SOLVED EXAMPLES

Ex 1 Find the differential equation whose solution represents the family : $c(y+c)^2 = x^3$

Sol. $c(y+c)^2 = x^3$ .....(i)

Differentiating, we get, $c.\left[2(y+c)\right]\frac{dy}{dx} = 3x^2$

Writing the value of c from (i), we have

$\frac{2x^3}{(y+c)^2}(y+c)\frac{dy}{dx} = 3x^2 \quad \Rightarrow \quad \frac{2x^3}{y+c}\frac{dy}{dx} = 3x^2$

i.e. $\frac{2x}{y+c}\frac{dy}{dx} = 3 \quad \Rightarrow \quad \frac{2x}{3}\left[\frac{dy}{dx}\right] = y+c$

Hence $c = \frac{2x}{3}\left[\frac{dy}{dx}\right] - y$

Substituting value of c in equation (i), we get $\left[\frac{2x}{3}\left(\frac{dy}{dx}\right) - y\right]\left[\frac{2x}{3}\frac{dy}{dx}\right]^2 = x^3$,

which is the required differential equation.

Ex. 2 Solve : $\frac{dy}{dx} = (x-3)(y+1)^{2/3}$

Sol. $\frac{dy}{dx} = (x-3)(y+1)^{2/3}$

$\int \frac{dy}{(y+1)^{2/3}} = \int (x-3)dx$

Integrate and solve for y : $3(y+1)^{1/3} = \frac{1}{2}(x-3)^2 + C$

$(y+1)^{1/3} = \frac{1}{6}(x-3)^2 + C_0$

$\Rightarrow \quad y+1 = \left(\frac{1}{6}(x-3)^2 + C_0\right)^3$

$\Rightarrow \quad y = \left(\frac{1}{6}(x-3)^2 + C_0\right)^3 - 1$

All of this looks routine. However, note that $y = -1$ is a solution to the original equation

$\frac{dy}{dx} = 0$ and $(x-3)(y+1)^{2/3} = 0$

However, we can not obtain $y = -1$ from $y = \left(\frac{1}{6}(x-3)^2 + C_0\right)^3 - 1$ by setting constant C_0 equal to any number.

(We need to find a constant which makes $\frac{1}{6}(x-3)^2 + C_0 = 0$ for all x.)

Two points emerge from this.

(i) We may sometime miss solutions while performing certain algebraic operations (in this case, division).

(ii) We don't always get every solution to a differential equation by assigning values to the arbitrary constants.

Ex. 3 Form a differential equation of family of circles touching x-axis at the origin

Sol. Equation of family of circles touching x-axis at the origin is

$x^2 + y^2 + \lambda y = 0$ ....(i) where λ is a parameter

$$2x + 2y\frac{dy}{dx} + \lambda\frac{dy}{dx} = 0 \quad \text{.........(ii)}$$

Eliminating 'λ' from (i) and (ii)

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

which is required differential equation.

Ex. 4 The solution of the differential equation $\frac{dy}{dx} = \frac{\sin y + x}{\sin 2y - x\cos y}$ is -

Sol. Here, $\frac{dy}{dx} = \frac{\sin y + x}{\sin 2y - x\cos y}$

$\Rightarrow \quad \cos y\frac{dy}{dx} = \frac{\sin y + x}{2\sin y - x}$, (put $\sin y = t$)

$\Rightarrow \quad \frac{dt}{dx} = \frac{t + x}{2t - x}$ (put $t = vx$)

$$\frac{xdv}{dx} + v = \frac{vx + x}{2vx - x} = \frac{v+1}{2v-1}$$

$\therefore \quad x\frac{dv}{dx} = \frac{v+1}{2v-1} - v = \frac{v + 1 - 2v^2 + v}{2v - 1}$

or $\quad \frac{2v - 1}{-2v^2 + 2v + 1}dv = \frac{dx}{x}$ on solving, we get

$$\sin^2 y = x\sin y + \frac{x^2}{2} + c.$$

Ex. 5 Solve $\frac{dy}{dx} = \frac{x + 2y + 3}{2x + 3y + 4}$

Sol. Put $x = X + h$, $y = Y + k$

We have $\frac{dY}{dX} = \frac{X + 2Y + (h + 2k + 3)}{2X + 3Y + (2h + 3k + 4)}$

To determine h and k, we write

$h + 2k + 3 = 0,\ 2h + 3k + 4 = 0 \quad \Rightarrow \quad h = 1,\ k = -2$

So that $\frac{dY}{dX} = \frac{X + 2Y}{2X + 3Y}$

Putting Y = VX, we get

$$V + X\frac{dV}{dX} = \frac{1 + 2V}{2 + 3V} \Rightarrow \frac{2 + 3V}{3V^2 - 1}dV = -\frac{dX}{X}$$

$$\Rightarrow \quad \left[\frac{2+\sqrt{3}}{2(\sqrt{3}V - 1)} - \frac{2-\sqrt{3}}{2(\sqrt{3}V + 1)}\right]dV = -\frac{dX}{X}$$

$$\Rightarrow \quad \frac{2+\sqrt{3}}{2\sqrt{3}}\log(\sqrt{3}\,V - 1) - \frac{2-\sqrt{3}}{2\sqrt{3}}\log(\sqrt{3}\,V + 1) = (-\log X + c)$$

$$\Rightarrow \quad \frac{2+\sqrt{3}}{2\sqrt{3}}\log(\sqrt{3}\,Y - X) - \frac{2-\sqrt{3}}{2\sqrt{3}}\log(\sqrt{3}\,Y + X) = A$$

where A is another constant and X = x – 1, Y = y + 2.

Ex. 6 Solve : $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

Sol. The equation can be written as

$$y - ay^2 = (x + a)\frac{dy}{dx}$$

$$\frac{dx}{x+a} = \frac{dy}{y - ay^2}$$

$$\frac{dx}{x+a} = \frac{1}{y(1-ay)}dy$$

$$\frac{dx}{x+a} = \left(\frac{1}{y} + \frac{a}{1-ay}\right)dy$$

Integrating both sides,

$$●n\,(x + a) = ●n\,y - ●n\,(1 - ay) + ●n\,c$$

$$●n\,(x + a) = ●n\left(\frac{cy}{1-ay}\right)$$

$$cy = (x + a)(1 - ay)$$

where 'c' is an arbitrary constant.

Ex.7 The solution of differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$ is -

Sol. The given differential equation is

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1} \quad \Rightarrow \quad \frac{dy}{dx} + \frac{2x}{x^2 - 1}y = \frac{1}{(x^2 - 1)^2} \qquad .....(i)$$

This is linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

$$\therefore \quad \text{I.F.} = e^{\int P\,dx} = e^{\int 2x/(x^2-1)dx} = e^{\log(x^2-1)} = (x^2 - 1)$$

multiplying both sides of **(i)** by I.F. $= (x^2 - 1)$, we get

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

integrating both sides we get

$$y(x^2 - 1) = \int \frac{1}{x^2 - 1}dx + C \qquad \left[\text{Using : } y\,(\text{I.F.}) = \int Q.(\text{I.F.})\,dx + C\right]$$

$$\Rightarrow \quad y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + C.$$

This is the required solution.

Ex. 8 Solve the differential equation $x\frac{dy}{dx} + y = x^3y^6$.

Sol. The given differential equation can be written as

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{xy^5} = x^2$$

Putting $y^{-5} = v$ so that

$$-5y^{-6}\frac{dy}{dx} = \frac{dv}{dx} \quad \text{or} \quad y^{-6}\frac{dy}{dx} = -\frac{1}{5}\frac{dv}{dx} \quad \text{we get}$$

$$-\frac{1}{5}\frac{dv}{dx} + \frac{1}{x}v = x^2 \Rightarrow \frac{dv}{dx} - \frac{5}{x}v = -5x^2 \qquad ......(i)$$

This is the standard form of the linear deferential equation having integrating factor

$$\text{I.F} = e^{\int -\frac{5}{x}dx} = e^{-5\log x} = \frac{1}{x^5}$$

Multiplying both sides of **(i)** by I.F. and integrating w.r.t. x

We get $v.\frac{1}{x^5} = \int -5x^2.\frac{1}{x^5}dx$

$$\Rightarrow \frac{v}{x^5} = \frac{5}{2}x^{-2} + c$$

$\Rightarrow y^{-5}x^{-5} = \frac{5}{2}x^{-2} + c$ which is the required solution.

Ex. 9 Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

Sol. $\frac{dy}{dx} = \sin(x + y)$

putting $x + y = t$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1$$

$\therefore \quad \frac{dt}{dx} - 1 = \sin t \quad \Rightarrow \quad \frac{dt}{dx} = 1 + \sin t \quad \Rightarrow \quad \frac{dt}{1+\sin t} = dx$

Integrating both sides,

$\int \frac{dt}{1+\sin t} = \int dx \quad \Rightarrow \quad \int \frac{1-\sin t}{\cos^2 t}\, dt = x + c \quad \Rightarrow \quad \int (\sec^2 t - \sec t \tan t)\, dt = x + c$

$$\tan t - \sec t = x + c$$

$$-\frac{1-\sin t}{\cos t} = x + c$$

$\Rightarrow \quad \sin t - 1 = x\cos t + c\cos t$ substituting the value of t

$$\sin(x + y) = x\cos(x + y) + c\cos(x + y) + 1$$

Ex. 10 Solve $2\frac{y}{x} + \left(\left(\frac{y}{x}\right)^2 - 1\right)\frac{dy}{dx}$

Sol. Putting $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$2v + (v^2 - 1)\left(v + x\frac{dv}{dx}\right) = 0$$

$$v + x\frac{dv}{dx} = -\frac{2v}{v^2 - 1}$$

$$x\frac{dv}{dx} = \frac{-v(1+v^2)}{v^2 - 1}$$

$$\int \frac{v^2 - 1}{v(1+v^2)}\, dv = -\int \frac{dx}{x}$$

$$\int \left(\frac{2v}{1+v^2} - \frac{1}{v}\right) dv = -\ell n\, x + c$$

$$\ell n\,(1 + v^2) - \ell n\, v = -\ell n\, x + c$$

$\ell n \left|\frac{1+v^2}{v}.\, x\right| = c \quad \Rightarrow \quad \ell n \left|\frac{x^2+y^2}{y}\right| = c$

$x^2 + y^2 = yc'$ where $c' = e^c$

Ex. 11 Prove that $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ are self orthogonal family of curves.

Sol. $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ ...**(i)**

Differentiating **(i)** with respect to x, we have

$$\frac{x}{a^2+\lambda}+\frac{y}{b^2+\lambda}\frac{dy}{dx}=0 \quad \text{...(ii)}$$

From **(i)** and **(ii)**, we have to eliminate λ.

Now, **(ii)** gives

$$\lambda=\frac{-\left[b^2x+a^2y\frac{dy}{dx}\right]}{x+y\frac{dy}{dx}}$$

$$\Rightarrow a^2+\lambda=\frac{(a^2-b^2)x}{x+y(dy/dx)},\quad b^2+\lambda=\frac{-(a^2-b^2)y(dy/dx)}{x+y(dy/dx)}$$

Substituting these values in **(i)**, we get

$$\left(x+y\frac{dy}{dx}\right)\left(x-y\frac{dx}{dy}\right)=a^2-b^2. \quad \text{....(iii)}$$

as the differential equation of the given family.

Changing dy/dx to –dx/dy in **(iii)**, we obtain

$$\left(x-y\frac{dx}{dy}\right)\left(x+y\frac{dy}{dx}\right)=a^2-b^2. \quad \text{...(iv)}$$

which is the same as **(iii)**. Thus we see that the family **(i)** as self-orthogonal, i.e., every member of the family **(i)** cuts every other member of the same family orthogonally.

