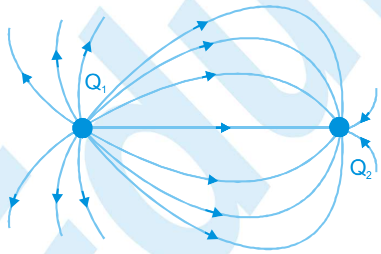
According to question

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'}{(R/2)^2} = \frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right) \qquad (2)^{a+3} = 32$$

Putting the value of Q and Q′ get

$a = 2$ **Ans. 2**

24.



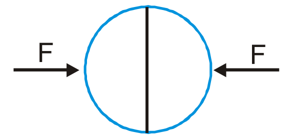
From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

No. of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude & opposite sign to that of Q_1.

25.



Electrostatics repulsive force ;

$$F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2 \ ; \ F = F_{ele} = \frac{\sigma^2 \pi R^2}{2\varepsilon_0}$$

26. In equilibrium,

$mg = qE$

In absence of electric field,

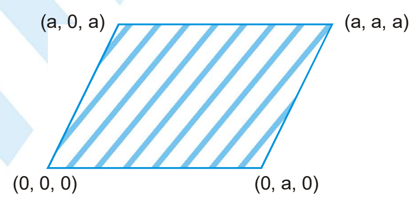
$mg = 6\pi\eta rv \quad \Rightarrow \quad qE = 6\pi qrv$

$$m = \frac{4}{3}\pi R r^3 d. = \frac{qE}{g} \Rightarrow \frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$$

After substituting value we get,

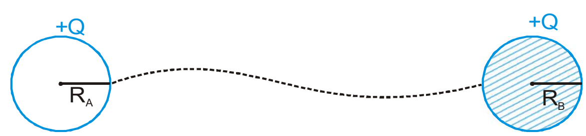
$q = 8 \times 10^{-19}$ C **Ans.**

27.



$$\text{flux} = (E_0 \cos 45°) \times \text{area}) = \frac{E_0}{\sqrt{2}} \times a \times \sqrt{2}a = E_0 a^2$$

28.



$Q_A + Q_B = 2Q$ ...**(i)**

$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$ ...**(ii)**

(i) and **(ii)** $\Rightarrow Q_A = Q_B\left(\frac{R_A}{R_B}\right)$

$$\& \ Q_B\left(1 + \frac{R_A}{R_B}\right) = 2Q \Rightarrow Q_B = \frac{2Q}{\left(1 + \frac{R_A}{R_B}\right)} = \frac{2Q\,R_B}{R_A + R_B}$$

& $Q_A = \frac{2Q\,R_A}{R_A + R_B} \Rightarrow Q_A > Q_B$

$\frac{\sigma_A}{\sigma_B} = \frac{Q_A / 4\pi R_A^2}{Q_B / 4\pi R_B^2} = \frac{R_B}{R_A}$ using **(ii)**

$E_A = \frac{\sigma_A}{\epsilon_0}$ & $E_B = \frac{\sigma_B}{\epsilon_0}$ ✈ $\sigma_A < \sigma_B$

$\Rightarrow E_A < E_B$ (at surface)

29. The frequency will be same $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

but due to the constant qE force, the equilibrium position gets shifted by $\frac{qE}{K}$ in forward direction.

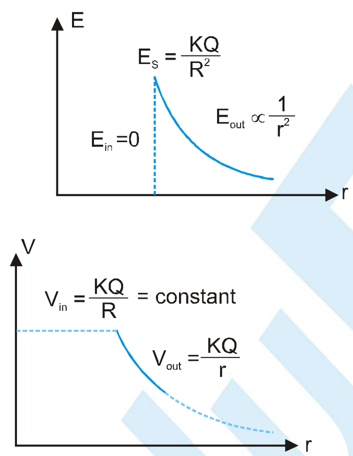
So Ans. will be **(A)**

30. $\phi = \int E ds = \frac{Kq}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$

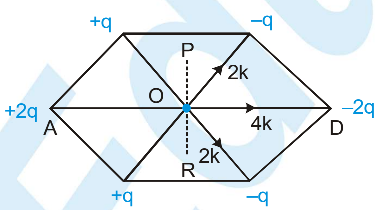
$W_{ext} = q(V_B - V_A)$

Comment : **(D)** is not crrect answer because it is not given that charge is moving slowly.

31.



32.



