SOLVED EXAMPLES

Ex. 1 A function f(x) satisfies the following property : $f(x + y) = f(x)\,f(y)$ Show that the function is continuous for all values of x if it is continuous at x = 1.

Sol. As the function is continuous at x = 1, we have

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$$

or $$\lim_{h\to0} f(1-h) = \lim_{h\to0} f(1+h) = f(1)$$

or $$\lim_{h\to0} f(1)\,f(-h) = \lim_{h\to0} f(1)\,f(h) = f(1) \quad [\text{Using } f(x+y) = f(x)\,f(y)]$$

or $$\lim_{h\to0} f(-h) = \lim_{h\to0} f(h) = 1 \quad \text{....(i)}$$

Now, consider any arbitrary point x = a.

LHL $= \lim_{h\to0} f(a-h)$

$= \lim_{h\to0} f(a)\,f(-h)$

$= f(a)\lim_{h\to0} f(-h) = f(a)$ [As $\lim_{h\to0} f(-h) = 1$, using (i)]

RHL $= \lim_{h\to0} f(a+h)$

$= \lim_{h\to0} f(a)\,f(h)$

$= f(a)\lim_{h\to0} f(h) = f(a)$ [As $\lim_{h\to0} f(h) = 1$, using (i)]

Hence, at any arbitrary point (x = a), LHL = RHL = f(a).
Therefore, the function is continuous for all values of x if its is continuous at 1.

Ex. 2 Find the points of discontinuity of the following functions.

(i) $f(x) = \dfrac{1}{2\sin x - 1}$

(ii) $f(x) = [[x]] - [x-1]$, where [.] represent the greatest integer function.

Sol. (i) $f(x) = \dfrac{1}{2\sin x - 1}$

f(x) is discontinuous when

$2\sin x - 1 = 0$

or $\sin x = \dfrac{1}{2}$, i.e., $x = 2n\pi + \dfrac{\pi}{6}$ or $x = 2n\pi + \dfrac{5\pi}{6}$, $n \in Z$

(ii) $f(x) = [[x]] - [x-1] = [x] - ([x]-1) = 1$

Therefore, f(x) is continuous $\forall\ x \in R$.

Ex. 3 Let $f(x) = \begin{cases} \dfrac{\log_e \cos x}{\sqrt[4]{1+x^2}-1}, & x > 0 \\ \dfrac{e^{\sin 4x}-1}{\log_e(1+\tan 2x)}, & x < 0 \end{cases}$

Find the value of f(0) which makes the function continuous at x = 0,

Sol. LHL $= \lim_{x\to 0^-} \dfrac{e^{\sin 4x}-1}{\log_e(1+\tan 2x)}$

$$= \lim_{x\to 0^-} \frac{\dfrac{e^{\sin 4x}-1}{\sin 4x}\sin 4x}{\dfrac{\log_e(1+\tan 2x)}{\tan 2x}\tan 2x} \quad \Rightarrow \quad \lim_{x\to 0^-}\frac{\sin 4x}{\tan 2x}$$

$= 2$

RHL $= \lim_{x\to 0^+}\left(\dfrac{\log_e \cos x}{\sqrt[4]{(1+x^2)}-1}\right)$

$$= \lim_{x\to 0^+}\left(\frac{-\tan x}{\frac{1}{4}(1+x^2)^{-\frac{3}{4}}2x}\right) \qquad \text{[Using L'HR]}$$

$= -2$

Here $f(0^-) \neq f(0^+)$

Hence f(x) cannot be defined.

Hence, f(x) has non-removable type of discontinuity.

Ex. 4 $f(x) = \begin{cases} \cos^{-1}\{\cot x\} & x < \dfrac{\pi}{2} \\ \pi[x]-1 & x \ge \dfrac{\pi}{2} \end{cases}$; find jump of discontinuity, where [ ] denotes greatest integer & { } denotes fractional part function.

Sol. $f(x) = \begin{cases} \cos^{-1}\{\cot x\} & x < \dfrac{\pi}{2} \\ \pi[x]-1 & x \ge \dfrac{\pi}{2} \end{cases}$

$$\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{x\to\frac{\pi^-}{2}} \cos^{-1}\{\cot x\} = \lim_{h\to 0}\cos^{-1}\left\{\cot\left(\frac{\pi}{2}-h\right)\right\} = \lim_{h\to 0}\cos^{-1}\{\tanh\} = \frac{\pi}{2}$$

$$= \lim_{x\to\frac{\pi^+}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} \pi[x]-1 = \lim_{h\to 0}\pi\left[\frac{\pi}{2}+h\right]-1 = \pi-1$$

$\therefore$ jump of discontinuity $= \pi - 1 - \dfrac{\pi}{2} = \dfrac{\pi}{2} - 1$

Ex. 5 Let f(x) be a function defined as $f(x) = \begin{cases} \dfrac{x^2-1}{x^2-2|x-1|-1}, & x \neq 1 \\ \dfrac{1}{2}, & x = 1 \end{cases}$

Discuss the continuity of the function at x = 1.

Sol.

$$f(1^+) = \lim_{x\to1^+} \frac{x^2-1}{x^2-2|x-1|-1}$$

$$= \lim_{x\to1^+} \frac{x^2-1}{x^2-2(x-1)-1}$$

$$= \lim_{x\to1^+} \frac{(x+1)}{(x+1)-2} = \infty$$

$$f(1^-) = \lim_{x\to1^-} \frac{x^2-1}{x^2-2|x-1|-1}$$

$$= \lim_{x\to1^-} \frac{x^2-1}{x^2-2(1-x)-1}$$

$$= \lim_{x\to1^-} \frac{(x+1)}{(x+1)+2} = \frac{1}{2}$$

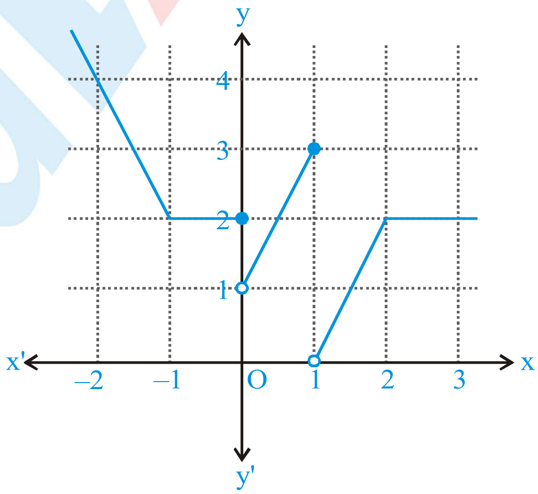
Hence, f(x) is discontinuous at x = 1.

Ex. 6 $f(x) = \begin{cases} |x+1|; & x \le 0 \\ x; & x > 0 \end{cases}$ and $g(x) = \begin{cases} |x|+1; & x \le 1 \\ -|x-2|; & x > 1 \end{cases}$

Draw its graph and discuss the continuity of f(x) + g(x).

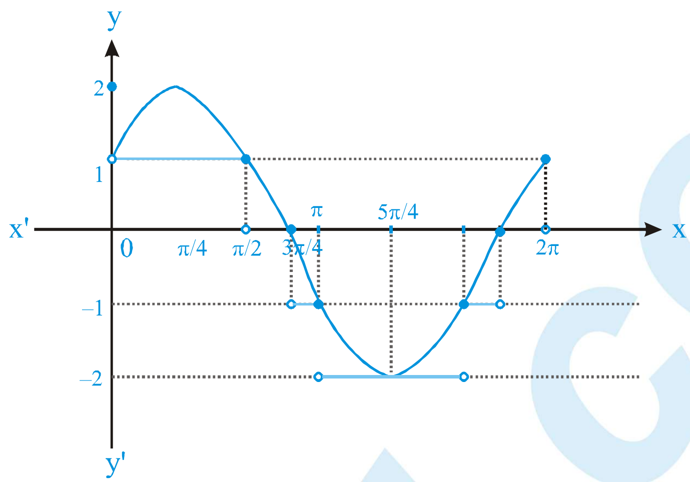
Sol. Since f(x) is discontinuous at x = 0 and g(x) is continuous at x = 0, f(x) + g(x) is discontinuous at x = 0.

Since f(x) is continuous at x = 1 and g(x) is discontinuous x = 1, f(x) + g(x) isdiscontinuous at x = 1.



Ex. 7 Draw the graph and discuss continuity of $f(x) = [\sin x + \cos x]$, $x \in [0, 2\pi]$, where [.] represents the greatest integer function.

Sol. $f(x) = [\sin x + \cos x] = [g(x)]$, where $g(x) = \sin x + \cos x$,



$$g(0) = 1,\ g\left(\frac{\pi}{4}\right) = \sqrt{2},\ g\left(\frac{\pi}{2}\right) = 1$$

$$g\left(\frac{3\pi}{4}\right) = 0,\ g(\pi) = -1,\ g\left(\frac{5\pi}{4}\right) = -\sqrt{2}$$

$$g\left(\frac{3\pi}{2}\right) = 1,\ g\left(\frac{7\pi}{4}\right) = 0,\ g(2\pi) = 1$$

Clearly, from the graph given in fig. f(x) is discontinuous at $x = 0, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, \frac{7\pi}{2}, \pi$

Ex. 8 Let $f(x) = \lim\limits_{n\to\infty} \frac{1}{1 + n\sin^2 x}$, then find $f\left(\frac{\pi}{4}\right)$ and also comment on the continuity at x = 0

Sol. Let $f(x) = \lim\limits_{n\to\infty} \frac{1}{1 + n\sin^2 x}$

$$f\left(\frac{\pi}{4}\right) = \lim_{n\to\infty} \frac{1}{1 + n \,.\, \sin^2 \frac{\pi}{4}} = \lim_{n\to\infty} \frac{1}{1 + n\left(\frac{1}{2}\right)} = 0$$

Now

$$f(0) = \lim_{n\to\infty} \frac{1}{n \,.\, \sin^2(0) + 1} = \frac{1}{1+0} = 1$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\lim_{n\to\infty} \frac{1}{1 + n\sin^2 x}\right] = 0$$

{here $\sin^2 x$ is very small quantity but not zero and very small quantity when multiplied with ∞ becomes ∞}

$\therefore$ f(x) is not continuous at x = 0

Ex. 9 Discuss the continuity of $f(x) = \begin{cases} x\{x\}+1, & 0 \le x < 1 \\ 2-\{x\}, & 1 \le x \le 2 \end{cases}$

where $\{x\}$ denotes the fractional part function.

