

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. $m^3 - 3m^2 - 4m + 12 = 0 \Rightarrow m = \pm 2, 3$

$m \in \mathbb{N}$ hence $m \in \{2, 3\} \Rightarrow$ (C)

4. $y \sin 2x - \cos x + (1 + \sin^2 x) \frac{dy}{dx} = 0$ where $y = f(x)$

$$\frac{dy}{dx} + \left(\frac{\sin 2x}{1 + \sin^2 x} \right) y = \frac{\cos x}{1 + \sin^2 x}$$

I.F. $= e^{\int \frac{\sin 2x}{1 + \sin^2 x} dx} = e^{\int \frac{dt}{t}} = e^{\ln(1 + \sin^2 x)} = 1 + \sin^2 x$

(by putting $1 + \sin^2 x = t$)

$$y(1 + \sin^2 x) = \int \cos x dx$$

$$y(1 + \sin^2 x) = \sin x + C ; (y(0) = 0) \Rightarrow C = 0$$

hence, $y = \frac{\sin x}{1 + \sin^2 x}$

$$y\left(\frac{\pi}{6}\right) = \frac{2}{5}$$

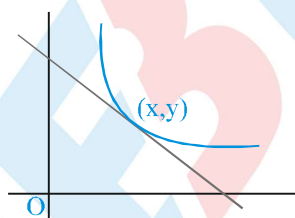
5. $Y - y = m(X - x)$

for X-intercept $Y = 0$

$$X = x - \frac{y}{m}$$

hence $x - \frac{y}{m} = y$

or $\frac{dy}{dx} = \frac{y}{x - y}$



put $y = Vx$

$$V + x \frac{dV}{dx} = \frac{V}{1 - V}$$

$$x \frac{dV}{dx} = \frac{V}{1 - V} - V = \frac{V - V + V^2}{1 - V}$$

$$\int \frac{1 - V}{V^2} dV = \int \frac{dx}{x}$$

$$-\frac{1}{V} - \ln V = \ln x + c$$

$$-\frac{x}{y} - \ln \frac{y}{x} = \ln x + c$$

$$-\frac{x}{y} = \ln y + c$$

$$x = 1, y = 1 \Rightarrow c = -1$$

$$1 - \frac{x}{y} = \ln y$$

$$y = e \cdot e^{-x/y}$$

$$e^{-x/y} = \frac{e}{y}$$

$$y e^{x/y} = e \Rightarrow$$
 (A)

6. put $xe^x = t$

$$(e^x + xe^x) \frac{dx}{dy} = \frac{dt}{dy}$$

$$\therefore \frac{dt}{dy} + (ye^y - t) = 0 \Rightarrow \frac{dt}{dy} - t + ye^y = 0$$

I.F. $e^{-\int dy} = e^{-y}$

$$t \cdot e^{-y} = - \int ye^y e^{-y} dy$$

$$x e^x e^{-y} = - \frac{y^2}{2} + C$$

$$f(0) = 0 \Rightarrow C = 0; 2x e^x e^{-y} + y^2 = 0$$

8. $\frac{dy}{dt} = -k\sqrt{y}$; when $t = 0$; $y = 4$

$$\int_4^0 \frac{dy}{\sqrt{y}} = -k \int_0^t dt$$

$$2\sqrt{y} \Big|_4^0 = -kt = -\frac{t}{15}$$

$$0 - 4 = -\frac{t}{15} \Rightarrow t = 60 \text{ minutes} \Rightarrow$$
 (C)

10. $y \cdot e^{-2x} = Ax e^{-2x} + B$

$$e^{-2x} \cdot y_1 - 2y e^{-2x} = A(e^{-2x} - 2x e^{-2x})$$

Cancelling e^{-2x} throughout

$$y_1 - 2y = A(1 - 2x) \dots (i)$$

differentiating again

$$y_2 - 2y_1 = -2A \Rightarrow A = \frac{2y_1 - y_2}{2}$$

hence substituting A in (1)

$$2(y_1 - 2y) = (2y_1 - y_2)(1 - 2x)$$

$$2y_1 - 4y = 2y_1(1 - 2x) - (1 - 2x)y_2$$

$$(1 - 2x) \frac{d}{dx} \left(\frac{dy}{dx} - 2y \right) + 2 \left(\frac{dy}{dx} - 2y \right) = 0$$

hence $k = 2$ and $l = -2$

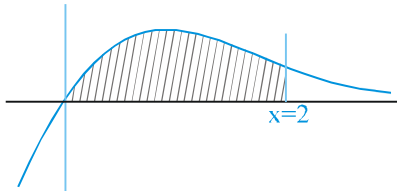
$\Rightarrow$ ordered pair $(k, l) \equiv (2, -2)$

11. $y = xe^{-x}$

$$y' = e^{-x} - x e^{-x} = (1 - x)e^{-x} \uparrow \text{ for } x < 1$$

$$y'' = -e^{-x} - [e^{-x} - x e^{-x}] = e^{-x}[-1 - 1 + x] \\ = (x - 2)e^{-x}$$

for point of inflection $y'' = 0 \Rightarrow x = 2$



$$A = \int_0^2 x e^{-x} dx = -x e^{-x} \Big|_0^2 + \int_0^2 e^{-x}$$

$$= (-2 e^{-2}) - (e^{-x}) \Big|_0^2$$

$$= -2 e^{-2} - (e^{-2} - 1) = 1 - e^{-2} - 2e^{-2} = 1 - 3e^{-2}$$

13. $\sin x \frac{dy}{dx} + y \cos x = 1$

$$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$$

$$\text{I.F.} = e^{\int \cot x dx} = e^{\ln(\sin x)} = \sin x$$

$$y \sin x = \int \operatorname{cosec} x \cdot \sin x dx$$

$$y \sin x = x + C$$

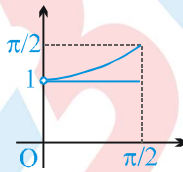
if $x = 0$, y is finite

$$\therefore C = 0$$

$$y = x (\operatorname{cosec} x) = \frac{x}{\sin x}$$

$$\text{Now } I < \frac{\pi^2}{4} \text{ and } I > \frac{\pi}{2}$$

$$\text{Hence } \frac{\pi}{2} < I < \frac{\pi^2}{4} \Rightarrow \text{(A)}$$



$$14. \int_0^x f(x) dx = y^3$$

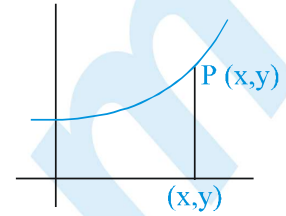
Differentiating

$$f(x) = 3y^2 \cdot \frac{dy}{dx}$$

$$y = 3y^2 \frac{dy}{dx} \Rightarrow y = 0 \text{ (rejected)}$$

$$\text{or } 3y dy = dx$$

$$\frac{3y^2}{2} = x + c \Rightarrow \text{parabola} \Rightarrow C$$



$$15. (x - h)^2 + (y - k)^2 = a^2 \quad \dots\dots(i)$$

$$2(x - h) + 2(y - k) \frac{dy}{dx} = 0 \quad \dots\dots(ii)$$

$$1 + \left(\frac{dy}{dx} \right)^2 + (y - k) \frac{d^2y}{dx^2} = 0 \quad \dots\dots(iii)$$

From (3) we have $(y - k)$, use in (2) to get $(x - h)$ and put $(x - h)$ and $(y - k)$ in (1)

$$16. \frac{dy}{dx} - y = y^2(\sin x + \cos x)$$

$$\frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} = \sin x + \cos x ;$$

$$\text{Let } -\frac{1}{y} = t \Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + t = \sin x + \cos x$$

I.F. e^x

$$\therefore t \cdot e^x = \int e^x (\sin x + \cos x) dx ; -\frac{1}{y} e^x = e^x \sin x + C$$

$$\text{if } x = 0, y = 1 \Rightarrow C = -1$$

$$-\frac{e^x}{y} = e^x \sin x - 1$$

$$\text{if } x = \pi \text{ then } -\frac{e^x}{y} = -1 \Rightarrow y = e^\pi$$

$$17. \frac{dy}{dx} = 100 - y$$

$$\Rightarrow \int \frac{dy}{100 - y} = \int dx \Rightarrow -\bullet n(100 - y) = x + c$$

since $y(0) = 50 \Rightarrow -\bullet n 50 = c$

$$\therefore -\bullet n(100 - y) = x - \bullet n 50$$

$$\Rightarrow \bullet n\left(\frac{50}{100 - y}\right) = x \Rightarrow \frac{50}{100 - y} = e^x$$

$$18. y = mx + c; \quad \frac{dy}{dx} = m; \quad \frac{d^2y}{dx^2} = 0$$

substituting in $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = -4x$

$$0 - 3m - 4(mx + c) = -4x$$

$$-3m - 4c - 4mx = -4x$$

$$-(3m + 4c) = 4x(m - 1)$$

....(i)

(i) is true for all real x if

$$m = +1 \text{ and } c = -3/4$$

$\Rightarrow$ (B)

$$19. \frac{dy}{dx} - \frac{y}{x^2} \tan \frac{1}{x} = -\sec \frac{1}{x} \cdot \frac{1}{x^2}$$

$$IF = e^{-\int \frac{1}{x^2} \tan \frac{1}{x} dx} = \sec \frac{1}{x}$$

$$\Rightarrow y \cdot \sec \frac{1}{x} = - \int \sec^2 \left(\frac{1}{x} \right) \cdot \frac{1}{x^2} dx = \tan \frac{1}{x} + c$$

if $y \rightarrow -1$ then $x \rightarrow \infty \Rightarrow c = -1$

$$\Rightarrow y = \sin \frac{1}{x} - \cos \frac{1}{x}$$

$$20. S_1: y^2 = 4ax$$

$$2yy' = 4a$$

$$y^2 = 2yy'x \Rightarrow \frac{dy}{dx} = \frac{y}{2x} \Rightarrow \frac{dy}{y} = \frac{dx}{2x}$$

$$S_2: x \cos \alpha + y \sin \alpha = p \quad \dots(1)$$

$$\cos \alpha + \sin \alpha y' = 0$$

$$y' = -\cot \alpha \quad \dots(2)$$

solving (1) & (2) we get we get 2 degree equation

$$S_3: ax^2 + by^2 + c = 0$$

$$x^2 + k_1 y^2 + k_2 = 0 \text{ two arbitrary constant}$$

$$\therefore \text{order} = 2$$

$$21. Y - y = -\frac{1}{m}(X - x)$$

when $m = \frac{dy}{dx}$

take, let $Y = 0$

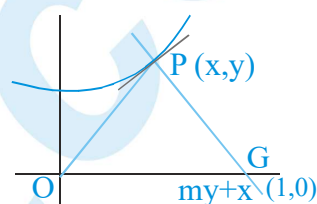
$$X = my + x$$

hence $x(my + x) = 2(x^2 + y^2)$

$$xy \frac{dy}{dx} = x^2 + 2y^2 \quad (\text{Step-A})$$

Now put $y = vx$

$$v + x \frac{dv}{dx} = \frac{x^2(1 + 2v^2)}{x^2v} = \frac{1 + 2v^2}{v}$$



$$x \frac{dv}{dx} = \frac{1 + 2v^2}{v} - v = \frac{1 + v^2}{v}$$

$$\int \frac{v dv}{1 + v^2} = \int \frac{dx}{x}$$

$$\frac{1}{2} \ln(1 + v^2) = \ln x + c$$

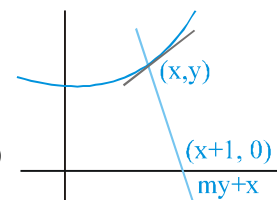
$$\ln\left(\frac{1 + v^2}{x^2}\right) = c$$

$$x^2 + y^2 = cx^4 \Rightarrow A$$

$$22. \text{Given } y \frac{dy}{dx} + x = x + 1$$

$$\frac{y^2}{2} = x + c$$

$$x = 0, y = 0 \Rightarrow c = 0$$



$$\therefore y^2 = 2x \Rightarrow \text{latus rectum} = 2 \Rightarrow B$$

$$23. \phi(x) = \phi'(x) \quad \phi(1) = 2$$

$$\frac{d\phi}{dx} = \phi$$

$$\bullet n \phi(x) = x + c$$

$$\bullet n 2 = 1 + c \Rightarrow c = \bullet n 2 - 1$$

$$\bullet n \phi(3) = 3 + c = 2 + \bullet n 2 \Rightarrow \phi(3) = 2e^2$$

27. I.F. = e^{-x}

$$\therefore ye^{-x} = \int e^{-x}(\cos x - \sin x) dx \quad \text{put } -x = t$$

$$= - \int e^t(\cos t + \sin t) dt$$

$$= -e^t \sin t + c$$

$$ye^{-x} = e^{-x} \sin x + c$$

since y is bounded when $x \rightarrow \infty \Rightarrow c = 0$

$$\therefore y = \sin x$$

$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x) dx = \sqrt{2} - 1 \Rightarrow \text{(A)}$$

29. $\ln c + \ln |x| = \frac{x}{y}$

$$\text{diff. w.r.t. } x, \quad \frac{1}{x} = \frac{y - x y_1}{y^2}$$

$$\frac{y^2}{x} = y - x \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2} \Rightarrow \text{(D)}$$

$$31. \frac{dy}{dx} = 2ax = 2x\left(\frac{y}{x^2}\right); \quad \frac{dy}{dx} = \frac{2y}{x};$$

$$\text{now } m = \frac{dy}{dx} = -1$$

$$\Rightarrow m = -\frac{x}{2y} \Rightarrow \frac{dy}{dx} = -\frac{x}{2y}$$

$$33. + \sin y \frac{dy}{dx} = +ae^{-x}$$

$$\sin y \frac{dy}{dx} = \cos y \Rightarrow \frac{dy}{dx} = \cot y$$

equation to the orthogonal trajectory will be obtained by solving the differential equation

$$-\frac{dy}{dx} = \cot y$$

$$c - x = \ln(\sin y) \Rightarrow \sin y = ce^{-x}$$

$$y^2 = -\frac{x^2}{2} + c$$

$$34. f'(x) - \frac{2x(x+1)}{x+1} f(x) = \frac{e^{x^2}}{(x+1)^2}$$

$$\text{I.F.} = e^{\int -2x dx} = e^{-x^2}$$

$$\therefore f(x) \cdot e^{-x^2} = \int \frac{dx}{(x+1)^2} \Rightarrow f(x) \cdot e^{-x^2} = -\frac{1}{x+1} + C$$

$$\text{at } x=0, f(0)=5 \Rightarrow C=6$$

$$\therefore f(x) = \left(\frac{6x+5}{x+1}\right) \cdot e^{x^2}$$

36. differentiate $xy(x) = x^2 y'(x) + 2xy(x)$

$$\text{or } xy(x) + x^2 y'(x) = 0$$

$$x \frac{dy}{dx} + y = 0$$

$$\ln y + \ln x = \ln c$$

$$xy = c \Rightarrow \text{(D)}$$

37. Let equation of St. Line

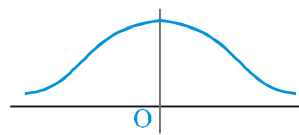
$$Y - y = m(X - x)$$

$$\text{Distance from origin} \Rightarrow \left| \frac{mx - y}{\sqrt{1+m^2}} \right| = 1$$

$$\therefore (mx - y)^2 = 1 + m^2$$

$$\left(y - \frac{dy}{dx} x\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