Ex. 12 Solve the differential equation

$t(1+t^2)\,dx=(x+xt^2-t^2)\,dt$ and it given that $x=-\pi/4$ at $t=1$

Sol. $t(1+t^2)\,dx=[x(1+t^2)-t^2]\,dt$

$$\frac{dx}{dt}=\frac{x}{t}-\frac{t}{(1+t^2)}$$

$$\frac{dx}{dt}-\frac{x}{t}=-\frac{t}{1+t^2} \qquad \text{which is linear in } \frac{dx}{dt}$$

Here, $P=-\frac{1}{t}$, $Q=-\frac{t}{1+t^2}$ IF $=e^{-\int\frac{1}{t}dt}=e^{-\bullet nt}=\frac{1}{t}$

$\therefore$ General solution is -

$$x\cdot\frac{1}{t}=\int\frac{1}{t}\cdot\left(-\frac{t}{1+t^2}\right)dt+c$$

$$\frac{x}{t} = -\tan^{-1} t + c$$

putting $x = -\pi/4, t = 1$

$-\pi/4 = -\pi/4 + c \quad \Rightarrow \quad c = 0$

$\therefore \quad x = -t\tan^{-1} t$

Ex.13 Solve $\frac{y + \sin x \cos^2(xy)}{\cos^2(xy)} dx + \left(\frac{x}{\cos^2(xy)} + \sin y\right) dy = 0.$

Sol. The given differential equation can be written as;

$$\frac{y\,dx + x\,dy}{\cos^2(xy)} + \sin x\,dx + \sin y\,dy = 0.$$

$\Rightarrow \quad \sec^{27}(xy)\, d(xy) + \sin x\,dx + \sin y\,dy = 0$

$\Rightarrow \quad d(\tan(xy)) + d(-\cos x) + d(-\cos y) = 0$

$\Rightarrow \quad \tan(xy) - \cos x - \cos y = c.$

Ex. 14 Solve $(y \log x - 1)\, y dx = x dy.$

Sol. The given differential equation can be written as

$$x\frac{dy}{dx} + y = y^2 \log x \qquad .....(i)$$

Divide by xy^2. Hence $\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = \frac{1}{x}\log x$

Let $\frac{1}{y} = v \quad \Rightarrow \quad -\frac{1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$ so that $\frac{dv}{dx} - \frac{1}{x}v = -\frac{1}{x}\log x \qquad .....(ii)$

(ii) is the standard linear differential equation with $P = -\frac{1}{x}$, $Q = -\frac{1}{x}\log x$

I.F. $= e^{\int p dx} = e^{\int -1/x\, dx} = 1/x$

The solution is given by

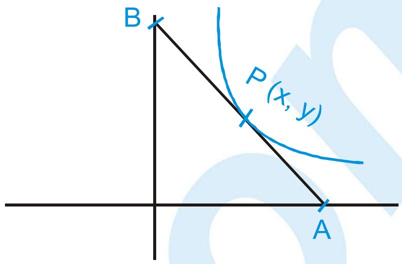
$$v.\frac{1}{x} = \int \frac{1}{x}\left(-\frac{1}{x}\log x\right) dx = -\int \frac{\log x}{x^2} dx = \frac{\log x}{x} - \int \frac{1}{x}.\frac{1}{x} dx = \frac{\log x}{x} + \frac{1}{x} + c$$

$\Rightarrow \quad v = 1 + \log x + cx = \log ex + cx$

or $\quad \frac{1}{y} = \log ex + cx$ or $y(\log ex + cx) = 1.$

Ex. 15 Find the curves for which the portion of the tangent included between the co-ordinate axes is isected at the point of contact.

Sol. Let P (x, y) be any point on the curve.



Equation of tangent at P (x, y) is -

$Y - y = m(X - x)$ where $m = \dfrac{dy}{dx}$

is slope of the tangent at P (x, y).

Co-ordinates of $A\left(\dfrac{mx - y}{m}, 0\right)$ & B (0, y–mx)

P is the middle point of A & B

$\therefore \quad \dfrac{mx - y}{m} = 2x \qquad \Rightarrow \quad mx - y = 2mx$

$\Rightarrow \quad mx = -y \qquad \Rightarrow \quad \dfrac{dy}{dx}x = -y$

$\Rightarrow \quad \dfrac{dx}{x} + \dfrac{dy}{y} = 0 \qquad \Rightarrow \quad \bullet nx + \bullet ny = \bullet nc$

$\therefore \quad xy = c$

Ex.16 For a certain curve y = f(x) satisfying $\dfrac{d^2y}{dx^2} = 6x - 4$, f(x) has a local minimum value 5 when x = 1. Find the equation of the curve and also the global maximum and global minimum values of f(x) given that $0 \le x \le 2$.

Sol. Integrating $\dfrac{d^2y}{dx^2} = 6x - 4$, we get $\dfrac{dy}{dx} = 3x^2 - 4x + A$

When x = 1, $\dfrac{dy}{dx} = 0$, so that A = 1.

Hence $\dfrac{dy}{dx} = 3x^2 - 4x + 1$ .....**(i)**

Integrating, we get $y = x^3 - 2x^2 + x + B$

When x = 1, y = 5, so that B = 5.

Thus we have $y = x^3 - 2x^2 + x + 5$.

From **(i)**, we get the critical points x = 1/3, x = 1

At the critical point $x = \dfrac{1}{3}$, $\dfrac{d^2y}{dx^2}$ is negative.

Therefore at x = 1/3, y has a local maximum.

At x = 1, $\dfrac{d^2y}{dx^2}$ is positive.

Therefore at x = 1, y has a local minimum.

Also $f(1) = 5$, $f\left(\dfrac{1}{3}\right) = \dfrac{139}{27}$. $f(0) = 5$, $f(2) = 7$

Hence the global maximum value = 7, and the global minimum value = 5.

Ex. 17 Solve $\frac{dy}{dx}$ = tany cotx – secy cosx.

Sol. $\frac{dy}{dx}$ = tany cotx – secy cosx.

Rearrange it :

(sin x – siny)cos x dx + sin x cosy dy = 0.

Put u = sin y, So, du = cos y dy :

Substituting, we get

$$(\sin x - u)\cos x\, dx + \sin x\, du = 0,\ \frac{du}{dx} - u\frac{\cos x}{\sin x} = -\cos x$$

The equation is first-order linear in u.

The integrating factor is

$$I = \exp\int -\frac{\cos x}{\sin x}dx = \exp\{-\ln(\sin x)\} = \frac{1}{\sin x}.$$

Hence, $u\frac{1}{\sin x} = -\int\frac{\cos x}{\sin x}dx = -\ln|\sin x| + C$.

Solve for u : u = –sin x ln |sin x|+ C sin x.

Put y back : siny = – sin x ln |sin x|+ C sin x.

Ex. 18 Show that (4x + 3y + 1) dx + (3x + 2y + 1) dy = 0 represents a hyperbola having the lines x + y = 0 and 2x + y + 1 = 0 as asymptotes

So. (4x + 3y + 1) dx + (3x + 2y + 1) dy = 0

4xdx + 3 (y dx + x dy) + dx + 2y dy + dy = 0

Integrating each term,

$2x^2 + 3xy + x + y^2 + y + c = 0$

$2x^2 + 3xy + y^2 + x + y + c = 0$

which is the equation of hyperbola when $h^2 > ab$ & $\Delta \neq 0$.

Now, combined equation of its asymptotes is -

$2x^2 + 3xy + y^2 + x + y + \lambda = 0$

which is pair of straight lines

$\therefore$ $\Delta = 0$

$\Rightarrow$ $2.1\,\lambda + 2 . \frac{1}{2}.\frac{1}{2}.\frac{3}{2} - 2 . \frac{1}{4} - 1 . \frac{1}{4} - \lambda\,\frac{9}{4} = 0$

$\Rightarrow$ $\lambda = 0$ $\therefore$ $2x^2 + 3xy + y^2 + x + y = 0$

$\Rightarrow$ $(x + y)(2x + y) + (x + y) = 0$ $\Rightarrow$ $(x + y)(2x + y + 1) = 0$

$\Rightarrow$ $x + y = 0$ or $2x + y + 1 = 0$

Ex. 19 Solve the equation $x\int_0^x y(t)dt = (x+1)\int_0^x t\,y(t)dt,\ x > 0$

Sol. Differentiating the equation with respect to x, we get

$$xy(x)+1.\int_0^x y(t)dt = (x+1)xy(x)+1.\int_0^x ty(t)dt$$

i.e., $\int_0^x y(t)dt = x^2 y(x)+\int_0^x ty(t)dt$

Differentiating again with respect to x, we get $y(x) = x^2 y'(x) + 2xy(x) + xy(x)$

i.e., $(1-3x)y(x) = \dfrac{x^2\,dy(x)}{dx}$

i.e., $\dfrac{(1-3x)dx}{x^2} = \dfrac{dy(x)}{y(x)}$, integrating we get

i.e., $y = \dfrac{c}{x^3}e^{-1/x}$

Ex. 20 The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1, 1)

Sol. Let P (x, y) be any point on the curve

Equation of tangent at 'P' is -

$$Y - y = m(X - x)$$

$$mX - Y + y - mx = 0$$

Now $\left(\dfrac{y-mx}{\sqrt{1+m^2}}\right) = x$

$$y^2 + m^2x^2 - 2mxy = x^2(1+m^2)$$

$\dfrac{y^2-x^2}{2xy} = \dfrac{dy}{dx}$ which is homogeneous equation

Putting $y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ $\quad\therefore\quad$ $v + x\dfrac{dv}{dx} = \dfrac{v^2-1}{2v}$

$x\dfrac{dv}{dx} = \dfrac{v^2-1-2v^2}{2v}$ $\quad\Rightarrow\quad$ $\displaystyle\int\frac{2v}{v^2+1}dv = -\int\frac{dx}{x}$

$$\ell n(v^2+1) = -\ell n\,x + \ell n\,c$$

$$x\left(\frac{y^2}{x^2}+1\right) = c$$

Curve is passing through (1, 1)

$\therefore$ $c = 2$

$$x^2 + y^2 - 2x = 0$$

Ex. 21 **(Discontinuous forcing)** Solve : $y'+\frac{3}{x}y = g(x)$, where $g(x)=\begin{cases}1 & \text{if } 0 \le x \le 1 \\ \frac{1}{x} & \text{if } x > 1\end{cases}$, and $y\left(\frac{1}{2}\right)=\frac{1}{8}$, and y(x) is continuous on $[0,\infty)$.

Sol. The idea is to solve the equation separately on $0 \le x \le 1$ and on $x > 1$, then match the pieces up at x = 1 to get a continuous solution.

$0 \le x \le 1 : y'+\frac{3}{x}y = 1$. The integrating factor is $I = \exp\int\frac{3}{x}dx = e^{3\ln x} = x^3$.

Then $yx^3 = \int x^3\,dx = \frac{1}{4}x^4 + C$.

The solution is $y = \frac{1}{4}x + \frac{C}{x^3}$

Plug in the initial condition $\frac{1}{8} = y\left(\frac{1}{2}\right) = \frac{1}{8} + 8C, C = 0$

The solution on the interval $0 \le x \le 1$ is $y = \frac{1}{4}x$.

Note that $y(1) = \frac{1}{4}$.

$x > 1 : y' + \frac{3}{x}y = \frac{1}{x}$. The integrating factor is the same as before, so $yx^3 = \int x^2\,dx = \frac{1}{3}x^3 + C$.

The solution is $y = \frac{1}{3} + \frac{C}{x^3}$.

In order, to get value of C, set $y(1) = \frac{1}{4} \Rightarrow \frac{1}{4} = y(1) = \frac{1}{3} + C$, $C = -\frac{1}{12}$

The solution on the interval x > 1 is $y = \frac{1}{3} - \frac{1}{12}\frac{1}{x^3}$

The complete solution is $y = \begin{cases}\frac{1}{4}x & \text{if } 0 \le x \le 1 \\ \frac{1}{3} - \frac{1}{12x^3} & \text{if } x > 1\end{cases}$

Ex. 22 Let $y = f(x)$ be a differentiable function $\forall x \in R$ and satisfies :

$f(x) = x + \int_0^1 x^2 z f(z)dz + \int_0^1 x z^2 f(z)\,dz$. Determine the function.