$E_0 = 6\,K$ (along OD)

$V_0 = 0$

Potential on line PR is zero

33. C

34. C

35. A

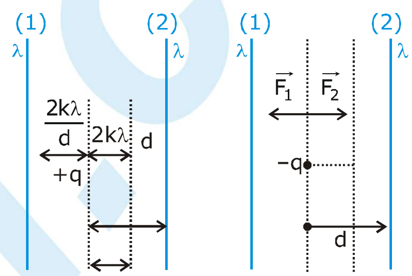
36. $360 \to \frac{2n\lambda}{r}$; $60 \to \frac{\frac{2n\lambda}{r}}{360} \times 60$

$\frac{R\lambda}{3r} = \frac{\lambda}{4\pi\varepsilon_0 \times 3r}$

$n = 6$

Since angle is 60° hence total flux will be 1/6th of total flux.

37.



Right $\Rightarrow$ d ↓ Right $\Rightarrow$ d ↓

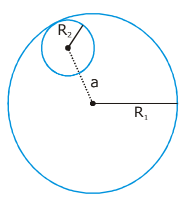
$\Rightarrow$ force due to (2) $\Rightarrow$ force due to (2)

↑ & due to (1) ↓ ↑ while due to (1) ↓.

Thus F_{net} is leftwards. This F_{net} is rightwards

∴ SHM $\Rightarrow$ No SHM

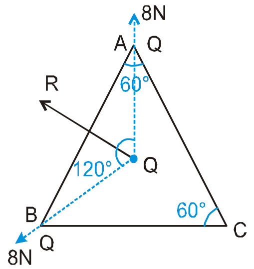
38.



$\vec{E}$ is uniform & both its magnitude & direction depends on $\vec{a}$ due to the vector nature of $\vec{E}$.

MOCK TEST : BASIC MATHS

1.

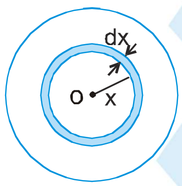


$$R = \sqrt{8^2 + 8^2 + 2.8.8\cos 120^\circ} = 8\,N$$

2. We can consider all the charge inside the sphere to be concentrated on the center of sphere

Consider an elementry shell of radius x and thickness dx.

$$E = \frac{K\int dq}{r^2} = \frac{K\int_0^r 4\pi x^2 dx\ (\alpha x)}{r^2}$$



$$= \frac{k4\pi\alpha}{r^2}\int_0^r x^3 dx = \frac{\alpha r^2}{4\varepsilon_0}$$

3. From the F.B.D. of charge, we have :

$$q\left(\frac{2k\lambda}{r}\right) = \frac{mv^2}{r}$$

so $v = \sqrt{\frac{2kq\lambda}{m}}$ Now, $T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

where, $k = \frac{1}{4\pi\varepsilon_0}$

4. $W_{net} = q\vec{E}.\vec{d}$ where ($\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{i}$ & $\vec{d} = (x_1 - x_2)\hat{i}$)

$\therefore$ $W_{net} = q\frac{\sigma}{2\varepsilon_0}.(x_1 - x_2)$

5. Electric field between the two cylinders = $\frac{2k\lambda}{r}$

$\therefore$ Force on charge q = $\frac{2k\lambda q}{r}$

This force is centripetal force

$$\therefore \frac{2k\lambda q}{r} = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{2\frac{1}{4\pi\epsilon_0}\frac{\lambda q}{m}} = \sqrt{\frac{\lambda q}{2\pi\epsilon_0 m}}$$

6. $V = V_1 + V_2 + V_3$

(where V_1, V_2 & V_3 are potentials due to the three parts of ring)

$$= \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{R}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}.\left(\frac{2Q}{R}\right) = \frac{Q}{2\pi\varepsilon_0 R}$$

7. $dV = -\vec{E}.d\vec{r} = -(-2x^3 \times 10^3\hat{i}).(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$= 2x^3 \times 10^3\,dx$

$$\Rightarrow \int_0^V dV = -\int_2^1 (2x^3) \times 10^3\,dx$$

$\Rightarrow V = 7.5 \times 10^3\,V$

8. Potential difference due to inner 10C charge

$$= K\,10\left(\frac{1}{\cdot 1} - \frac{1}{\cdot 2}\right) = 9 \times 10^{10}(5)$$

$= 45 \times 10^{10} = 4.5 \times 10^{11} V$

Potential difference due to outer charge

$$= \left(\frac{K \times 20}{0.2} - \frac{K \times 20}{0.2}\right) = 0\,V$$

$\therefore$ P.d. $= 4.5 \times 10^{11} V$

9. $V_A - V_B$ = work done by electric field on + 1 coul. charge from A to B = E Rθ