Sol. $f(0) = f(0^+) = 1$

$f(2) = 2$ and $f(2^-) = 1$

Hence, f(x) is discontinuous at x = 2. Also,

$f(1^+) = 2$, $f(1^-) = 1 + 1 = 2$, and $f(1) = 2$

Hence, f(x) is continuous at x = 1.

Ex. 10 $f(x) = \begin{cases} \text{sgn}(x-2) \times [\log_e x], & 1 \le x \le 3 \\ \{x^2\}, & 3 < x \le 3.5 \end{cases}$

where [.] denotes the greatest integer function and {.} represent the fractional part function. Find the point where the continuity of f(x) should be checked. Hence, find the points of discontinuity.

Sol. **(A)** Continuity should be checked at the endpoints of intervals of each definition, i.e., x = 1, 3, 3, 5.

(B) For $\{x^2\}$, continuity should be checked when $x^2 = 10, 11, 12$ or $x = \sqrt{10}, \sqrt{11}, \sqrt{12}$. $\{x^2\}$ is discontinuous for those values of x where x^2 is an integer

(note, here x^2 is monotonic for given domain).

(C) for sgn (x – 2), continuity should be checked when x – 2 = 0 or x = 2.

(D) for $[\log_e x]$, continuity should be checked when $\log_e x = 1$ or $x = e$ ($\in [1, 3]$).

Hence, the overall continuity must be checked at $x = 1, 2, e, 3, \sqrt{10}, \sqrt{11}, \sqrt{12}, 3.5$

Further, f(1) = 0 and

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \text{sgn}(x-2) \times [\log_e x] = 0$$

Hence, f(x) is continuous at x = 1.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \text{sgn}(x-2) \times [\log_e x] = (-1) \times 0 = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \text{sgn}(x-2) \times [\log_e x] = (1) \times 0 = 0$$

Also, $f(2) = 0$.

Hence, f(x) is continuous at x = 2.

$$\lim_{x\to e^-} f(x) = \lim_{x\to e^-} \text{sgn}(x-2) \times [\log_e x] = (1) \times 0 = 0$$

$$\lim_{x\to e^+} f(x) = \lim_{x\to e^+} \text{sgn}(x-2) \times [\log_e x] = (1) \times (1) = 1$$

Hence f(x) is discontinuous at x = e.

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \text{sgn}(x-2) \times [\log_e x] = 1$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \{x^2\} = 0$$

Hence, f(x) is discontinuous at x = 3.

Also, $\{x^2\}$, and hence, f(x) is discontinuous at $x = \sqrt{10}, \sqrt{11}, \sqrt{12}$.

$$\lim_{x\to 3.5^-} f(x) = \lim_{x\to 3.5^-} \{x^2\} = 0.25 = f(3.5)$$

Hence, f(x) is discontinuous at $x = e, 3, \sqrt{10}, \sqrt{11}, \sqrt{12}$.

Ex. 11 If $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{1}{x-2}$, then discuss the continuity of f(x), g(x) and fog(x).

Sol. $f(x) = \frac{x+1}{x-1}$

Thus, f is not defined at x = 1 and f is discontinuous at x = 1.

Now, $g(x) = \frac{1}{x-2}$

g(x) is not defined at x = 2. Therefore, g is discontinuous at x = 2.

Now, fog(x) will be discontinuous at

(A) $x = 2$ [point of discontinuity of g(x)]

(B) $g(x) = 1$ [when g(x) = point of discontinuity of f(x)]

For $g(x) = 1$, $\frac{1}{x-2} = 1$ or $x = 3$.

Therefore, fog(x) is discontinuous at x = 2 and x = 3.

Also, $fog(x) = \frac{\frac{1}{x-2}+1}{\frac{1}{x-2}-1}$

Here, fog(2) is not defined.

$$\lim_{x\to 2} fog(x) = \lim_{x\to 2} \frac{\frac{1}{x-2}+1}{\frac{1}{x-2}-1} = \lim_{x\to 2} \frac{1+x-2}{1-x+2} = 1$$

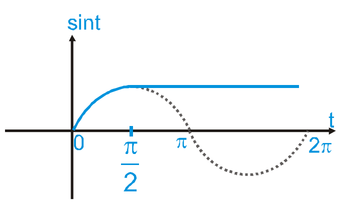
Therefore, fog(x) is discontinuous at x = 2 and it has a removable discontinuity at x = 2. For continuity at x = 3,

$$\lim_{x\to 3^+} fog(x) = \lim_{x\to 3^+} \frac{\frac{1}{x-2}+1}{\frac{1}{x-2}-1} = -\infty \qquad \lim_{x\to 3^-} fog(x) = \lim_{x\to 3^-} \frac{\frac{1}{x-2}+1}{\frac{1}{x-2}-1} = \infty$$

Therefore, fog(x) is discontinuous at x = 3, and it is a non-removable discontinuity at x = 3.

Ex. 12 $f(x) = \text{maximum}(\sin t, 0 \le t \le x), 0 \le x \le 2\pi$ discuss the continuity of this function at $x = \frac{\pi}{2}$

Sol. $f(x) = \text{maximum}(\sin t, 0 \le t \le x), 0 \le x \le 2\pi$



if $x \in \left[0, \frac{\pi}{2}\right]$, sint is increasing function

Hence if $t \in [0, x]$, sint will attain its maximum value at $t = x$.

$\therefore$ $f(x) = \sin x$ if $x \in \left[0, \frac{\pi}{2}\right]$

if $x \in \left(\frac{\pi}{2}, 2\pi\right]$ and $t \in [0, x]$

then sint will attain its maximum value when $t = \frac{\pi}{2}$

$\therefore \quad f(x) = \sin\frac{\pi}{2} = 1$ if $x \in \left(\frac{\pi}{2}, 2\pi\right]$ $\quad \therefore \quad f(x) = \begin{cases} \sin x, & \text{if } x \in \left[0, \frac{\pi}{2}\right] \\ 1, & \text{if } x \in \left(\frac{\pi}{2}, 2\pi\right] \end{cases}$

Now $\quad f\left(\frac{\pi}{2}\right) = 1$

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \sin x = 1$$

$$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} 1 = 1$$

as $\quad f(\pi/2) =$ L.H.S. = R.H.S. $\quad \therefore \quad f(x)$ is continuous at $x = \frac{\pi}{2}$

Ex. 13 Let f be continuous on the interval [0, 1] to R such that f(0) = f(1). Prove that there exists a point c in $\left[0, \frac{1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

Sol. Consider a continuous function $g(x) = f\left(x + \frac{1}{2}\right) - f(x)$ $\quad \left(g \text{ is continuous } \forall x \in \left[0, \frac{1}{2}\right]\right)$

$g(0) = f\left(\frac{1}{2}\right) - f(0) = f\left(\frac{1}{2}\right) - f(1)$ $\quad$ [As f(0) = f(1)]

and $g\left(\frac{1}{2}\right) = f(1) - f\left(\frac{1}{2}\right) = -\left[f\left(\frac{1}{2}\right) - f(1)\right]$

Since g is continuous and g(0) and g(1/2) have opposite signs, the equation g(x) = 0 must have at least one root in [0, 1/2].

Hence, for some $c \in \left[0, \frac{1}{2}\right]$, $g(c) = 0$ implies $f\left(c + \frac{1}{2}\right) = f(c)$.

Ex. 14 Let $f : [0, 1] \xrightarrow{\text{onto}} [0, 1]$ be a continuous function, then prove that f(x) = x for atleast one x ∈ [0, 1]

Sol. Consider g(x) = f(x) – x

$g(0) = f(0) - 0 = f(0) \geq 0$ $\quad \{\because 0 \leq f(x) \leq 1\}$

$g(1) = f(1) - 1 \leq 0$ $\quad \Rightarrow \quad g(0) \cdot g(1) \leq 0$

$\Rightarrow$ g(x) = 0 has atleast one root in [0, 1] $\quad \Rightarrow \quad$ f(x) = x for atleast one x ∈ [0, 1]

Ex. 15 Using intermediate value there, prove that exists a number x such that $x^{2005} + \frac{1}{(1+\sin^2 x)^2} = 2005$.

Sol. Let $\quad f(x) = x^{2005} + (1 + \sin^2 x)^{-2}$.

Thus, f is continuous and f(0) = 1 < 2005 and $f(2) > 2^{2005}$, which is much greater than 2005. Hence, from the intermediate value theorem, there exists a number c in (0, 2) such that f(c) = 2005.

Exercise # 1 [Single Correct Choice Type Questions]

1. If $f(x)=\begin{cases} x+2 & , \text{ when } x<1 \\ 4x-1 & , \text{ when } 1\le x\le 3 \\ x^2+5 & , \text{ when } x>3 \end{cases}$, then correct statement is -

(A) $\lim_{x\to 1} f(x)=\lim_{x\to 3} f(x)$ **(B)** f(x) is continuous at x = 3

(C) f(x) is continuous at x = 1 **(D)** f(x) is continuous at x = 1 and 3

2. If $f(x)=\dfrac{x-e^x+\cos 2x}{x^2}$, $x\neq 0$ is continuous at x = 0, then

(A) $f(0)=\dfrac{5}{2}$ **(B)** [f(0)] = –2 **(C)** {f(0)} = –0.5 **(D)** [f(0)] . {f(0)} = –1.5

where [x] and {x} denotes greatest integer and fractional part function

3. Let $f(x)=\dfrac{g(x)}{h(x)}$, where g and h are cotinuous functions on the open interval (a, b). Which of the following statements is true for a < x < b?