Sol. We have , $f(x) = x + x^2\int_0^1 z f(z)\,dz + x\int_0^1 z^2 f(z)dz$

Let $f(x) = x + x^2\lambda_1 + x\lambda_2$

Now $\lambda_1 = \int_0^1 z f(z)dz = \int_0^1 ((1+\lambda_2)z + z^2\lambda_1)z\,dz = \frac{1+\lambda_2}{3} + \frac{\lambda_1}{4} \Rightarrow 9\lambda_1 - 4\lambda_2 = 4$ .....**(i)**

also $\lambda_2 = \int_0^1 z^2 f(z)dz = \int_0^1 ((1+\lambda_2)z^3 + z^4\lambda_1)dz = \frac{(1+\lambda_2)}{4} + \frac{\lambda_1}{5} \Rightarrow 15\lambda_2 - 4\lambda_1 = 5$ ....**(ii)**

from **(i)** and **(ii)**;

$\lambda_1 = \frac{80}{119}$ and $\lambda_2 = \frac{61}{119} \Rightarrow f(x) = x + \frac{80}{119}x^2 + \frac{61}{119}x = \frac{20x}{119}(4 + 9x)$

Exercise # 1 [Single Correct Choice Type Questions]

1. The order and degree of the differential equation $\sqrt[3]{\frac{dy}{dx}} - 4\frac{d^2y}{dx^2} - 7x = 0$ are a and b, then a + b is -

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

2. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$, is

(A) 0 **(B)** 1 **(C)** 2 **(D)** more than 2

3. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} + f(x)\,y = 0$, then a solution of differential equation $\frac{dy}{dx} + f(x)\,y = r(x)$ is

(A) $\frac{1}{y(x)}\int y_1(x)\,dx$ **(B)** $y_1(x)\int \frac{r(x)}{y_1(x)}dx$ **(C)** $\int r(x)y_1(x)\,dx$ **(D)** none of these

4. A function $y = f(x)$ satisfies the differential equation $f(x)\cdot\sin 2x - \cos x + (1 + \sin^2 x)\,f'(x) = 0$ with initial condition $y(0) = 0$. The value of $f(\pi/6)$ is equal to

(A) 1/5 **(B)** 3/5 **(C)** 4/5 **(D)** 2/5

5. The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is

(A) $y e^{\frac{x}{y}} = e$ **(B)** $x e^{\frac{x}{y}} = e$ **(C)** $x e^{\frac{y}{x}} = e$ **(D)** $y e^{\frac{y}{x}} = e$

6. The solution of the differential equation, $e^x(x + 1)dx + (ye^y - xe^x)dy = 0$ with initial condition $f(0) = 0$, is

(A) $xe^x + 2y^2e^y = 0$ **(B)** $2xe^x + y^2e^y = 0$ **(C)** $xe^x - 2y^2e^y = 0$ **(D)** $2xe^x - y^2e^y = 0$

7. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\sin(x + C_3) - C_4\, e^{x+C_5}$ is

(A) 5 **(B)** 4 **(C)** 2 **(D)** 3

8. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality k > 0 depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \frac{1}{15}$ then the time to drain the tank if the water is 4 meter deep to start with is

(A) 30 min **(B)** 45 min **(C)** 60 min **(D)** 80 min

9. The equation of the curve passing through origin and satisfying the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ is -

(A) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$ **(B)** $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 + 3\tan 4x}\right) - \frac{5x}{3}$

(C) $y = \frac{1}{3}\tan^{-1}\left(\frac{3 + \tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$ **(D)** none of these

10. If the differential equation of the family of curve given by $y = Ax + Be^{2x}$ where A and B are arbitrary constant is of the form $(1-2x)\frac{d}{dx}\left(\frac{dy}{dx}+ly\right)+k\left(\frac{dy}{dx}+ly\right)=0$ then the ordered pair (k, l) is

(A) (2, – 2) **(B)** (–2, 2) **(C)** (2, 2) **(D)** (–2, – 2)

11. The area bounded by the curve $y = x\,e^{-x}$; $xy = 0$ and $x = c$ where c is the x-coordinate of the curve's inflection point, is

(A) $1 - 3e^{-2}$ **(B)** $1 - 2e^{-2}$ **(C)** $1 - e^{-2}$ **(D)** 1

12. The slope of a curve at any point is the reciprocal of twice the ordinate at that point and it passes through the point (4, 3). The equation of the curve is

(A) $x^2 = y + 5$ **(B)** $y^2 = x - 5$ **(C)** $y^2 = x + 5$ **(D)** $x^2 = y + 5$

13. A function $y = f(x)$ satisfies the condition $f'(x)\sin x + f(x)\cos x = 1$, $f(x)$ being bounded when $x \to 0$. If $I = \int_0^{\pi/2} f(x)dx$ then

(A) $\frac{\pi}{2} < I < \frac{\pi^2}{4}$ **(B)** $\frac{\pi}{4} < I < \frac{\pi^2}{2}$ **(C)** $1 < I < \frac{\pi}{2}$ **(D)** $0 < I < 1$

14. A curve is such that the area of the region bounded by the co-ordinate axes, the curve & the ordinate of any point on it is equal to the cube of that ordinate. The curve represents

(A) a pair of straight lines **(B)** a circle

(C) a parabola **(D)** an ellipse

15. The differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is a constant)

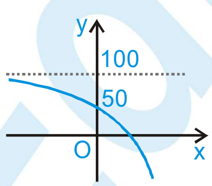
(A) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$ **(B)** $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

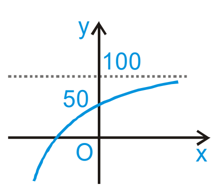
(C) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ **(D)** none of these

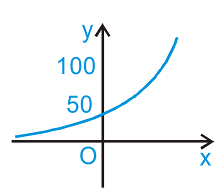
16. If the differentiable equation $\frac{dy}{dx} - y = y^2(\sin x + \cos x)$ with $y(0) = 1$ then $y(\pi)$ has the value equal to

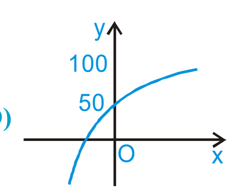
(A) e^{π} **(B)** $-e^{\pi}$ **(C)** $e^{-\pi}$ **(D)** $-e^{-\pi}$

17. Which one of the following curves represents the solution of the initial value problem Dy = 100 – y, where y(0) = 50

(A) 

(B) 

(C) 

(D) 

18. The value of the constant 'm' and 'c' for which $y = mx + c$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$.

(A) is $m = -1; c = 3/4$
(B) is $m = 1; c = -3/4$
(C) no such real m, c
(D) is $m = 1;\ c = 3/4$

19. The solution of the differential equation, $x^2 \frac{dy}{dx} .\cos\frac{1}{x} - y\sin\frac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$ is

(A) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$
(B) $y = \frac{x+1}{x\sin\frac{1}{x}}$
(C) $y = \cos\frac{1}{x} + \sin\frac{1}{x}$
(D) $y = \frac{x+1}{x\cos\frac{1}{x}}$

20. S_1 : The differential equation of parabolas having their vertices at origin and foci on the x-axis is a equation whose variables are separable

S_2 : Straight lines which are at a fixed distance p from origin is a differential equation of degree 2

S_3 : All conics whose axes coincide with the axes of coordinates is a equation of order 2

(A) TTT **(B)** TFT **(C)** FFT **(D)** TTF

21. The equation of a curve passing through (1, 0) for which the product of the abscissa of a point P & the intercept made by a normal at P on the x-axis equals twice the square of the radius vector of the point P, is

(A) $x^2 + y^2 = x^4$ **(B)** $x^2 + y^2 = 2x^4$ **(C)** $x^2 - y^2 = 4x^4$ **(D)** $x^2 - y^2 = x^4$

22. The latus rectum of the conic passing through the origin and having the property that normal at each point (x, y) intersects the $x-$axis at $((x+1), 0)$ is

(A) 1 **(B)** 2 **(C)** 4 **(D)** none

23. If $\phi(x) = \phi'(x)$ and $\phi(1) = 2$, then $\phi(3)$ equals

(A) e^2 **(B)** $2\ e^2$ **(C)** $3\ e^2$ **(D)** $2\ e^3$

24. The order & the degree of the differential equation whose general solution is, $y = c(x - c)^2$, are respectively

(A) 1, 1 **(B)** 1, 2 **(C)** 1, 3 **(D)** 2, 1

25. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k =

(A) –1 **(B)** 0 **(C)** 1 **(D)** 2

26. In a chemical reaction a substance changes into another such that the rate of decomposition of a chemical substance x present at instant t is proportional to x itself i.e. amount of unchanged substance still present. If half of the substance present initially has been converted at the end of 1 minute then the time t in minutes at the end of which 99 % of the substance will have changed lies in the interval

(A) 5 and 6 **(B)** 6 and 7 **(C)** 7 and 8 **(D)** more than 10

27. A function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} - y = \cos x - \sin x$, with initial condition that y is bounded when $x \to \infty$. The area enclosed by $y = f(x)$, $y = \cos x$ and the y-axis in the 1[st] quadrant

(A) $\sqrt{2} - 1$ **(B)** $\sqrt{2}$ **(C)** 1 **(D)** $\frac{1}{\sqrt{2}}$

28. The solution of the differential equation $(x^2 \sin^3 y - y^2 \cos x)\, dx + (x^3 \cos y \sin^2 y - 2y \sin x)\, dy = 0$ is

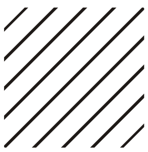
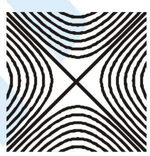
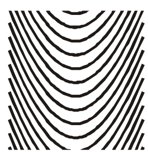
(A) $x^3 \sin^3 y = 3y^2 \sin x + C$ **(B)** $x^3 \sin^3 y + 3y^2 \sin x = C$

(C) $x^2 \sin^3 y + y^3 \sin x = C$ **(D)** $2x^2 \sin y + y^2 \sin x = C$

29. If $y = \frac{x}{\ln|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \varphi\left(\frac{x}{y}\right)$ then the function $\varphi\left(\frac{x}{y}\right)$ is :

(A) $\frac{x^2}{y^2}$ **(B)** $-\frac{x^2}{y^2}$ **(C)** $\frac{y^2}{x^2}$ **(D)** $-\frac{y^2}{x^2}$

30. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following ?

(A)  **(B)**  **(C)**  **(D)** 

31. The equation to the orthogonal trajectories of the system of parabolas $y = ax^2$ is

(A) $\frac{x^2}{2} + y^2 = c$ **(B)** $x^2 + \frac{y^2}{2} = c$ **(C)** $\frac{x^2}{2} - y^2 = c$ **(D)** $x^2 - \frac{y^2}{2} = c$

32. If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then function y is

(A) $e^{(1-x)^2/2}$ **(B)** $e^{(1+x)^2/2} - 1$ **(C)** $\log_e(1+x) - 1$ **(D)** $1 + x$

33. The equation of the trajectories which is orthogonal to the family of curves $\cos y = ae^{-x}$ is

(A) $\sin y = c\, e^x$ **(B)** $\cos y = c\, e^x$ **(C)** $\sin y = c\, e^{-x}$ **(D)** None

34. A function $y = f(x)$ satisfies $(x+1) . f'(x) - 2(x^2 + x) f(x) = \frac{e^{x^2}}{(x+1)}$, $\forall\ x > -1$

If $f(0) = 5$, then $f(x)$ is

(A) $\left(\frac{3x+5}{x+1}\right).e^{x^2}$ **(B)** $\left(\frac{6x+5}{x+1}\right).e^{x^2}$ **(C)** $\left(\frac{6x+5}{(x+1)^2}\right).e^{x^2}$ **(D)** $\left(\frac{5-6x}{x+1}\right).e^{x^2}$

35. The differential equations of all conics whose centre lie at the origin is of order :

(A) 2 (B) 3 (C) 4 (D) none of these

36. A curve passing through (2, 3) and satisfying the differential equation $\int_0^x t\,y(t)\,dt = x^2 y(x), \ (x > 0)$ is

(A) $x^2 + y^2 = 13$ (B) $y^2 = \frac{9}{2}x$ (C) $\frac{x^2}{8} + \frac{y^2}{18} = 1$ (D) $xy = 6$

37. The differential equation for all the straight lines which are at a unit distance from the origin is

(A) $\left(y - x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$ (B) $\left(y + x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

(C) $\left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$ (D) $\left(y + x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

38. A curve $y = f(x)$ passing through the point $\left(1, \frac{1}{\sqrt{e}}\right)$ satisfies the differential equation $\frac{dy}{dx} + x\,e^{-\frac{x^2}{2}} = 0$. Then which of the following does not hold good ?

(A) $f(x)$ is differentiable at $x = 0$.

(B) $f(x)$ is symmetric w.r.t. the origin.

(C) $f(x)$ is increasing for $x < 0$ and decreasing for $x > 0$.

(D) $f(x)$ has two inflection points.

39. The solution of $\frac{x\,dy}{x^2 + y^2} = \left(\frac{y}{x^2 + y^2} - 1\right) dx$ is -

(A) $y = x\cot(c - x)$ (B) $\cos^{-1} y/x = -x + c$

(C) $y = x\tan(c - x)$ (D) $y^2/x^2 = x\tan(c - x)$

40. A function $f(x)$ satisfying $\int_0^1 f(tx)\,dt = n f(x)$, where $x > 0$, is

(A) $f(x) = c \cdot x^{\frac{1-n}{n}}$ (B) $f(x) = c \cdot x^{\frac{n}{n-1}}$ (C) $f(x) = c \cdot x^{\frac{1}{n}}$ (D) $f(x) = c \cdot x^{(1-n)}$

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Which one of the following is homogeneous function ?