$\therefore$ $V_B = V_A - ER\theta = V - ER\theta$

10. By argument of symmetry, it will be half of the potential produced by the full sphere

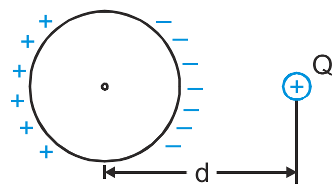
$\because$ Charge on hemisphere = Q,

so charge on sphere = 2Q

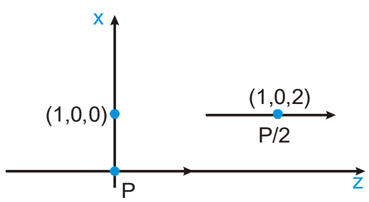
$$\Rightarrow \frac{1}{2}.\frac{K(2Q)}{R} = \frac{KQ}{R}$$

$$V = \frac{KQ}{R} = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{15 \times 10^{-2}} = 300\,V$$

11. Net potential of the sphere due to the induced charge is zero. Therefore potential is due to the point charge only, equal to
potential at the centre of the sphere.



12. The given point is at axis of $\frac{\vec{P}}{2}$ dipole and at equatorial line of $\vec{P}$ dipole so
total field at given point is.

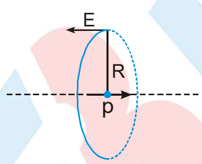


$$\vec{E} = -\frac{k\vec{P}}{(1)^3} + \frac{2k(\vec{P}/2)}{(2)^3} = k\vec{P}\left(-1+\frac{1}{8}\right) = \frac{-7\vec{P}}{32\pi\varepsilon_0}$$

13. Electric field at each point on the surface of ring

due to dipole is $E = \frac{k\,p}{R^3}$

in direction opposite to the dipole moment. (figure below)



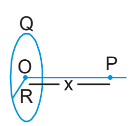
Hence net force on ring is $F = QE = \frac{kpQ}{R^3}$

Alternate solution

Electric field due to ring at point P on its axis at a distance x from centre O of ring is

$$E = k\,.\,\frac{Qx}{(x^2+R^2)^{3/2}}\ ;\ \left.\frac{dE}{dx}\right]_{\text{at } x=0} = \frac{kQ}{R^3}$$

$\therefore$ Force on dipole $= \frac{dE}{dx} = \frac{k\,Qp}{R^3}$

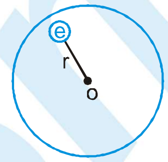


14. After covering with a hemispherical shell ; $\phi_{shell} + \phi_{disc} = 0$ (from Gauss law)

$\therefore\ \phi_{shell} = -\phi_{disc} = -\phi$

15. $eE = m_e\,\omega^2 r$ (Balancing of forces on e^-)

Also, $V_O - V_R = -\int_R^0 E.dr = \int_0^R E.dr = \frac{m_e\omega^2}{e}\int_0^R r\,dr$



So, $V = \frac{m_e\omega^2 R^2}{2e}$

16. $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{\varepsilon_0 K^2 Q^2}{2r^4} \Rightarrow v = \frac{KQ}{r}$

$$\frac{u}{v^2} = \frac{\frac{1}{2}\varepsilon_0 K^2\frac{Q^2}{r^4}}{\frac{K^2Q^2}{r^2}} = \frac{1}{2}\frac{\varepsilon_0}{r^2}$$

because $\frac{u}{v^2} \propto \frac{1}{r^2}$ so the correct option is B.

17. $\phi = \vec{E}\cdot\vec{ds}$

since r<< R so we can consider electric field is constant throughout the surface of smaller ring, hence

$$\phi \propto E \propto \frac{x}{(R^2+x^2)^{3/2}}$$

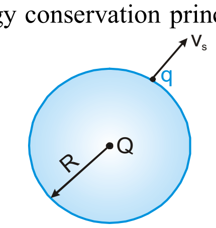
So, the best represnted graph is C.

18. Let q be positive.

If it escapes then from energy conservation principle,

$\frac{1}{2}mv_s^2 + \frac{KQq}{R} = 0$

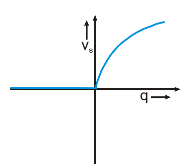
$v = \sqrt{\frac{-2KQq}{Rm}}$



{Note that Q is negative, therefore the quantity within the root is positive.}

$\therefore\ v \propto \sqrt{q}$

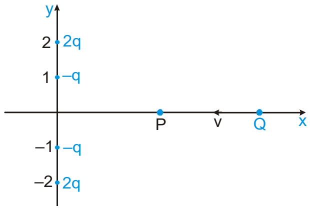
When q is negative, escape velocity will be zero due to electrostatic repulsion from negative Q.



19. $\vec{E} = -\nabla v = +\frac{1}{x^2}\hat{i} + \frac{1}{y^2}\hat{j} + \frac{1}{z^2}\hat{k} = \hat{i}+\hat{j}+\hat{k}$

20. There exists a point P on the x-axis (other than the origin), where net electric field is zero.Once the charge Q reaches point P, attractive forces of the two –ve charge will dominate

and automatically cause the charge Q to cross the origin.