38. $y = f(x) = e^{-\frac{x^2}{2}}$. The graph is as shown



$$39. \frac{x}{x^2 + y^2} dy = \left(\frac{y}{x^2 + y^2} - 1\right) dx$$

$$\Rightarrow \frac{xdy - ydx}{x^2 + y^2} = -dx$$

$$\Rightarrow \tan^{-1} \frac{y}{x} = -x + c \Rightarrow y = x \tan(c - x)$$

$$40. \int_0^1 f(tx) dt = n \cdot f(x)$$

put $x = y \Rightarrow dt = \frac{1}{x} dy$

$$\therefore \frac{1}{x} \int_0^x f(y) dy = n f(x)$$

$$\therefore \int_0^x f(y) dy = x \cdot n \cdot f(x)$$

Differentiating

$$f(x) = n [f(x) + x f'(x)]$$

$$f(x) (1 - n) = n x f'(x)$$

$$\therefore \frac{f'(x)}{f(x)} = \frac{1-n}{n x}$$

Integrating $\ln f(x) = \left(\frac{1-n}{n} \right) \ln c x = \ln (c x)^{\frac{1-n}{n}}$

$$\therefore f(x) = c x^{\frac{1-n}{n}}$$

EXERCISE - 2

Part # I : Multiple Choice

3. $y = e^{-x} \cos x$
 $y_1 = -e^{-x} \cos x - e^{-x} \sin x = -e^{-x} (\sin x + \cos x)$

$$= -\sqrt{2} e^{-x} \left(\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} \right)$$

$$y_1 = -\sqrt{2} e^{-x} \cos \left(x - \frac{\pi}{4} \right)$$

$$y_2 = +\sqrt{2} e^{-x} \cos \left(x - \frac{\pi}{4} \right) + \sqrt{2} e^{-x} \sin \left(x - \frac{\pi}{4} \right)$$

$$= \sqrt{2} e^{-x} \left(\cos \left(x - \frac{\pi}{4} \right) + \sin \left(x - \frac{\pi}{4} \right) \right)$$

$$= \sqrt{2} \cdot \sqrt{2} e^{-x} \cos \left(x - \frac{\pi}{4} - \frac{\pi}{4} \right)$$

similarly

$$= \left(\sqrt{2} \right)^2 e^{-x} \cos \left(x - \frac{2\pi}{4} \right)$$

$$y_3 = - \left(\sqrt{2} \right)^3 e^{-x} \cos \left(x - \frac{3\pi}{4} \right)$$

$$y_4 = + \left(\sqrt{2} \right)^4 e^{-x} \cos \left(x - \frac{4\pi}{4} \right)$$

$$= - \left(\sqrt{2} \right)^4 e^{-x} \cos x$$

$$y_4 + 4y = 0$$

Similarly

$$y_8 = \left(\sqrt{2} \right)^8 e^{-x} \cos \left(x - \frac{8\pi}{4} \right) = \left(\sqrt{2} \right)^8 e^{-x} \cos (x - 2\pi)$$

$$= 16 e^{-x} \cos x$$

$$y_8 - 16y = 0$$

5. $\frac{dy}{1+y^2} + \frac{dx}{\sqrt{1-x^2}} = 0$

$$\Rightarrow \tan^{-1} y + \sin^{-1} x + c = 0$$

$$\Rightarrow \cot^{-1} \frac{1}{y} + \cos^{-1} \sqrt{1-x^2} + c = 0$$

9. $f(x) = \frac{\sin x}{x}$

13. $\frac{dy}{dx} + y = f(x)$

I.F. = e^x

$ye^x = \int e^x f(x) dx + C$

now if $0 \leq x \leq 2$ then $ye^x = \int e^x e^{-x} dx + C$

$\Rightarrow ye^x = x + C$

$x = 0, y(0) = 1, C = 1$

$\therefore ye^x = x + 1$ (1)

$y = \frac{x+1}{e^x}; y(1) = \frac{2}{e}$ Ans. $\Rightarrow$ (A) is correct

$y' = \frac{e^x - (x+1)e^x}{e^{2x}}$

$y'(1) = \frac{e - 2e}{e^2} = \frac{-e}{e^2} = -\frac{1}{e}$ Ans.

$\Rightarrow$ (B) is correct

if $x > 2$

$ye^x = \int e^{x-2} dx$

$ye^x = e^{x-2} + C$

$y = e^{-2} + Ce^{-x}$

as y is continuous

$\therefore \lim_{x \rightarrow 2} \frac{x+1}{e^x} = \lim_{x \rightarrow 2} (e^{-2} + Ce^{-x})$

$3e^{-2} = e^{-2} + Ce^{-2} \Rightarrow C = 2$

$\therefore$ for $x > 2$

$y = e^{-2} + 2e^{-x}$ hence $y(3) = 2e^{-3} + e^{-2} = e^{-2}(2e^{-1} + 1)$

$y' = -2e^{-x}$

$y'(3) = -2e^{-3}$ Ans. $\Rightarrow$ (D) is correct

15. $x^2 y_1^2 + xy_1 - 6y^2 = 0$

It is quadratic equation in y_1

$y_1 = \frac{-xy \pm \sqrt{x^2 y^2 + 24y^2 x^2}}{2x^2} = \frac{-xy \pm 5xy}{2x^2}$

$y_1 = -\frac{3y}{x} \quad | \quad y_1 = \frac{2y}{x}$

$\frac{dy}{dx} = -\frac{3y}{x} \quad | \quad \frac{dy}{dx} = \frac{2y}{x}$

$-\frac{dy}{y} = 3 \frac{dx}{x}$

$-\bullet n y = 3 \bullet nx + \bullet nc$
 $x^3 y = C$

$\frac{dy}{dx} = \frac{2y}{x}$

$\ln y = 2 \bullet nx + \bullet nc$
 $y = cx^2$

Option (C)

$\frac{1}{2} \log y = c + \log x$

$\log y = \bullet n c_1 + \log x^2$

$y = C_1 x^2$

18. We have equation of tangent is $(Y - y) = m(X - x)$

Put $X = 0$, we get $Y = y - mx$.

Now $(OT)^2 = (PT)^2$ (given)

$\Rightarrow (y - mx)^2 = x^2 + m^2 x^2$

$\Rightarrow y^2 + m^2 x^2 - 2mxy = x^2 + m^2 x^2$

$\therefore \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y^2 = t \Rightarrow 2y \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow x \frac{dt}{dx} = t - x^2$

$\Rightarrow \frac{dt}{dx} - \frac{t}{x} = -x$

$\therefore$ I.F. = $e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$

The solution is given by $t \left(\frac{1}{x} \right) = \int -dx = -x + C$

$\Rightarrow t = -x^2 + Cx$

Hence $y^2 + x^2 = Cx$.

If this is passing through $(2, 2) \Rightarrow C = 4 \Rightarrow x^2 + y^2 = 4x$

$\Rightarrow (x - 2)^2 + y^2 = 4$

It's director circle will be $(x - 2)^2 + y^2 = 8$.

Put $x = 0, y^2 = 4 \Rightarrow y = 2$ or -2

Intercept on y-axis is 4.

This represents a family of circles with centre at y-axis and passing through origin.

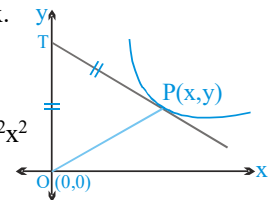
Verify other alternatives.

22. Given that $\left| y \frac{dy}{dx} \right| = \left| \frac{4y}{\frac{dy}{dx}} \right|$ at $(2, 1)$

$\Rightarrow \frac{dy}{dx} = \pm 2$ at $(2, 1)$

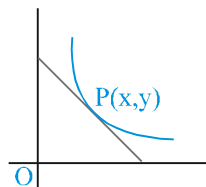
$\therefore$ Equation of tangent having positive slope is $y = 2x - 3$.

Now verify alternatives.



Part # II : Assertion & Reason

2. Equation of tangent $Y - y = m(X - x)$
put $X=0$, $Y = y - mx$



hence initial ordinate is

$$y - mx = x - 1 \quad \Rightarrow \quad mx - y = 1 - x$$

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{1-x}{x} \text{ which is a linear differential equation}$$

Hence statement-1 is correct and its degree is 1

⇒ statement-2 is also correct. Since every 1st degree differential equation need not be linear hence statement-2 is not the correct explanation of statement-1.

3. Statement-2 $\bullet_n \frac{x}{y} - \frac{1}{xy} = c$

i.e $\bullet_{nx} - \bullet_{ny} - \frac{1}{xv} = c$

$$\text{diff.} \quad \frac{1}{x} - \frac{1}{y} \frac{dy}{dx} + \frac{y+x \frac{dy}{dx}}{(xy)^2} = 0$$

$$xy^2 dx - x^2y dy + ydx + xdy = 0$$

$$(1 + xy) ydx + x(1 - xy) dy = 0$$

$\therefore$ statement is true.

Statement-1 $x - \frac{y^2}{2} + \frac{1}{3}(x^2 + y^2)^{3/2} = c$

$$1 - y \frac{dy}{dx} + \frac{1}{2} \sqrt{x^2 + y^2} \left(2x + 2y \frac{dy}{dx} \right) = 0$$

$$dx - y dy + x \sqrt{x^2 + y^2} dx + y \sqrt{x^2 + y^2} dy = 0$$

$$(1 + x \sqrt{x^2 + y^2}) dx + y(-1 + \sqrt{x^2 + y^2}) dy = 0$$

6. S-1: order is 2

$$7. \frac{1}{1 + \frac{y^2}{x^2}} \cdot \left(x \frac{dy}{dx} - y \right) = mx + c \Rightarrow$$

$$\frac{x^2}{x^2 + y^2} \cdot \left(x \frac{dy}{dx} - y \right) = mx + c \Rightarrow \frac{x \frac{dy}{dx} - y}{x^2 + y^2} = mx + c$$

statement-1 is false

statement-2 is linear form of differential equation which is true

- ## 8. Integral curves are

$$y = cx - x^2$$

The DE does not represent all the parabolas passing through origin but it represents all parabolas through origin with axis of symmetry parallel to y-axis and coefficient of x^2 as -1 , hence statement-1 is false.

Statement-2 is universally true.]

10. Line touches the curve at $(0, b)$ and $\left. \frac{dy}{dx} \right|_{x=0}$ also exists

but even if $\frac{dy}{dx}$ fails to exist, tangents line can be drawn.

$$13. C_1: 2x - \frac{2y}{3} \frac{dy}{dx} = 0 \Rightarrow \left. \frac{dy}{dx} \right|_{x_1, y_1} = \frac{3x_1}{y_1} = m_1$$

$$C_2: 3xy^2 \frac{dy}{dx} + y^3 = 0 \Rightarrow \left. \frac{dy}{dx} \right|_{x_1, y_1} = -\frac{y_1}{3x_1} = m_2$$

$$\therefore m_1 \cdot m_2 = -1 \quad \Rightarrow \quad C_1 \text{ and } C_2 \text{ are orthogonal}$$

15. We have $f(x) = \int_0^x f(t) \sin t \, dt - \int_0^x \sin(t-x) \, dt$
(King property)

$$\Rightarrow f(x) = \int_0^x f(t) \sin t \, dt + \int_0^x \sin t \, dt$$

$$\therefore f'(x) = f(x) \sin x + \sin x$$

$$\frac{dy}{dx} - y \sin x = \sin x \quad \text{which is linear}$$

EXERCISE - 3

Part # I : Matrix Match Type

3. (A) $xy = ydx + y^2 dy$

$$\Rightarrow \frac{x dy - y dx}{y^2} = dy \Rightarrow -d\left(\frac{x}{y}\right) = dy$$

$$-\frac{x}{y} = y + c \quad \text{put } x=1 \quad y=1 \Rightarrow c=-2$$

$$-\frac{x}{y} = y - 2 \Rightarrow \frac{x}{3} = -5$$

(B) $\frac{dy}{dt} - \frac{t}{t+1} y = \frac{1}{t+1}$

$$\text{I.F} = e^{-\int \frac{t+1-1}{t+1} dt} = e^{-t \ln(t+1)} = (t+1)e^{-t}$$

$$\text{solution is } (t+1)e^{-t} y = -e^{-t} + c$$

put $t=0$

$$\text{and } y=-1 \Rightarrow c=0$$

$$\therefore 2e^{-1} y = -e^{-1}$$

$$y = -\frac{1}{2}$$

(C) $(x^2 + y^2) dy = xy dx$

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

put $y=vx \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\Rightarrow \bullet n v - \frac{1}{2v^2} = -\bullet n x + c$$

$$\therefore c = -\frac{1}{2}$$

$$\therefore \bullet n \frac{y}{x} - \frac{1}{2} \frac{x^2}{y^2} = -\bullet n x - \frac{1}{2} \quad \text{put } y=e$$

$$\therefore x = \sqrt{3} e$$

(D) $\frac{dy}{dx} + 2\frac{y}{x} = 0$

$$x^2 y = C \quad \text{put } x=1, y=1 \quad \text{and we get } C=1$$

$$\text{put } x=2$$

$$\Rightarrow y = \frac{1}{4}$$

Part # II : Comprehension

Comprehension # 1

For reservoir A

$$\frac{dV_A}{dt} \propto -V_A \Rightarrow \frac{dV_A}{dt} = -k_1 V_A$$

$$\Rightarrow \int_{V_{A_0}}^{V_A} \frac{dV_A}{V_A} = -k_1 \int_0^t dt \Rightarrow \log \frac{V_A}{V_{A_0}} = -k_1 t$$

$$\Rightarrow V_A = V_{A_0} e^{-k_1 t}$$

$$\text{Similarly } V_B = V_{B_0} e^{-k_2 t}$$

$$\text{so } \frac{V_A}{V_B} = \frac{V_{A_0}}{V_{B_0}} e^{-(k_1 - k_2)t}$$

$$\text{At } t=0, \quad V_{A_0} = 2V_{B_0}$$

$$\text{and at } t=1, V_A = 1.5 V_B$$

$$\text{so } \frac{3}{2} = 2e^{-(k_1 - k_2)} \therefore e^{-(k_1 - k_2)} = \frac{3}{4}$$

1. At $t = \frac{1}{2}, \quad V_A = kV_B$

$$\text{so } k = 2\left(\frac{3}{4}\right)^{1/2} \Rightarrow k = \sqrt{3}$$

2. Let at $t = t_0$ both the reservoirs have same quantity of water, then

$$V_A = V_B \Rightarrow 2e^{-(k_1 - k_2)t_0} = 1$$

$$\Rightarrow \left(\frac{3}{4}\right)^{t_0} = \frac{1}{2} \therefore t_0 = \log_{3/4} \left(\frac{1}{2}\right)$$

$$\Rightarrow t_0 = \log_{4/3} 2$$

$$\text{and also } t_0 = \frac{1}{\log_2 \frac{4}{3}} = \frac{1}{2 - \log_2 3}$$

3. Now $\frac{V_A}{V_B} = 2 e^{-(k_1 - k_2)t} \Rightarrow f(t) = 2 e^{-(k_1 - k_2)t}$

$$f(t) = -2(k_1 - k_2) e^{-(k_1 - k_2)t} = 2 \ln \frac{3}{4} e^{-(k_1 - k_2)t}$$

$$\Rightarrow f(t) \text{ is decreasing.}$$

Comprehension # 2

1-3

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2} \right) y = \frac{4x^2}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\ln(1+x^2)} = (1+x^2)$$

$$\therefore y(1+x^2) = \int 4x^2 dx = \frac{4x^3}{3} + C$$

$$\text{passing through } (0, 0) \Rightarrow C = 0$$

$$\therefore y = \frac{4x^3}{3(1+x^2)}$$

$$\frac{dy}{dx} = \frac{4}{3} \left[\frac{(1+x^2)3x^2 - x^3 \cdot 2x}{(1+x^2)^2} \right] = \frac{4}{3} \left[\frac{3x^2 + x^4}{(1+x^2)^2} \right]$$

$$= \frac{4x^2(3+x^2)}{3(1+x^2)^2}$$

$$\text{hence } \frac{dy}{dx} > 0 \quad \forall x \neq 0;$$

$$\frac{dy}{dx} = 0 \text{ at } x = 0 \text{ and it does not change sign } \Rightarrow x = 0 \text{ is}$$

the point of inflection **Ans.**

$y = f(x)$ is increasing for all $x \in \mathbb{R}$

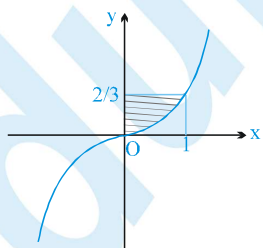
$$x \rightarrow \infty; y \rightarrow \infty; \quad x \rightarrow -\infty; y \rightarrow -\infty$$