(A) $f(x, y) = \dfrac{x - y}{x^2 + y^2}$ **(B)** $f(x, y) = x^{\frac{1}{3}}.y^{-\frac{2}{3}} \tan^{-1}\dfrac{x}{y}$

(C) $f(x, y) = x(\ln\sqrt{x^2 + y^2} - \ln y) + ye^{x/y}$ **(D)** $f(x, y) = x\left[\ln\dfrac{2x^2 + y^2}{x} - \ln(x + y)\right] + y^2 \tan\dfrac{x + 2y}{3x - y}$

2. The equation of the curve such that the distance between the origin & the tangent line at an arbitrary point is equal to the distance between the origin & the normal at the same point is :

(A) $x^2 + y^2 = c \, . \, e^{\arctan(y/x)}$ **(B)** $\sqrt{x^2 + y^2} = c \, . \, e^{-\arctan(y/x)}$

(C) $\sqrt{x^2 + y^2} \; c \, . \, e^{\arctan(y/x)}$ **(D)** $x^2 + y^2 = c \, . \, e^{-\arctan(y/x)}$

3. If $y = e^{-x} \cos x$ and $y_n + k_n y = 0$, where $y_n = \dfrac{d^n y}{dx^n}$ and k_n, $n \in N$ are constants.

(A) $k_4 = 4$ **(B)** $k_8 = -16$ **(C)** $k_{12} = 20$ **(D)** $k_{16} = -24$

4. The value of the constant 'm' and 'c' for which $y = mx + c$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$

(A) is $m = -1$ **(B)** is $c = 3/4$ **(C)** is $m = 1$ **(D)** is $c = -3/4$

5. Solution of the differential equation $\dfrac{dy}{dx} + \dfrac{1 + y^2}{\sqrt{1 - x^2}} = 0$ is

(A) $\tan^{-1} y + \sin^{-1} x = c$ **(B)** $\tan^{-1} x + \sin^{-1} y = c$

(C) $\tan^{-1} y \, . \, \sin^{-1} x = c$ **(D)** $\cot^{-1}\dfrac{1}{y} + \cos^{-1}\sqrt{1 - x^2} = c$

6. Identify the statement(s) which is/are True.

(A) $f(x, y) = e^{y/x} + \tan\dfrac{y}{x}$ is a homogeneous function of degree zero

(B) $x \, . \, \ln\dfrac{y}{x} \, dx + \dfrac{y^2}{x} \sin^{-1}\dfrac{y}{x} \, dy = 0$ is a homogeneous differential equation of degree one

(C) $f(x, y) = x^2 + \sin x \, . \cos y$ is not homogeneous function.

(D) $(x^2 + y^2) \, dx - (xy^2 - y^3) \, dy = 0$ is a homegeneous differential equation.

7. The graph of the function $y = f(x)$ passing through the point (0, 1) and satisfying the differential equation $\dfrac{dy}{dx} + y\cos x = \cos x$ is such that -

(A) it is a constant function **(B)** it is periodic

(C) it is neither an even nor an odd function **(D)** it is continuous & differentiable for all x.

8. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is -

(A) $\log\left(\frac{x}{y}\right) = cy$ **(B)** $\log\left(\frac{y}{x}\right) = cx$ **(C)** $y = xe^{cx}$ **(D)** $x = ye^{cx}$

9. A function $y = f(x)$ satisfying the differential equation $\frac{dy}{dx} . \sin x - y\cos x + \frac{\sin^2 x}{x^2} = 0$ is such that, $y \to 0$ as $x \to \infty$ then the statement which is correct is

(A) $\underset{x \to 0}{\text{Limit}}\ f(x) = 1$ **(B)** $\int_0^{\pi/2} f(x)\,dx$ is less than $\frac{\pi}{2}$

(C) $\int_0^{\pi/2} f(x)\,dx$ is greater than unity **(D)** f(x) is an odd function

10. The solution of $(x + y + 1)\,dy = dx$ are

(A) $x + y + 2 = Ce^y$ **(B)** $x + y + 4 = = C\log y$

(C) $\log(x + y + 2) = Cy$ **(D)** $\log(x + y + 2) = C + y$

11. The differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y + x^2 = 0$ is of the following type -

(A) linear **(B)** homogeneous **(C)** order two **(D)** degree one

12. Solutions of the differential equation $x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - 6y^2 = 0$ -

(A) $y = cx^2$ **(B)** $x^3 y = c$ **(C)** $xy^3 = c$ **(D)** $y = cx$

13. Let $\frac{dy}{dx} + y = f(x)$ where y is a continuous function of x with y (0) = 1 and $f(x) = \left[\begin{array}{ll} e^{-x} & \text{if } 0 \le x \le 2 \\ e^{-2} & \text{if } x > 2 \end{array}\right.$.
Which of the following hold(s) good ?

(A) $y(1) = 2e^{-1}$ **(B)** $y'(1) = -e^{-1}$ **(C)** $y(3) = -2e^{-3}$ **(D)** $y'(3) = -2e^{-3}$

14. The function f(x) satisfying the equation, $f^2(x) + 4f'(x) . f(x) + [f'(x)]^2 = 0$ is -

(A) $f(x) = c . e^{(2-\sqrt{3})x}$ **(B)** $f(x) = c . e^{(2+\sqrt{3})x}$ **(C)** $f(x) = c . e^{(\sqrt{3}-2)x}$ **(D)** $f(x) = c . e^{-(2+\sqrt{3})x}$

15. The solution of $x^2 y_1^2 + xy\,y_1 - 6y^2 = 0$ are

(A) $y = Cx^2$ **(B)** $x^2 y = C$ **(C)** $\frac{1}{2}$ ●n y = C+ log x **(D)** $x^3 y = C$

16. The solution the differential equation $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}\left(e^x + e^{-x}\right) + 1 = 0$ is are -

(A) $y + e^{-x} = c$ **(B)** $y - e^{-x} = c$ **(C)** $y + e^x = c$ **(D)** $y - e^x = c$

17. The equation of the curve passing through (3, 4) & satisfying the differential equation,

$y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$ can be -

(A) $x - y + 1 = 0$ **(B)** $x^2 + y^2 = 25$

(C) $x^2 + y^2 - 5x - 10 = 0$ **(D)** $x + y - 7 = 0$

18. The tangent at any point P on a curve $f(x, y) = 0$ cuts the y-axis at T. If the distance of the point T from P equals the distance of T from the origin then the curve with this property represents a family of circles. Which of the following is/are correct ?

(A) Any arbitrary line $y = mx$ cuts every member of this family at the points where the slopes of these members are equal.

(B) $f(x, y) = 0$ is orthogonal to the family of circles $x^2 + y^2 - ky = 0 \ \forall \ k \in R$

(C) If $f(x, y) = 0$ passes through (2, 2) then the intercept made by its director circle on the y-axis is equal to 8.

(D) If $f(x, y) = 0$ passes through (–1, 1) then image of its centre in the line $y = x$, is (1, 0)

19. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represent a parabola if -

(A) $a = -2, b = 0$ **(B)** $a = -2, b = 2$ **(C)** $a = 0, b = 2$ **(D)** $a = 0, b = 0$

20. Let $y = (A + Bx)e^{3x}$ be a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0$, $m, n \in I$, then -

(A) $m + n = 3$ **(B)** $n^2 - m^2 = 64$ **(C)** $m = -6$ **(D)** $n = 9$

21. Let C be the family of curves $f(x, y, c) = 0$ (no member of C is x-axis) such that length of subnormal at any point P(x, y) on the curve C is equal to four times that of the length of subtangent at the same point. Which of the following statement(s) is(are) correct ?

(A) Equation of the line with positive y-intercept passing through (4, 2) and perpendicular to the curve C is $x + 2y = 8$.

(B) Orthogonal trajectory of C is family of parallel lines having gradient ± 2.

(C) Order and degree of the differential equation of family of curves C are 1 and 2 respectively.

(D) Differential equation of family of curves is $2y' \pm x = 0$.

22. A normal is drawn at a point P(x, y) of a curve. It meets the x-axis and the y-axis in point A and B, respectively, such that $\frac{1}{OA} + \frac{1}{OB} = 1$, where O is the origin, the equation of such a curve is a circle which passes through (5, 4) and has -

(A) centre (1, 1) **(B)** centre (2, 1) **(C)** radius 5 **(D)** radius 4

23. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{4}{x}$ are -

(A) $9(y + c)^2 = x^3$ **(B)** $y + c = \frac{-x^{3/2}}{3}$ **(C)** $y + c = \frac{x^{3/2}}{3}$ **(D)** all of these

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** The order of the differential equation whose primitive is $y = A + \ell nBx$ is 2

Statement-II : If there are 'n' independent arbitrary constants in a family of curve then the order of the corresponding differential equation is 'n'.

2. A curve C has the property that its initial ordinate of any tangent drawn is less than the abscissa of the point of tangency by unity.

Statement-I: Differential equation satisfying the curve is linear.

Statement-II: Degree of differential equation is one

3. **Statement-I:** Solution of $(1 + x\sqrt{x^2+y^2})\,dx + y(-1 + \sqrt{x^2+y^2})\,dy = 0$ is $x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} + c = 0$

Statement-II : Solution of $(1 + xy)\,y\,dx + (1 - xy)\,x\,dy = 0$ is $\ell n\frac{x}{y} - \frac{1}{xy} = c$

4. **Statement-I :** The orthogonal trajectory to the curve $(x - a)^2 + (y - b)^2 = r^2$ is $y = mx + b - am$ where a and b are fixed numbers and r & m are parameters.

Statement-II : In a plane, the line that passes through the centre of circle is normal to the circle.

5. **Statement-I :** The equation of the curve passing through (3, 9) which satisfies differential equation

$$\frac{dy}{dx} = x + \frac{1}{x^2} \text{ is } 6xy = 3x^3 + 29x - 6$$

Statement-II : The solution of D.E. $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$

6. **Statement-I:** Differential equation corresponding to all lines, $ax + by + c = 0$ has the order 3.

Statement-II: General solution of a differential equation of n^{th} order contains n independent arbitrary constants.

7. **Statement-I :** The solution of D.E. $\dfrac{x\frac{dy}{dx} - y}{\sqrt{x^2 - y^2}} = mx^2$ is given by $\tan^{-1}\frac{y}{x} = \frac{mx^2}{2} + c$

Statement-II : The solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is $x(y + \cos x) = \sin x + c$

8. **Statement-I :** Integral curves denoted by the first order linear differential equation $\frac{dy}{dx} - \frac{1}{x}y = -x$ are family of parabolas passing through the origin.

Statement-II : Every differential equation geometrically represents a family of curve having some common property.

9. **Statement-I :** $\sin x\frac{d^2y}{dx^2}+\cos x\frac{dy}{dx}+\tan x=0$ is not a linear differential equation.

Statement-II : A differential equation is said to be linear if dependent variable and its differential coefficients occurs in first degree and are not multiplied together.

10. **Statement-I :** The line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at some point $x=x_0$.

Statement-II : $\frac{dy}{dx}$ exists at $x=x_0$.

11. **Statement-I :** Solution of the differential equation $y-x\frac{dy}{dx}=y^2+\frac{dy}{dx}$ is $y=c(1-y)(x+1)$

Statement-II : D.E. $\frac{dy}{dx}=f(x).g(y)$ can be solved by seperating variables. $\frac{dy}{g(y)}=f(x)\,dx$

12. **Statement-I :** The solution of $(y\,dx-x\,dy)\cot\left(\frac{x}{y}\right)=ny^2\,dx$ is $\sin\left(\frac{x}{y}\right)=ce^{nx}$

Statement- II : Such type of differential equations can only be solved by the substitution x = vy.

13. Consider the curves $C_1: x^2-\frac{y^2}{3}=a^2$ and $C_2: xy^3=c$

Statement-I : C_1 and C_2 are orthogonal curves.

Statement-II : C_1 and C_2 intersect at right angles everywhere wherever they intersect.

14. Consider the differential equation $(xy-1)\frac{dy}{dx}+y^2=0$

Statement-I : The solution of the equation is xy = logy + c.

Statement-II : The given differential equation can be expressed as $\frac{dx}{dy}+Px=Q$, whose integrating factor is ●ny.

15. Consider a differentiable function y = f (x) which satisfies $f(x)=\int_0^x (f(t)\sin t-\sin(t-x))dt$

Statement-I : The differential equation corresponding to y = f (x) is a first order linear differential equation.

Statement-II : The differential equation corresponding to y = f (x) is of degree one.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1.

Column - I		Column - II	
(A)	Solution of $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$ is	(p)	$xy^2 = 2y^5 + c$
(B)	Solution of $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is	(q)	$\sec y = x + 1 + ce^x$
(C)	Solution of $\sec^2 y\, dy + \tan y\, dx = dx$ is	(r)	$(x + 1)(1 - y) = cy$
(D)	Solution of $\sin y \frac{dy}{dx} = \cos y\,(1 - x\cos y)$ is	(s)	$\tan y = 1 + ce^{-x}$

2. Match the properties of the curves given in column-I with the corresponding curve(x) given in the column-II.

Column-I		Column-II	
(A)	A curve passing through (2, 3) having the property that length of the radius vector of any of its point P is equal to the length of the tangent drawn at this point, can be	(p)	Straight line
(B)	A curve passing through (1, 1) having the property that any tangent intersects the y-axis at the point which is equidistant from the point of tangency and the origin, can be	(q)	Circle
(C)	A curve passing through (1, 0) for which the length of normal is equal to the radius vector, can be	(r)	Parabola
(D)	A curve passes through the point (2, 1) and having the property that the segment of any of its tangent between the point of tangency and the x-axis is bisected by the y-axis, can be	(s)	Hyperbola

3.