(A) f is continuous at all x for which x is not zero.

(B) f is continuous at all x for which $g(x)=0$

(C) f is continuous at all x for which $g(x)$ is not equal to zero.

(D) f is continuous at all x for which $h(x)$ is not equal to zero.

4. If $f(x)=\begin{cases} \dfrac{x^2-(a+2)x+2a}{x-2} & , \; x\neq 2 \\ 2 & , \; x=2 \end{cases}$ is continuous at x = 2, then a is equal to -

(A) 0 **(B)** 1 **(C)** –1 **(D)** 2

5. A function f(x) is defined as below $f(x)=\dfrac{\cos(\sin x)-\cos x}{x^2}$, $x\neq 0$ and f(0) = a

f(x) is continuous at x = 0 if 'a' equals

(A) 0 **(B)** 4 **(C)** 5 **(D)** 6

6. If $f(x)=\begin{cases} \dfrac{1}{e^{1/x}+1} & , \; x\neq 0 \\ 0 & , \; x=0 \end{cases}$, then -

(A) $\lim_{x\to 0+} f(x)=1$ **(B)** $\lim_{x\to 0-} f(x)=0$

(C) f(x) is discontinuous at x = 0 **(D)** f(x) is continuous

7. Consider the function $f(x) = \begin{cases} x\{x\}+1 & 0 \le x < 1 \\ 2-\{x\} & 1 \le x \le 2 \end{cases}$ where $\{x\}$ denotes the fractional part function. Which one of the following statements is NOT correct?

(A) $\lim_{x \to 1} f(x)$ exists
(B) $f(0) \ne f(2)$
(C) $f(x)$ is continuous in $[0, 2]$
(D) Rolles theorem is not applicable to $f(x)$ in $[0, 2]$

8. If $f(x) = \begin{cases} [x]+[-x], & x \ne 2 \\ \lambda, & x = 2 \end{cases}$, f is continuous at $x = 2$ then λ is (where [.] denotes greatest integer) -

(A) –1 **(B)** 0 **(C)** 1 **(D)** 2

9. Given $f(x) = \dfrac{e^x - \cos 2x - x}{x^2}$ for $x \in R - \{0\}$

$$g(x) \begin{cases} = f(\{x\}) & \text{for } n < x < n + \frac{1}{2} \\ = f(1-\{x\}) & \text{for } n + \frac{1}{2} \le x < n+1, n \in I \\ = \frac{5}{2} & \text{otherwise} \end{cases} \quad \left\{ \begin{array}{l} \text{where } \{x\} \text{ denotes} \\ \text{fractional part function} \end{array} \right.$$

then g (x) is

(A) discontinuous at all integral values of x only
(B) continuous everywhere except for x = 0
(C) discontinuous at $x = n + \frac{1}{2}$; $n \in I$ and at some $x \in I$
(D) continuous everywhere

10. Function $f(x) = \dfrac{1}{\log|x|}$ is discontinuous at -

(A) one point **(B)** two points **(C)** three points **(D)** infinite number of points

11. Let $f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right|$, when $-2 \le x \le 2$. where [.] represents greatest integer function. Then

(A) f(x) is continuous at x = 2
(B) f(x) is continuous at x = 1
(C) f(x) is continuous at x = – 1
(D) f(x) is discontinuous at x = 0

12. $f(x) = \dfrac{2\cos x - \sin 2x}{(\pi - 2x)^2}$; $g(x) = \dfrac{e^{-\cos x} - 1}{8x - 4\pi}$

$$h(x) \begin{cases} = f(x) & \text{for } x < \pi/2 \\ = g(x) & \text{for } x > \pi/2 \end{cases}$$

then which of the following holds ?

(A) h is continuous at $x = \pi/2$
(B) h has an irremovable discontinuity at $x = \pi/2$
(C) h has a removable discontinuity at $x = \pi/2$
(D) $f\left(\frac{\pi^+}{2}\right) = g\left(\frac{\pi^-}{2}\right)$

13. If function $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$, is continuous function, then f(0) is equal to -

(A) 2 **(B)** 1/4 **(C)** 1/6 **(D)** 1/3

14. Let $[x]$ denote the integral part of $x \in R$. $g(x) = x - [x]$. Let $f(x)$ be any continuous function with $f(0) = f(1)$ then the function $h(x) = f(g(x))$

(A) has finitely many discontinuities **(B)** is discontinuous at some x = c

(C) is continuous on R **(D)** is a constant function.

15. If $f(x) = \begin{cases} \dfrac{\log(1+2ax) - \log(1-bx)}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$, is continuous at x = 0, then k is equal to -

(A) 2a + b **(B)** 2a – b **(C)** b – 2a **(D)** a + b

16. Let f(x) = Sgn(x) and g(x) = x (x^2 – 5x + 6). The function f(g(x)) is discontinuous at

(A) infinitely many points **(B)** exactly one point

(C) exactly three points **(D)** no point

17. If $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & , x < 0 \\ a & , x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} & , x > 0 \end{cases}$, then correct statement is -

(A) f(x) is discontinuous at x = 0 for any value of a **(B)** f(x) is continuous at x = 0 when a = 8

(C) f(x) is continuous at x = 0 when a = 0 **(D)** none of these

18. Let $f(x) = \begin{cases} \dfrac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} & \text{if } x > 2 \\ \dfrac{x^2-4}{x - \sqrt{3x-2}} & \text{if } x < 2 \end{cases}$ then

(A) f(2) = 8 $\Rightarrow$ f is continuous at x = 2 **(B)** f(2) = 16 $\Rightarrow$ f is continuous at x = 2

(C) $f(2^-) \neq f(2^+)$ $\Rightarrow$ f is discontinuous **(D)** f has a removable discontinuity at x = 2

19. If $f(x) = \dfrac{1-\tan x}{4x-\pi}$, $x \neq \dfrac{\pi}{4}$, $x \in \left[0, \dfrac{\pi}{2}\right)$ is a continuous functions, then f(π/4) is equal to -

(A) –1/2 **(B)** 1/2 **(C)** 1 **(D)** –1

20. $f(x) = \begin{cases} \dfrac{\sqrt{(1+px)} - \sqrt{(1-px)}}{x} & , \; -1 \le x < 0 \\ \dfrac{2x+1}{x-2} & , \; 0 \le x \le 1 \end{cases}$ is continuous in the interval $[-1, 1]$, then 'p' is equal to:

(A) -1 **(B)** $-1/2$ **(C)** $1/2$ **(D)** 1

21. Let $f(x) = \dfrac{x(1 + a\cos x) - b\sin x}{x^3}$, $x \ne 0$ and $f(0) = 1$. The value of a and b so that f is a continuous function are -

(A) 5/2, 3/2 **(B)** 5/2, –3/2 **(C)** –5/2, –3/2 **(D)** none of these

22. Let $f(x) = \begin{cases} a\sin^{2n} x & \text{for } x \ge 0 \text{ and } n \to \infty \\ b(\cos^{2m} x) - 1 & \text{for } x < 0 \text{ and } m \to \infty \end{cases}$ then

(A) $f(0^-) \ne f(0^+)$ **(B)** $f(0^+) \ne f(0)$
(C) $f(0^-) = f(0)$ **(D)** f is continuous at $x = 0$

23. If $f(x + y) = f(x) + f(y) + c$, for all real x and y and f(x) is continuous at $x = 0$ and $f'(0) = 1$ then $f'(x)$ equals to

(A) c **(B)** –1 **(C)** 0 **(D)** 1

24. Which of the following functions has finite number of points of discontinuity in R
(where [.] denotes greatest integer)

(A) $\tan x$ **(B)** $|x|/x$ **(C)** $x + [x]$ **(D)** $\sin[\pi x]$

25. If $y = \dfrac{1}{t^2 + t - 2}$ where $t = \dfrac{1}{x-1}$, then the number of points of discontinuities of $y = f(x)$, $x \in R$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** infinite

26. The value of f(0), so that function, $f(x) = \dfrac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$ becomes continuous for all x, is given by -

(A) $a\sqrt{a}$ **(B)** $-\sqrt{a}$ **(C)** $\sqrt{a}$ **(D)** $-a\sqrt{a}$

27. Let f (x) be the continuous function such that $f(x) = \dfrac{1 - e^x}{x}$ for $x \ne 0$ then

(A) $f'(0^+) = \dfrac{1}{2}$ and $f'(0^-) = -\dfrac{1}{2}$ **(B)** $f'(0^+) = -\dfrac{1}{2}$ and $f'(0^-) = \dfrac{1}{2}$

(C) $f'(0^+) = f'(0^-) = \dfrac{1}{2}$ **(D)** $f'(0^+) = f'(0^-) = -\dfrac{1}{2}$

28. 'f' is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$ is -

(A) $\frac{2(\sqrt{3}-2)}{\sqrt{3}}$ **(B)** $2(1 - \sqrt{3})$ **(C)** zero **(D)** cannot be determined

29. The equation $2\tan x + 5x - 2 = 0$ has

(A) no solution in $[0, \pi/4]$ **(B)** at least one real solution in $[0, \pi/4]$

(C) two real solution in $[0, \pi/4]$ **(D)** None of these

30. Consider the piecewise defined function $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 0 & \text{if } 0 \le x \le 4 \\ x - 4 & \text{if } x > 4 \end{cases}$ choose the answer which best describes the continuity of this function -

(A) the function is unbounded and therefore cannot be continuous

(B) the function is right continuous at $x = 0$

(C) the function has a removable discontinuity at 0 and 4, but is continuous on the rest of the real line

(D) the function is continuous on the entire real line

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. f(x) is continuous at x=0, then which of the following are always true ?