Area enclosed by $y = f^{-1}(x)$, x -axis and ordinate at $x = \frac{2}{3}$

$$A = \frac{2}{3} - \frac{4}{3} \int_0^1 \frac{x^3}{1+x^2} dx$$

$$\text{put } 1+x^2 = t \Rightarrow 2x dx = dt$$

$$A = \frac{2}{3} - \frac{2}{3} \int_1^2 \frac{(t-1)}{t} dt = \frac{2}{3} - \frac{2}{3} \int_1^2 \left(1 - \frac{1}{t} \right) dt$$



$$= \frac{2}{3} - \frac{2}{3} [t - \ln t]_1^2 = \frac{2}{3} - \frac{2}{3} [(2 - \ln 2) - 1]$$

$$= \frac{2}{3} - \frac{2}{3} [1 - \ln 2] = \frac{2}{3} \ln 2$$

Comprehension # 3

$$1. \quad 2x^3 dx + 2y^3 dy - (xy^2 dx + x^2 y dy) = 0$$

$$d \left(\frac{x^4}{2} \right) + d \left(\frac{y^4}{2} \right) - \frac{1}{2} d(x^2 y^2) = 0$$

$$\Rightarrow d(x^4 + y^4 - x^2 y^2) = 0 \Rightarrow x^4 + y^4 - x^2 y^2 = c$$

$$2. \quad \frac{xdy - ydx}{x^2 + y^2} + dx = 0 \Rightarrow \frac{\frac{xdy - ydx}{x^2}}{1 + \left(\frac{y}{x} \right)^2} + dx = 0$$

$$\Rightarrow \frac{d \left(\frac{y}{x} \right)}{1 + \left(\frac{y}{x} \right)^2} + dx = 0 \Rightarrow d \left(\tan^{-1} \left(\frac{y}{x} \right) \right) + dx = 0$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) + x = c$$

$$3. \quad e^y dx + x e^y dy - 2y dy = 0$$

$$d(xe^y) - d(y^2) = 0$$

$$\text{Solution is } xe^y - y^2 = c$$

Comprehension # 4

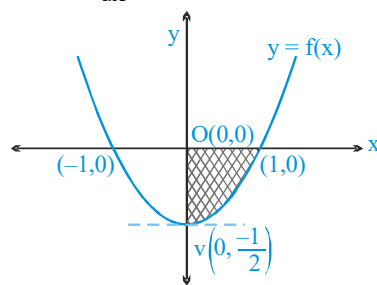
1-3

$$\text{Equation of normal (N): } Y - y = \frac{-1}{m} (X - x)$$

$$\text{Put } X = 0 \Rightarrow Y = y + \frac{x}{m}$$

$$\text{hence } y\text{-intercept of the normal is } y + \frac{x}{\frac{dy}{dx}}$$

$$\text{So, } y + \frac{x}{\frac{dy}{dx}} = \sqrt{x^2 + y^2} \Rightarrow \int \frac{y dy + x dx}{\sqrt{x^2 + y^2}} = \int dy$$



$$\Rightarrow \frac{1}{2} \int \frac{d(x^2 + y^2)}{\sqrt{x^2 + y^2}} = \int dy \Rightarrow \sqrt{x^2 + y^2} = y + k$$

As $M(-\sqrt{3}, 1)$ satisfy it, so $k = 1$

$$\Rightarrow x^2 + y^2 = y^2 + 2y + 1 \Rightarrow x^2 = 2\left(y + \frac{1}{2}\right)$$

1. From graph $\left. \frac{dy}{dx} \right|_{(1,0)} = 1 \Rightarrow \tan \theta = 1 \Rightarrow \theta = \tan^{-1}(1)$ Ans.

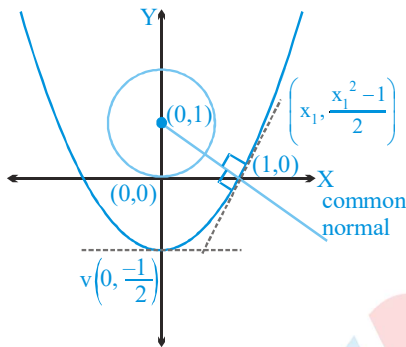
2. Clearly, required area

$$= \left| \int_0^1 \frac{x^2 - 1}{2} dx \right| = \left| \frac{1}{2} \left(\frac{x^3}{3} - x \right) \right|_0^1 = \left| \frac{1}{2} \left(\frac{1}{3} - 1 \right) \right| = \frac{1}{3}$$

(square units).

3. Clearly, $\frac{-1}{x_1} = \frac{\left(\frac{x_1^2 - 1}{2} \right) - 1}{x_1 - 0}$

$$\Rightarrow x_1^3 - x_1 = 0 \Rightarrow x_1(x_1^2 - 1) = 0$$



$$\therefore x_1 = -1, 0, 1$$

Now, distance of $(1, 0)$ from $(0, 1) = \sqrt{2}$

So, shortest distance between the curve C and the circle $= \sqrt{2} - 1$ (units).

Comprehension # 5

1. We have

$$f(x) = 2x^3 + 3\left(1 - \frac{3a}{2}\right)x^2 + 3(a^2 - a)x + b$$

So,

$$f'(x) = 6x^2 + 6\left(1 - \frac{3a}{2}\right)x + 3(a^2 - a)$$

Now, for $f(x)$ to have negative point of local minimum, we must have both roots of the equation $f'(x) = 0$ real (unequal) and negative.

$$\therefore \text{Discriminant} > 0 \Rightarrow 36 \left[\left(1 - \frac{3a}{2}\right)^2 - 2(a^2 - a) \right] > 0$$

$$\Rightarrow \left(\frac{a}{2} - 1 \right)^2 > 0$$

$$\Rightarrow a \in \mathbb{R} - \{2\} \quad \dots\dots(1)$$

Also, sum of roots < 0

$$\Rightarrow \frac{-6\left(1 - \frac{3a}{2}\right)}{6} < 0 \Rightarrow \frac{3a}{2} - 1 < 0 \Rightarrow a < \frac{2}{3} \quad \dots\dots(2)$$

And product of roots > 0

$$\Rightarrow \frac{3(a^2 - a)}{6} > 0$$

$$\Rightarrow a(a - 1) > 0 \Rightarrow a \in (-\infty, 0) \cup (1, \infty) \quad \dots\dots(3)$$

$\therefore$ From $(1) \cap (2) \cap (3)$, we get $a \in (-\infty, 0)$

2. We have

$$g(x) = \frac{f'(x)}{6} = x^2 + \left(1 - \frac{3a}{2}\right)x + \left(\frac{a^2 - a}{2}\right)$$

$$\text{As discriminant of } g(x) = \left(\frac{a}{2} - 1\right)^2 > 0 \quad \forall a \in \mathbb{R} - \{2\}$$

$\Rightarrow$ Ordinate of the vertex of parabola $y = g(x)$ lies below x-axis.

So, $a \in \mathbb{R} - \{2\}$

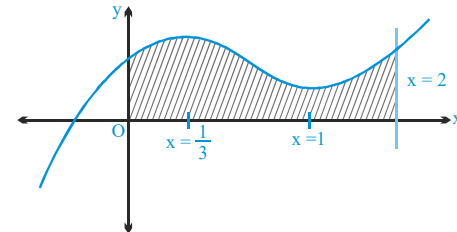
3. We have

$$h''(x) = 6x - 4 \Rightarrow h'(x) = 3x^2 - 4x + c$$

$$\text{As } h'(1) = 0, \text{ so } 0 = 3 - 4 + c \Rightarrow c = 1$$

$$\therefore h'(x) = 3x^2 - 4x + 1$$

$$\Rightarrow h(x) = x^3 - 2x^2 + x + k$$



$$\text{As } h(1) = 5, \text{ so } 5 = 1 - 2 + 1 + k \Rightarrow k = 5.$$

$$\text{Hence, } h(x) = x^3 - 2x^2 + x + 5$$

Clearly, required area

$$= \int_{1/3}^2 h(x) dx = \int_{1/3}^2 (x^3 - 2x^2 + x + 5) dx$$

$$= \left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + 5x \right)_{1/3}^2 = \frac{32}{3} \text{ (square units).}$$

EXERCISE - 4

Subjective Type

1. (i) (2, 2) (ii) (3, 2) (iii) 1, 1
 (iv) 1, 2
 (v) 3, degree is not applicable (vi) 3, 2
 (vii) 2, degree is not applicable

2.

(II) $x^2 + y^2 + 2gx + 2fy + c = 0$

Differentiation, we get,

$$\Rightarrow 2x + 2yy' + 2g + 2fy' = 0$$

Again differentiating,

$$\Rightarrow 1 + (y')^2 + yy'' + fy'' = 0 \quad \dots\dots(i)$$

Again differentiating,

$$\Rightarrow 2(y')y'' + y'y'' + yy''' + fy''' = 0$$

$$\Rightarrow 3y'y'' + y''' \left[y - \frac{1}{y''} - \frac{(y')^2}{y''} - y \right] = 0 \text{ (from (i))}$$

$$\Rightarrow 3y'(y'')^2 - y'''[1 + (y')^2] = 0$$

(III) $(y' - 1)(x^2 + y^2 - 1) + 2(x + yy')(x - y) = 0$

(IV) $y = c_1 e^{3x} + c_2 e^{2x} + c_3 e^x \quad \dots\dots(1)$

$$\frac{dy}{dx} = 3c_1 e^{3x} + 2c_2 e^{2x} + c_3 e^x \quad \dots\dots(2)$$

$$\frac{d^2y}{dx^2} = 9c_1 e^{3x} + 4c_2 e^{2x} + c_3 e^x \quad \dots\dots(3)$$

$$\frac{d^3y}{dx^3} = 27c_1 e^{3x} + 8c_2 e^{2x} + c_3 e^x \quad \dots\dots(4)$$

Apply (4) - 6 × (3) + 11 × (2) - 6 × (1)

$$\Rightarrow \frac{d^3y}{dx^3} - 6 \frac{d^2y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0$$

3. (i) $\frac{dT}{dt} = -k(T - S)$

$$\Rightarrow \bullet n(T - S) = -kt + C$$

$$\bullet n(S_0 - S) = C$$

$$\Rightarrow \ln \left(\frac{T - S}{S_0 - S} \right) = -kt$$

(ii) $\frac{d}{dt} (4\pi r^2) = kt$

$$\int d(4\pi r^2) = \int kt dt \Rightarrow 4\pi r^2 = \frac{kt^2}{2} + c$$

$$t = 0; r = 3 \Rightarrow c = 36\pi$$

$$t = 2; r = 5 \Rightarrow k = 32\pi$$

$$\therefore r^2 = 4t^2 + 9$$

$$\Rightarrow r = \sqrt{4t^2 + 9}$$

(iii) $\frac{dy}{dx} = \frac{\lambda y}{x}$

$$\bullet ny = \lambda \bullet nx + C$$

$$\therefore y = kx^\lambda$$

4. (A) $\frac{du}{dx} + Pu = Q; \frac{dv}{dx} + Pv = Q$

$$\Rightarrow \frac{d}{dx} (u - v) = -P(u - v)$$

$$\frac{d(u - v)}{u - v} = -P dx$$

$$\Rightarrow \bullet n(u - v) = - \int P dx$$

$$\frac{dy}{dx} + Py = Qx$$

$$\text{I.F.} = \frac{1}{u - v}$$

$$y \cdot \frac{1}{u - v} = \int \frac{Q}{u - v} + c$$

$$\Rightarrow \frac{u}{u - v} = \int \frac{Q}{u - v} + c' \quad [u \text{ satisfies it}]$$

$$\therefore \frac{y}{u - v} = \frac{u}{u - v} + k$$

$$\Rightarrow y = u + k(u - v) \dots (1)$$

(B) If $y = \alpha u + \beta v$ is a particular solution then compare with (1)

$$\alpha = k + 1, \beta = -k$$

$$\Rightarrow \alpha + \beta = 1$$

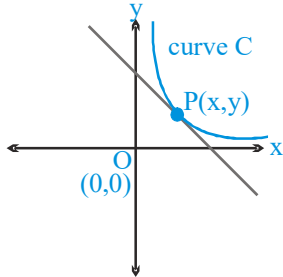
(C) If ω is a particular solution then it satisfies (1)

$$\Rightarrow \omega = u + k(u - v)$$

$$\frac{u - v}{\omega - u} = \text{constant}$$

5. Let $P(x, y)$ be any point on the curve C .

Now, $\frac{dy}{dx} = \frac{1}{y}$



$$\Rightarrow ydy = dx$$

$$\Rightarrow \frac{y^2}{2} = x + k$$

Since the curve passes through $M(2,2)$, so $k = 0$

$$\Rightarrow y^2 = 2x$$

Hence required area

$$= 2 \int_0^2 \sqrt{2x} dx = 2\sqrt{2} \times \frac{2}{3} (x^{3/2})_0^2 = \frac{4}{3} \sqrt{2} \times 2\sqrt{2} = \frac{16}{3}$$

(square unit)

$$\Rightarrow p + q = 19$$

6. $y - x = m(X - x)$

$$X_{int} = x - \frac{y}{m}$$

$$\frac{x - \frac{y}{m} + x}{2} = 0$$

$$2x = \frac{y}{m}$$

$$\frac{dy}{dx} = \frac{y}{2x}$$

$$\bullet n y = \frac{1}{2} \bullet n x + c \Rightarrow \frac{y^2}{x} = k$$

$$\Rightarrow y^2 = kx$$

7.

$$(V) [1 + y(1 + x^2)] dx = -x(1 + x^2) dy$$

$$\frac{dy}{dx} + \frac{y}{x} = -\frac{1}{x(1 + x^2)}$$

$$\text{I.F. } e^{\int 1/x} = x$$

$$\therefore yx = -\int \frac{1}{1 + x^2} dx$$

$$xy = -\tan^{-1} x + c$$

$$(VI) y - xy' = b + bx^2 y'$$

$$\Rightarrow y - b = (x + bx^2) \frac{dy}{dx}$$

$$\Rightarrow \frac{dx}{x(1 + bx)} = \frac{dy}{y - b}$$

$$\Rightarrow \int \left(\frac{1}{x} - \frac{b}{1 + bx} \right) dx = \log(y - b) + \log c$$

$$\Rightarrow \log x - \log(1 + bx) = \log(y - b) + \log c$$

$$\Rightarrow \frac{x}{c} = (1 + bx)(y - b)$$

$$\Rightarrow b + \left(\frac{1}{c} + b^2 \right) x = y(1 + bx)$$

$$\Rightarrow b + kx = y(1 + bx)$$

$$(X) \frac{dy}{dx} - \frac{y}{x} = -\frac{y^4 \cos x}{x^3}$$

$$\frac{1}{y^4} \frac{dy}{dx} - \frac{1}{y^3 x} = -\frac{\cos x}{x^3}$$

$$\text{Put } -\frac{1}{y^3} = t \Rightarrow \frac{3}{y^4} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + \frac{3t}{x} = \frac{-3 \cos x}{x^3}$$

$$\text{I.F. } e^{\int 3/x} = x^3$$

$$tx^3 = -\int \frac{3 \cos x}{x^3} x^3 dx$$

$$\Rightarrow -\frac{1}{y^3} x^3 = -3 \sin x + c$$

$$\Rightarrow x^3 y^{-3} = 3 \sin x - c$$

$$(XI) \frac{dy}{dx} - y = \frac{2xy^2}{e^x}$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} = \frac{2x}{e^x}$$

$$\text{Put } -\frac{1}{y} = t$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + t = \frac{2x}{e^x}$$

$$\text{I.F. is } e^{\int dx} = e^x$$

$$te^x = \int \frac{2x}{e^x} e^x dx$$

$$\Rightarrow -\frac{1}{y} e^x = x^2 + c$$

$$\Rightarrow y^{-1} e^x = -x^2 - c$$

$$(XIII) x(x^2 + 1) \frac{dy}{dx} + (x^2 - 1)y = x^3 \bullet nx$$

$$\frac{dy}{dx} + \frac{(x^2 - 1)y}{x(x^2 + 1)} = \frac{x^2 \bullet nx}{x^2 + 1}$$

$$\text{I.F.} = e^{\int \frac{x^2 - 1}{x(x^2 + 1)} dx} = \frac{x^2 + 1}{x}$$

$$\left(\frac{x^2 + 1}{x} \right) y = \int \frac{x^2 + 1}{x} \times \frac{x^2}{x^2 + 1} \bullet nx dx$$

$$\Rightarrow \left(\frac{x^2 + 1}{x} \right) y = \int x \bullet nx dx$$

$$\Rightarrow \left(\frac{x^2 + 1}{x} \right) y = \bullet nx \cdot \frac{x^2}{2} - \frac{x^2}{4} + c$$

$$\Rightarrow 4(x^2 + 1)y + x^3(1 - 2\bullet nx) = cx$$

$$8. \frac{y^2 + y\sqrt{y^2 - x^2}}{x^2} = \ln \left| y + \sqrt{y^2 - x^2} \right| \frac{c^2}{x^3}, \text{ where same sign}$$

has to be taken.