Column - I		Column - II	
(A)	$xdy = y(dx + ydy)$, $y(1) = 1$ and $y(x_0) = -3$, then $x_0 =$	(p)	$\frac{1}{4}$
(B)	If y(t) is solution of $(t + 1)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, then $y(1) =$	(q)	-15
(C)	$(x^2 + y^2)\,dy = xydx$ and $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$	(r)	$-\frac{1}{2}$
(D)	$\frac{dy}{dx} + \frac{2y}{x} = 0$, $y(1) = 1$, then $y(2) =$	(s)	$\sqrt{3}\,e$

4. **Column – I** | **Column – II**

Column – I		Column – II	
(A)	The degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} - \frac{5dy}{dx} = 0$ is :	**(p)**	1
(B)	Find degree of $x^3 \frac{d^2y}{dx^2} - \ln\left(x.\frac{dy}{dx}\right) + 1 = 0$	**(q)**	2
(C)	Order of the differential equation $y\frac{dy}{dx} + x^3\left(\frac{d^2y}{dx^2}\right) + xy = \cos x$	**(r)**	3
(D)	Order of the differential equation $y = \frac{ax}{bx+c}$, where a, b, c are arbitrary constants.	**(s)**	none

Part # II [Comprehension Type Questions]

Comprehension # 1

A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B . Both the reservoirs are filled completely with water , their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released , the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B.

Let V_A & V_B represents volume of reservoir A & B at any time t, then :

On the basis of above information, answer the following questions :

1. If after 1/2 an hour $V_A = kV_B$, then k is -

(A) 3 **(B)** 3/4 **(C)** $\sqrt{3}$ **(D)** none of these

2. After how many hours do both the reservoirs have the same quantity of water ?

(A) $\log_{4/3} 2$ hrs **(B)** $\log_{(4/3)} 4$ hrs **(C)** 2 hrs **(D)** $\frac{1}{2-\log_2 3}$ hrs

3. If $\frac{V_A}{V_B} = f(t)$, where 't' is time. Then f(t) is -

(A) increasing **(B)** decreasing **(C)** non-monotonic **(D)** data insufficient.

Comprehension # 2

A curve y = f (x) satisfies the differential equation $(1 + x^2)\frac{dy}{dx} + 2yx = 4x^2$ and passes through the origin.

1. The function y = f (x)

(A) is strictly increasing $\forall\ x \in R$.
(B) is such that it has a minima but no maxima.
(C) is such that it has a maxima but no minima.
(D) has no inflection point.

2. The area enclosed by $y = f^{-1}(x)$, the x-axis and the ordinate at x = 2/3 is

(A) $2\,\ln 2$ **(B)** $\frac{4}{3}\ln 2$ **(C)** $\frac{2}{3}\ln 2$ **(D)** $\frac{1}{3}\ln 2$

3. For the function y = f (x) which one of the following does not hold good ?

(A) f (x) is a rational function
(B) f (x) has the same domain and same range.
(C) f (x) is a transcidental function
(D) y = f (x) is a bijective mapping.

Comprehension # 3

Differential equations are solved by reducing them to the exact differential of an expression in x & y i.e., they are reduced to the form d(f(x, y)) = 0

e.g. : $\frac{xdx + ydy}{\sqrt{x^2+y^2}} = \frac{ydx - xdy}{x^2}$

$$\Rightarrow \frac{1}{2}\frac{2xdx + 2ydy}{\sqrt{x^2+y^2}} = -\frac{xdy - ydx}{x^2} \Rightarrow d\left(\sqrt{x^2+y^2}\right) = -d\left(\frac{y}{x}\right) \Rightarrow d\left(\sqrt{x^2+y^2} + \frac{y}{x}\right) = 0$$

$\therefore$ solution is $\sqrt{x^2+y^2} + \frac{y}{x} = c$.

Use the above method to answer the following question (1 to 3)

1. The general solution of $(2x^3 - xy^2)\,dx + (2y^3 - x^2y)\,dy = 0$ is

(A) $x^4 + x^2y^2 - y^4 = c$
(B) $x^4 - x^2y^2 + y^4 = c$
(C) $x^4 - x^2y^2 - y^4 = c$
(D) $x^4 + x^2y^2 + y^4 = c$

2. General solution of the differential equation $\frac{xdy}{x^2+y^2} + \left(1 - \frac{y}{x^2+y^2}\right)dx = 0$ is

(A) $x + \tan^{-1}\left(\frac{y}{x}\right) = c$ **(B)** $x + \tan^{-1}\frac{x}{y} = c$ **(C)** $x - \tan^{-1}\left(\frac{y}{x}\right) = c$ **(D)** none of these

3. General solution of the differential equation $e^y\,dx + (xe^y - 2y)\,dy = 0$ is

(A) $xe^y - y^2 = c$
(B) $ye^x - x^2 = c$
(C) $ye^y + x = c$
(D) $xe^y - 1 = cy^2$

Comprehension # 4

Let C be a curve $y = f(x)$ passing through $M\left(-\sqrt{3}, 1\right)$ such that the y-intercept of the normal at any point P(x, y) on the curve C is equal to the distance of P from the origin.

1. Which one of the following statement is correct ?

(A) f(x) has exactly two points of inflection.

(B) f(x) has maxima but no minima.

(C) The equation $f(x) = -2$ has two distinct real solutions.

(D) Inclination of the tangent line drawn to f(x) at (1, 0) is $\tan^{-1}1$.

2. The area enclosed by the curve C and the co-ordinate axes is equal to

(A) $\frac{1}{6}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{1}{3}$

3. The shortest distance between the curve C and the circle $x^2 + y^2 - 2y = 0$ is equal to

(A) $\frac{1}{2}$ **(B)** $\sqrt{2}-1$ **(C)** $\sqrt{6}-1$ **(D)** $\sqrt{5}-1$

Comprehension # 5

Consider three real-valued functions f, g and h defined on R (the set of real numbers).

Let $f(x) = 2x^3 + 3\left(1-\frac{3a}{2}\right)x^2 + 3(a^2-a)x + b$ where $a, b \in R$ and $g(x) = \frac{f'(x)}{6}$.

Also h(x) is such that $h''(x) = 6x - 4$ and h(x) has a local minimum value 5 at $x = 1$.

1. The true set of values of a for which f(x) has negative point of local minimum, is

(A) $(-\infty, 0)$ **(B)** $(1, \infty)$ **(C)** $(0, 1)$ **(D)** $(1, \infty) - \{2\}$

2. The complete set of values of a for which vertex of parabola $y = g(x)$ has negative ordinate, is

(A) $\left(\frac{1}{2}, 1\right)$ **(B)** $(0, \infty)$ **(C)** $R - \{2\}$ **(D)** $(1, \infty) - \{2\}$

3. The area bounded by $y = h(x)$ between $x = 0$ and $x = 2$, is

(A) $\frac{23}{3}$ **(B)** $\frac{20}{3}$ **(C)** $\frac{40}{3}$ **(D)** $\frac{32}{3}$

Comprehension # 6

In order to solve the differential equation of the form $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + ... + a_n y = 0$, where a_0, a_1, a_2 are constants.

We take the auxiliary equation $a_0 D^n + a_1 D^{n-1} + ... + a_n = 0$

Find the roots of this equation and then solution of the given differential equation will be as given in the following table.

Roots of the auxiliary equation		Corresponding complementary function
1.	One real root α_1	$c_1 e^{\alpha_1 x}$
2.	Two real and different roots α_1 and α_2	$c_1 e^{\alpha_1 x} + c_2 e^{\alpha_2 x}$
3.	Two real and equal roots α_1 and α_1	$(c_1 + c_2 x)\ e^{\alpha_1 x}$
4.	Three real and equal roots $\alpha_1, \alpha_1, \alpha_1$	$(c_1 + c_2 x + c_3 x^2)\ e^{\alpha_1 x}$
5.	One pair of imaginary roots $\alpha \pm i\beta$	$(c_1 \cos \beta x + c_2 \sin \beta x)\ e^{\alpha x}$
6.	Two pair of equal imaginary roots $\alpha \pm i\beta$ and $\alpha \pm i\beta$	$[(c_1 + c_2 x) \cos \beta x + (c_1 + c_2 x) \sin \beta x]\ e^{\alpha x}$

Solution of the given differential equation will be y = sum of all the corresponding parts of the complementary functions.

1. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$

(A) $y = (c_1 + c_2 x)e^x$ **(B)** $y = (c_1 e^x + c_2 e^x)$ **(C)** $y = (c_1 x)e^x$ **(D)** none of these

2. Solve $\frac{d^2y}{dx^2} + a^2 y = 0.$

(A) $y = (c_1 \cos ax + c_2 \sin ax)e^{ax}$ **(B)** $y = c_1 \cos ax + c_2 \sin ax$

(C) $y = c_1 e^{ax} + c_2 e^{-ax}$ **(D)** none of these

3. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

(A) $y = (c_1 + c_2 x + c_3 x^2)\quad e^x$ **(B)** $y = x\,(c_1 e^x + c_2 e^{2x} + c_3 e^{3x})$

(C) $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$ **(D)** none of these

Comprehension # 7

Let y = f(x) and y = g(x) be the pair of curves such that

(i) the tangents at point with equal abscissae intersect on y-axis.

(ii) the normals drawn at points with equal abscissae intersect on x-axis and

(iii) curve f(x) passes through (1, 1) and g(x) passes through (2, 3) then

On the basis of above information, answer the following questions :

1. The curve f(x) is given by -

(A) $\frac{2}{x} - x$ **(B)** $2x^2 - \frac{1}{x}$ **(C)** $\frac{2}{x^2} - x$ **(D)** none of these

2. The curve g(x) is given by -

(A) $x - \frac{1}{x}$ **(B)** $x + \frac{2}{x}$ **(C)** $x^2 - \frac{1}{x^2}$ **(D)** none of these

3. The value of $\int_1^2 (g(x) - f(x))dx$ is -

(A) 2 **(B)** 3 **(C)** 4 **(D)** $4\ \ell n2$

Exercise # 4 [Subjective Type Questions]

1. Find the order and degree of the following differential equations -

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y^4 = 0$

(ii) $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 = y$

(iii) $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

(iv) $\left(\frac{dy}{dx}\right) + y = \frac{1}{\frac{dy}{dx}}$

(v) $e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$

(vi) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/2} = x\frac{d^3y}{dx^3}$

(vii) $\frac{d^2y}{dx^2} = \sin\left(x + \frac{dy}{dx}\right)$

2.

(I) Form a differential equation for the family of curves represented by $ax^2 + by^2 = 1$, where a & b are arbitrary constants.

(II) Obtain the differential equation of the family of circles $x^2 + y^2 + 2gx + 2fy + c = 0$; where g , f & c are arbitrary constants.

(III) Form the differential equation of circles passing through the points of intersection of unit circle with centre at the origin and the line bisecting the first quadrant.

(IV) Obtain the differential equation associated with the primitive , $y = c_1e^{3x} + c_2e^{2x} + c_3e^x$, where c_1 , c_2 , c_3 are arbitrary constants.

3.

(i) The temperature T of a cooling object drops at a rate which is proportional to the difference T – S, where S is constant temperature of the surrounding medium.

Thus, $\frac{dT}{dt} = -k(T - S)$, where $k > 0$ is a constant and t is the time. Solve the differential equation if it is given that $T(0) = 150$.

(ii) The surface area of a spherical balloon, being inflated changes at a rate proportional to time t. If initially its radius is 3 units and after 2 seconds it is 5 units, find the radius after t seconds.

(iii) The slope of the tangent at any point of a curve is λ times the slope of the straight line joining the point of contact to the origin. Formulate the differential equation representing the problem and hence find the equation of the curve.

4. Consider the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$

(A) If two particular solutions of given equation u(x) and v(x) are known, find the general solution of the same equation in term of u(x) and v(x).

(B) If α and β are constants such that the linear combinations $\alpha.u(x) + \beta.v(x)$ is a solution of the given equation, find the relation between α and β.

(C) If w(x) is the third particular solution different from u(x) and v(x) then find the ratio $\frac{v(x) - u(x)}{w(x) - u(x)}$.

5. Let C be a curve passing through M (2, 2) such that the slope of the tangent at any point to the curve is reciprocal of the ordinate of the point. If the area bounded by curve C and line $x = 2$ is expressed as a rational $\frac{p}{q}$ (where p and q are in their lowest form), then find $(p + q)$.