Now if Q is projected with just enough velocity to reach P,

its K.E. at P is zero.

But while being attracted towards origin it acquires K.E. & hence its net energy at the origin is positive. (P.E. at origin = zero).

21. The electric field inside the inner shell is zero

So, the potential on inner shell and all the points inside it will be constant.

22. $V = \frac{K\,\vec{p}.\vec{r}}{r^3} = \frac{9\times10^9\times(2\hat{i}-3\hat{j}+4\hat{k}).(2\hat{i}+2\hat{j}-\hat{k})}{[2^2+2^2+1^2]^{3/2}}$

$= \frac{9\times10^9\times(4-6-4)}{27}$

$V = -2\times109$ volts **Ans.**

23. Interaction energy of system of charges is

$= \frac{1}{2}[U_1 + ... + U_6] = \frac{1}{2}[6U_1] = 3U_1$

$= 3kq^2\left(-\frac{2}{a}+\frac{2}{a\sqrt{3}}-\frac{1}{2a}\right) = \frac{q^2}{\pi\in_0 a}\left(\frac{\sqrt{3}}{2}-\frac{15}{8}\right)$

24. The distribution of charge on the outer surface, depends only on the charges outside, and it distributes itself such that the net, electric field inside the outer surface due to the charge on outer surface and all the outer charges is zero. Similarly the distribution of charge on the inner surface, depends only on the charges inside the inner surface, and it distributes itself such that the net, electric field outside the inner surface due to the charge on inner surface and all the inner charges is zero.

Also the force on charge inside the cavity is due to the charge on the inner surface. Hence answer is option

25. Using the formula for electric field produced by large sheet $E = \frac{Q}{2A\in_0}$

We get ;

$E_A = \frac{4Q}{2A\varepsilon_0}(-\hat{i})$; $E_B = \frac{2Q}{2A\varepsilon_0}.(-\hat{i})$; $E_C = \frac{4Q}{2A\varepsilon_0}.(+\hat{i})$

26. $V_B - V_A = -\int E_x\,dx = -$ [Area under $E_x - x$ curve]

$V_B - 10 = -\frac{1}{2}.2.(-20) = 20$

$V_B = 30$ V.

27. $F = \frac{kq^2}{r^2}$; $(k = \frac{1}{4\pi\varepsilon_0}) \Rightarrow \frac{kq^2}{r^2} = \frac{mv^2}{R_C}$

$\Rightarrow R_C = \frac{mv^2r^2}{kq^2} \Rightarrow R_C = \frac{4\pi\varepsilon_0 v^2 r^2 m}{q^2}$

28. Speed will be maximum when acceleration becomes zero. ie when $Kx = EQ$

$\Rightarrow x = \frac{EQ}{K}$

By work–energy theorem : $w_{all} = \Delta KE$

$\Rightarrow EQX - \frac{1}{2}KX^2 = \frac{1}{2}mv^2$

Substituting $x = EQ/K$,

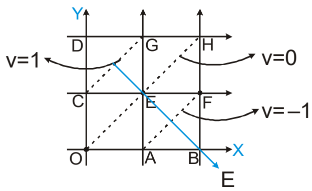
$V_{max} = QE/\sqrt{mK}$

Compression will be maximum when velocity becomes zero.

$W_{all} = \Delta KE \Rightarrow EQX - \frac{1}{2}KX^2 = 0$; $X_{max} = \frac{2EQ}{K}$

29. OEH is an equipotential surface, the uniform E.F. must be perpendicular to it pointing from higher to lower potential as shown

Hence, $\hat{E} = \left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$.

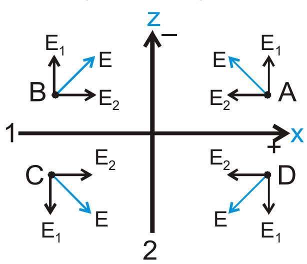


$$E = \frac{(V_E - V_B)}{EB} = \frac{0-(-2)}{\sqrt{2}} = \sqrt{2}\ \text{V/m}$$

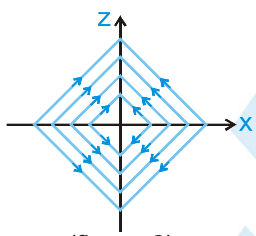
$$\therefore \quad \vec{E} = E.\hat{E} = \sqrt{2}\,\frac{(\hat{i}-\hat{j})}{\sqrt{2}} = (\hat{i}-\hat{j})\ \text{V/m}$$

30. $V_p = \frac{kq}{r'} + V_{in} = V_C = \frac{kq}{r} \qquad \therefore \quad V_{in} = \frac{kq}{r} - \frac{kq}{r'}$

31. The electric field intensity due to each uniformly charged infinite plane is uniform. The electric field intensity at points A, B, C and D due to plane 1, plane 2 and both planes are given by E_1, E_2 and E as shown in figure 1. Hence the electric lines of forces are as given in figure 2.