$$9. e^{-2 \tan^{-1} \frac{y+2}{x-3}} = c \cdot (y+2)$$

$$10. \frac{|y - mx|}{\sqrt{1 + m^2}} = \frac{|my + x|}{\sqrt{1 + m^2}}$$

$$|y - mx| = |my + x|$$

$$\Rightarrow y - mx = \pm(my + x) \quad \dots(i)$$

by taking positive sign

$$y - mx = my + x$$

$$y - x = m(x + y)$$

$$\Rightarrow m = \frac{dy}{dx} = \frac{y - x}{y + x} \quad \text{let } y = tx \quad y = tx$$

$$t + x \frac{dt}{dx} = \frac{t - 1}{t + 1}$$

$$- \int \frac{t dt}{1 + t^2} - \int \frac{dt}{1 + t^2} = \bullet nx + c$$

$$- \frac{1}{2} \bullet n(1 + t^2) - \tan^{-1} t = \bullet nx + c$$

$$\bullet n \sqrt{x^2 + y^2} = -\tan^{-1} \frac{y}{x} + c$$

$$\Rightarrow \sqrt{x^2 + y^2} = c_1 e^{-\tan^{-1} y/x}$$

from equ. (i) by taking negative sign

$$\frac{dy}{dx} = \frac{\frac{y}{x} + 1}{\frac{-y}{x} + 1} \quad \text{put } y = tx$$

$$\Rightarrow t + x \frac{dt}{dx} = \frac{t + 1}{-t + 1}$$

$$\Rightarrow \frac{1 - t}{1 + t^2} dt = \frac{dx}{x}$$

$$\Rightarrow \tan^{-1} t - \frac{1}{2} \bullet n(1 + t^2) = \bullet nx + c$$

$$\bullet n \sqrt{x^2 + y^2} = +\tan^{-1} \frac{y}{x} + c$$

$$\Rightarrow \sqrt{x^2 + y^2} = c_2 e^{\tan^{-1} y/x}$$

then final solution $\sqrt{x^2 + y^2} = c e^{\pm \tan^{-1} y/x}$

11. (A) $c(x-y)^{2/3}(x^2+xy+y^2)^{1/6} = \exp\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x+2y}{x\sqrt{3}}\right]$

where $\exp x \equiv e^x$

(B) $y^2 - x^2 = c(y^2 + x^2)^2$

12. $x^2 + y^2 - 2x = 0, x = 1$

13. (i) $y' + P(x)y = Q(x)$

y_1 & y_2 are two solution of above equation so

$y_1' + P(x)y_1 = Q(x) \quad \dots(1)$

$y_2' + P(x)y_2 = Q(x) \quad \dots(2)$

Now if $y = y_1 + C(y_2 - y_1)$ is a solution of diff. equation then

$\frac{d}{dx}(y_1 + c(y_2 - y_1)) + P(x)(y_1 + c(y_2 - y_1)) = Q(x)$

$\Rightarrow y_1' + c(y_2' - y_1') + P(x)(y_1 + c(y_2 - y_1)) = Q(x) \quad \dots(3)$

from equation (1) & (2)

$(y_2' - y_1') = P(x)(y_1 - y_2)$

Now from equation (3)

$\Rightarrow y_1' + P(x)c(y_1 - y_2) + P(x)y_1 + P(x)C(y_2 - y_1) = Q(x)$

$\Rightarrow y_1' + P(x)y_1 = Q(x)$

which is true by equation (1)

so

$y = y_1 + c(y_2 - y_1)$ is a general solution of given diff. equation

(ii) multiply equation (1) by α and equation (2) by β then add

$(\alpha y_1' + \beta y_2') + P(x)(\alpha y_1 + \beta y_2) = Q(x)(\alpha + \beta)$

let $y = \alpha y_1 + \beta y_2$

$y' + P(x)y = Q(x)(\alpha + \beta)$

for y to be solution of diff. equation

$\alpha + \beta = 1$

14. $f(x) > 0 \quad \forall x \geq 2$

$\frac{d}{dx}(x f(x)) \leq -k f(x)$

$x \frac{dy}{dx} + y \leq -ky \quad [f(x) = y]$

$x \frac{dy}{dx} \leq -y(k+1)$

$\frac{dy}{dx} + \frac{(k+1)}{x} y \leq 0$

I.F. = $e^{\int \frac{k+1}{x} dx} = e^{\ln x^{k+1}} = x^{k+1}$

$x^{k+1} \cdot \frac{dy}{dx} + (k+1) \cdot x^k y \leq 0$

$\frac{d}{dx}(y \cdot x^{k+1}) \leq 0$

$\Rightarrow g(x) = y \cdot x^{k+1}$ decreases $\forall x \geq 2$

$\therefore g(x) \leq g(2) \cdot 2^{k+1}$

$f(x) \cdot x^{k+1} \leq f(2) \cdot 2^{k+1}$

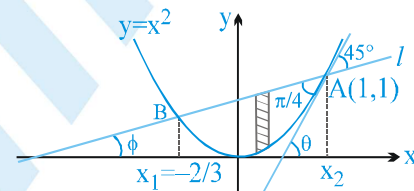
$f(x) \leq A \cdot x^{-k-1}$

15. $y = x^2$

$\left[\frac{dy}{dx}\right]_{A(1,1)} = 2x = 2; \quad \text{hence, } \tan \theta = 2 \quad \dots(1)$

Let the line l makes an angle ϕ with the x -axis

then, $\phi = \theta - \frac{\pi}{4}; \quad m_l = \tan \phi = \frac{\tan \theta - 1}{1 + \tan \theta} = \frac{1}{3}$



$\therefore$ equation of the line l is,

$y - 1 = \frac{1}{3}(x - 1)$

$\Rightarrow 3y - 3 = x - 1 \quad \Rightarrow \quad 3y = x + 2$

solving line and parabola $y = x^2$,

$3x^2 = x + 2 \quad \Rightarrow \quad 3x^2 - x - 2 = 0$

$x_1 x_2 = -\frac{2}{3}; \quad \text{but } x_2 = 1;$

$\therefore x_1 = -\frac{2}{3}$

$\therefore A = \int_{-2/3}^1 \left(\frac{x+2}{3} - x^2 \right) dx$

$= \frac{1}{3} \int_{-2/3}^1 (x + 2 - 3x^2) dx = \frac{1}{3} \left[\frac{x^2}{2} + 2x - x^3 \right]_{-2/3}^1$

$= \frac{1}{3} \left[\left(\frac{1}{2} + 2 - 1 \right) - \left(\frac{2}{9} - \frac{4}{3} + \frac{8}{27} \right) \right]$

$$= \frac{1}{3} \left[\frac{3}{2} - \left(\frac{6-36+8}{27} \right) \right] = \frac{1}{3} \left[\frac{3}{2} + \frac{22}{27} \right]$$

$$= \frac{1}{3} \left[\frac{81+44}{54} \right] = \frac{125}{162} = \frac{a_1}{a_2}$$

$$\Rightarrow a_1 + a_2 = 287$$

16. (A) $x^2 + 2y^2 = c$, (B) $\sin y = ce^{-x}$

17. $\frac{x^2 - y^2}{x^2 + y^2}$ let the line from origin be $y = mx$

$$\frac{dy}{dx} = \frac{1-m^2}{1+m^2} \text{ which is constant and independent of } x, y$$

$$\text{Hence } f'(x_1) = g'(x_1)$$

18. $27 \frac{7}{9}$ minutes

19.

(I). $y^2 + x \bullet \ln x = 0$

(II). $y^2 = 3x^2 - 6x - x^3 + ce^{-x} + 4$

(III). $x \bullet ny = e^x(x-1) + c$

(IV). $\sin y = (e^x + c)(1+x)$

(V). $cx^2 + 2xe^{-y} = 1$

(VI) $\left(\frac{dy}{dx} - y \right) \left(\frac{dy}{dx} - x \right) = 0$

$$\frac{dy}{dx} = y \quad \text{or} \quad \frac{dy}{dx} = x$$

$$\Rightarrow \ln y = x + c \quad \text{or} \quad y = \frac{x^2}{2} + c$$

$$\Rightarrow y = ke^x$$

(VII). $y^2 = -1 + (x+1) \ln \frac{c}{x+1}$ or $x + (x+1) \ln \frac{c}{x+1}$

(VIII) $xy = t \Rightarrow x \frac{dy}{dx} + y = \frac{dt}{dx}$

$$\Rightarrow x \frac{dy}{dx} = \frac{dt}{dx} - \frac{t}{x}$$

$$\Rightarrow (1 - t + t^2) = x \left(\frac{dt}{dx} - \frac{t}{x} \right)$$

$$\Rightarrow \frac{(1 - t + t^2)}{x} + \frac{t}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1+t^2}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{1+t^2} = \frac{dx}{x}$$

$$\Rightarrow \tan^{-1} t = \bullet \ln |x| + \bullet \ln c$$

$$\Rightarrow \tan^{-1}(xy) = \bullet \ln |cx|$$

$$xy = \tan \bullet \ln |xc|$$

(IX) $e^y = c \cdot \exp(-e^x) + e^x - 1$

(X) $y^2 = \frac{2}{3} \sin x + \frac{c}{\sin^2 x}$

22. $y^2 = \frac{x^4 + c^4}{2x^2}$ or $y^2 + 2x^2 \bullet \ln x = cx^2$

23. $\frac{y^2 \text{ int}}{\text{LSN}} = \frac{xy}{m^2 \text{ LST}}$

$$\frac{(y-mx)^2}{|my|} = \frac{xy}{m^2 \left(\frac{y}{m} \right)}$$

$$m \neq 0 \quad y \neq 0$$

$$[\text{as } y=0 \quad \text{or } y=c \text{ is not a solution}]$$

$$(y-mx)^2 = xy$$

$$y - mx = \pm \sqrt{xy}$$

$$y \pm xy = mx \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y \pm \sqrt{xy}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \pm \sqrt{\frac{y}{x}}$$

put $\frac{y}{x} = t^2$

$$\Rightarrow \frac{dy}{dx} = t^2 + 2xt \frac{dt}{dx}$$

now diff. eq.

$$t^2 + 2xt \frac{dt}{dx} = t^2 \pm \sqrt{t}$$

$$\text{after solving } x = e^{2\sqrt{y/x}}; x = e^{-2\sqrt{y/x}}$$

24. $f(x) = e^{2x}$

25. We have $f'(x) = \ln x + 2 - \frac{f(x)}{x \ln x}$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x \ln x} = \ln x + 2 \quad \dots\dots(1)$$

Integrating factor = $e^{\int \frac{1}{x \ln x} dx} = \ln x$

$\therefore$ Solution of the differential equation (1) is

$$y \ln x = \int \ln x (\ln x + 2) dx + C$$

$$\Rightarrow y \ln x = \int (\ln^2 x + 2 \ln x) dx + C$$

$$\left(\int (f(x) + x f'(x)) dx = x f(x) + C \right)$$

$$\Rightarrow y \ln x = x \ln^2 x + C$$

As, $x = 1$

$$\Rightarrow C = 0$$

Hence, $y = x \ln x$

$$f(x) = x \ln x \Rightarrow f'(x) = x \left(\frac{1}{x} \right) + \ln x$$

$$f(e) = e, f'(e) = 1 + 1 = 2$$

$$f(e) - f'(e) = e - 2$$

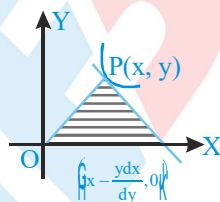
$$\Rightarrow [f(e) - f'(e)] = 0$$

26. Equation of tangent

$$Y - y = \frac{dy}{dx} (X - x)$$

when $Y = 0, X = x - y \frac{dx}{dy}$

$$\left| \frac{1}{2} \left(x - y \frac{dx}{dy} \right) y \right| = a^2$$



$$xy - y^2 \frac{dx}{dy} = \pm 2a^2$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = m \frac{2a^2}{y^2}$$

$$\Rightarrow \frac{x}{y} = m \int \frac{2a^2}{y^2} \frac{1}{y} dy$$

$$\Rightarrow \frac{x}{y} = \pm \frac{a^2}{y^2} + c \Rightarrow x = cy \pm \frac{a^2}{y}$$

27. $x^2 + y^2 - 2x = 0$

29. Equation of tangent $Y - y = \frac{dy}{dx} (X - x)$

$$\text{distance from origin} = \frac{\left| -x \frac{dy}{dx} + y \right|}{\sqrt{1 + \left(\frac{dy}{dx} \right)^2}}$$

Equation of normal $Y - y = \frac{-1}{\frac{dy}{dx}} (X - x)$

$$\Rightarrow Y \frac{dy}{dx} - y \frac{dy}{dx} = -X + x$$

$$\text{distance from origin} = \frac{\left| x + y \frac{dy}{dx} \right|}{\sqrt{1 + \left(\frac{dy}{dx} \right)^2}}$$

Now, $\left| -x \frac{dy}{dx} + y \right| = \left| x + y \frac{dy}{dx} \right|$

either $-x \frac{dy}{dx} + y = x + y \frac{dy}{dx}$

or $x \frac{dy}{dx} - y = x + y \frac{dy}{dx}$

$$\Rightarrow (x + y) \frac{dy}{dx} = y - x$$

30. $\frac{dy_1}{dx} + Py_1 = Q, \frac{dy_2}{dx} + Py_2 = Q$

Put $y_2 = y_1 z$

$$\Rightarrow \frac{dy_2}{dx} = y_1 \frac{dz}{dx} + z \frac{dy_1}{dx}$$

$$\Rightarrow y_1 \frac{dz}{dx} + z \frac{dy_1}{dx} = Q - Py_2$$

$$\Rightarrow y_1 \frac{dz}{dx} + z \frac{dy_1}{dx} + Py_1 z = Q$$

$$\Rightarrow y_1 \frac{dz}{dx} + z Q = Q \Rightarrow y_1 \frac{dz}{dx} = Q(1 - z)$$

$$\Rightarrow \int \frac{dz}{1-z} = \int \frac{Q}{y_1} dx$$

$$\Rightarrow \bullet n |z-1| = - \int \frac{Q}{y_1} dx + \lambda$$

$$\Rightarrow z = 1 + a e^{-\int \frac{Q}{y_1} dx}$$

31. $y = f(x) f(x) \geq 0 f(0) = 0$

$$\int_0^x f(x) dx = K f(x)^{n+1}$$

$$f(x) = (n+1) K f(x)^n f'(x)$$

$$y^{1-n} = (n+1) K \frac{dy}{dx}$$

$$\int dx = (n+1) K \int y^{n-1} dy$$

$$\Rightarrow x + c = (n+1) K \frac{y^n}{n}$$

$$x=0; y=0 \Rightarrow$$

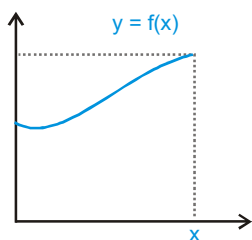
$$c=0$$

$$x=1, y=1 \Rightarrow$$

$$K = \frac{n}{n+1}$$

$$x = y^n \Rightarrow$$

$$y = (x)^{1/n}$$



32. $\frac{dm}{dt} = -\lambda m$

$$\frac{1}{m} dm = -\lambda dt$$

$$\bullet n m = -\lambda t + c$$

$$m = ke^{-\lambda t} \quad (\text{at } t=0, m=m_0)$$

$$\Rightarrow k = m_0$$

$$m = m_0 e^{-\lambda t} \quad (\text{at } t=t_0, m = m_0 - \frac{\alpha m_0}{100})$$

$$\Rightarrow \lambda = \frac{-1}{t_0} \ln \left(1 - \frac{\alpha}{100} \right)$$

$$\text{or } (x-y) \frac{dy}{dx} = x+y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-x}{y+x} \quad \text{or} \quad \frac{dy}{dx} = \frac{x+y}{x-y}$$

$$\text{Put } y = vx$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v-1}{v+1} \quad \text{or} \quad v + x \frac{dv}{dx} = \frac{1+v}{1-v}$$

$$\Rightarrow \int \frac{v+1}{1+v^2} dv = \int \frac{-dx}{x} \quad \text{or} \quad \int \frac{v-1}{1+v^2} dv = \int \frac{-dx}{x}$$

$$\Rightarrow \frac{1}{2} \bullet n |1+v^2| + \tan^{-1} v = - \bullet nx + \bullet nc$$

$$\text{or } \frac{1}{2} \bullet n |1+v^2| - \tan^{-1} v = \bullet nc - \bullet nx$$

Hence solution will be

$$\frac{1}{2} \bullet n |1+v^2| + \bullet nx = \pm \tan^{-1} v + \bullet nc$$

$$x \sqrt{1+v^2} = ke^{\pm \tan^{-1} v}$$

$$\Rightarrow \sqrt{x^2 + y^2} = ce^{\pm \tan^{-1} y/x}$$

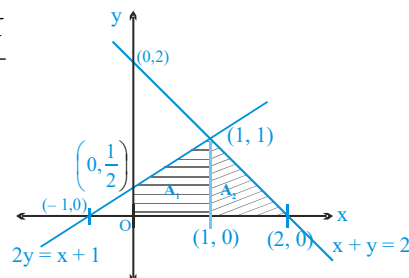
33. We have, $\left| x - y \frac{dx}{dy} \right| = 2 \left| \frac{dy}{dx} \right|$

$$\Rightarrow x - y \frac{dx}{dy} = \pm 2 \frac{dy}{dx}$$

Taking positive sign,

$$\Rightarrow 2 \left(\frac{dy}{dx} \right)^2 - x \left(\frac{dy}{dx} \right) + y = 0 \Rightarrow \frac{dy}{dx}$$

$$= \frac{x \pm \sqrt{x^2 - 8y}}{4}$$



But no solution exist, as it does not pass through (1, 1).