6. Find all the curves possessing the following property; the segment of the tangent between the point of tangency & the x–axis is bisected at the point of intersection with the y–axis .

7. Solve the following differential equations (I - XV)

(I) $(x + \tan y)\, dy = \sin 2y\, dx$

(II) $\frac{dy}{dx} + \frac{x}{1+x^2}\, y = \frac{1}{2x(1+x^2)}$

(III) $(1-x^2)\frac{dy}{dx} + 2xy = x(1-x^2)^{1/2}$

(IV) $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$

(V) $(1 + y + x^2y)dx + (x + x^3)dy = 0$

(VI) $y - x\,Dy = b(1 + x^2Dy)$

(VII) $\frac{dy}{dx} + \frac{y}{x}\,\ell n y = \frac{y}{x^2}(\ell n y)^2$

(VIII) $\frac{dy}{dx} + xy = y^2e^{x^2/2}.\sin x$

(IX) $2\frac{dy}{dx} - y\sec x = y^3\tan x$

(X) $x^2y - x^3\frac{dy}{dx} = y^4\cos x$

(XI) $y(2xy + e^x)\,dx - e^x\,dy = 0$

(XII) $\sin x\frac{dy}{dx} + 3y = \cos x$

(XIII) $x(x^2+1)\frac{dy}{dx} = y(1-x^2) + x^3.\ell n x$

(XIV) $x\frac{dy}{dx} - y = 2x^2\operatorname{cosec} 2x$

(XV) $(1 + y^2)\,dx = (\tan^{-1}y - x)dy$

8. Find the equation of a curve such that the projection of its ordinate upon the normal is equal to its abscissa.

9. Solve : $\frac{dy}{dx} = \frac{2(y+2)^2}{(x+y-1)^2}$

10. Find the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point.

11. Solve : **(A)** $\frac{dy}{dx} = \frac{x^2 + xy}{x^2 + y^2}$ **(B)** $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$

12. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve.

13. Let y_1 and y_2 are two different solutions of the equation $y' + P(x)\,.\,y = Q(x)$.

(i) Prove that $y = y_1 + C(y_2 - y_1)$ is the general solution of the same equation (C is a constant)

(ii) Find the relationship between the constants α and β, such that the linear combination $\alpha y_1 + \beta y_2$ be a solution of the given equation.

14. Let the function $\ell n f(x)$ is defined where f(x) exists for $x \geq 2$ & k is fixed positive real number, prove that if $\frac{d}{dx}(x.f(x)) \leq -kf(x)$ then $f(x) \leq A\,x^{-1-k}$ where A is independent of x.

15. Let l be the line created by rotating the tangent line to the parabola $y = x^2$ at the point A(1, 1), about A by an angle of $(-\pi/4)$. Let B be the other intersection of the line l with $y = x^2$. If the area enclosed by the line l and the parabola is $\frac{a_1}{a_2}$ where a_1 and a_2 are coprime, find $(a_1 + a_2)$.

16. Find the orthogonal trajectories for the given family of curves when 'a' is the parameter.

(A) $y = ax^2$ **(B)** $\cos y = ae^{-x}$

17. Let c_1 and c_2 be two integral curves of the differential equation $\frac{dy}{dx} = \frac{x^2 - y^2}{x^2 + y^2}$. A line passing through origin meets c_1 at $P(x_1, y_1)$ and c_2 at $Q(x_2, y_2)$. If $c_1 : y = f(x)$ and $c_2 : y = g(x)$ prove that $f'(x_1) = g'(x_2)$.

18. A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of soluble lawn fertilizer runs into the tank at the rate of 1 lit/min and the mixture in pumped out of the tank at the rate of 3 litres/min. Find the time when the amount of fertilizer in the tank is maximum.

19. Solve the following differential equations (I - X)

(I) $(x - y^2)\,dx + 2xy\,dy = 0$ **(II)** $(x^3 + y^2 + 2)\,dx + 2y\,dy = 0$

(III) $x\frac{dy}{dx} + y \ln y = xye^x$ **(IV)** $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)\,e^x \sec y$

(V) $\frac{dy}{dx} = \frac{e^y}{x^2} - \frac{1}{x}$ **(VI)** $\left(\frac{dy}{dx}\right)^2 - (x+y)\frac{dy}{dx} + xy = 0$

(VII) $\frac{dy}{dx} = \frac{y^2 - x}{2y(x+1)}$ **(VIII)** $(1 - xy + x^2y^2)\,dx = x^2\,dy$

(IX) $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ **(X)** $yy' \sin x = \cos x\,(\sin x - y^2)$

20. The light rays emanating from a point source situated at origin when reflected from the mirror of a search light are reflected as beam parallel to the x-axis. Show that the surface is parabolic, by first forming the differential equation and then solving it.

21. Let $f(x, y, c_1) = 0$ and $f(x, y, c_2) = 0$ define two integral curves of a homogeneous first order differential equation. If P_1 and P_2 are respectively the points of intersection of these curves with an arbitrary line, $y = mx$ then prove that the slopes of these two curves at P_1 and P_2 are equal.

22. Find the curve such that the ordinate of any of its points is the geometric mean between the abscissa and the sum of the abscissa and subnormal at the point.

23. A curve passing through (1 , 0) such that the ratio of the square of the intercept cut by any tangent off the y–axis to the subnormal is equal to the ratio of the product of the co–ordinates of the point of tangency to the product of square of the slope of the tangent and the subtangent at the same point. Determine all such possible curves.

24. Given two curves $y = f(x)$, where $f(x) > 0$, passing through the points (0, 1) & $y = \int_{-\infty}^{x} f(t)\,dt$ passing through the points (0, 1/2). The tangents drawn to both curves at the points with equal abscissas intersect on the x-axis. Find the curve f(x).

25. Let y(x) be a real-valued differentiable function on the interval $(0, \infty)$ such that $y(1) = 0$ and satisfies $y'(x) = \ln x + 2 - \frac{y(x)}{x \ln x}$. Find the value of $[y(e) - y'(e)]$.

26. Find the curve for which the area of the triangle formed by the x–axis, the tangent line and radius vector of the point of tangency is equal to a^2.

27. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1 , 1).

28. Find the curve for which sum of the lengths of the tangent and subtangent at any of its point is proportional to the product of the co-ordinates of the point of tangency, the proportionality factor is equal to k.

29. Show that the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point, $\sqrt{x^2 + y^2} = c\,e^{\pm \tan^{-1}\frac{y}{x}}$

30. If y_1 & y_2 be solutions of the differential equation $\frac{dy}{dx} + Py = Q$, where P & Q are functions of x alone, and $y_2 = y_1 z$, then prove that $z = 1 + a\,e^{-\int \frac{Q}{y_1} dx}$, 'a' being an arbitrary constant.

31. Find the curve $y = f(x)$ where $f(x) \geq 0$, $f(0) = 0$, bounding a curvilinear trapezoid with the base [0, x] whose area is proportional to $(n + 1)^{th}$ power of f(x). It is known that $f(1) = 1$

32. It is known that the decay rate of radium is directly proportional to its quantity at each given instant. Find the law of variation of a mass of radium as a function of time if at $t = 0$, the mass of the radius was m_0 and during time t_0 α % of the original mass of radium decay.

33. Let C_1 and C_2 be two curves which satisfy the differential equation $\left|x - y\frac{dx}{dy}\right| = 2\left|\frac{dy}{dx}\right|$ and passes through M (1, 1). If the area enclosed by curves C_1, C_2 and co-ordinate axes is $\frac{m}{n}$ $(m, n \in N)$ then find the least value of $(m+n)$.

34. A tank consists of 50 liters of fresh water. Two liters of brine each litre containing 5 gms of dissolved salt are run into tank per minute ; the mixture is kept uniform by stirring , and runs out at the rate of one litre per minute. If 'm' grams of salt are present in the tank after t minute, express 'm' in terms of t and find the amount of salt present after 10 minutes.

35. Find all functions f(x) defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with real values and has a primitive F(x) such that $f(x) + \cos x \,.\, F(x) = \frac{\sin 2x}{(1 + \sin x)^2}$. Find f(x).

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The solution of the differential equation $(x^2 - y^2)dx + 2xy\,dy = 0$ is- **[AIEEE-2002]**

(1) $x^2 + y^2 = cx$ (2) $x^2 - y^2 + cx = 0$ (3) $x^2 + 2xy = y^2 + cx$ (4) $x^2 + y^2 = 2xy + cx^2$

2. The differential equation, which represents the family of plane curves $y = e^{cx}$, is- **[AIEEE-2002]**

(1) $y' = cy$ (2) $xy' - \log y = 0$ (3) $x\log y = yy'$ (4) $y\log y = xy'$

3. The equation of the curve through the point (1, 0), whose slope is $\frac{y-1}{x^2+x}$ is- **[AIEEE-2002]**

(1) $(y - 1)(x + 1) + 2x = 0$ (2) $2x(y - 1) + x + 1 = 0$

(3) $x(y - 1)(x + 1) + 2 = 0$ (4) $x(y + 1) + y(x + 1) = 0$

4. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively- **[AIEEE-2003]**

(1) 2, 3 (2) 2, 1 (3) 1, 2 (4) 3, 2

5. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$, is - **[AIEEE-2003]**

(1) $xe^{2\tan^{-1}y} = e^{\tan^{-1}y} + k$ (2) $(x - 2) = ke^{-\tan^{-1}y}$

(3) $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$ (4) $xe^{\tan^{-1}y} = \tan^{-1}y + k$

6. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is- **[AIEEE-2004]**

(1) $2(x^2 - y^2)y' = xy$ (2) $2(x^2 + y^2)y' = xy$ (3) $(x^2 - y^2)y' = 2xy$ (4) $(x^2 + y^2)y' = 2xy$

7. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is- **[AIEEE-2004]**

(1) $-\frac{1}{xy} = C$ (2) $-\frac{1}{xy} + \log y = C$ (3) $\frac{1}{xy} + \log y = C$ (4) $\log y = Cx$

8. The differential representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows- **[AIEEE-2005, IIT-1999]**

(1) order 1, degree 2 (2) order 1, degree 1 (3) order 1, degree 3 (4) order 2, degree 2

9. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is- **[AIEEE-2005]**

(1) $y\log\left(\frac{x}{y}\right) = cx$ (2) $x\log\left(\frac{y}{x}\right) = cy$ (3) $\log\left(\frac{y}{x}\right) = cx$ (4) $\log\left(\frac{x}{y}\right) = cy$

10. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of- **[AIEEE-2006]**

(1) first order and second degree (2) first order and first degree

(3) second order and first degree (4) second order and second degree

11. The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE-2007]**

(1) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$ (3) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$

12. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x+y}{x}$ satisfying the condition y(1) = 1 is- **[AIEEE-2008]**

(1) $y = \ell nx + x$ (2) $y = x\ell nx + x^2$ (3) $y = xe^{(x-1)}$ (4) $y = x\ell nx + x$

13. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is- **[AIEEE-2008]**

(1) $(x-2)y'^2 = 25 - (y-2)^2$ (2) $(y-2)y'^2 = 25 - (y-2)^2$
(3) $(y-2)^2y'^2 = 25 - (y-2)^2$ (4) $(x-2)^2y'^2 = 25 - (y-2)^2$

14. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is :- **[AIEEE-2009]**

(1) yy" = y' (2) $yy'' = (y')^2$ (3) $y' = y^2$ (4) y" = y'y

15. Solution of the differential equation cos x dy = y(sin x – y)dx, $0 < x < \dfrac{\pi}{2}$ is - **[AIEEE-2010]**

(1) sec x = (tan x + c) y (2) y sec x = tan x + c
(3) y tan x = sec x + c (4) tan x = (sec x + c) y

16. If $\dfrac{dy}{dx} = y + 3 > 0$ and y(0) = 2, then y(ln 2) is equal to **[AIEEE-2011]**

(1) 13 (2) –2 (3) 7 (4) 5

17. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\dfrac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is **[AIEEE-2011]**

(1) $I - \dfrac{k(T-t)^2}{2}$ (2) e^{-kT} (3) $T^2 - \dfrac{I}{k}$ (4) $I - \dfrac{kT^2}{2}$

18. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by : **[AIEEE-2011]**

(1) $\left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{3}\right)^2 = 2$ (2) 2y – 3x = 0 (3) $y = \dfrac{6}{x}$ (4) $x^2 + y^2 = 13$

19. Consider the differential equation $y^2dx + \left(x - \dfrac{1}{y}\right)dy = 0$. It y(1) =1, then x is given by : **[AIEEE-2011]**

(1) $1 - \dfrac{1}{y} + \dfrac{e^{\frac{1}{y}}}{e}$ (2) $4 - \dfrac{2}{y} - \dfrac{e^{\frac{1}{y}}}{e}$ (3) $3 - \dfrac{1}{y} + \dfrac{e^{\frac{1}{y}}}{e}$ (4) $1 + \dfrac{1}{y} - \dfrac{e^{\frac{1}{y}}}{e}$

20. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\ p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is : **[AIEEE-2012]**

(1) ln18 **(2)** 2 ln18 **(3)** ln9 **(4)** $\frac{1}{2}$ ln18

21. At present a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is : **[JEE (Main)-2013]**

(1) 2500 **(2)** 3000 **(3)** 3500 **(4)** 4500

22. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If p(0) = 100, then p(t) equals : **[Main 2014]**

(1) $400 - 300\ e^{t/2}$ **(2)** $300 - 200\ e^{-t/2}$ **(3)** $600 - 500\ e^{t/2}$ **(4)** $400 - 300\ e^{-t/2}$

23. Let y(x) be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then y (e) is equal to :

(1) 2 **(2)** 2e **(3)** e **(4)** 0 **[Main 2015]**

24. If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, y(1 + xy) dx = x dy, then $f\left(-\frac{1}{2}\right)$ is equal to : **[Main 2016]**

(1) $-\frac{4}{5}$ **(2)** $\frac{2}{5}$ **(3)** $\frac{4}{5}$ **(4)** $-\frac{2}{5}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. A country has a food deficit of 10% . Its population grows continuously at a rate of 3% per year . Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after 'n' years, where 'n' is the smallest integer bigger than or equal to, $\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$. **[JEE 2000 (Mains)]**

2.(A) Let f(x), x $\geq$ 0, be a nonnegative continuous function, and let $F(x) = \int_0^x f(t)dt,\ x \geq 0$. If for some c > 0, $f(x) \leq cF(x)$ for all x $\geq$ 0, then show that $f(x) = 0$ for all x $\geq$ 0.