(A) $\lim_{x\to 0} f(x)=0$
(B) f(x) is non continuous at x=1
(C) $g(x)=x^2f(x)$ is continuous at x = 0
(D) $\lim_{x\to 0^+} (f(x)-f(0))=0$

2. The value(s) of x for which $f(x)=\dfrac{e^{\sin x}}{4-\sqrt{x^2-9}}$ is continuous, is (are) -

(A) 3
(B) –3
(C) 5
(D) all $x \in (-\infty,-3] \cup [3,\infty)$

3. Which of the following function(s) not defined at x = 0 has/have removable discontinuity at x=0?

(A) $f(x)=\dfrac{1}{1+2^{\cot x}}$
(B) $f(x)=\cos\left(\dfrac{|\sin x|}{x}\right)$
(C) $f(x)=x\sin\dfrac{\pi}{x}$
(D) $f(x)=\dfrac{1}{\ln|x|}$

4. $f(x)=\dfrac{2\cos x-\sin 2x}{(\pi-2x)^2}$; $g(x)=\dfrac{e^{-\cos x}-1}{8x-4\pi}$

$$h(x)\begin{cases} =f(x) & \text{for } x<\pi/2 \\ =g(x) & \text{for } x>\pi/2 \end{cases}$$

then which of the followings does not holds ?

(A) h is continuous at x = π/2
(B) h has an irremovable discontinuity at x=π/2
(C) h has a removable discontinuity at x = π/2
(D) $f\left(\dfrac{\pi^+}{2}\right)=g\left(\dfrac{\pi^-}{2}\right)$

5. If $f(x)=\cos\left[\dfrac{\pi}{x}\right]\cos\left(\dfrac{\pi}{2}(x-1)\right)$; where [x] is the greatest integer function of x, then f(x) is continuous at -

(A) x = 0
(B) x = 1
(C) x = 2
(D) none of these

6. Let $f(x)=\dfrac{|x+\pi|}{\sin x}$, then

(A) $f(-\pi^+)=-1$
(B) $f(-\pi^-)=1$
(C) $\lim_{x\to-\pi} f(x)$ does not exist
(D) $\lim_{x\to\pi} f(x)$ does not exist

7. On the interval I = [–2, 2], the function $f(x)=\begin{cases}(x+1)e^{-\left[\frac{1}{|x|}+\frac{1}{x}\right]} & (x\neq 0)\\ 0 & (x=0)\end{cases}$

then which one of the following hold good ?

(A) is continuous for all values of $x \in I$
(B) is continuous for $x \in I-(0)$
(C) assumes all intermediate values from f(–2) & f(2)
(D) has a maximum value equal to 3/e

8. Let $f(x) = [x]$ & $g(x) = \begin{cases} 0; & x \in Z \\ x^2; & x \in R - Z \end{cases}$, then (where [.] denotes greatest integer function) -

(A) $\lim_{x \to 1} g(x)$ exists, but g(x) is not continuous at x= 1.

(B) $\lim_{x \to 1} f(x)$ does not exist and $f(x)$ is not continuous at x=1.

(C) gof is continuous for all x.

(D) fog is continuous for all x.

9. Which of the following function(s) not defined at $x = 0$ has/have non-removable discontinuity at the point x = 0 ?

(A) $f(x) = \dfrac{1}{1 + 2^{\frac{1}{x}}}$ **(B)** $f(x) = \text{arc}\tan\dfrac{1}{x}$ **(C)** $f(x) = \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ **(D)** $f(x) = \dfrac{1}{\ln|x|}$

10. Indicate all correct alternatives if, $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0,\pi]$

(A) $\tan(f(x))$ & $\dfrac{1}{f(x)}$ are both continuous **(B)** $\tan(f(x))$ & $\dfrac{1}{f(x)}$ are both discontinuous

(C) $\tan(f(x))$ & $f^{-1}(x)$ are both continuous **(D)** $\tan(f(x))$ is continuous but $\dfrac{1}{f(x)}$ is not

11. Which of the following function(s) defined below has/have single point continuity.

(A) $f(x) = \begin{bmatrix} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$ **(B)** $g(x) = \begin{bmatrix} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{bmatrix}$

(C) $h(x) = \begin{bmatrix} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{bmatrix}$ **(D)** $k(x) = \begin{bmatrix} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{bmatrix}$

12. Which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{1 + 2^{\cot x}}$ **(B)** $f(x) = \cos\left(\dfrac{|\sin x|}{x}\right)$

(C) $f(x) = x\sin\dfrac{\pi}{x}$ **(D)** $f(x) = \dfrac{1}{\ln|x|}$

13. The function, $f(x) = [\,|x|\,] - |\,[x]\,|$ where $[x]$ denotes greatest integer function

(A) is continuous for all positive integers

(B) is discontinuous for all non positive integers

(C) has finite number of elements in its range

(D) is such that its graph does not lie above the x – axis.

14. If $f(x) = \dfrac{1}{2}x - 1$, then on the interval $[0, \pi]$

(A) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both continuous **(B)** $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both discontinuous

(C) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous **(D)** $\tan(f(x))$ is continuous but $\dfrac{1}{f(x)}$ is not.

15. Given $f(x)=\begin{cases}3-\left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right] & \text{for} \quad x>0\\ \left\{x^2\right\}\cos\left(e^{1/x}\right) & \text{for} \quad x<0\end{cases}$ where { } & [ ] denotes the fractional part and the integral part functions respectively, then which of the following statement does not hold good -

(A) $f(0^-)=0$ **(B)** $f(0^+)=3$

(C) $f(0)=0 \Rightarrow$ continuity of f at $x=0$ **(D)** irremovable discontinuity of f at $x=0$

16. If $f(x)=\dfrac{1}{x^2-17x+66}$, then $f\left(\dfrac{2}{x-2}\right)$ is discontinuous at x =

(A) 2 **(B)** $\dfrac{7}{3}$ **(C)** $\dfrac{24}{11}$ **(D)** 6,11

17. f is a continous function in [a, b]; g is a continuous functin in [b, c]

A function h (x) is defined as

$$h(x)\begin{cases}=f(x) & \text{for } x\in[a,b]\\ =g(x) & \text{for } x\in(b,c]\end{cases}$$

if $f(B)=g(B)$, then

(A) h(x) has a removable discontinuity at x=b. **(B)** h(x) may or may not be continuous in [a, c]

(C) $h(b^-)=g(b^+)$ and $h(b^+)=f(b^-)$ **(D)** $h(b^+)=g(b^-)$ and $h(b^-)=f(b^+)$

18. Function whose jump (non-negative difference of LHL & RHL) of discontinuity is greater than or equal to one, is/are -

(A) $f(x)=\begin{cases}\dfrac{(e^{1/x}+1)}{(e^{1/x}-1)} & ; \quad x<0\\ \dfrac{(1-\cos x)}{x} & ; \quad x>0\end{cases}$

(B) $g(x)=\begin{cases}\dfrac{x^{1/3}-1}{x^{1/2}-1} & ; \quad x>1\\ \dfrac{\ln x}{(x-1)} & ; \quad \dfrac{1}{2}<x<1\end{cases}$

(C) $u(x)=\begin{cases}\dfrac{\sin^{-1}2x}{\tan^{-1}3x} & ; \quad x\in\left(0,\dfrac{1}{2}\right]\\ \dfrac{|\sin x|}{x} & ; \quad x<0\end{cases}$

(D) $v(x)=\begin{cases}\log_3(x+2) & ; \quad x>2\\ \log_{1/2}(x^2+5) & ; \quad x<2\end{cases}$

19. Let f(x) = [x] + $\sqrt{x-[x]}$, where [.] denotes the greatest integer function. Then

(A) f(x) is continuous on R^+ **(B)** f(x) is continuous on R

(C) f(x) is continuous on R – I **(D)** discontinuous at x = 1

20. Let 'f' be a continuous function on R. If $f(1/4^n)=(\sin e^n)e^{-n^2}+\dfrac{n^2}{n^2+1}$ then f(0) is -

(A) not unique **(B)** 1

(C) data sufficient to find f(0) **(D)** data insufficient to find f(0)

21. Let $f(x+y) = f(x) + f(y)$ for all $x, y \in R$. Then

(A) $f(x)$ must be continuous $\forall x \in R$ **(B)** $f(x)$ may be continuous $\forall x \in R$

(C) $f(x)$ must be discontinuous $\forall x \in R$ **(D)** $f(x)$ may be discontinuous $\forall x \in R$

22. Let f(x) and g(x) be defined by f(x) = [x] and $g(x) = \begin{cases} 0 \ , & x \in I \\ x^2 \ , & x \in R - I \end{cases}$

(where [.] denotes the greatest integer function), then

(A) $\lim\limits_{x \to 1} g(x)$ exists, but g is not continuous at x = 1

(B) $\lim\limits_{x \to 1} f(x)$ does not exist and f is not continuous at x = 1

(C) gof is continuous for all x **(D)** fog is continuous for all x

23. Given $f(x) \begin{cases} = b([x]^2 + [x]) + 1 & \text{for } x \geq -1 \\ = \sin(\pi(x+a)) & \text{for } x < -1 \end{cases}$

where [x] denotes the integral part of x, then for what values of a, b the function is continuous at x = – 1 ?

(A) $a = 2n + (3/2) \ ; \ b \in R \ ; \ n \in I$ **(B)** $a = 4n + 2 \ ; \ b \in R \ ; \ n \in I$

(C) $a = 2n + (3/2) \ ; \ b \in R \ ; \ n \in I$ **(D)** $a = 4n + 1 \ ; \ b \in R^+ \ ; \ n \in I$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. Let $f(x) = \begin{cases} \dfrac{\cos x - e^{-x^2/2}}{x^3} \ , & x \neq 0 \\ 0 \ , & x = 0 \end{cases}$ then

Statement - I $f(x)$ is continuous at x = 0.