Now, taking negative sign,

$$\Rightarrow 2 \left(\frac{dy}{dx} \right)^2 + x \left(\frac{dy}{dx} \right) - y = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x \pm \sqrt{x^2 + 8y}}{4}$$

$$\text{Let } x^2 + 8y = v^2 \Rightarrow 2x + 8 \frac{dy}{dx} = 2v \frac{dv}{dx}$$

$$\text{So, } v \frac{dv}{dx} - x = -x \pm v \Rightarrow \frac{dv}{dx} = \pm 1$$

$$\Rightarrow v = \pm x + c \Rightarrow \sqrt{x^2 + 8y} = \pm x + C$$

So, curves are

$$\sqrt{x^2 + 8y} = x + 2 \quad \text{and} \quad \sqrt{x^2 + 8y} = -x + 4$$

∴ On squaring, we get $2y = x + 1$ and $x + y = 2$.

∴ Clearly, required area $= A_1 + A_2 = \frac{3}{4} + \frac{1}{2} = \frac{5}{4} = \frac{m}{n}$
(given)

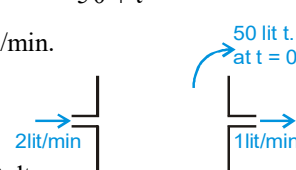
So, $(m+n)_{\text{Least}} = 5 + 4 = 9$

34. Input rate = 10 gm/min

After time t volume of tank $= 50 + (2 - 1)t$

Concentration of salt in time $t = \frac{m}{50 + t}$ gm/lit

Output rate $= \frac{m}{50 + t} \cdot 1$ gm/min.

$$\frac{dm}{dt} = \frac{-m}{50 + t} + 10$$


$$m(50 + t) = \int 10(50 + t) dt$$

$$\Rightarrow m(50 + t) = 500t + 5t^2 + c$$

$$\Rightarrow (\text{at } t = 0, m = 0 \Rightarrow c = 0)$$

$$\Rightarrow m(50 + t) = 5(100t + t^2)$$

$$\Rightarrow m = 5t \left(\frac{100 + t}{50 + t} \right) \text{ gm}$$

35. $\int f(x) dx = F(x) + c$

$$f(x) = F'(x)$$

$$\text{let } F(x) = y$$

$$\frac{dy}{dx} + \cos x \cdot y = \frac{\sin 2x}{(1 + \sin x)^2}$$

$$\text{I.F.} = e^{\sin x}$$

$$y \cdot e^{\sin x} = 2 \int \frac{e^{\sin x} \cdot \sin x \cos x}{(1 + \sin x)^2} dx + c$$

$$y \cdot e^{\sin x} = 2 \int \frac{e^t (t + 1 - 1)}{(1 + t)^2} dt \quad [\text{Put } \sin x = t]$$

$$= 2 \int e^t \left\{ \frac{1}{t + 1} + \frac{-1}{(1 + t)^2} \right\} dt + c$$

$$y \cdot e^{\sin x} = \frac{2e^t}{t + 1} + c \Rightarrow y \cdot e^{\sin x} = \frac{2e^{\sin x}}{\sin x + 1} + c$$

$$\Rightarrow y = \frac{2}{\sin x + 1} + c \cdot e^{-\sin x}$$

$$f(x) = \frac{dy}{dx} = \frac{-2 \cos x}{(\sin x + 1)^2} - c \cdot e^{-\sin x} \cos x$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2x \cdot vx} = \frac{v^2 - 1}{2v}$$

$$\text{or } \frac{xdv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x \frac{dv}{dx} = \frac{-v^2 - 1}{2v} \quad \text{or} \quad -\int \frac{2v dv}{1 + v^2} = \int \frac{dx}{x}$$

$$-\log(1 + v^2) = \log x + c$$

$$\log x + \log(1 + v^2) = \log c$$

$$\log x \cdot \left(1 + \frac{y^2}{x^2} \right) = \log c \quad \text{or} \quad x \left(\frac{x^2 + y^2}{x^2} \right) = c$$

$$\frac{x^2 + y^2}{x} = c \quad \text{or} \quad x^2 + y^2 = cx$$

2. $y = e^{cx}$

$$\log y = cx \quad \dots (i)$$

$$\frac{1}{y} y' = c \Rightarrow y' = cy$$

$$c = \frac{y'}{y} \text{ put in equation (i) } \log y = \frac{y'}{y} \cdot x$$

$$\text{or } y \log y = xy'$$

3. Given $\frac{dy}{dx} = \frac{y-1}{x(x-1)}$ or $\int \frac{dy}{y-1} = \int \frac{dx}{x(x+1)}$

$$\log(y-1) = \log \left(\frac{x}{x+1} \right) + \log C$$

$$\text{or } y-1 = \frac{cx}{x+1} \quad \dots (i)$$

Equation (i) passes through (1, 0)

$$-1 = \frac{C}{2} \Rightarrow C = -2 \text{ Put in (i)}$$

$$(y-1) = \frac{-2x}{x+1} \quad (y-1)(x+1) + 2x = 0$$

4. Equation of given parabola is $y^2 = Ax + B$ where A and B are parameters

$$2y \frac{dy}{dx} = A \quad y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

This is the equation of given parabola order = 2, degree 1

5. $(1 + y^2) = (e^{\tan^{-1}y} - x) \frac{dy}{dx}$ or $(1 + y^2) \frac{dx}{dy} + x = e^{\tan^{-1}y}$

$$\frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1}y}}{(1 + y^2)}$$

Now I.F. = $e^{\int \frac{dy}{1 + y^2}} = e^{\tan^{-1}y}$

$$\therefore \text{solution } x(e^{\tan^{-1}y}) = \int \frac{e^{\tan^{-1}y}}{1 + y^2} e^{\tan^{-1}y} dy + C$$

$$xe^{\tan^{-1}y} = \frac{e^{2 \tan^{-1}y}}{2} + C$$

or $2xe^{\tan^{-1}y} = e^{2 \tan^{-1}y} + K$

6. Given family of curves is

$$x^2 + y^2 - 2ay = 0 \quad \dots (1)$$

$$2x + 2yy' - 2ay' = 0 \quad \dots (2)$$

Now put the value of 2a from (1) to in (2)

$$2x + 2yy' - \frac{x^2 + y^2}{y} \cdot y' = 0$$

$$2xy + (y^2 - x^2)y' = 0 \quad \text{or} \quad (x^2 - y^2)y' = 2xy$$

7. $ydx + (x + x^2y)dy = 0$ $ydx + xdy = -x^2ydy$

$$\int \frac{d(xy)}{(xy)^2} = - \int \frac{dy}{y} \Rightarrow -\frac{1}{xy} = -\log y + c$$

$$\Rightarrow \frac{-1}{xy} + \log y = c$$

8. $y^2 = 2c(x + \sqrt{c}) \quad \dots (1)$

$$y^2 = 2cx + 2c\sqrt{c}$$

$$2y \frac{dy}{dx} = 2c$$

$$\Rightarrow yy_1 = C \text{ Put in equation (1)}$$

$$\Rightarrow y^2 = 2yy_1(x + \sqrt{yy_1})$$

$$y^2 = -2yy_1x = 2yy_1\sqrt{yy_1}$$

or $(y^2 - 2yy_1x)^2 = 4y^3y_1^3$

Degree = 3 order = 1

9. $\frac{dy}{dx} = \frac{y}{x} \left(\log \frac{y}{x} + 1 \right)$ which is homogeneous equ.

Put $y = vx$, $\frac{dy}{dx} = v + \frac{xdv}{dx}$

$$v + x \frac{dv}{dx} = \frac{vx}{x} \left(\log \frac{vx}{x} + 1 \right)$$

$$\frac{xdv}{dx} = v(\log v + 1) - v = v \log v + v - v$$

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x} \Rightarrow \log(\log v) = \log x + \log c$$

$$\Rightarrow \log \frac{y}{x} = cx$$

10. Given $Ax^2 + By^2 = 1$ Divide by B

$$\frac{A}{B}x^2 + y^2 = \frac{1}{B} \text{ Differentiate w.r.t } x$$

$$2x \frac{A}{B} + 2y \frac{dy}{dx} = 0 \quad \dots (i)$$

Again Differentiate w.r.t. x

$$2 \frac{A}{B} + 2 \left[y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] = 0 \quad \dots (ii)$$

Put $\frac{A}{B} = - \left[y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right]$ in equation (i)

$$-2x \left[y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] + 2y \frac{dy}{dx} = 0$$

or $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$

It have second order and first degree.

11. Let the centre of circle is (h, 0) and radius will be also h

$$\therefore \text{equation of circle } (x - h)^2 + (y - 0)^2 = h^2$$

$$\Rightarrow x^2 - 2hx + h^2 + y^2 = h^2$$

$$\Rightarrow x^2 - 2hx + y^2 = 0 \quad \dots (i)$$

Equation (i) passes through origin differentiating it w.r.t. x

$$2x - 2h + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow h = x + y \frac{dy}{dx} \text{ put in equation (i)}$$

$$x^2 - 2x \left(x + y \frac{dy}{dx} \right) + y^2 = 0 \Rightarrow y^2 = x^2 + 2xy \frac{dy}{dx}$$

12. $\frac{dy}{dx} = 1 + \frac{y}{x}$ put $y = vx$, $\frac{dy}{dx} = v + x \frac{dv}{dx}$

$$v + x \frac{dv}{dx} = 1 + \frac{vx}{x} \Rightarrow x \frac{dv}{dx} = 1$$

$$\int dv = \int \frac{dx}{x}$$

$$\Rightarrow v = \log x + c \text{ or } \frac{y}{x} = \log x + c \dots (i)$$

Given $y(1) = 1 \Rightarrow 1 = \log 1 + c \Rightarrow c = 1$ put (i)

$$y = x \log x + x$$

13. Equation of circle $(x-h)^2 + (y-2)^2 = 25 \dots (i)$

Differentiate w.r.t. x

$$2(x-h) + 2(y-2) \frac{dy}{dx} = 0$$

$$(x-h) = -(y-2) \frac{dy}{dx} \text{ put in (i)}$$

$$(y-2)^2 \left(\frac{dy}{dx} \right)^2 + (y-2)^2 = 25$$

$$\text{or } (y-2)^2 (y')^2 + (y-2)^2 = 25$$

14. $y = c_1 e^{c_2 x} \dots (1)$

$$y' = c_1 c_2 e^{c_2 x} \dots (2)$$

$$y'' = c_1 c_2^2 e^{c_2 x}$$

$$y'' = c_2 y' \dots (3)$$

Now $\frac{(2)}{(1)}$

$$\frac{y'}{y} = c_2$$

$$\Rightarrow \text{Put in (3)}$$

$$y'' = \frac{y'}{y} \cdot y' \Rightarrow y'' y = (y')^2$$

15. $\cos x \, dy = y(\sin x - y) dx$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin x - y^2}{\cos x} \Rightarrow \frac{dy}{dx} = y \tan x - y^2 \sec x$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{1}{y} \tan x - \sec x$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} \tan x = -\sec x$$

$$\Rightarrow -\frac{1}{y^2} \frac{dy}{dx} + \frac{1}{y} \tan x = \sec x \dots (1)$$

Put $\frac{1}{y} = t$ in equation (1)

$$\Rightarrow -\frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx} \dots (2)$$

From equation (1) & (2), we get,

$$\Rightarrow \frac{dt}{dx} + t \cdot \tan x = \sec x$$

$$\therefore \text{I.F.} = e^{\int \tan x \, dx}$$

$$= e^{\log |\sec x|} = \sec x$$

$\therefore$ Solution of differential equation is :

$$t \cdot \sec x = \int \sec x \cdot \sec x \cdot dx + c$$

$$\frac{1}{y} \sec x = \tan x + c$$

$$\sec x = y (\tan x + c)$$

16. $\frac{dy}{dx} = y + 3 > 0 \quad y(0) = 2, y(\log 2) = ?$

$$\int \frac{dy}{y+3} = \int dx$$

$$\log |y+3| = x + c$$

$$y(0) = 2$$

$$\log |2+3| = 0 + c \Rightarrow c = \log 5.$$

$$y(\log 2) = ?$$

$$\log |y+3| = \log 2 + \log 5$$

$$\log |y+3| = \log 10$$

$$y+3 = 10$$

$$y = 7$$

17. $\frac{dV}{dt} = -k(T-t)$

$$\int dV = \int -K(T-t) dt$$

$$V = -K \left[Tt - \frac{t^2}{2} \right] + C$$

$$\text{At } t=0 \, V=I \Rightarrow C=I$$

$$V = -Kt \left(T - \frac{t}{2} \right) + I$$

$$V(T) = -KT \left(T - \frac{T}{2} \right) + I = \frac{-KT^2}{2} + I$$

18. Equation of tangent at (x_1, y_1) is

$$y - y_1 = \frac{dy_1}{dx_1} (x - x_1)$$

$$x\text{-intercept} = x_1 - y_1 \frac{dx_1}{dy_1}$$

According to question

$$x_1 = \frac{x_1 - y_1 \frac{dx_1}{dy_1}}{2}$$

$$\Rightarrow x_1 = -y_1 \frac{dx_1}{dy_1}$$

$$\int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$\Rightarrow \bullet ny = -\bullet nx + \bullet nc$$

$$\Rightarrow y = \frac{c}{x} \Rightarrow xy = c$$

Now at $x = 2, y = 3$

$$\Rightarrow c = 6$$

$$\therefore xy = 6 \Rightarrow y = \frac{6}{x}$$

$$19. y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$$

$$\Rightarrow y^2 \frac{dx}{dy} + x = \frac{1}{y} \Rightarrow \frac{dx}{dy} + \frac{x}{y^2} + \frac{1}{y^3}$$

$$\therefore \text{Integrating factor (I.F.)} = e^{\int \frac{1}{y^2} dy} = e^{-1/y}$$

$\therefore$ General solution is –

$$x \cdot e^{-1/y} = \int \frac{1}{y^3} e^{-1/y} dy + c$$

$$\text{Let } I_1 = \int \frac{1}{y^3} e^{-1/y} dy$$

$$\text{put } \frac{-1}{y} = t$$

$$y^2 dy = dt$$

$$\therefore I_1 = -\int t e^t dt$$

$$= -e^t (t - 1)$$

$$= e^t (1 - t)$$

$\therefore$ General solution is

$$x e^{-1/y} = e^{-1/y} \left(1 + \frac{1}{y}\right) + C$$

$$\Rightarrow x = 1 + \frac{1}{y} + C e^{1/y}$$

Put $x = 1, y = 1$

$$\therefore 1 = 1 + \frac{1}{1} + C e^{1/1}$$

$$\Rightarrow C = -1/e$$

$$\therefore x = 1 + \frac{1}{y} - \frac{e^{1/y}}{e}$$

$$20. \frac{dP(t)}{dt} = \frac{1}{2} P(t) - 450$$

integrate

$$\int \frac{dP}{P - 900} = \int \frac{1}{2} dt$$

$$\bullet \ln|(P - 900)| = \frac{1}{2} t + C \dots (1)$$

given $t = 0 \rightarrow P = 850$

$$\therefore C = \bullet \ln 50$$

from (1)

$$\ln|(P - 900)| = \frac{1}{2} t + \bullet \ln 50$$

$$\frac{1}{2} t = \ln \left| \frac{P - 900}{50} \right|$$

$$t = 2 \ln \left| \frac{P - 900}{50} \right|$$

at $P = 0$

$$t = 2 \bullet \ln \frac{900}{50}$$

$$t = 2 \bullet \ln 18$$

$$21. P = 100x - 12 x^{3/2} \cdot \frac{2}{3} + C$$

$$x = 0, P = 2000$$

$$C = 2000$$

$$P_{(x=25)} = 2500 - 1000 + 2000 = 3500$$

24. $y(1 + xy) dx = x dy$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + y^2 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = y^2$$

Bernoulli's DE

$$n = 2$$

$$LF = \int \frac{1}{x} \left(-\frac{1}{x} \right) dx = \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$\text{solution } y^{1-2} x = \int (1-2) \cdot x \cdot 1 \cdot dx$$

$$\Rightarrow \frac{x}{y} = -\frac{x^2}{2} + C$$

$$\text{Given } f(1) = -1$$

$$\Rightarrow \frac{1}{-1} = -\frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$$

$$\therefore \text{equation } \frac{x}{y} = -\frac{x^2}{2} - \frac{1}{2}$$

$$\text{when } x = -\frac{1}{2}, \text{ we have } -\frac{1}{2y} = -\frac{1}{4 \times 2} - \frac{1}{2}$$

$$\Rightarrow -\frac{1}{y} = -\frac{5}{4} \Rightarrow y = \frac{4}{5}$$

Part # II : IIT-JEE ADVANCED

1. Let X_0 be initial population of the country and Y_0 be its initial food production.

Let the average consumption be a units. Therefore, food required initially aX_0 . It is given

$$Y_0 = aX_0 \left(\frac{90}{100} \right) = 0.9 aX_0 \quad \dots (1)$$

Let X be the population of the country in year t .