(B) A hemispherical tank of radius 2 meters is initially full of water and has an outlet of 12 cm^2 cross sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law V(t) =0.6 $\sqrt{2gh(t)}$, where V(t) and h(t) are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. **[JEE 2001 (Mains)]**

3. If y(t) is a solution of $(1 + t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then y(1) is equal to -

(A) 1/2 **(B)** e + 1/2 **(C)** e – 1/2 **(D)** –1/2 **[JEE 2003]**

4. Let p(x) be a polynomial such that p(1) = 0 and $\dfrac{d}{dx}(p(x)) > p(x)$ for all $x \geq 1$ show that $p(x) > 0$, for all $x > 1$.

[JEE 2003 (mains)]

5. A conical flask of height H has pointed bottom and circular top of radius R. It is completely filled with a volatile liquid. The rate of evaporation of the liquid is proportional to the surface area of the liquid in contact with air, with the constant of proportionality K > 0. Neglecting the thickness of the flask, find the time it takes for the liquid to evaporate completely. **[JEE 2003 (mains)]**

6. If y = y (x) and $\dfrac{2+\sin x}{y+1}\left(\dfrac{dy}{dx}\right) = -\cos x$, y(0) = 1, then $y\left(\dfrac{\pi}{2}\right)$ equals - **[JEE 2004, (Screening)]**

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{2}{3}$ **(C)** $-\dfrac{1}{3}$ **(D)** 1

7. A curve passes through (2, 0) and slope at point P (x, y) is $\dfrac{(x+1)^2 + (y-3)}{(x+1)}$. Find equation of curve and area between curve and x-axis in 4^{th} quadrant. **[JEE - 2004 (Mains)]**

8. (A) The solution of primitive integral equation $(x^2 + y^2)\, dy = xy\, dx$, is y = y (x). If y (1) = 1 and $y(x_0) = e$, then x_0 is - **[JEE 2005, (Screening)]**

(A) $\sqrt{2\,(e^2-1)}$ **(B)** $\sqrt{2\,(e^2+1)}$ **(C)** $\sqrt{3}\,e$ **(D)** none of these

(B) For the primitive integral equation $y\,dx + y^2 dy = x\, dy$; $x \in R$, $y > 0$, y = y (x), y (1) = 1, then y (–3) is -

(A) 3 **(B)** 2 **(C)** 1 **(D)** 5

9. If length of tangent at any point on the curve y = f(x) intercepted between the point and the x-axis is of length 1. Find the equation of the curve. **[JEE 2005 (Mains)]**

10. A tangent drawn to the curve y = f(x) at P(x, y) cuts the x-axis and y-axis at A and B respectively such that BP : AP = 3 : 1, given that f(1) = 1, then - **[JEE 2006]**

(A) equation of the curve is $x\dfrac{dy}{dx} - 3y = 0$ **(B)** normal at (1, 1) is x + 3y = 4

(C) curve passes through (2, 1/8) **(D)** equation of the curve is $x\dfrac{dy}{dx} + 3y = 0$

11. (A) Let f(x) be differentiable on the interval $(0, \infty)$ such that f(1) = 1, and $\lim\limits_{t\to x}\dfrac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each x > 0. Then f(x) is - **[JEE 2007]**

(A) $\dfrac{1}{3x} + \dfrac{2x^2}{3}$ **(B)** $\dfrac{-1}{3x} + \dfrac{4x^2}{3}$ **(C)** $\dfrac{-1}{x} + \dfrac{2}{x^2}$ **(D)** $\dfrac{1}{x}$

(B) The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines family of circles with

(A) variable radii and a fixed centre at (0, 1)

(B) variable radii and a fixed centre at (0, –1)

(C) fixed radius 1 and variable centres along the x-axis.

(D) fixed radius 1 and varialble centres along the y-axis.

12. Let a solution y = y(x) of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

Statement-1 : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$ **[JEE 2008]**

Statement-2 : y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

13. Match the statements/ expressions in **Column I** with the open intervals in **Column II.** **[JEE 2009]**

Column I		Column II	
(A)	Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$	**(p)**	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B)	Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$	**(q)**	$\left(0, \frac{\pi}{2}\right)$
		(r)	$\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(C)	Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	**(s)**	$\left(0, \frac{\pi}{8}\right)$
(D)	Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	**(t)**	$(-\pi, \pi)$

14. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x,y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to **[JEE 2010]**

15. (A) Let $f : [1,\infty) \to [2,\infty)$ be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)dt = 3x\,f(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is **[JEE 2011]**

(B) Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\frac{d\,f(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then the value of y(2) is **[JEE 2011]**

16. If y(x) satisfies the differential equation $y' - y\tan x = 2x \sec x$ and y(0) = 0, then **[JEE 2012]**

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ **(B)** $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$ **(C)** $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{9}$ **(D)** $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

17. Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval **[JEE Ad.]**

(A) (2e – 1, 2e) **(B)** (e – 1, 2e – 1) **(C)** $\left(\frac{e-1}{2}, e-1\right)$ **(D)** $\left(0, \frac{e-1}{2}\right)$

18. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is **[JEE Ad. 2013]**

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ **(B)** $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ **(D)** $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Paragraph for Question 19 and 20

Let $f : [0,1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0,1]$.

19. If the function $e^{-x}f(x)$ assumes its minimum in the interval [0,1] at $x = \frac{1}{4}$, which of the following is true ? **[JEE Ad. 2013]**

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$ **(B)** $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$ **(D)** $f'(x) < f(x), \frac{3}{4} < x < 1$

20. Which of the following is true for $0 < x < 1$? **[JEE Ad. 2013]**

(A) $0 < f(x) < \infty$ **(B)** $-\frac{1}{2} < f(x) < \frac{1}{2}$ **(C)** $-\frac{1}{4} < f(x) < 1$ **(D)** $-\infty < f(x) < 0$

21. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$.

Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)\,dx$ is **[JEE Ad. 2014]**

(A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ **(B)** $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ **(C)** $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ **(D)** $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

22. Let y(x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true ? **[JEE Ad. 2015]**

(A) $y(-4) = 0$
(B) $y(-2) = 0$
(C) y(x) has a critical point in the interval $(-1, 0)$
(D) y(x) has no critical point in the interval $(-1, 0)$

23. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family ofcircles is represented by the differential equation $Py'' + Qy' + 1 = 0$, y' (here $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true **[JEE Ad. 2015]**

(A) $P = y + x$ **(B)** $P = y - x$
(C) $P + Q = 1 - x + y + y' + (y')^2$ **(D)** $P - Q = x + y - y' - (y')^2$

24. A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\frac{dy}{dx} = y^2 = 0$, $x > 0$, passes through the point (1, 3). Then the solution curve

(A) intersects $y = x + 2$ exactly at one point.
(B) intersects $y = x + 2$ exactly at two points
(C) intersects $y = (x + 2)^2$
(D) does NOT intersect $y = (x + 3)^2$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. Solution of differential equation $x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + ...$ is

(A) $y^2 = x^2(\ell n\, x^2 - 1) + c$ **(B)** $y = x^2(\ell n\, x - 1) + c$

(C) $y^2 = x(\ell n\, x - 1) + c$ **(D)** $y = x^2 e^{x^2} + c$

2. If gradient of a curve at any point P(x, y) is $\frac{x+y+1}{2y+2x+1}$ and it is passes through origin, then curve is

(A) $2(x+3y) = \ell n\left|\frac{3x+3y+2}{2}\right|$ **(B)** $x + 3y = \ell n\left|\frac{3x+3y+2}{2}\right|$

(C) $3y + x = \ell n\,(3x + 2y + 1)$ **(D)** $6y - 3x = \ell n\left|\frac{3x+3y+2}{2}\right|$

3. The solution of the differential equation $y_1\, y_3 = 3y_2^{\,2}$ is

(A) $x = A_1 y^2 + A_2\, y + A_3$ **(B)** $x = A_1\, y + A_2$

(C) $x = A_1\, y^2 + A_2\, y$ **(D)** none of these

4. The equation of the curve satisfying the differential equation $y_2\,(x^2 + 1) = 2xy_1$ passing through the point (0, 1) and having slope of tangnet at x = 0 as 3, is

(A) $y = x^2 + 3x + 2$ **(B)** $y^2 = x^2 + 3x + 1$ **(C)** $y = x^3 + 3x + 1$ **(D)** none of these

5. If the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$ is $x = ce^{\sin y} - k\,(1 + \sin y)$, then k =

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

6 The differential equation of all parabola having their axis of symmetry coinciding with the axis of X is

(A) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ **(B)** $y\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$ **(C)** $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ **(D)** none of these

7. The differential equation of all parabola having their axis of symmetry coinciding with the x-axis is

(A) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ **(B)** $y\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$ **(C)** $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ **(D)** none of these

8. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

9. The equation of curve passing through (3, 4) and satisfying the differential equation

$y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$ can be

S_1 : $x - y + 1 = 0$ S_2 : $x + y - 7 = 0$

S_3 : $x^2 + y^2 = 25$ S_4 : $x^2 + y^2 - 5x = 10$

(A) TFTF (B) TTFF (C) TTFT (D) FTFT

10. The solution of differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$ is

(A) $y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

(B) $y(x^2 + 1) = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

(C) $y(x^2 - 1) = \frac{5}{2}\log\left|\frac{x-1}{x+1}\right| + C$

(D) None of these

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $y = e^{-x}\cos x$ and $y_n + k_n y = 0$, where $y_n = \frac{d^n y}{dx^n}$ and k_n, $n \in N$ are constants.

(A) $k_4 = 4$ (B) $k_8 = -16$ (C) $k_{12} = 20$ (D) $k_{16} = -24$

12. The differential equation $\frac{d^2 y}{dx^2} + y + \cot^2 x = 0$ must be satisfied by

(A) $y = 2 + c_1\cos x + \sqrt{c_2}\,\sin x$

(B) $y = \cos x . \ell n\left(\tan\frac{x}{2}\right) + 2$

(C) $y = 2 + c_1\cos x + c_2\sin x + \cos x \log\left(\tan\frac{x}{2}\right)$

(D) all the above

13. The differential equation $\frac{d^2 y}{dx^2} + y + \cot^2 x = 0$ must be satisfied by

(A) $y = 2 + c_1\cos x + \sqrt{c_2}\,\sin x$

(B) $y = \cos x . \ell n\left(\tan\frac{x}{2}\right) + 2$

(C) $y = 2 + c_1\cos x + c_2\sin x + \cos x \log\left(\tan\frac{x}{2}\right)$

(D) all the above

14. The differential equation of all circles in a plane must be $\left(y_1 = \frac{dy}{dx}, y_2 = \frac{d^2y}{dx^2}, ...etc.\right)$

(A) $y_3\left(1 + y_1^2\right) - 3y_1y_2^2 = 0$

(B) of order 3 and degree 1

(C) of order 3 and degree 2

(D) $y_3^2\left(1 - y_1^2\right) - 3y_1y_2^2 = 0$

15. The solution of $\left(\frac{dy}{dx}\right)(x^2y^3 + xy) = 1$ is

(A) $1/x = 2 - y^2 + C\,e^{-y^2/2}$

(B) The solution of an equation which is reducible to linear equation.