Statement - II $\lim\limits_{x \to 0} \dfrac{\cos x - e^{-x^2/2}}{x^4} = \dfrac{-1}{12}$.

2. **Statement - I** Range of $f(x) = x\left(\dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\right) + x^2 + x^4$ is not R.

Statement - II Range of a continuous even function can not be R.

3. **Statement - I** $f(x) = \{\tan x\} - [\tan x]$ is continuous at $x = \dfrac{\pi}{3}$,

where [.] and {.} represent greatest integral function and fractional part function respecitvely.

Statement - II If $y = f(x)$ & $y = g(x)$ are continuous at $x = a$ then $y = f(x) \pm g(x)$ are continuous at $x = a$

4. **Statement - I** $f(x) = \sin x + [x]$ is discontinuous at $x = 0$

Statement - II If $g(x)$ is continuous & $h(x)$ is discontinuous at $x = a$, then $g(x) + h(x)$ will necessarily be discontinuous at $x = a$

5. Let $f(x) = x - x^2$ and $g(x) = \{x\}\ \forall\ x \in R$. Where $\{\cdot\}$ denotes fractional part function.

Statement - I $f(g(x))$ will be continuous $\forall\ x \in R$.

Statement - II $f(0) = f(1)$ and $g(x)$ is periodic with period 1.

6. **Statement - I** $\lim\limits_{x\to 0}\left[\dfrac{\sin x}{x}\right] \neq \left[\lim\limits_{x\to 0}\dfrac{\sin x}{x}\right]$, where [.] represents greatest integer function.

Statement - II $\lim\limits_{x\to a} h(g(x)) = h\left(\lim\limits_{x\to a} g(x)\right)$, if $y = h(x)$ is continuous at $x = \lim\limits_{x\to a} g(x)$.

7. **Statement - I** The equation $\dfrac{x^3}{4} - \sin \pi x + 3 = 2\dfrac{1}{3}$ has atleast one solution in $[-2, 2]$

Statement - II If $f:[a, b] \to R$ be a function & let 'c' be a number such that $f(a) < c < f(b)$, then there is atleast one number $n \in (a, b)$ such that $f(n) = c$.

8. **Statement - I** $f(x) = |x - 2| + \dfrac{x^2 - 5x + 6}{x - 1} + \tan x$ is continuous function within the domain of $f(x)$.

Statement - II All absolute valued polynomial function, Rational polynomial function, trigonometric functions are continuous within their domain.

9. Let $f(x) = \begin{cases} -ax^2 - b|x| - c & -\alpha \le x < 0 \\ ax^2 + b|x| + c & 0 \le x \le \alpha \end{cases}$ where a, b, c are positive and $\alpha > 0$, then

Statement - I The equation $f(x) = 0$ has atleast one real root for $x \in [-\alpha, \alpha]$

Statement - II Values of $f(-\alpha)$ and $f(\alpha)$ are opposite in sign.

10. Consider $f(x) = \begin{cases} 2\sin(a\cos^{-1} x) & \text{if } x \in (0,1) \\ \sqrt{3} & \text{if } x = 0 \\ ax + b & \text{if } x < 0 \end{cases}$

Statement-I : If $b = \sqrt{3}$ and $a = \dfrac{2}{3}$ then $f(x)$ is continuous in $(-\infty, 1)$

Statement-II : If a function is defined on an interval I and limit exist at every point of interval I then function is continuous in I.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1.

Column - I	Column - II
(A) If $f(x) = 1/(1-x)$, then the points at which the function fofof(x) is discontinuous	**(p)** $\frac{1}{2}$
(B) $f(u) = \frac{1}{u^2+u-2}$, where $u = \frac{1}{x-1}$. The values of x at which 'f' is discontinuous	**(q)** 0
(C) $f(x) = u^2$, where $u = \begin{cases} x-1, x \geq 0 \\ x+1, x < 0 \end{cases}$ The number of values of x at which 'f' is discontinuous	**(r)** 2
(D) The number of value of x at which the function $f(x) = \frac{2x^5 - 8x^2 + 11}{x^4 + 4x^3 + 8x^2 + 8x + 4}$ is discontinuous	**(s)** 1

2.

Column - I	Column - II
(A) If $f(x) = \begin{cases} \sin\{x\}; & x < 1 \\ \cos x + a; & x \geq 1 \end{cases}$ where {.} denotes the fractional part function, such that f(x) is continuous at x = 1. If $\lvert k \rvert = \frac{a}{\sqrt{2}\sin\frac{(4-\pi)}{4}}$ then k is	**(p)** 1
(B) If the function $f(x) = \frac{(1-\cos(\sin x))}{x^2}$ is continuous at x = 0, then f(0) is	**(q)** 0
(C) $f(x) = \begin{bmatrix} x & , x \in Q \\ 1-x, & x \notin Q \end{bmatrix}$, then the values of x at which f(x) is continuous	**(r)** –1
(D) If $f(x) = x + \{-x\} + [x]$, where [x] and {x} represents integral and fractional part of x, then the values of x at which f(x) is discontinuous	**(s)** $\frac{1}{2}$

Part # II [Comprehension Type Questions]

Comprehension # 1

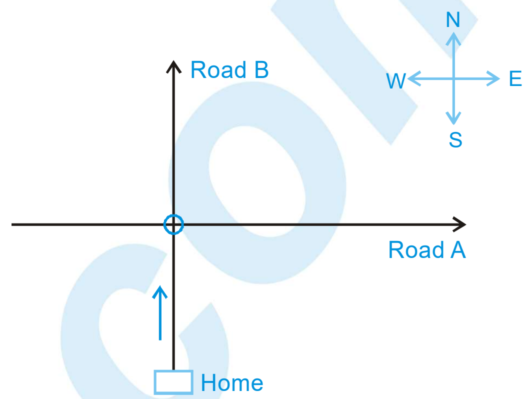
A man leaves his home early in the morning to have a walk. He arrives at a junction of road A & road B as shown in figure. He takes the following steps in later journey :

(A) 1 km in north direction

(B) changes direction & moves in north-east direction for $2\sqrt{2}$ kms.

(C) changes direction & moves southwards for distance of 2 km.

(D) finally he changes the direction & moves in south-east direction to reach road A again.



Visible/Invisible path :- The path traced by the man in the direction parallel to road A & road B is called invisible path, the remaining path traced is visible.

Visible points :- The points about which the man changes direction are called visible points except the point from where he changes direction last time

Now if road A & road B are taken as x-axis & y-axis then visible path & visible point represents the graph of y = f(x).

On the basis of above information, answer the following questions :

1. The value of x at which the function is discontinuous -

(A) 2 **(B)** 0 **(C)** 1 **(D)** 3

2. The value of x at which fof(x) is discontinuous -

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

3. If f(x) is periodic with period 3, then f(19) is -

(A) 2 **(B)** 3 **(C)** 19 **(D)** none of these

Comprehension # 2

If both $\lim\limits_{x\to c^-} f(x)$ and $\lim\limits_{x\to c^+} f(x)$ exist finitely and are equal , then the function f is said to have removable discontinuity at x = c

If both the limits i.e. $\lim\limits_{x\to c^-} f(x)$ and $\lim\limits_{x\to c^+} f(x)$ exist finitely and are not equal, then the function f is said to have non-removable discontinuity at x = c and in this case $\left|\lim\limits_{x\to c^+} f(x) - \lim\limits_{x\to c^-} f(x)\right|$ is called jump of the discontinuity.

1. Which of the following function has non-removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{\ln|x|}$ **(B)** $f(x) = x \sin\dfrac{\pi}{x}$ **(C)** $f(x) = \dfrac{1}{1+2^{\cot x}}$ **(D)** $f(x) = \cos\left(\dfrac{|\sin x|}{x}\right)$

2. Which of the following function not defined at x = 0 has removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{1+2^{\frac{1}{x}}}$ **(B)** $f(x) = \tan^{-1}\dfrac{1}{x}$ **(C)** $f(x) = \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$ **(D)** $f(x) = \dfrac{1}{\ln|x|}$

3. If $f(x) = \begin{cases} \tan^{-1}(\tan x); & x \le \dfrac{\pi}{4} \\ \pi[x]+1 \quad ; & x > \dfrac{\pi}{4} \end{cases}$, then jump of discontinuity is

(where [.] denotes greatest integer function)

(A) $\dfrac{\pi}{4} - 1$ **(B)** $\dfrac{\pi}{4} + 1$ **(C)** $1 - \dfrac{\pi}{4}$ **(D)** $-1 - \dfrac{\pi}{4}$

Comprehension # 3

If $S_n(x) = \dfrac{x}{x+1} + \dfrac{x^2}{(x+1)(x^2+1)} + \ldots\ldots + \dfrac{x^{2^n}}{(x+1)(x^2+1)\ldots\ldots(x^{2^n}+1)}$ and $x > 1$

$\lim\limits_{n\to\infty} S_n(x) = 1$

$$g(x) = \begin{cases} \dfrac{\sqrt{ax+b}-1}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$$

$h: R \to R \quad h(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 7$

On the basis of above information, answer the following questions :

1. If g(x) is continuous at x = 0 then a + b is equal to -

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

2. If g(x) is continuous at x = 0 then g'(0) is equal to -

(A) ● **(B)** $\dfrac{h(6)}{2}$ **(C)** $a - 2b$ **(D)** does not exist

3. Identify the incorrect option -

(A) h(x) is surjective **(B)** domain of g(x) is $[-1/2, \infty)$

(C) h(x) is bounded **(D)** ● = 1

Exercise # 4 [Subjective Type Questions]

1. Let $f(x)=\begin{cases}-2\sin x & \text{for } -\pi\le x\le -\frac{\pi}{2}\\ a\sin x+b & \text{for } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ \cos x & \text{for } \frac{\pi}{2}\le x\le \pi\end{cases}$. If f is continuous on $[-\pi,\pi]$ then find the values of a & b.