Then $\frac{dX}{dt}$ = rate of change of population

$$= \frac{3}{100} X = 0.03 X$$

$$\therefore \frac{dX}{X} = 0.03 dt$$

$$\text{Integrating } \int \frac{dX}{X} = \int 0.03 dt$$

$$\Rightarrow \log X = 0.03t + c$$

$$\Rightarrow X = A \cdot e^{0.03t} \text{ where } A = e^c$$

$$\text{At } t = 0, X = X_0, \text{ thus } X_0 = A$$

$$\therefore X = X_0 e^{0.03t}$$

Let Y be the food production in year t .

$$\text{Then } Y = Y_0 \left(1 + \frac{4}{100} \right)^t = 0.9aX_0 (1.04)^t$$

$$(\rightarrow Y_0 = 0.9aX_0 \text{ from (1)})$$

Food consumption in the year t is $aX_0 e^{0.03t}$.

Again for no food deficit, $Y - X \geq 0$

$$\Rightarrow 0.9 X_0 a (1.04)^t > a X_0 e^{0.03t}$$

$$\Rightarrow \frac{(1.04)^t}{e^{0.03t}} > \frac{1}{0.9} = \frac{10}{9}$$

Taking log on both sides,

$$t[\bullet n(1.04) - 0.03] \geq \bullet n 10 - \bullet n 9$$

$$\Rightarrow t \geq \frac{1 \bullet n 10 - 1 \bullet n 9}{1 \bullet n(1.04) - 0.03}$$

Thus the least integral values of the year n , when the country becomes self sufficient, is the smallest integer

$$\text{greater than or equal to } \frac{1 \bullet n 10 - 1 \bullet n 9}{1 \bullet n(1.04) - 0.03}$$

$$3. \frac{dy}{dt} - \frac{t}{1+t} y = \frac{1}{1+t} \text{ I.F.} = e^{\int \frac{t}{1+t} dt} = e^{-t}(1+t)$$

$$\therefore \text{solution is } ye^{-t}(1+t) = \int e^{-t}(1+t) \frac{1}{1+t} dt + C$$

$$ye^{-t}(1+t) = -e^{-t} + c \text{ given } y(0) = -1 \Rightarrow c = 0$$

$$y(1+t) = -1 \text{ or } y = -\frac{1}{1+t}$$

$$\text{and } y(1) = -\frac{1}{1+1} = -\frac{1}{2}$$

5. Given : liquid evaporates at a rate proportional to its surface area.

$$\Rightarrow \frac{dv}{dt} \propto -S \quad \dots(1)$$

We know, volume of liquid = $\frac{1}{3} \pi r^2 h$

and surface area = πr^2 (of liquid in contact with air)

$$\text{or } V = \frac{1}{3} \pi r^2 h \text{ and } S = \pi r^2 \quad \dots(2)$$

$$\text{Also, } \tan \theta = \frac{R}{H} = \frac{r}{h} \quad \dots(3)$$

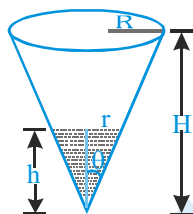
From (2) and (3),

$$V = \frac{1}{3} \pi r^3 \cot \theta \text{ and } S = \pi r^2 \quad \dots(4)$$

Substituting (4) in (1), we get

$$\frac{1}{3} \pi \cot \theta \cdot 3r^2 \cdot \frac{dr}{dt} = -K\pi r^2$$

$$\Rightarrow \cot \theta \int_R^0 dr = -K \int_0^T dt$$



where T is required time after which the cone is empty.

$$\Rightarrow \cot \theta (0 - R) = -K(T - 0)$$

$$\Rightarrow R \cot \theta = KT$$

$$\Rightarrow H = KT \quad (\text{using (3)})$$

$$\Rightarrow T = \frac{H}{K}$$

$$6. \frac{dy}{dx} = \frac{-\cos x(1+y)}{2+\sin x} \text{ or } \int \frac{dy}{1+y} = \int \frac{-\cos x dx}{2+\sin x}$$

$$\log(1+y) = -\log(2+\sin x) + c$$

$$\text{or } \log(1+y) + \log(2+\sin x) = c$$

Given $y(0) = 1$ means when $x = 0, y = 1$

$$\Rightarrow \log 2 + \log 2 = c \log c \Rightarrow c = 4$$

$$\Rightarrow 1+y = \frac{4}{2+\sin x} \text{ or } y = \frac{4}{2+\sin x} - 1$$

$$\text{or } y\left(\frac{\pi}{2}\right) = \frac{4}{3} - 1 = \frac{1}{3}$$

$$8. (A) \frac{dy}{dx} = \frac{xy}{x^2+y^2} \text{ This is homogeneous so put}$$

$$y = vx, \frac{dy}{dx} = v + \frac{dv}{dx} \cdot x$$

$$v + \frac{dv}{dx} x = \frac{vx \cdot x}{x^2 + (vx)^2} = \frac{v}{1+v^2}$$

$$\text{or } x \frac{dv}{dx} = \frac{v}{1+v^2} - \frac{v}{1} = \frac{-v^3}{1+v^2}$$

$$\int -\frac{(1+v^2)dv}{v^3} = \int \frac{dx}{x} \text{ or } -\int \frac{dv}{v^3} - \int \frac{dv}{v} = \int \frac{dx}{x}$$

$$\text{or } \frac{1}{2v^2} - \log v = \log x + c$$

$$\text{put } v = \frac{y}{x}$$

$$\Rightarrow \frac{x^2}{2y^2} = \log \frac{y}{x} + \log x + c = \log y + c \text{ given } y(1) = 1$$

$$\frac{1}{2} = c \Rightarrow c = 2 \Rightarrow \frac{x^2}{2y^2} = \log y + 2$$

Now put $x = x_0, y = e$

$$\frac{x_0^2}{2e^2} = 1 + \frac{1}{2} = \frac{3}{2} \text{ or } x_0^2 = \frac{6e^2}{2} = 3e^2$$

$$\Rightarrow x_0 = \sqrt{3} \cdot e$$

$$(B) \frac{xdy-ydx}{y^2} = dy \text{ or } \frac{ydx-xdy}{y^2} = -dy$$

$$\text{or } \int d\left(\frac{x}{y}\right) = -\int dy$$

$$\frac{x}{y} = -y + c \text{ Given } y(1) = 1$$

$$\Rightarrow 1 = -1 + c \Rightarrow c = 2$$

$$\frac{x}{y} = -y + 2 \text{ Now } y(-3), \frac{-3}{y} = -y + 2$$

$$\text{or } y^2 - 2y - 3 = 0$$

$$y^2 - 3y + y - 3 = 0 = (y-3)(y+1) = 0$$

$$y = 3$$

$$\text{or } y = -1 \text{ But } y > 0$$

$$\therefore y = 3$$

11. (A) $\lim_{t \rightarrow x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Applying L-hospital, we get

$$\Rightarrow x^2 f'(x) - 2x f(x) + 1 = 0$$

$$\Rightarrow \frac{x^2 f'(x) - 2x f(x)}{(x^2)^2} + \frac{1}{x^4} = 0$$

$$\Rightarrow \frac{d}{dx} \left(\frac{f(x)}{x^2} \right) = -\frac{1}{x^4} \Rightarrow f(x) = cx^2 + \frac{1}{3x}$$

Also $f(1) = 1 \Rightarrow c = \frac{2}{3}$

$$\Rightarrow f(x) = \frac{2x^2}{3} + \frac{1}{3x}$$

(B) $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$$\Rightarrow \int \frac{y}{\sqrt{1-y^2}} dy = \int dx$$

$$\Rightarrow -\sqrt{1-y^2} = x + c \Rightarrow (x+c)^2 + y^2 = 1$$

Centre $(-c, 0)$; radius = 1

12. $\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{dy}{y\sqrt{y^2-1}}$

$$\sec^{-1}x = \sec^{-1}y + c \rightarrow y(2) = \frac{2}{\sqrt{3}} \therefore c = \frac{\pi}{6}$$

$$\sec^{-1}x = \sec^{-1}y + \frac{\pi}{6} \Rightarrow y = \sec(\sec^{-1}x - \frac{\pi}{6})$$

Now $\cos^{-1} \frac{1}{x} = \cos^{-1} \frac{1}{y} + \frac{\pi}{6}$

$$\cos^{-1} \frac{1}{y} = \cos^{-1} \frac{1}{x} - \cos^{-1} \frac{\sqrt{3}}{2}$$

$$\frac{1}{y} = \frac{\sqrt{3}}{2x} - \sqrt{1 - \frac{1}{x^2}} \left(\frac{1}{2} \right)$$

$$\frac{2}{y} = \frac{\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$$

Hence S(I) is true and S(II) is false.

13. (A) $\frac{dy}{dx} = -\frac{y}{(x-3)^2}$

$$\Rightarrow \ln y = \frac{1}{x-3} + c \Rightarrow y = e^{\frac{1}{x-3}+c}, x \neq 3.$$

(B) $I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5) dx$

Applying $x \rightarrow 6-x$

$$I = \int_1^5 (5-x)(4-x)(3-x)(2-x)(1-x) dx = -I$$

$$\Rightarrow I = 0.$$

(C) $f(x) = \cos^2 x + \sin x$

$$f'(x) = -2\cos x \sin x + \cos x$$

$$\Rightarrow \cos x (-2 \sin x + 1) = 0$$

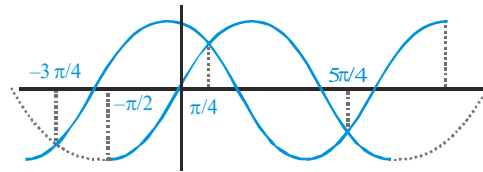
$$\cos x = 0 \text{ or } \sin x = \frac{1}{2}$$

sign of $f'(x)$ changes from -ve to +ve while $f(x)$ passes

$$\text{through } x = \frac{\pi}{6}, \frac{5\pi}{6}.$$

(D) $f(x) = \tan^{-1}(\sin x + \cos x)$

$$f(x) = \frac{\cos x - \sin x}{1 + (\sin x + \cos)^2} > 0$$



$$x \in (-3\pi/4, \pi/4)$$

14. Given $y = f(x)$

Tangent at point $P(x, y)$

$$Y - y = \left(\frac{dy}{dx} \right)_{(x,y)} (X - x)$$

Now y -intercept

$$\Rightarrow Y = y - x \frac{dy}{dx}$$

Given that, $y - x \frac{dy}{dx} = x^3$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -x^2$ is a linear differential equation

with I.F. = $e^{\int -\frac{1}{x} dx} = e^{-\ln x} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$

Hence, solution is $\frac{y}{x} = \int -x^2 \cdot \frac{1}{x} dx + C$

or $\frac{y}{x} = -\frac{x^2}{2} + C$

Given $f(1) = 1$

Substituting we get, $C = \frac{3}{2}$

so $y = -\frac{x^3}{2} + \frac{3}{2}x$

Now $f(-3) = \frac{27}{2} - \frac{9}{2} = 9$

15. (A) (Bonus)

(Comment : The given relation does not hold for $x=1$, therefore it is not an identity. Hence there is an error in given question. The correct identity must be-)

$\int_1^x f(t) dt = 3xf(x) - x^3 - 5, \forall x \geq 1$

Now applying Newton Leibnitz theorem

$6f(x) = 3xf'(x) - 3x^2 + 3f(x)$

$\Rightarrow 3f(x) = 3xf'(x) - 3x^2$

Let $y = f(x)$

$\Rightarrow x \frac{dy}{dx} - y = x^2 \Rightarrow \frac{xdy - ydx}{x^2} = dx$

$\Rightarrow \int d\left(\frac{y}{x}\right) = \int dx$

$\Rightarrow \frac{y}{x} = x + C$ (where C is constant)

$\Rightarrow y = x^2 + Cx$

$\therefore f(x) = x^2 + Cx$

Given $f(1) = 2 \Rightarrow C = 1$

$\therefore f(2) = 2^2 + 2 = 6$

(B) Given $y(0) = 0, g(0) = g(2) = 0$

Let $y'(x) + y(x) \cdot g'(x) = g(x) g'(x)$

$\Rightarrow y'(x) + (y(x) - g(x)) g'(x) = 0$

$\Rightarrow \frac{y'(x)}{g'(x)} + y(x) = g(x)$

$\Rightarrow \frac{dy(x)}{dg(x)} + y(x) = g(x)$

$\Rightarrow \text{I.F.} = e^{\int d(g(x))} = e^{g(x)}$

$\Rightarrow y(x) \cdot e^{g(x)} = \int e^{g(x)} g(x) \cdot dg(x)$

$y(x) \cdot e^{g(x)} = g(x) \cdot e^{g(x)} - e^{g(x)} + c$

put $x = 0$

$\Rightarrow 0 = 0 - 1 + c \Rightarrow c = 1$

$\Rightarrow y(2) \cdot e^{g(2)} = g(2) e^{g(2)} - e^{g(2)} + 1$

$\Rightarrow y(2) = 0 - e^0 + 1 \Rightarrow y(2) = 0$

16. $\frac{dy}{dx} - y \tan x = 2x \sec x$

I.F. = $e^{\int -\tan x dx} = \cos x$

$\therefore$ Equation reduces to

$y \cdot \cos x = \int 2x \cdot \sec x \cdot \cos x dx$

$\Rightarrow y \cos x = x^2 + C$

$\rightarrow y(0) = 0 \Rightarrow 0 = 0 + C$

$\therefore y \cos x = x^2 \Rightarrow y(x) = x^2 \sec x$

$\therefore y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} \sqrt{2} = \frac{\pi^2}{8\sqrt{2}} \quad (\therefore \text{(A) is correct})$

$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} \cdot 2 = \frac{2\pi^2}{9} \quad (\therefore \text{(C) is wrong})$

Also $y'(x) = 2x \sec x + x^2 \sec x \tan x$

$\Rightarrow y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} \cdot \sqrt{2} + \frac{\pi^2 \sqrt{2}}{16} \quad (\therefore \text{(B) is wrong})$

and $y'\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{3} \cdot 2 + \frac{\pi^2}{9} \cdot 2 \cdot \sqrt{3}$

$= \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}} \quad (\therefore \text{(D) is correct})$

17. $f'(x) - 2f(x) < 0$

Multiply both side by e^{-2x}

$$e^{-2x} f'(x) - 2e^{-2x} f(x) < 0$$

$$\frac{d}{dx}(e^{-2x} f(x)) < 0$$

Now, $g(x) = e^{-2x} f(x)$

$\therefore g(x)$ is a decreasing function.