(figure 1)



(figure 2)

Aliter :

Electric lines of forces originate from positively charged plane and terminate at negatively charged plane. Hence the correct representation of ELOF is as shown figure 2.

32. The acceleration of centre of mass of system of particles is

$$a_{cm} = \frac{(q_1+q_2)}{2m}E$$

$\therefore$ x-coordinate of centre of mass at t = 2 second is

$$x_{cm} = \frac{1}{2}a_{cm}t^2 = \frac{1}{2}\frac{(q_1+q_2)}{2m}E \times 2^2 = \frac{q_1+q_2}{m}E$$

Let the x-coordinates of q_1 and q_2 at t = 2 sec be x_1 and x_2 ; [$x_1 = 2a$ at t = 2 sec.]

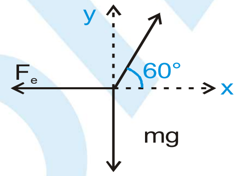
$$\therefore \quad x_{cm} = \frac{mx_1 + mx_2}{2m} = \frac{x_1+x_2}{2}$$

or $\quad x_2 = 2x_{cm} - x_1 = 2\frac{(q_1+q_2)}{m}E - 2a$

33. The bowl exerts a normal force N on each bead, directed along the radial line or at 60° above the horizontal. Consider the free-body diagram of the bead on the left with the electric force F_e applied.

$\Sigma F_y = N\sin 60° - mg = 0, \quad \Rightarrow N = mg/\sin 60°$

$\Sigma F_x = -F_e + N\cos 60° = 0,$



$$\Rightarrow \frac{Kq^2}{R^2} = N\cos 60°$$

$$= \frac{mg}{\tan 60°} = \frac{mg}{\sqrt{3}}$$

34. The potential at centre of sphere in which q charge is uniformly distributed throughout the volume is -

$$V_C = \frac{1}{4\pi\varepsilon_O}\frac{3q}{2R}$$

By symmetry the potential at centre due to half sphere will be half of the complete sphere.

$$\therefore V_C = \frac{1}{4\pi\varepsilon_O}\frac{3q/2}{2R} = \frac{1}{4\pi\varepsilon_O}\frac{3Q}{2R} \qquad \left[\because \frac{q}{2} = Q\right]$$

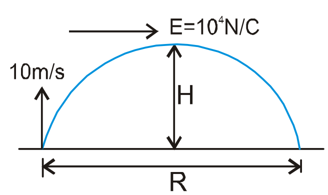
35. Increasing the accelerating voltage means increasing speed of the electron, thereby decreasing time spent between the plates. It will reduce X.

Increasing deflecting voltage means increasing electric field between the plates, making acceleration of electron greater. Increasing distance once again will change electric field between the plates.

36. Time of flight (t) $= \frac{2u}{g} = \frac{2\times 10}{10} = 2$ sec

$$H = \frac{u^2}{2g} = \frac{10\times 10}{2\times 10} = 5\text{m}$$

$$R = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$



$$= \frac{1}{2} \times \frac{10^{-3}\times 10^4 \times 2 \times 2}{2} = 10\text{ m}$$

37. Charge on $a_1 = (r_1\,\theta)\,\lambda$
Charge on $a_2 = (r_2\,\theta)\,\lambda$

$\therefore$ Ratio of charges $= \dfrac{r_1}{r_2}$

E_1, Field produced by $a_1 = \dfrac{K[(r_1\,\theta)\lambda]}{r_1^2} = \dfrac{KQ_1}{r_1^2}$

E_2, Field produced by $a_2 = \dfrac{KQ_2}{r_2^2}$

So, $\dfrac{E_1}{E_2} = \dfrac{KQ_1}{r_1^2} \times \dfrac{r_2^2}{KQ_2} = \dfrac{r_2}{r_1}$

So, $\dfrac{E_1}{E_2} = \dfrac{KQ_1}{r_1^2} \times \dfrac{r_2^2}{KQ_2} = \dfrac{r_2}{r_1}$

As $r_2 > r_1$

Therefore $E_1 > E_2$

i.e. Net field at A is towards a_2.

$V_1 = \dfrac{K.(r_1\theta)\lambda}{r_1} = K\theta\lambda$

$V_2 = \dfrac{K.(r_2\,\theta)\lambda}{r_2} = K\theta\lambda \qquad \Rightarrow V_1 = V_2$.