2. If $f(x)=\{x\}$ & $g(x)=[x]$ (where $\{.\}$ & $[.]$ denotes the fractional part and the integral part functions respectively), then discuss the continuity of :

(i) $h(x)=f(x).\,g(x)$ at $x=1$ and 2

(ii) $h(x)=f(x)+g(x)$ at $x=1$

(iii) $h(x)=f(x)-g(x)$ at $x=1$

(iv) $h(x)=g(x)+\sqrt{f(x)}$ at $x=1$ and 2

3. Determine the kind of discontinuity of the function $y=-\frac{2^{1/x}-1}{2^{1/x}+1}$ at the point $x=0$

4. Let $f(x)=\begin{cases}1+x^3, & x<0\\ x^2-1, & x\ge 0\end{cases}$; $g(x)=\begin{cases}(x-1)^{1/3}, & x<0\\ (x+1)^{1/2}, & x\ge 0\end{cases}$ Discuss the continuity of $g(f(x))$.

5. Find the values of 'a' & 'b' so that the function, $f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x} & , x<\pi/2\\ a & , x=\pi/2\\ \frac{b(1-\sin x)}{(\pi-2x)^2} & , x>\pi/2\end{cases}$ is continuous at $x=\pi/2$.

6. Consider the function $g(x)=\begin{cases}\frac{1-a^x+xa^x\ln a}{a^x x^2} & \text{for } x<0\\ \frac{2^x a^x-x\ln 2-x\ln a-1}{x^2} & \text{for } x>0\end{cases}$ where $a>0$.

Find the value of 'a' & 'g(0)' so that the function g(x) is continuous at $x=0$.

7. The function $f(x)=\begin{cases}\left(\frac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} & , 0<x<\frac{\pi}{2}\\ b+2 & , x=\frac{\pi}{2}\\ \left(1+|\cos x|\right)^{\left(\frac{a|\tan x|}{b}\right)} & , \frac{\pi}{2}<x<\pi\end{cases}$ Determine the values of 'a' & 'b', if f is continuous at $x=\pi/2$.

8. Determine the values of a,b & c for which the function $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & \text{for } x<0\\ c & \text{for } x=0\\ \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} & \text{for } x>0\end{cases}$

is continuous at $x=0$

9. Let $f(x) = \begin{cases} \dfrac{\left(\dfrac{\pi}{2} - \sin^{-1}\left(1 - \{x\}^2\right)\right).\sin^{-1}\left(1 - \{x\}\right)}{\sqrt{2}\left(\{x\} - \{x\}^3\right)} & \text{for } x \neq 0 \\ \dfrac{\pi}{2} & \text{for } x = 0 \end{cases}$ where $\{x\}$ is the fractional part of x.

Consider another function g(x); such that

$$g(x) = \begin{cases} f(x) & \text{for } x \geq 0 \\ 2\sqrt{2}\ f(x) & \text{for } x < 0 \end{cases}$$

Discuss the continuity of the functions f(x) & g(x) at x = 0.

10. If $f(x) = \underset{n \to \infty}{\text{Limit}} \dfrac{\log(x+2) - x^{2n} \sin x}{x^{2n} + 1}$ $(n \in N)$, examine the continuity of f(x) at x = 1.

11. Examine the continuity at x = 0 of the sum function of the infinite series:

$$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\ldots\ldots\ldots\infty\,.$$

12. Draw the graph of the function $f(x) = x - |x - x^2|, -1 \leq x \leq 1$ & discuss the continuity or discontinuity of f in the interval $-1 \leq x \leq 1$.

13. Suppose that $f(x) = x^3 - 3x^2 - 4x + 12$ and $h(x) = \begin{cases} \dfrac{f(x)}{x-3} & , x \neq 3 \\ K & , x = 3 \end{cases}$, then

(A) find all zeros of f

(B) find the value of K that makes h continuous at x = 3

(C) using the value of K found in (b), determine whether h is an even function.

14. If g : [a, b] onto [a, b] is continuous show that there is some $c \in [a, b]$ such that g(c) = c.

15. If $f(x) = \dfrac{\sin 3x + A \sin 2x + B \sin x}{x^5}$ $(x \neq 0)$ is continuous at x = 0, then find A & B. Also find f(0).

16. Given $f(x) = \sum_{r=1}^{n} \tan\left(\dfrac{x}{2^r}\right)\sec\left(\dfrac{x}{2^{r-1}}\right)$; r, n ∈ N

$$g(x) = \begin{cases} \underset{n\to\infty}{\text{Lim}} \dfrac{\ln\left(f(x) + \tan\dfrac{x}{2^n}\right) - \left(f(x) + \tan\dfrac{x}{2^n}\right)^n . \left[\sin\left(\tan\dfrac{x}{2}\right)\right]}{1 + \left(f(x) + \tan\dfrac{x}{2^n}\right)^n} & ; \quad x \neq \pi/4 \\ K & ; \quad x = \pi/4 \end{cases}$$

where [ ] denotes the greatest integer function and the domain of g(x) is $\left(0, \dfrac{\pi}{2}\right)$. Find the value of k, if possible, so that g(x) is continuous at x = π/4. Also state the points of discontinuity of g(x) in (0, π/4), if any.

17. Find interval for which the function given by the following expressions are continuous :

(i) $f(x)=\dfrac{3x+7}{x^2-5x+6}$ **(ii)** $f(x)=\dfrac{1}{|x|-1}-\dfrac{x^2}{2}$

(iii) $f(x)=\dfrac{\sqrt{x^2+1}}{1+\sin^2 x}$ **(iv)** $f(x)=\tan\left(\dfrac{\pi x}{2}\right)$

18. If $f(x)=\begin{cases}2x^2+12x+16, & -4\le x\le -2\\ 2-|x|, & -2<x\le 1\\ 4x-x^2-2, & 1<x\le 3\end{cases}$, then comment on continuity of

(i) $|f(x)|$ **(ii)** $f(|x|)$

19. $f(x)=\begin{cases}\dfrac{a^{\sin x}-a^{\tan x}}{\tan x-\sin x} & \text{for } x>0\\ \dfrac{\ln(1+x+x^2)+\ln(1-x+x^2)}{\sec x-\cos x} & \text{for } x<0\end{cases}$ if 'f' is continuous at $x = 0$, find 'a'

now if $g(x)=\ln\left(2-\dfrac{x}{a}\right).\cot(x-a)$ for $x\ne a$, $a\ne 0$, $a>0$. If 'g' is continuous at $x = a$ then show that $g(e^{-1})=-e$

20. If $f(x) = x + \{-x\} + [x]$, where [.] is the integral part & { . } is the fractional part function. Discuss the continuity of f in $[-2, 2]$. Also find nature of each discontinuity.

21. If $f(x)=\begin{cases}-x^2, & \text{when } x\le 0\\ 5x-4, & \text{when } 0<x\le 1\\ 4x^2-3x, & \text{when } 1<x<2\\ 3x+4, & \text{when } x\ge 2\end{cases}$, discuss the continuity of f(x) in R.

22. Discuss the continuity of f in [0,2] where $f(x)=\begin{cases}|4x-5|\,[x] & \text{for } x>1\\ [\cos\pi x] & \text{for } x\le 1\end{cases}$; (where [x] is the greatest integer not greater than x). Also draw the graph.

23. Let $y_n(x)=x^2+\dfrac{x^2}{1+x^2}+\dfrac{x^2}{(1+x^2)^2}+\ldots\ldots\dfrac{x^2}{(1+x^2)^{n-1}}$ and $y(x)=\lim\limits_{n\to\infty} y_n(x)$. Discuss the continuity of $y_n(x)$ $(n = 1, 2, 3, \ldots.. n)$ and $y(x)$ at $x = 0$

24. If $f(x . y) = f(x). f(y)$ for all x, y and f(x) is continuous at $x = 1$. Prove that f(x) is continuous for all x except possibly at $x = 0$. Given $f(1)\ne 0$.

25. If $f(x)=\dfrac{x^2+1}{x^2-1}$ and $g(x) = \tan x$, then discuss the continuity of fog (x).

26. Show that :

(A) a polynomial of an odd degree has at least one real root

(B) a polynomial of an even degree has at least two real roots if it attains at least one value opposite in sign to the coefficient of its highest-degree term.

27. Let [x] denote the greatest integer function & f(x) be defined in a neighbourhood of 2 by

$$f(x)=\begin{cases}\dfrac{\left(\exp\left\{(x+2)\ln 4\right\}\right)^{\frac{[x+1]}{4}}-16}{4^{x}-16}, & x<2\\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)}, & x>2\end{cases}$$

Find the value of A & f(2) in order that f(x) may be continuous at x = 2.

28. Let $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$. Determine the composite function g(x) = f (f(x)) & hence find the point of discontinuity of g , if any.