$$x > \frac{1}{2}$$

$$g(x) < g\left(\frac{1}{2}\right)$$

$$\Rightarrow e^{-2x} f(x) < \frac{1}{e} \Rightarrow f(x) < e^{2x-1}$$

$$\begin{aligned} \Rightarrow \int_{1/2}^1 f(x) dx &< \frac{1}{e} \int_{1/2}^1 e^{2x} dx \\ &= \left[\frac{1}{2e} e^{2x} \right]_{1/2}^1 = \frac{1}{2e} (e^2 - e) = \frac{1}{2} (e - 1) \end{aligned}$$

$$\Rightarrow \int_{1/2}^1 f(x) dx < \frac{e-1}{2}$$

obviously $f(x)$ is positive

$$\therefore \int_{1/2}^1 f(x) dx > 0$$

18. $\frac{dy}{dx} = \frac{y}{x} + \sec \frac{y}{x}$

Let $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + \frac{xdv}{dx} = v + \sec v$$

$$\cos v dv = \frac{dx}{x}$$

$$\sin v = \ln x + c$$

$$\sin\left(\frac{y}{x}\right) = \ln x + c$$

→ passing through $\left(1, \frac{\pi}{6}\right)$

$$\Rightarrow \sin \frac{\pi}{6} = c \Rightarrow c = \frac{1}{2}$$

$$\therefore \sin \frac{y}{x} = \ln x + \frac{1}{2}$$

Paragraph for Question 19 and 20

19. $e^{-x}(f''(x) - 2f'(x) + f(x)) \geq 1$

$$D((f'(x) - f(x))e^{-x}) \geq 1$$

$$\Rightarrow D((f'(x) - f(x))e^{-x}) \geq 0$$

$$\Rightarrow (f'(x) - f(x))e^{-x} \text{ is an increasing function.}$$

As we know that $e^{-x}f(x)$ has local minima at $x = \frac{1}{4}$

$$e^{-x}(f'(x) - f(x)) = 0 \text{ at } x = \frac{1}{4}$$

$$\text{Let } F(x) = e^{-x}(f'(x) - f(x))$$

$$F(x) < 0 \text{ in } \left(0, \frac{1}{4}\right)$$

$$e^{-x}(f'(x) - f(x)) < 0 \text{ in } \left(0, \frac{1}{4}\right)$$

$$f'(x) < f(x) \text{ in } \left(0, \frac{1}{4}\right)$$

option C

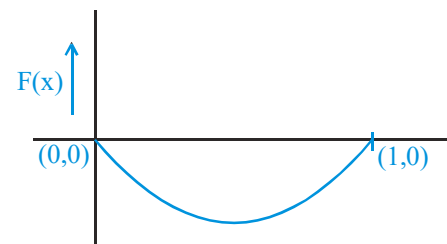
20. $D(e^{-x}(f'(x) - f(x))) \geq 0 \forall x \in (0, 1)$

$$D(D(e^{-x}f(x))) \geq 0 \forall x \in (0, 1)$$

$$D^2(e^{-x}f(x)) \geq 0$$

$$\text{Let } F(x) = e^{-x}f(x)$$

$F''(x) > 0$ means it is concave upward.



$$F(0) = F(1) = 0$$

$$F(x) < 0 \forall x \in (0, 1)$$

$$e^{-x}f(x) < 0 \forall x \in (0, 1)$$

$$f(x) < 0$$

Option D is possible

$$(x+2)^2 + y(x+2) = y^2 \cdot \frac{dx}{dy}$$

$$\frac{dx}{dy} = \frac{(x+2)^2}{y^2} + \frac{x+2}{y}$$

$$\frac{1}{(x+2)^2} \frac{dx}{dy} = \frac{1}{y^2} + \frac{1}{y(x+2)}$$

$$\therefore \frac{1}{(x+2)^2} \frac{dx}{dy} - \frac{1}{(x+2)y} = \frac{1}{y^2}$$

$$-\frac{dt}{dy} - \frac{t}{y} = \frac{1}{y^2}$$

$$\therefore \text{Put } \frac{1}{x+2} = t, -\frac{1}{(x+2)^2} \frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{dt}{dy} + \frac{t}{y} = -\frac{1}{y^2} \quad \text{I.F.} = e^{\int \frac{1}{y} dy} = y$$

$$t \cdot y = C + \int y \left(-\frac{1}{y^2} \right) dy$$

$$t \cdot y = C - \log y$$

$$\therefore \frac{1}{x+2} \cdot y = C - \log y$$

$$\text{It passes } (1, 3) \Rightarrow 1 = C + \log 3 \Rightarrow C = 1 + \log(3)$$

$$\frac{y}{x+2} = 1 + \log 3 - \log y$$

[A] option is correct.

For option (C)

$$\frac{(x+2)^2}{x+2} = 1 - \log\left(\frac{y}{3}\right)$$

$$x+1 = \log\left(\frac{y}{3}\right)$$

$$\therefore y = 3e^{x-1}$$

$\Rightarrow$ Intersect

For option (D)

$$\frac{(x+3)^2}{4+2} - 1 = -\log\left(\frac{(x+3)^2}{3}\right)$$

$$\therefore \frac{(x+3)^2 - 1}{x+2} = -\log\left\{\frac{(x+3)^2}{3}\right\}$$

$$3e^{\left(\frac{(x+3)^2 - 1}{-x-2}\right)} = (x+3)^2$$

$\Rightarrow$ will intersect.

$\Rightarrow$ (D) is not correct.

MOCK TEST

1. (A)

$$x^2 = e^{\left(\frac{x}{y}\right)^{-1} \left(\frac{dy}{dx}\right)} \Rightarrow x^2 = e^{\left(\frac{y}{x}\right) \left(\frac{dy}{dx}\right)}$$

$$\Rightarrow \ln x^2 = \frac{y}{x} \frac{dy}{dx}$$

$$\Rightarrow \int x \ln x^2 dx = \int y dy$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt$$

$$\frac{1}{2} \int \ln t dt = \frac{y^2}{2}$$

$$\Rightarrow c + t \bullet \ln t - t = y^2$$

$$\Rightarrow y^2 = x^2 \bullet \ln x^2 - x^2 + c$$

2. (D)

$$\frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$$

$$\text{put } x+y = t, 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} = \frac{t+1}{2t+1} + 1 = \frac{3t+2}{2t+1}$$

$$\Rightarrow \int \frac{2t+1}{3t+2} dt = \int dx$$

$$\Rightarrow \frac{2t}{3} - \frac{1}{9} \bullet \ln(3t+2) = x + c$$

$$\Rightarrow 6(x+y) - \bullet \ln(3x+3y+2) = 9x + c$$

$$\Rightarrow \bullet \ln(3x+3y+2) = 6y - 3x + c$$

Since it passes through (0, 0) hence equation of curve is

$$6y - 3x = \bullet \ln \left| \frac{3x+3y+2}{2} \right|$$

3. (A)

$$y_1 y_3 = 3y_2^2 \Rightarrow \int \frac{y_3}{y_2} = \int \frac{3y_2}{y_1}$$

$$\Rightarrow \bullet \ln y_2 = 3 \bullet \ln y_1 + \bullet \ln c$$

$$\Rightarrow y_2 = c y_1^3 \Rightarrow \int \frac{y_2}{y_1^2} = \int c y_1$$

$$\Rightarrow -\frac{1}{y_1} = c y + d$$

$$\Rightarrow -dx = (cy + d) dy$$

$$\Rightarrow -x = \frac{cy^2}{2} + d y + e$$

4. $\frac{d^2y}{dx^2}(x^2+1) = 2x \frac{dy}{dx}$

$$\Rightarrow \int \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} dx = \int \frac{2x}{x^2+1} dx$$

$$\Rightarrow \ln\left(\frac{dy}{dx}\right) = \ln(x^2+1) + \ln c$$

$$\Rightarrow \frac{dy}{dx} = c(x^2+1) \Rightarrow c = 3 \text{ as at } x=0, \frac{dy}{dx} = 3$$

$$\Rightarrow \frac{dy}{dx} = 3(x^2+1) dx$$

$$\Rightarrow y = x^3 + 3x + 1$$

5. (B)

$$\frac{dy}{dx} = \frac{1}{x \cos y + 2 \sin y \cos y}$$

$$\therefore \frac{dx}{dy} = x \cos y + 2 \sin y \cos y$$

$$\Rightarrow \frac{dx}{dy} + (-\cos y)x = 2 \sin y \cos y$$

$$\therefore \text{I.F.} = e^{-\int \cos y dy} = e^{-\sin y}$$

$\therefore$ The solution is

$$x \cdot e^{-\sin y} = 2 \int e^{-\sin y} \cdot \sin y \cos y dy$$

$$\Rightarrow x \cdot e^{-\sin y} = -2 \sin y e^{-\sin y} - 2 \int (-e^{-\sin y}) \cos y dy$$

$$\Rightarrow x \cdot e^{-\sin y} = -2 \sin y e^{-\sin y} + 2 \int e^{-\sin y} \cos y dy$$

$$= -2 \sin y e^{-\sin y} - 2 e^{-\sin y} + c$$

$$\Rightarrow x = -2 \sin y - 2 + c e^{\sin y} = c e^{\sin y} - 2(1 + \sin y)$$

$$\therefore k = 2$$

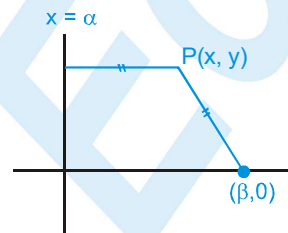
6. (A)

Directrix $\perp$ to x axis, Let $x = \alpha$ and focus on x axis Let

$(\beta, 0)$, Now

$$(x - \beta)^2 + y^2 = (x - \alpha)^2$$

$$\beta^2 - 2\beta x + y^2 = \alpha^2 - 2\alpha x$$



$$y^2 = 2(\beta - \alpha)x + \alpha^2 - \beta^2$$

In general $y^2 = mx + c$ (Two arbitrary constant m and c)

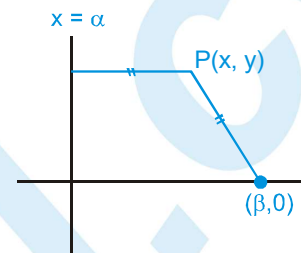
$$2y \frac{dy}{dx} = m$$

$$2y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

7. Directrix $\perp$ to x axis, Let $x = \alpha$ and focus on x axis Let $(\beta, 0)$, Now

$$(x - \beta)^2 + y^2 = (x - \alpha)^2$$

$$\beta^2 - 2\beta x + y^2 = \alpha^2 - 2\alpha x$$



$$y^2 = 2(\beta - \alpha)x + \alpha^2 - \beta^2$$

In general $y^2 = mx + c$ (Two arbitrary constant m and c)

$$2y \frac{dy}{dx} = m$$

$$2y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

8. (A)

Put $x = \sin \theta$, $y = \sin \phi$, so that the given equation is reduced to

$$\cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$\Rightarrow 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2} = 2a \cos \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2}$$

$$\Rightarrow \cot \frac{\theta - \phi}{2} = a$$

$$\Rightarrow \theta - \phi = 2 \cot^{-1} a$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$$

Differentiating we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

So the degree is one.

9. (A)

$$y \left(\frac{dy}{dx} \right)^2 + x \frac{dy}{dx} - y \frac{dy}{dx} - x = 0$$

$$y \frac{dy}{dx} \left(\frac{dy}{dx} - 1 \right) + x \left(\frac{dy}{dx} - 1 \right) = 0$$

$$\left(y \frac{dy}{dx} + x \right) \left(\frac{dy}{dx} - 1 \right) = 0$$

∴ either $y dy + x dx = 0$ or $y - x = c$
since the curves pass through the point (3, 4)
∴ $x^2 + y^2 = 25$ or $x - y + 1 = 0$

10. (A)

The given differential equation is

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{x^2 - 1} y = \frac{1}{(x^2 - 1)^2} \quad \dots\dots(i)$$

This is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = (x^2 - 1)$$

$$\therefore \text{Solution is } y (\text{I.F.}) = \int Q (\text{I.F.}) dx$$

$$\Rightarrow y(x^2 - 1) = \int \frac{1}{x^2 - 1} dx$$

$$\Rightarrow y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

This is the required solution.

11. (A, B)

$$y = e^{-x} \cos x$$

$$y_1 = -e^{-x} \cos x - e^{-x} \sin x = -\sqrt{2} e^{-x} \cos \left(x - \frac{\pi}{4} \right)$$

$$y_2 = + \left(\sqrt{2} \right)^2 e^{-x} \cos \left(x - \frac{\pi}{2} \right)$$

$$y_3 = \left(-\sqrt{2} \right)^3 e^{-x} \cos \left(x - \frac{3\pi}{4} \right)$$

$$y_4 = + \left(\sqrt{2} \right)^4 e^{-x} \cos (x - \pi) = -4 e^{-x} \cos x$$

$$\Rightarrow y_4 + 4y = 0 \quad k_4 = 4$$

Differentiating it again 4 times

$$y_8 + 4y_4 = 0 \quad \Rightarrow y_8 - 16y = 0 \quad \Rightarrow k_8 = -16$$

$$y_{12} + 4y_8 = 0 \quad \Rightarrow y_{12} + 64y = 0 \quad \Rightarrow k_{12} = 64$$

Similarly $k_{16} = -256$

$$12. (A) \quad \frac{dy}{dx} = -c_1 \sin x + \sqrt{c_2} \cos x$$

$$\frac{d^2 y}{dx^2} = -c_1 \cos x - \sqrt{c_2} \sin x = 2 - y$$

$$\frac{d^2 y}{dx^2} + y + 2 = 0$$

$$(B) \quad \frac{dy}{dx} = \cos x \frac{\sec^2 x / 2}{2 \tan \frac{x}{2}} - \sin x \bullet n \left(\tan \frac{x}{2} \right)$$

$$\frac{dy}{dx} = \cot x - \sin x \bullet n \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2 y}{dx^2} = -\operatorname{cosec}^2 x - \left(\sin x \cdot \frac{1}{\sin x} + \cos x \ln \left(\tan \frac{x}{2} \right) \right)$$

$$\frac{d^2 y}{dx^2} = -\cot^2 x - 2 - \cos x \bullet n \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2 y}{dx^2} + y + \cot^2 x = 0$$

$$(C) \quad \frac{dy}{dx} = -c_1 \sin x + c_2 \cos x + \frac{d}{dx} \left(\cos x \ln \left(\tan \frac{x}{2} \right) \right)$$

$$\frac{d^2 y}{dx^2} = -(c_1 \cos x + c_2 \sin x) + \frac{d^2}{dx^2} \left(\cos x \ln \left(\tan \frac{x}{2} \right) \right)$$

$$\frac{d^2 y}{dx^2} = -c_1 \cos x - c_2 \sin x - \cot^2 x - 2 - \cos x \bullet n \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2 y}{dx^2} + y + \cot^2 x = 0$$

13. (B,C)

(A) $\frac{dy}{dx} = -c_1 \sin x + \sqrt{c_2} \cos x$

$$\frac{d^2y}{dx^2} = -c_1 \cos x - \sqrt{c_2} \sin x = 2 - y$$

$$\frac{d^2y}{dx^2} + y - 2 = 0$$

(B) $\frac{dy}{dx} = \cos x \cdot \frac{\sec^2 x / 2}{2 \tan \frac{x}{2}} - \sin x \cdot \ln \left(\tan \frac{x}{2} \right)$

$$\frac{dy}{dx} = \cot x - \sin x \cdot \ln \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x - \left(\sin x \cdot \frac{1}{\sin x} + \cos x \ln \left(\tan \frac{x}{2} \right) \right)$$

$$\frac{d^2y}{dx^2} = -\cot^2 x - 2 - \cos x \cdot \ln \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2y}{dx^2} + y + \cot^2 x = 0$$

(C) $\frac{dy}{dx} = -c_1 \sin x + c_2 \cos x + \frac{d}{dx} \left(\cos x \ln \left(\tan \frac{x}{2} \right) \right)$

$$\frac{d^2y}{dx^2} = -(c_1 \cos x + c_2 \sin x) + \frac{d^2}{dx^2} \left(\cos x \ln \left(\tan \frac{x}{2} \right) \right)$$

$$\frac{d^2y}{dx^2} = -c_1 \cos x - c_2 \sin x - \cot^2 x - 2 - \cos x \cdot \ln \left(\tan \frac{x}{2} \right)$$

$$\frac{d^2y}{dx^2} + y + \cot^2 x = 0$$

14. $x^2 + y^2 + 2gx + 2fy + c = 0$

(There are three arbitrary constants g, f and c)

$$2x + 2y \cdot \frac{dy}{dx} + 2g + 2f \cdot \frac{dy}{dx} = 0 \quad \text{..... (i)}$$

$$2 + 2y_1^2 + 2yy_2 + 2fy_2 = 0 \quad \text{..... (ii)}$$

$$4y_1y_2 + 2y_1y_2 + 2yy_3 + 2fy_3 = 0 \quad \text{..... (iii)}$$

from (i), (ii) and (iii)

$$y_3(1 + y_1^2) - 3y_1y_2^2 = 0$$

15. (A, B, D)

$$\frac{dy}{dx} (x^2y^3 + xy) = 1 \Rightarrow \frac{dx}{dy} = x^2y^3 + xy$$

$$\Rightarrow \frac{dx}{dy} - xy = x^2y^3 \Rightarrow -\frac{1}{x^2} \frac{dx}{dy} + \frac{y}{x} = -y^3$$

put, $\frac{1}{x} = t$ and differentiating with respect to $y - \frac{1}{x^2}$

$$\frac{dx}{dy} = \frac{dt}{dy}$$

$$\Rightarrow \frac{dt}{dy} + ty = -y^3 \Rightarrow e^{\frac{y^2}{2}} t = \int -y^3 e^{y^2/2} dy$$

$$\Rightarrow \frac{1}{x} = 2 - y^2 + c e^{-\frac{y^2}{2}}$$

16. (B)

Statement-II $\bullet \ln \frac{x}{y} - \frac{1}{xy} = c$

i.e. $\bullet \ln x - \bullet \ln y - \frac{1}{xy} = c$

diff.

$$\frac{1}{x} - \frac{1}{y} \frac{dy}{dx} + \frac{y+x}{(xy)^2} \frac{dy}{dx} = 0$$

$$xy^2 dx - x^2y dy + ydx + xdy = 0$$

$$(1 + xy) ydx + x(1 - xy) dy = 0$$

$\therefore$ statement is true.