38. $0 \le x \le a : V_x = \left[-\int_0^x E_x dx\right] + V_{(0)} = 0 \text{ (as } E_x = 0)$

$x \ge a\,;\, V_x = -\int_a^x E_x dx + V_{(a)} = \left[-\int_a^x \dfrac{\sigma}{\epsilon_0} dx\right] + V_{(a)}$

$= -\dfrac{\sigma}{\epsilon_0}(x-a) \quad ; \quad (\text{As, } V_a = 0)$

$x \le 0\,;\, V_x = -\int_0^x E_x dx + V_{(0)} = -\left(-\dfrac{\sigma}{\epsilon_0}.x\right) + V_{(0)}$

$= \dfrac{\sigma}{\epsilon_0}.x \;\; ; (\text{As, } V_0 = 0)$

39. V at origin $\neq 0$

$E(r = 2\text{ m}) = \dfrac{K(-q)r}{(R_1^2 + r^2)^{3/2}} + \dfrac{K.Q.r}{(R_2^2 + r^2)^{3/2}}$

$= K.rq\left[-\dfrac{1}{10^{3/2}} + \dfrac{2\sqrt{2}}{2^{3/2}.10^{3/2}}\right] = 0$

From origin to r = 2, field is towards origin.

40. Charge is distributed over the surface of conductor in such a way that net field due to this charge and outside charge q is zero inside. Field due to only q is non-zero.

41. Potential at a point is zero does not imply that electric field at same point should be zero. For instance in the equatorial plane of a dipole, potential at any point is zero but electric field is not zero. Hence statement 1 is false.

No electric field in space means , potential at all points in space is same. Hence potential difference between any two points is zero. Hence statement 2 is true.

Statement–1 is False, Statement–2 is True.

42. The electric field inside the cavity depends only on point charge q. Hence $V_A - V_B$ remains constant even if point charge Q is shifted. Here statement 2 is correct explanation of statement 1.

43. For a non-uniformly charged thin circular ring with net zero charge, electric potential at each point on its axis is zero. Hence electric field at each point on its axis must be perpendicular to the axis. Therefore statement 1 is false and statement 2 is true.

44. The electric field due to disc is superposition of electric field due to its constituent ring as given in statement-2. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

45. From work energy theorem

Final K.E. = Initial K.E. + work done by non-uniform electric field.

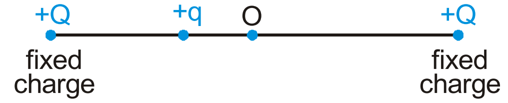
✈ initial K.E. = 0 and final K.E. cannot be negative.

$\therefore$ Work done by non-uniform electric field on a charged particle starting from rest is non-negative.

Hence statement I is true.

Consider a situation in which two point charges +Q are fixed some distance apart. At some distance left of equilibrium point O, a charge +q is released from rest. After the charge +q moving towards right crosses O, it experiences a force towards left.

Hence statement II is false.



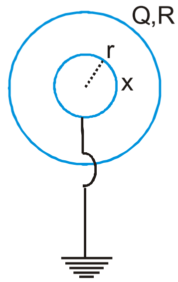
Statement-1 is True, Statement-2 is False.

46. **(C)**

47. $V_{ball} = 0$

$$\frac{KX}{r} + \frac{KQ}{R} = 0$$

$$\Rightarrow x = -\frac{Qr}{R}$$



Potential difference

$$V_s - V_b = \frac{KQ}{R} + \frac{Kx}{R}$$

because potential difference depend only on charge on inner surface after
electrostatic condition is reached after grounding.

$$V_s - V_b = \frac{KQ}{R}\left[1 - \frac{r}{R}\right]$$

48. Let the speed of charges A and B be V_A and V_B when the separation between them is $\bullet_o$. Then from conservation of momentum

$-mV_A + 2\,mV_B = 0$ or $V_A = 2\,V_B$

Applying conservation of energy, as the separation increases from $\bullet_o$ to $2\bullet_o$.