(C) $2/x = 1 - y^2 + e^{-y/2}$

(D) $\frac{1-2x}{x} = -y^2 + C\,e^{-y^2/2}$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement -I :** Solution of $(1 + x\sqrt{x^2+y^2})\,dx + y(-1 + \sqrt{x^2+y^2})\,dy = 0$ is $x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} + c = 0$

Statement -II : Solution of $(1 + xy)\,y\,dx + (1 - xy)\,x\,dy = 0$ is $\bullet n\,\frac{x}{y} - \frac{1}{xy} = c$

(A) Statement-I is True, Statement-IIis True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement -I :** The equation of the curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$

Statement -II : The solution of D.E. $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$

(A) Statement-I is True, Statement-IIis True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement -I :** The D.E. of all circles in a plane must be of order 3.
Statement -II : There is only one circle passing through three non-collinear points.
(A) Statement-I is True, Statement-IIis True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement -I :** The differential equation $y^3\,dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$
Statement -II : All differential equation of first order first degree becomes homogeneous if we put $y = tx$
(A) Statement-I is True, Statement-IIis True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement -I :** Order of differential equation represents number of arbitrary constants in the general solution.
Statement -II : Degree of differential equation represents number of family of curves.
(A) Statement-I is True, Statement-IIis True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. Find the solution of the following differential equation

Column – I		Column – II	
(A)	$(\sin x + \cos x)\, dy + (\cos x - \sin x)\, dx = 0$	(p)	$\sec y + \tan y = c\,(\operatorname{cosec} x + \cot x)$
(B)	$\sin x\, dy + \cos y\, dx = 0$	(q)	$2\,(x^2 + y^2) + 2\,(x \sin 2x + y \sin 2y) + (\cos 2x + \cos 2y) = c$
(C)	$x^{-1} \cos^2 y\, dy + y^{-1} \cos^2 x\, dx = 0$	(r)	$y = c - \log \lvert\sin x + \cos x\rvert$
(D)	$\tan x \sec^2 y\, dy + \tan y \sec^2 x\, dx = dx$	(s)	$\tan x \tan y = x + c$

22.

Column - I		Column - II	
(A)	$xdy = y(dx + ydy),\ y > 0$, $y(1) = 1$ and $y(x_0) = -3$, then $x_0 =$	(p)	$\frac{1}{4}$
(B)	If $y(t)$ is solution of $(t + 1)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, then $y(1) =$	(q)	-15
(C)	$(x^2 + y^2)\, dx = xydx$ and $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$	(r)	$-\frac{1}{2}$
(D)	$\frac{dy}{dx} + \frac{2y}{x} = 0,\ y(1) = 1$, then $y(2) =$	(s)	16
		(t)	$\sqrt{3}\, e$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Differential equation $\frac{dy}{dx} = f(x)\, g(y)$ can be solved by seperating variable $\frac{dy}{g(y)} = f(x)\, dx$

1. The equation of the curve to the point (1, 0) which satisfies the differential equation $(1 + y^2)\, dx - xy\, dy = 0$ is

(A) $x^2 + y^2 = 1$ (B) $x^2 - y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x^2 - y^2 = 2$

2. Solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$ is

(A) $\tan^{-1} y + \sin^{-1} x = c$ (B) $\tan^{-1} x + \sin^{-1} y = c$ (C) $\tan^{-1} y \cdot \sin^{-1} x = c$ (D) $\tan^{-1} y - \sin^{-1} x = c$

3. If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then $y =$

(A) $e^{\frac{(1-x)^2}{2}}$ (B) $e^{\frac{(1+x)^2}{2}} - 1$ (C) $\ln(1 + x) - 1$ (D) $1 + x$

24. Read the following comprehension carefully and answer the questions.

The rate at which a body undergoes a change in temperature is proportional to the difference between its temperature and temperature of the surrounding medium. If $y = f(t)$ is the temperature of the body at time t and M(t) denotes the temperature of the surrounding medium. Newton's law leads to the differential equation

$$y' = -k(y - M(t))$$

Where k is a positive constant. This first order linear equation is the mathematical model we use for cooling problems. The unique solution of the equation satisfying the initial condition f(a) = b is given by the formula

$$f(t) = be^{-kt} + e^{-kt}\int_a^t k\ M(z)\ e^{kz}dz$$

1. A body cools from 200° to 100° in 40 minutes while immersed in a medium where temperature is kept constant. Let M(t) = 10°. If we measure t in minutes and f(t) in degree then f(t) must be equal to

(A) $10 + 180\ e^{-kt}$ **(B)** $10 + 140\ e^{-kt}$ **(C)** $10 + 100\ e^{-kt}$ **(D)** $10 + 190\ e^{-kt}$

2. The value of k must be

(A) $\frac{(\log 19 - \log 9)}{100}$ **(B)** $\frac{(\log 19 - \log 9)}{10}$ **(C)** $\frac{\log 19 - \log 9}{40}$ **(D)** none of these

3. Suppose in the same system a body cools from 400° to 200° with M(t) = 10°, then time taken for cooling must be equal to

(A) 40 log 19 **(B)** 40 log 9 **(C)** $40\left(\frac{\log 19 - \log 9}{\log 39 - \log 19}\right)$ **(D)** $40\left(\frac{\log 39 - \log 19}{\log 19 - \log 9}\right)$

25. Read the following comprehension carefully and answer the questions.

A differential equation of the form $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$ is called linear differential equation where a_1 and a_2 are function of x only. In case a_1 and a_2 are constants, the solution of the linear differential equation can be easily written by following facts –

(i) y = 0 is a solution of differential equation.

(ii) If y = f(x) is a solution then y = c f(x) is also a solution.

(iii) If $y = f_1(x)$ & $y = f_2(x)$ are two solution then $y = f_1(x) + f_2(x)$ will also be a solution

(iv) If distinct roots of quadratic equation $m^2 + a_1 m + a_2 = 0$ are m_1 and m_2 (real or imaginary) then solution of differential equation is $y = c_1e^{m_1x} + c_2e^{m_2x}$

In case roots are complex the solution can be transformed to the form $e^{ax}(c_1 \cos bx + c_2 \sin bx)$ by using Euler's theorem.

(v) In case roots of $m^2 + a_1 m + a_2 = 0$ are equal (say m_1) the differential equation can be make linear by putting $\frac{dy}{dx} - m_1 y = v$

The linear differential equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = f(x)$ can also be satisfied by some other functions which are not of the above type such functions are called particular integrals

1. $y = e^x$ is a solution of differential equation $\frac{d^2y}{dx^2} - y = 0$, then which of the following is not a solution.

(A) e^{-x} **(B)** $e-x$ **(C)** $ae^x + be^{-x}$ **(D)** $e^x + c$

2. Which of the following is solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

(A) $c_1 + c_2 x$ **(B)** $(c_1 + c_2 x)\, e^{3x}$

(C) $c_1 \cos 3x + c_2 \sin 3x$ **(D)** none of these

3. A particular integral solution of the equation $\frac{d^2y}{dx^2} + a^2 y + \frac{\cos ax}{a} = 0$ is

(A) $c_1 \cos(ax) + c_2 \sin(ax)$ **(B)** $\frac{x \sin ax}{2a^2} + \frac{x \cos ax}{2a^2}$

(C) $\frac{x \sin ax}{2a^2} + \frac{\cos ax}{2a^2}$ **(D)** $\frac{-x \sin ax}{2a^2} - \frac{\cos ax}{2a^2}$

SECTION - VI : INTEGER TYPE

26. By eliminating the constant in the following equation $x^2 - y^2 = c(x^2 + y^2)^2$ its differential equation is $y' = \frac{x(\lambda y^2 - x^2)}{y(\lambda x^2 - y^2)}$, then find the value of λ.

27. The curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$ is $y^2 = \lambda x + 1 - e^{2x}$, then find the value of λ.

28. A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. After $(\lambda \log_{4/3} 2)$ hours both the reservoirs have the same quantity of water, then find the value of λ

29. Let the curve $y = f(x)$ passes through $(4, -2)$ satisfy the differential equation,

$$y(x + y^3)\,dx = x(y^3 - x)\,dy \;\&\; y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt,$$

$0 \le x \le \frac{\pi}{2}$. The area of the region bounded by curves, $y = f(x)$, $y = g(x)$ and $x = 0$ is $\frac{\lambda}{8}\left(\frac{3\pi}{16}\right)^4$, then find the value of λ.

30. The differential equation, $(x^2 + 4y^2 + 4xy)\,dy = (2x + 4y + 1)\,dx$ has solution

$$y = \ln\left((x+2y)^2 + \lambda(x+2y) + 2\right) - \frac{3}{2\sqrt{2}} \ln\left|\frac{x + 2y + 2 - \sqrt{2}}{x + 2y + 2 + \sqrt{2}}\right| + c$$, then find the value of λ.

ANSWER KEY

EXERCISE - 1

1. C **2.** C **3.** B **4.** D **5.** A **6.** B **7.** D **8.** C **9.** A **10.** A **11.** A **12.** C **13.** A
14. C **15.** B **16.** A **17.** B **18.** B **19.** A **20.** A **21.** A **22.** B **23.** B **24.** C **25.** C **26.** D
27. A **28.** A **29.** D **30.** B **31.** A **32.** B **33.** C **34.** B **35.** B **36.** D **37.** C **38.** B **39.** C
40. A

EXERCISE - 2 : PART # I

1. ABC **2.** BC **3.** AB **4.** CD **5.** AD **6.** ABC **7.** ABD **8.** BCD **9.** ABC
10. AD **11.** CD **12.** AB **13.** ABD **14.** CD **15.** ACD **16.** AD **17.** AB **18.** AB
19. AC **20.** ACD **21.** AC **22.** AC **23.** ABCD

PART - II

1. D **2.** B **3.** B **4.** A **5.** C **6.** D **7.** D **8.** D **9.** D **10.** B **11.** A **12.** C **13.** A
14. C **15.** B

EXERCISE - 3 : PART # I

1. $A\to r$ $B\to p$ $C\to s$ $D\to q$ **2.** $A\to p,s$ $B\to q$ $C\to q,s$ $D\to r$ **3.** $A\to q$ $B\to r$ $C\to s$ $D\to p$
4. $A\to q$ $B\to s$ $C\to q$ $D\to q$

PART - II

Comprehension # 1 : 1. C **2.** AD **3.** B
Comprehension # 2 : 1. A **2.** C **3.** C
Comprehension # 3 : 1. B **2.** A **3.** A
Comprehension # 4 : 1. D **2.** D **3.** B
Comprehension # 5 : 1. A **2.** C **3.** D
Comprehension # 6 : 1. A **2.** B **3.** C
Comprehension # 7 : 1. A **2.** B **3.** B

EXERCISE - 5 : PART # I

1. 1 **2.** 4 **3.** 1 **4.** 3 **5.** 3 **6.** 3 **7.** 2 **8.** 3 **9.** 3 **10.** 3 **11.** 3 **12.** 4 **13.** 3
14. 2 **15.** 1 **16.** 3 **17.** 4 **18.** 3 **19.** 4 **20.** 2 **21.** 3 **22.** 1 **23.** 1 **24.** 3

PART - II

2. **(B)** $\frac{7\pi x10^5}{135\sqrt{g}}$ sec **3.** D **5.** $\frac{H}{K}$ **6.** A **7.** $(x-3)(x+1)=y-3\,;\ \frac{4}{3}$ units **8.** **(A)** C **(B)** A

9. $\sqrt{1-y^2}+\ln\left|\frac{1-\sqrt{1-y^2}}{y}\right|=\pm x+c$ **10.** A, B, C, D **11. (A)** A **(B)** C **12.** C

13. **A** $\to$ p, q, s **B** $\to$ p, t **C** $\to$ p, q, r, t **D** $\to$ s **14.** 9 **21.** B **22.** AC **23.** BC **24.** AC

MOCK TEST

1. A	**2.** D	**3.** A	**4.** C	**5.** B	**6.** A	**7.** A	**8.** A	**9.** A
10. A	**11.** AB	**12.** BC	**13.** BC	**14.** AB	**15.** ABD	**16.** B	**17.** B	**18.** B
19. C	**20.** B	**21.** $A \to r$ $B \to p$ $C \to q$ $D \to s$			**22.** $A \to q$ $B \to r$ $C \to t$ $D \to p$			
23. 1. B	**2.** A	**3.** B	**24. 1.** D	**2.** C	**3.** D	**25. 1.** B	**2.** B	**3.** D
26. 3	**27.** 2	**28.** 1	**29.** 1	**30.** 4				