Statement-I $x - \frac{y^2}{2} + \frac{1}{3} (x^2 + y^2)^{3/2} + c = 0$

$$1 - y \frac{dy}{dx} + \frac{1}{2} \sqrt{x^2 + y^2} \left(2x + 2y \frac{dy}{dx} \right) = 0$$

$$dx - y dy + x \sqrt{x^2 + y^2} dx + y \sqrt{x^2 + y^2} dy = 0$$

$$(1 + x \sqrt{x^2 + y^2}) dx + y(-1 + \sqrt{x^2 + y^2}) dy = 0$$

$\therefore$ Statement is true

17. (B)

$$\frac{dy}{dx} = x + \frac{1}{x^2}$$

$$dy = xdx + \frac{1}{x^2} \cdot dx$$

$$y = \frac{x^2}{2} - \frac{1}{x} + c$$

$$x = 3, y = 9$$

$$9 = \frac{9}{2} - \frac{1}{3} + c \Rightarrow \frac{9}{2} + \frac{1}{3} = c$$

$$c = \frac{27 + 2}{6} = \frac{29}{6}$$

$$y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$$

Statement-I is true



Statement-II

$$\frac{dy}{dx} = t$$

$$t^2 - t(e^x + e^{-x}) + 1 = 0$$

$$e^x t^2 - t(e^{2x} + 1) + e^x = 0$$

$$e^x t(t - e^x) - (t - e^x) = 0$$

$$(t - e^x)(te^x - 1) = 0$$

$$dy = e^x dx$$

$$y = e^x \quad \dots\dots(i)$$

$$e^x \frac{dy}{dx} = 1$$

$$y = -e^{-x} \quad \dots\dots(ii)$$

$y = c_1 e^x + c_2 e^{-x}$ can not be the solution of given diff. Equation.

18. (B)

Statement-I : Equation of all circles can be given by $x^2 + y^2 + 2gx + 2fy + c = 0$, will be of order 3

Statement-II : is obviously true but it does not explain statement-I

19. (C)

Statement-I $y^3 \frac{dy}{dx} + (x + y^2) = 0$

$$y^2 = t$$

$$2y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{2} \frac{dt}{dx} \cdot t + x + t = 0$$

is homogenous equation

Statement-II is obviously false

20. (B)

Statement-1 is obviously true

Statement-2 is also obviously true

but statement-2 does not explain statement-1

21. (A) Variable separable

(B) Variable separable

(C) Variable separable

(D) Put $\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$

$$\tan x \frac{dt}{dx} + t \sec^2 x = 1$$

(Linear Diff. equation)

22. (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (t), (D) $\rightarrow$ (p)

(A) $xdy = ydx + y^2 dy \Rightarrow \frac{x dy - y dx}{y^2} = dy$

$$\Rightarrow -d\left(\frac{x}{y}\right) = dy$$

$$-\frac{x}{y} = y + c \quad \text{put } x = 1, y = 1 \Rightarrow c = -2$$

$$-\frac{x}{y} = y - 2 \quad \text{Now } \frac{x_0}{3} = -5 \Rightarrow x_0 = -15$$

(B) $\frac{dy}{dt} - \frac{t}{t+1} y = \frac{1}{t+1}$

$$I.f = e^{-\int \frac{t+1}{t+1} dt} = e^{-t+1n(t+1)} = (t+1)e^{-t}$$

solution is $(t+1)e^{-t} y = -e^{-t} + c$

$$\text{put } t = 0 \quad \text{and } y = -1 \Rightarrow c = 0$$

Now at $t = 1$

$$\therefore 2e^{-1} y = -e^{-1}$$

$$y = -\frac{1}{2}$$

(C) $(x^2 + y^2) dy = xy dx$

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2} \quad \text{put } y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore \frac{v}{1+v^2} = v + x \frac{dv}{dx} \Rightarrow \bullet n v - \frac{1}{2v^2} = -\bullet n x + c$$

$$\text{put } x = 1, y = 1, v = 1, \text{ then } c = -\frac{1}{2}$$

$$\therefore \bullet n \frac{y}{x} - \frac{1}{2} \frac{x^2}{y^2} = -\bullet n x - \frac{1}{2} \quad \text{put } y = e$$

$$\therefore x_0 = \sqrt{3} e$$

(D) $\frac{dy}{dx} + 2\frac{y}{x} = 0$

$$x^2 y = C \quad \text{put } x = 1, y = 1$$

and we get $C = 1$

$$\text{put } x = 2 \Rightarrow y = \frac{1}{4}$$

23.

1. (B)

$$\frac{dx}{x} = \frac{y dy}{1+y^2} \Rightarrow \bullet n x = \frac{1}{2} \cdot \bullet n (1+y^2) + c$$

from the given condition $c = 0 \therefore x^2 - y^2 = 1$



2. (A)

$$\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0 \Rightarrow \frac{dy}{1+y^2} + \frac{dx}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \tan^{-1} y + \sin^{-1} x = c$$

3. (B) $\frac{dy}{dx} = (1+x) \cdot (1+y)$ gives $y = e^{\frac{(1+x)^2}{2}} - 1$

24.

$$1. \int_{200}^{f(t)} \frac{dy}{10-y} = \int_0^t k \, dt$$

$$[\ln |10-y|]_{f(t)}^{200} = kt$$

$$\ln(190) - \ln |10-f(t)| = kt$$

$$\ln |10-f(t)| = \ln 190 - kt$$

$$|10-f(t)| = e^{\ln 190 - kt}$$

$$f(t) - 10 = 190e^{-kt} \quad (f(t) > 0 \text{ as } M(t) = 10)$$

$$f(t) = 10 + 190e^{-kt}$$

$$2. \frac{dy}{dt} + ky = kM(t)$$

$$\text{Let } M(t) = 10^0 \quad (\text{constant})$$

$$\int_{200}^{100} \frac{dy}{10-y} = \int_0^{40} k \, dt$$

$$[\ln |(10-y)|]_{100}^{200} = k[t]_0^{40}$$

$$\ln(190) - \ln(90) = 40k$$

$$k = \frac{\ln(19) - \ln(9)}{40}$$

$$3. \int_{400}^{200} \frac{dy}{10-y} = k \int_0^t dt$$

$$[\ln (y-10)]_{200}^{400} = kt$$

$$t = \frac{\ln 390 - \ln 190}{k}$$

$$t = 40 \left(\frac{\ln 39 - \ln 19}{\ln 19 - \ln 9} \right)$$

25.

1. (B)

Obviously option B is not a solution

2. (B)

Among the four option B is the solution of the differential equation

3. (D)

Among the four option D is satisfying

26. (3)

The given equation contains one constant.

Differentiating the equation once, we get

$$2x - 2yy' = 2c(x^2 + y^2)(2x + 2yy')$$

$$\text{But } c = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

Substituting for c, we get

$$(x - yy') = \frac{(x^2 + y^2)(x^2 - y^2)}{(x^2 + y^2)^2} \cdot 2(x + yy')$$

$$\Rightarrow (x^2 + y^2)(x - yy') = 2(x^2 - y^2)(x + yy')$$

$$\Rightarrow yy'[(x^2 + y^2) + 2(x^2 - y^2)] = x(x^2 + y^2) - 2x(x^2 - y^2)$$

$$\Rightarrow yy'(3x^2 - y^2) = x(3y^2 - x^2)$$

$$\Rightarrow y' = \frac{x(3y^2 - x^2)}{y(3x^2 - y^2)}$$

27. (2)

Suppose the point at which normal is drawn is $p \equiv (x, y)$

$$\text{Equation of the normal is } Y - y = -\frac{dx}{dy}(X - x)$$

$$\text{at } X \text{ axis } Y = 0$$

$$\therefore \text{ coordinate of } X = \frac{y \, dy}{dx} + x$$

$$\therefore \text{ Point where it cuts the } X \text{ axis are } \left(y \frac{dy}{dx} + x, 0 \right),$$

say A

$$\text{Mid point of PA} = \left(x + \frac{y \frac{dy}{dx}}{2}, \frac{y}{2} \right), \text{ which lies on the curve } 2y^2 = x$$

$$\therefore y^2 = 2x + y \frac{dy}{dx}$$

$$\Rightarrow y \frac{dy}{dx} + 2x = y^2 \quad \dots\dots(i)$$

$$\text{put } y^2 = t$$

$$\text{and differentiating with respect to } x \quad 2y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\text{Now from equation (i) we get } \frac{1}{2} \frac{dt}{dx} + 2x = t$$

$$\Rightarrow \frac{dt}{dx} - 2t = -4x$$

$$\text{I.F.} = e^{\int -2 dx} = e^{-2x} \Rightarrow te^{-2x} = - \int e^{-2x} 4x dx + c$$

$$\Rightarrow y^2 e^{-2x} = - \left\{ \frac{4x e^{-2x}}{-2} + \frac{1}{2} \int 4e^{-2x} dx \right\} + c$$

$$\Rightarrow y^2 e^{-2x} = + 2xe^{-2x} + e^{-2x} + c$$

$$\text{at } x=0 \quad y=0 \Rightarrow c = -1$$

$$\therefore y^2 = 2x - e^{2x} + 1$$

28. (1)

For reservoir A,

$$\frac{dV_A}{dt} = -k_1 V_A$$

$$\Rightarrow \frac{dV_A}{V_A} = -k_1 dt \Rightarrow \ln V_A = -k_1 t + c$$

$$\text{initially } t=0, V_A = V_{OA}$$

$$\therefore \ln V_{OA} = c \Rightarrow \ln V_A = -k_1 t + \ln V_{OA}$$

$$\Rightarrow \ln \left(\frac{V_A}{V_{OA}} \right) = -k_1 t \Rightarrow V_A = V_{OA} e^{-k_1 t} \quad \dots\dots(ii)$$

For reservoir B,

$$\frac{dV_B}{dt} = -k_2 V_B$$

$$\text{Similarly } V_B = V_{OB} e^{-k_2 t} \quad \dots\dots(iii)$$

$$\text{Given that at } t=0, V_{OA} = 2V_{OB}$$

$$\text{At } t=1 \text{ hr.}, V_A = \frac{3}{2} V_B$$

$$\Rightarrow V_{OA} e^{-k_1} = \frac{3}{2} V_{OB} e^{-k_2}$$

$$\Rightarrow 2V_{OB} e^{-k_1} = \frac{3}{2} V_{OB} e^{-k_2} \quad (\rightarrow V_{OA} = 2V_{OB})$$

$$\Rightarrow e^{(k_1 - k_2)} = \frac{4}{3}$$

$$\Rightarrow (k_1 - k_2) = \ln 4/3 \quad \dots\dots(iv)$$

$$\text{After time } t: V_A = V_B$$

$$\Rightarrow V_{OA} e^{-k_1 t} = V_{OB} e^{-k_2 t}$$

$$\Rightarrow 2V_{OB} e^{-k_1 t} = V_{OB} e^{-k_2 t} \quad (\rightarrow V_{OA} = 2V_{OB})$$

$$\Rightarrow 2 = e^{(k_1 - k_2)t}$$

$$\Rightarrow \ln 2 = (k_1 - k_2)t \quad \dots\dots(v)$$

$$\text{from (iii) and (iv) we get } t = \frac{\ln 2}{\ln (4/3)}$$

$$\Rightarrow t = \log_{4/3} 2$$

29. (1)

Since given differential equation is

$$y(x + y^3) dx = x(y^3 - x) dy$$

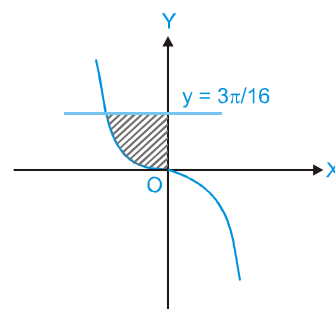
$$\Rightarrow (xy dx + x^2 dy) + y^4 dx - y^3 x dy = 0$$

$$\Rightarrow x(y dx + x dy) + y^3(y dx - x dy) = 0$$

$$\Rightarrow xd(xy) = y^3(x dy - y dx)$$

$$\Rightarrow xd(xy) = x^2 y^3 d\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{d(xy)}{(xy)^2} = \left(\frac{y}{x}\right) d\left(\frac{y}{x}\right)$$



$$\text{On integrating } -\frac{1}{xy} = \frac{1}{2} \left(\frac{y}{x}\right)^2 + c$$

$$\text{at } (4, -2)$$

$$\Rightarrow \frac{1}{8} = \frac{1}{2} \left(-\frac{2}{4}\right)^2 + c$$

$$\Rightarrow c = 0 \quad \therefore y^3 = -2x \quad \Rightarrow y = (-2x)^{1/3}$$

Since $y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1} \sqrt{t} \, dt + \int_{1/8}^{\cos^2 x} \cos^{-1} \sqrt{t} \, dt$

$\therefore \frac{dy}{dx} = g'(x) = x \cdot \sin 2x + x \cdot (-\sin 2x) = 0$

$\therefore y = c_1$ (constant)

put $\sin x = \cos x = \frac{1}{\sqrt{2}}$

$\therefore c_1 = \int_{1/8}^{1/2} (\sin^{-1} \sqrt{t} + \cos^{-1} \sqrt{t}) \, dt$

$= \frac{\pi}{2} \left(\frac{1}{2} - \frac{1}{8} \right) = \frac{3\pi}{16} \quad \therefore y = g(x) = \frac{3\pi}{16}$

Hence area between $y = \frac{3\pi}{16}$ and $y = (-2x)^{1/3}$

$= \int_0^{3\pi/16} x \, dy = \left| \int_0^{3\pi/16} \left(-\frac{y^3}{2} \right) dy \right| = \frac{1}{8} \cdot \left(\frac{3\pi}{16} \right)^4$ sq. units.

30. (4)

$(x^2 + 4y^2 + 4xy) \, dy = (2x + 4y + 1) \, dx.$

$\frac{dy}{dx} = \frac{2(x+2y)+1}{(x+2y)^2}$ (i)

put, $x + 2y = t$ and differentiating with respect to x

$1 + \frac{2dy}{dx} = \frac{dt}{dx}$

From equation (i) we get $\left(\frac{dt}{dx} - 1 \right) \frac{1}{2} = \frac{2t+1}{t^2}$

$\Rightarrow \frac{dt}{dx} - 1 = \frac{4t+2}{t^2}$

$\Rightarrow \int \frac{t^2}{t^2 + 4t + 2} \, dt = \int dx$

$\Rightarrow \int dt - \int \frac{4t+2}{t^2 + 4t + 2} \, dt = x + c_1$

$\Rightarrow t - \int \frac{2(2t+4)-6}{t^2 + 4t + 2} \, dt = x + c_1$

$\Rightarrow t - 2 \ln(t^2 + 4t + 2) + 6 \frac{1}{2\sqrt{2}} \ln \left| \frac{t+2-\sqrt{2}}{t+2+\sqrt{2}} \right| = x + c_1$

$\Rightarrow y = \ln((x+2y)^2 + 4(x+2y) + 2) - \frac{3}{2\sqrt{2}} \ln$

$\left| \frac{x+2y+2-\sqrt{2}}{x+2y+2+\sqrt{2}} \right| + c$, where $c = c_1/2$

dictionary definition of physics is "the study of matter, energy,