Gain in K.E of system of charges = Loss in electrostatic potential energy of the system of charges.

$$\frac{1}{2}mV_A^2 + \frac{1}{2}2m\,V_B^2 = \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{l_o} - \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{2l_o}$$

or $$\frac{1}{2}mV_A^2 + \frac{1}{2}2m\left(\frac{V_A}{2}\right)^2 = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{l_o}.$$

Solving we get the speed of charge A is

$$V_A = \sqrt{\frac{1}{3\pi\varepsilon_o}\frac{q^2}{ml_o}}.$$

49. The work done by electrostatic force on charge A, from work energy theorem, in the given duration is
= Final kinetic energy of charge A – Initial kinetic energy of charge A = $\frac{1}{2}mV_A^2 - 0$

$= \frac{1}{6\pi\varepsilon_o}\frac{q^2}{l_o}$. The sign of work done is positive

50. The net work done by electrostatic force on system of two charged particle is equal to change in electrostatic potential energy of the system

$$= \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{l_o} - \frac{1}{4\pi\varepsilon_o}\frac{2q^2}{2l_o} = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{l_o}.$$

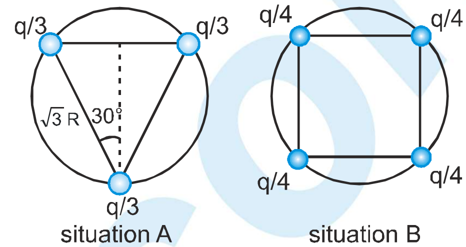
The sign of work done is positive

51. (B), 52. (C), 53. (D))

Potentials at the centre

$$v_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}; \qquad v_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Potential energy in situation I is

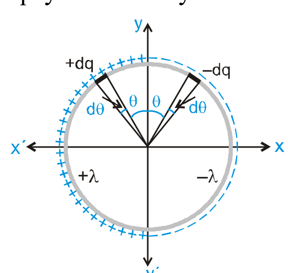


$$U_1 = 3 \times \frac{1}{4\pi\varepsilon_o}\frac{(q/3)^2}{(\sqrt{3}R)} = \frac{1}{12\sqrt{3}\pi\varepsilon_o}\frac{q^2}{R}$$

When one charge is removed, the field intensity at the centre is due to the removed charge only.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q/3}{r^2} \Rightarrow E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q/4}{r^2} \therefore \frac{E_1}{E_2} = \frac{4}{3}$$

54. Consider two small elements of ring having charges +dq and – dq symmetrically located about y-axis.



The potential due to this pair at any point on y-axis is zero. The sum of potential due to all such possible pairs is zero at all points on y-axis.

Hence potential at P(0, $\frac{R}{2}$) is zero.

55. Since all charge lies in x-y plane, hence direction of electric field at point P should be in x-y plane
Also y-axis is an equipotential (zero potential) line. Hence direction of electric field at all point on y-axis should be normal to y-axis.
$\therefore$ The direction of electric field at P should be in x-y plane and normal to y-axis. Hence direction of electric field is along positive-x direction.

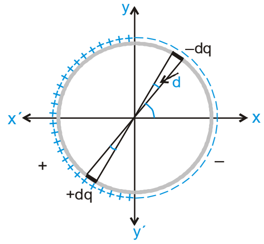
56. Consider two small elements of ring having charge +dq and –dq as shown in figure.

The pair constitutes a dipole of dipole moment.

$dp = dq\,2R = (\lambda R d\theta)\,2R$

The net dipole moment of system is vector sum of dipole moments of all such pairs of elementary charges.

By symmetry, the resultant dipole moment is



along negative x-direction.

$\therefore$ net dipole moment =

$$\int_{-\pi/2}^{+\pi/2} (dp\cos\theta)\ \hat{i} = -\int_{-\pi/2}^{+\pi/2} (2\lambda R^2 \cos\theta\, d\theta)\ \hat{i} = -4R^2\lambda\,\hat{i}$$

57.

(A) Electrostatic potential energy

$$= \frac{1}{4\pi\epsilon_0}\frac{(-Q)^2}{2a} = \frac{Q^2}{8\pi\epsilon_0 a}$$

(B) Electrostatic potential energy

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-Q)\times(-Q)}{5a/2} + \frac{(-Q)^2}{2(5a/2)}\right] = \frac{3}{20}\frac{Q^2}{\pi\epsilon_0 a}$$

(C) Electrostatic potential energy

$$= \frac{1}{4\pi\epsilon_0}\frac{3Q^2}{5a} = \frac{3}{20}\frac{Q^2}{\pi\epsilon_0 a}$$

(D) Electrostatic potential energy

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3Q^2}{5a} + \frac{(-Q)^2}{2(2a)} + \frac{(-Q)\times(-Q)}{2a}\right] = \frac{27Q^2}{80\pi\epsilon_0 a}$$

58. In situation A, B and C, shells I and II are not at same potential. Hence charge shall flow from sphere I to sphere II till both acquire same potential.

If charge flows, the potential energy of system decreases and heat is produced.

In situations A and B charges shall divide in some fixed ratio , but in situation C complete charge shall be transferred to shell II for potential of shell I and II to be same.