29. Find the point of discontinuity of y = f(u), where $f(u)=\dfrac{3}{2u^{2}+5u-3}$ and $u=\dfrac{1}{x+2}$.

30. If f(x . y) = f(x) . f(y) for all x, y and f(x) is continuous at x = 1. Prove that f(x) is continuous for all x except at x = 0. Given $f(1)\neq 0$.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $f(x)=\begin{cases} x & x\in Q \\ -x & x\notin Q \end{cases}$, then f is continuous at- **[AIEEE 2002]**

(1) Only at zero **(2)** only at 0, 1 **(3)** all real numbers **(4)** all rational numbers

2. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & , x\neq 0 \\ 0 & , x=0 \end{cases}$ then f(x) is- **[AIEEE 2003]**

(1) discontinuous everywhere
(2) continuous as well as differentiable for all x
(3) continuous for all x but not differentiable at x=0
(4) neither differentiable nor continuous at x = 0

3. Let $f(x)=\frac{1-\tan x}{4x-\pi}, x\neq\frac{\pi}{4}, x\in\left[0,\frac{\pi}{2}\right]$, If f(x) is continuous in $\left[0,\frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is- **[AIEEE 2004]**

(1) 1 **(2)** 1/2 **(3)** –1/2 **(4)** –1

4. The function $f: R/\{0\}\to R$ given by $f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$ can be made continuous at x = 0 by defining f(0) as- **[AIEEE 2007]**

(1) 2 **(2)** –1 **(3)** 0 **(4)** 1

5. The values of p and q for which the function $f(x)=\begin{cases} \frac{\sin(p+1)x+\sin x}{x} & , x<0 \\ q & , x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}} & , x>0 \end{cases}$ is continuous for all x in R, are:- **[AIEEE 2011]**

(1) $p=-\frac{3}{2}, q=\frac{1}{2}$ **(2)** $p=\frac{1}{2}, q=\frac{3}{2}$ **(3)** $p=\frac{1}{2}, q=-\frac{3}{2}$ **(4)** $p=\frac{5}{2}, q=\frac{1}{2}$

6. Define F(x) as the product of two real functions $f_1(x)=x, x\in IR$, and $f_2(x)=\begin{cases} \sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ as follows:

$F(x)=\begin{cases} f_1(x).f_2(x) & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ **[AIEEE 2011]**

Statement - 1 F(x) is continuous on IR.
Statement - 2 $f_1(x)$ and $f_2(x)$ are continuous on IR.
(1) Statemen-1 is false, statement-2 is true.
(2) Statemen-1 is true, statement-2 is true; Statement-2 is correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is true, statement-2 is not a correct explanation for statement-1
(4) Statement-1 is true, statement-2 is false

7. Consider the function, $f(x) = |x - 2| + |x - 5|, x \in R$.

Statement – 1 $f'(4) = 0$.

Statement – 2 f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5). **[AIEEE 2012]**

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Discuss the continuity of the function $f(x) = \begin{cases} \dfrac{e^{1/(x-1)} - 2}{e^{1/(x-1)} + 2}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ at x = 1.

[JEE 2001]

2. For every integer n, let a_n and b_n be real numbers. Let function $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for} \quad x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for} \quad x \in (2n-1, 2n) \end{cases}, \text{ for all integers n.}$$

If f is continuous, then which of the following holds(s) for all n ? **[JEE 2012]**

(A) $a_{n-1} - b_{n-1} = 0$ **(B)** $a_n - b_n = 1$ **(C)** $a_n - b_{n+1} = 1$ **(D)** $a_{n-1} - b_n = -1$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The value of f(0), so that the function $f(x)=\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ $(a>0)$ becomes continuous for all x, is given by -

(A) $a\sqrt{a}$ **(B)** $\sqrt{a}$ **(C)** $-\sqrt{a}$ **(D)** $-a\sqrt{a}$

2. If $f(x)=[x](\sin kx)^p$ is continuous for real x, then

(A) $k\in\{n\pi, n\in I\}, p>0$

(B) $k\in\{2n\pi, n\in I\}, p>0$

(C) $k\in\{n\pi, n\in I\}, p\in R-\{0\}$

(D) $k\in\{n\pi, n\in I, n\neq 0\}, p\in R-\{0\}$

3. If $f(x)=\begin{cases}\dfrac{\sin\{\cos x\}}{x-\pi/2} & , x\neq\dfrac{\pi}{2}\\ 1 & , x=\dfrac{\pi}{2}\end{cases}$, where {.} represents the fractional part function, then

(A) f(x) is continuous at $x=\dfrac{\pi}{2}$

(B) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists, but f is not continuous at $x=\dfrac{\pi}{2}$

(C) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exist

(D) $\lim\limits_{x\to\frac{\pi}{2}} f(x)=1$

4. $f(x)=\begin{cases}\dfrac{e^{e/x}-e^{-e/x}}{e^{1/x}+e^{-1/x}} & , x\neq 0\\ k & , x=0\end{cases}$

(A) f is continuous at x = 0, when k = 0

(B) f is not continuous at x = 0 for any real k.

(C) $\lim\limits_{x\to 0} f(x)$ exist infinitely.

(D) None of these

5. If $f(x)=\begin{cases}\dfrac{e^{[x]+|x|}-2}{[x]+|x|} & , x\neq 0\\ -1 & , x=0\end{cases}$, (where [.] denotes G.I.F.) then

(A) f(x) is continuous at x = 0

(B) $\lim\limits_{x\to 0^+} f(x)=-1$

(C) $\lim\limits_{x\to 0^-} f(x)=1$

(D) None of these

6. The correct statement for the function $f(x)=\begin{cases}x & , x\in Q\\ -x & , x\in R\sim Q\end{cases}$ is

(A) continuous every where

(B) f(x) is a periodic function

(C) discontinuous every where except at x = 0

(D) f(x) is an even function

7. Let $f(x) = [\cos x + \sin x]$, $0 < x < 2\pi$, where [.] denotes G.I.F. The number of points of discontinuity of f(x) is-

(A) 6 **(B)** 5 **(C)** 4 **(D)** 3

8. If $f(x) = \text{sgn}(x)$ and $g(x) = x(1 - x^2)$, then the number of points of discontinuity of function f(g(x)) is

(A) exact two
(B) exact three
(C) finite and more than 3
(D) infinitely many

9. The function $f(x) = \begin{cases} x^2\left[\dfrac{1}{x^2}\right] & , \; x \neq 0 \\ 0 & , \; x = 0 \end{cases}$, is (where [.] denotes G.I.F.)

(A) Continuous at $x = 1$
(B) Continuous at $x = -1$
(C) Discontinuous at $x = 0$
(D) Continuous at $x = 2$

10. **S_1 :** If f is continuous and g is discontinuous at x = a, then f(x). g(x) is discontinuous at x = a.

S_2 : $f(x) = \sqrt{2-x} + \sqrt{x-2}$ is not continuous at x = 2.

S_3 : $e^{-|x|}$ is differentiable at x = 0.

S_4 : If f(x) is differentiable every where, then $|f|^2$ is differentiable every where.

(A) TTFF **(B)** TTFT **(C)** FTFT **(D)** FFFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $f(x) = \dfrac{2\cos x - \sin 2x}{(\pi - 2x)^2}$; $g(x) = \dfrac{e^{-\cos x} - 1}{8x - 4\pi}$

$$f(x) = \begin{cases} f(x) & \text{for } x < \pi/2 \\ g(x) & \text{for } x > \pi/2 \end{cases}$$

then which of the followings does not holds ?

(A) h is continuous at $x = \pi/2$
(B) h has an irremovable discontinuity at $x=\pi/2$
(C) h has a removable discontinuity at $x = \pi/2$
(D) $f\left(\dfrac{\pi^+}{2}\right) = g\left(\dfrac{\pi^-}{2}\right)$

12. Which of the following function(s) has/have removable discontinuity at x = 1.

(A) $f(x) = \dfrac{1}{\ln|x|}$ **(B)** $f(x) = \dfrac{x^2 - 1}{x^3 - 1}$ **(C)** $f(x) = 2^{-2^{\frac{1}{1-x}}}$ **(D)** $f(x) = \dfrac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$

13. On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x+1)e^{-\left[\frac{1}{|x|} + \frac{1}{x}\right]} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

then which one of the following hold good ?

(A) is continuous for all values of $x \in I$
(B) is continuous for $x \in I - (0)$
(C) assumes all intermediate values from f(–2) & f(2)
(D) has a maximum value equal to 3/e

14. Which of the following function(s) defined below has/have single point continuity.

(A) $f(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$
(B) $g(x) = \begin{cases} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{cases}$

(C) $h(x) = \begin{cases} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$
(D) $k(x) = \begin{cases} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{cases}$

15. Given $f(x) = \begin{cases} b([x]^2 + [x]) + 1 & \text{for } x \geq -1 \\ \sin(\pi(x+a)) & \text{for } x < -1 \end{cases}$

where [x] denotes the integral part of x, then for what values of a, b the function is continuous at x = – 1 ?

(A) $a = 2n + (3/2);\ b \in R\ ;\ n \in I$
(B) $a = 4n + 2\ ;\ b \in R\ ;\ n \in I$

(C) $a = 4n + (3/2)\ ;\ b \in R^+\ ;\ n \in I$
(D) $a = 4n + 1\ ;\ b \in R^+\ ;\ n \in I$

SECTION - III : ASSERTION AND REASON TYPE

16. Consider $f(x) = \begin{cases} 2\sin(a\cos^{-1} x) & \text{if } x \in (0,1) \\ \sqrt{3} & \text{if } x = 0 \\ ax + b & \text{if } x < 0 \end{cases}$

Statement-I : If $b = \sqrt{3}$ and $a = \frac{2}{3}$ then $f(x)$ is continuous in $(-\infty, 1)$

Statement-II : If a function is defined on an interval I and limit exist at every point of interval I then function is continuous in I.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** $f(x) = \sin x + [x]$ is discontinuous at $x = 0$.

Statement-II : If g(x) is continuous & h(x) is discontinuous at x = a, then g(x) + h(x) will necessarily be discontinuous at x = a.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** The equation $\frac{x^3}{4} - \sin \pi x + 3 = 2\frac{1}{3}$ has atleast one solution in [–2, 2]

Statement-II : If f:[a, b] → R be a function & let 'c' be a number such that f(a) < c < f(b), then there is atleast one number n ∈ (a, b) such that f(n) = c.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement-I :** If $f : R \to R$ is a continuous function such that $f(x) = f(3x)\ \forall\ x \in R$, then f is constant function.

Statement-II : If f is continuous at $x = \lim_{x\to a} g(x)$, then $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** Range of $f(x) = x\left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\right) + x^2 + x^4$ is not R.

Statement-II : Range of a continuous even function can not be R.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column I	Column II
(A) If function $f(x) = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$, is continuous function, then f(0) is equal to -	**(p)** 0
(B) The number of points where $f(x) = [\sin x + \cos x]$ (where [ ] denotes the greatest integer function), $x \in (0,\ 2\pi)$ is not continuous is -	**(q)** 5
(C) If $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right)$ is a continuous functions, then $f(\pi/4)$ is equal to -	**(r)** 1/6
(D) If $f(x) = \begin{cases} \frac{x^2 - (a+2)x + 2a}{x-2} & , \ x \neq 2 \\ 2 & , \ x = 2 \end{cases}$ is continuous at $x = 2$, then a is equal to	**(s)** –1/2

22.

Column I		Column II	
(A)	If $f(x) = \dfrac{\tan\left(\dfrac{\pi}{4} - \pi\right)}{\cot 2x}$, $(x \neq \pi/4)$, is continuous at $x = \pi/4$, then the value of $f\left(\dfrac{\pi}{4}\right)$ is	**(p)**	–3/2
(B)	Let $f(x) = \begin{cases} \dfrac{x-4}{\lvert x-4 \rvert} + a, & x < 4 \\ a + b, & x = 4 \\ \dfrac{x-4}{\lvert x-4 \rvert} + b, & x > 4 \end{cases}$ Then f(x) is continuous at x = 4, then the value of a & b	**(q)**	1
(C)	If $f(x) = \dfrac{x - e^x + \cos 2x}{x^2}$, $x \neq 0$, is continuous at $x = 0$, then $[f(0)]\ \{f(0)\} =$	**(r)**	1/2
(D)	Let f(x) be defined in the interval at [0, 4] such that $f(x) = \begin{cases} 1 - x, & 0 \le x \le 1 \\ x + 2, & 1 < x < 2 \\ 4 - x, & 2 \le x \le 4 \end{cases}$	**(s)**	–1
	Then the number of points where f(f(x)) is discontinuous is	**(t)**	2

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Consider $f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n\to\infty} \dfrac{f(x) + x^{2n} g(x)}{1 + x^{2n}}$,

1. If F(x) is continuous at x = 1, then

(A) $b = a + 3$ **(B)** $b = a - 1$ **(C)** $a = b - 2$ **(D)** None of these

2. If F(x) is continuous at x = –1, then

(A) $a + b = -2$ **(B)** $a - b = 3$ **(C)** $a + b = 5$ **(D)** None of these

3. If F(x) is continuous at $x = \pm 1$, then f(x) = g(x) has

(A) imaginary roots **(B)** both the roots positive

(C) both the roots negative **(D)** roots of opposite signs

24. **Read the following comprehension carefully and answer the questions.**

There are two systems S_1 and S_2 of definitions of limit and continuity. In system S_1 the definition are as usual. In system S_2 the definition of limit is as usual but the continuity is defined as follows :

A function f(x) is defined to be continuous at x = a if

(i) $\left|\lim_{x\to a^-} f(x) - \lim_{x\to a^+} f(x)\right| \le 1$ and

(ii) f(a) lies between the values of $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ if $\lim_{x\to a^-} f(x) \ne \lim_{x\to a^+} f(x)$

else $f(a) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

1. If $f(x) = \begin{cases} x+2.7 & , \ x<0 \\ 2.9 & , \ x=0 \\ 2x+3 & , \ x>0 \end{cases}$ and $g(x) = \begin{cases} 3x+3 & , \ x<0 \\ 2.8 & , \ x=0 \\ -x^2+2.7 & , \ x>0 \end{cases}$, then consider statements

(i) f(x) is discontinuous under the system S_1

(ii) f(x) is continuous under the system S_2

(iii) g(x) is continuous under the system S_2

which 7of the following option is correct

(A) only (i) is true
(B) only (i) and (ii) are true
(C) only (ii) and (iii) are true
(D) all (i), (ii), (iii) are true

2. If each of f(x) and g(x) is continuous at x = a in S_2, then in S_2 which of the following is continuous

(A) f + g **(B)** f – g **(C)** f . g **(D)** None of these

3. Which of the following is incorrect

(A) a continuous function under the definition in S_1 must also be continuous under the definition in S_2

(B) A continuous function under the definition in S_2 must also be continuous under the definition in S_1

(C) A discontinuous function under the definition in S_1 must also be discontinuous under the definition in S_2

(D) A discontinuous function under the definition in S_1 must be continuous under the definition in S_2

25. **Read the following comprehension carefully and answer the questions.**

Given the continuous function

$$y = f(x) = \begin{cases} x^2+10x+8, & x \le -2 \\ ax^2+bx+c, & -2<x<0, \ a\ne 0 \\ x^2+2x, & x\ge 0 \end{cases}$$

If a line L touches the graph of y = f(x) at the points, then

1. The slope of the line L is equal to

(A) 1 **(B)** 2 **(C)** 4 **(D)** 6

2. The value of (a + b + c) is equal to

(A) $5\sqrt{2}$ **(B)** 5 **(C)** 6 **(D)** 7

3. If y = f(x) is differentiable at x = 0, then the value of b is -

(A) –1 **(B)** 2 **(C)** 4 **(D)** cannot be determined

SECTION - VI : INTEGER TYPE

26. If $g(x)=\begin{cases}\dfrac{1-a^x+xa^x.\ln a}{x^2a^x} & , \quad x<0\\ \dfrac{(2a)^x-x\ln 2a-1}{x^2} & , \quad x>0\end{cases}$

(where $a>0$), then find 'a' and $g(0)$ so that $g(x)$ is continuous at $x=0$.

27. Find the value of f(0) so that the function

$$f(x)=\frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x\neq 0$$

($\{x\}$ denotes fractional part of x) becomes continuous at $x=0$

28. The function $f(x)=\begin{cases}\left(\dfrac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} & , \quad 0<x<\dfrac{\pi}{2}\\ b+2 & , \quad x=\dfrac{\pi}{2}\\ \left(1+|\cos x|\right)^{\left(\frac{a|\tan x|}{b}\right)} & , \quad \dfrac{\pi}{2}<x<\pi\end{cases}$. Determine the values of 'a' & 'b', if f is continuous at $x=\pi/2$.

29. Let f be a continuous function on R such that $f\left(\dfrac{1}{4x}\right)=\left(\sin e^x\right)e^{-x^2}+\dfrac{x^2}{x^2+1}$, then find the value of f(0).

30. If the function f(x) defined as $f(x)=\begin{cases}(\sin x+\cos x)^{\text{cosec}x} & , \quad -\dfrac{\pi}{2}<x<0\\ a & , \quad x=0\\ \dfrac{e^{\frac{1}{x}}+e^{\frac{2}{x}}+e^{\frac{3}{x}}}{ae^{-2+\frac{1}{x}}+be^{-1+\frac{3}{x}}} & , \quad 0<x<\dfrac{\pi}{2}\end{cases}$

is continuous at $x=0$, then find a & b.

ANSWER KEY

EXERCISE - 1

1. C **2.** D **3.** D **4.** A **5.** A **6.** C **7.** C **8.** A **9.** D **10.** C **11.** D **12.** B **13.** C
14. C **15.** A **16.** C **17.** B **18.** C **19.** A **20.** B **21.** C **22.** A **23.** D **24.** B **25.** C **26.** B
27. D **28.** B **29.** B **30.** D

EXERCISE - 2 : PART # I

1. CD **2.** AB **3.** BCD **4.** ACD **5.** BC **6.** ABCD **7.** BCD **8.** ABC **9.** ABC
10. CD **11.** BCD **12.** BCD **13.** ABCD **14.** CD **15.** BD **16.** ABC **17.** AC **18.** ACD
19. ABC **20.** BC **21.** BD **22.** ABC **23.** AC

PART - II

1. A **2.** A **3.** A **4.** A **5.** A **6.** A **7.** C **8.** A **9.** D **10.** C

EXERCISE - 3 : PART # I

1. **A** $\rightarrow$ q,s **B** $\rightarrow$ p,r,s **C** $\rightarrow$ q **D** $\rightarrow$ q **2.** **A** $\rightarrow$ p,r **B** $\rightarrow$ s **C** $\rightarrow$ s **D** $\rightarrow$ p,q,r

PART - II

Comprehension # 1 : **1.** A **2.** BC **3.** CA **Comprehension # 2 :** **1.** C **2.** D **3.** C
Comprehension # 3 : **1.** D **2.** B **3.** C

EXERCISE - 5 : PART # I

1. 1 **2.** 3 **3.** 3 **4.** 4 **5.** 1 **6.** 4 **7.** 4

PART - II

1. Discontinuous at $x = 1$; $f(1^+) = 1$ and $f(1^-) = -1$ **2.** B, D

MOCK TEST

1. C **2.** A **3.** C **4.** B **5.** D **6.** C **7.** B **8.** B **9.** C **10.** C **11.** ACD **12.** BD

13. BCD **14.** BCD **15.** AC **16.** C **17.** A **18.** C **19.** A **20.** A

21. **A** $\to$ r **B** $\to$ q **C** $\to$ s **D** $\to$ p **22.** **A** $\to$ r **B** $\to$ q,s **C** $\to$ p **D** $\to$ t

23. **1.** A **2.** C **3.** D **24.** **1.** D **2.** D **3.** B **25.** **1.** A **2.** D **3.** B

26. $\frac{1}{\sqrt{2}}, \frac{1}{8}(\ell n\, 2)^2$ **27.** no value of f(0) **28.** a = 0, b = –1 **29.** 1 **30.** a = e, b = 1