(A) → p, q , (B) → p, q , (C) → p, q, s

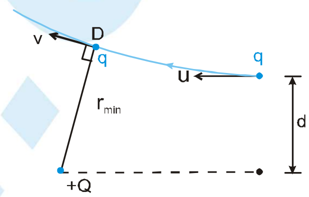
In situation D both the shells are at same potential, hence no charge flows through connecting wire.

$\therefore$ (D) → r, s

59. The path of the particle will be as shown in the figure. At the point of minimum distance (D) the velocity of the particle will be ⊥ to its position vector w.r.t. to +Q.

Now by conservation of energy :-

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + \frac{KQq}{r_{min}} \qquad \text{......(1)}$$



∵ Torque on q about Q is zero hence angular momentum about Q will be conserved

$\Rightarrow$ $m v r_{min} = m u d$ ......**(2)**

by **(2)** in **(1)** $\Rightarrow$ $\frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{ud}{r_{min}}\right)^2 + \frac{KQq}{r_{min}}$

$$\Rightarrow \frac{1}{2}mu^2 \left(1 - \frac{d^2}{r_{min}^2}\right) = \frac{mu^2 d}{r_{min}} \qquad \{\because KQq = mu^2 d \text{ (given)}\}$$

$\Rightarrow r_{min}^2 - 2r_{min}d - d^2 = 0$

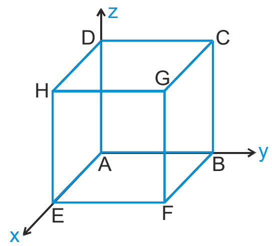
$$\Rightarrow r_{min} = \frac{2d \pm \sqrt{4d^2 + 4d^2}}{2} = d(1 \pm \sqrt{2})$$

∵ distance cannot be negative

$\therefore$ $r_{min} = d(1 + \sqrt{2})$ **Ans.**

60. Flux through ABCD.

$\phi_1 = \vec{E}\cdot\vec{A} = (x^2\hat{i} + y\hat{j}).(-a^2\hat{i}) = 0$ as $x = 0$



Flux through EFGH

$\phi_2 = (x^2\hat{i} + y\hat{j}).(+a^2\hat{i}) = x^2.a^2 = a^4 = 1.0 \times 10^{-4}$ Nm²/C

Flux through BCGF

$\phi_3 = (x^2\hat{i} + y\hat{j}).(a^2\hat{j}) = a^3 = 1.0 \times 10^{-3}$ Nm²/C

Flux through EADH

$\phi_4 = (x^2\hat{i} + y\hat{j}).(-a^2\hat{j}) = 0$ as $y = 0$

Flux through ABFE

$\phi_5 = (x^2\hat{i} + y\hat{j}).(-a^2\hat{k}) = 0$

Flux through CDHG

$\phi_6 = 0$

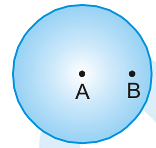
Net flux = $(1.0 \times 10^{-4} + 1.0 \times 10^{-3})$ N-m²/C

$= 11 \times 10^{-4}$ N-m²/C

61. Assume a solid sphere without cavity.

Potential at A due to this solid sphere

$$\Rightarrow V_A' = \frac{3}{2}\cdot\frac{1}{4\pi\epsilon_0}\frac{\left(\frac{4}{3}\pi R^2\rho\right)}{R} = \frac{\rho R^2}{2\epsilon_0}$$



Electric field at 'C' due to this solid sphere = $\vec{E}'_C$

$$= \frac{\rho}{3\epsilon_0}\vec{AC}$$

Now consider the cavity filled with negative charge

$$V_A'' = \frac{1}{4\pi\epsilon_0}\frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3(-\rho)}{\frac{R}{2}} = \frac{-\rho R^2}{12\epsilon_0}$$

$$\vec{E}''_C = \frac{-\rho}{3\epsilon_0}\vec{BC}$$

Now net values for the solid sphere with the cavity can be given by superposition of the above two cases

Hence, $V_A = V_A' + V_A'' = \frac{\rho R^2}{\epsilon_0}\left(\frac{1}{2} - \frac{1}{12}\right) = \frac{5\rho R^2}{12\epsilon_0}$

$$\vec{E}_C = \vec{E}'_C + \vec{E}''_C = \frac{\rho}{3\epsilon_0}(\vec{AC} - \vec{BC}) = \frac{\rho}{3\epsilon_0}\vec{AB}$$

$$\therefore\ E_C = \frac{\rho}{3\epsilon_0}\left(\frac{R}{2}\right) = \frac{\rho R}{6\epsilon_0}$$

Ans. $VA = \frac{5\rho R^2}{12\epsilon_0}$, $EC = \frac{\rho R}{6\epsilon_0}$