

EXERCISE-I

Integral power of iota, Algebraic operations and Equality of complex numbers

- 1.** If $i = \sqrt{-1}$, then $1 + i^2 + i^3 - i^6 + i^8$ is equal to
 (A) $2 - i$ (B) 1
 (C) 3 (D) -1
- 2.** If $i^2 = -1$, then the value of $\sum_{n=1}^{200} i^n$
 (A) 50 (B) -50
 (C) 0 (D) 100
- 3.** The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals
 (A) i (B) $i - 1$
 (C) $-i$ (D) 0
- 4.** The least positive integer n which will reduce $\left(\frac{i-1}{i+1}\right)^n$ to a real number, is
 (A) 2 (B) 3
 (C) 4 (D) 5
- 5.** The value of $i^{1+3+5+\dots+(2n+1)}$ is
 (A) i if n is even, $-i$ if n is odd
 (B) 1 if n is even, -1 if n is odd
 (C) 1 if n is odd, -1 if n is even
 (D) i if n is even, -1 if n is odd
- 6.** $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right) \left(\frac{3+4i}{2-4i}\right) =$
 (A) $\frac{1}{2} + \frac{9}{2}i$ (B) $\frac{1}{2} - \frac{9}{2}i$
 (C) $\frac{1}{4} - \frac{9}{4}i$ (D) $\frac{1}{4} + \frac{9}{4}i$
- 7.** Additive inverse of $1 - i$ is
 (A) $0 + 0i$ (B) $-1 - i$
 (C) $-1 + i$ (D) None of these

- 8.** $\operatorname{Re} \frac{(1+i)^2}{3-i} =$
 (A) $-1/5$ (B) $1/5$
 (C) $1/10$ (D) $-1/10$
- 9.** If $(1-i)x + (1+i)y = 1 - 3i$, then $(x, y) =$
 (A) $(2, -1)$ (B) $(-2, 1)$
 (C) $(-2, -1)$ (D) $(2, 1)$
- 10.** $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be real, if $\theta =$
 (A) $2n\pi$ (B) $n\pi + \frac{\pi}{2}$
 (C) $n\pi$ (D) None of these
 [Where n is an integer]
- 11.** If $z \neq 0$ is a complex number, then
 (A) $\operatorname{Re}(z) = 0 \Rightarrow \operatorname{Im}(z^2) = 0$
 (B) $\operatorname{Re}(z^2) = 0 \Rightarrow \operatorname{Im}(z^2) = 0$
 (C) $\operatorname{Re}(z) = 0 \Rightarrow \operatorname{Re}(z^2) = 0$
 (D) None of these
- 12.** If $\frac{5(-8+6i)}{(1+i)^2} = a + ib$, then (a, b) equals
 (A) $(15, 20)$ (B) $(20, 15)$
 (C) $(-15, 20)$ (D) None of these
- 13.** The true statement is
 (A) $1 - i < 1 + i$ (B) $2i + 1 > -2i + 1$
 (C) $2i > 1$ (D) None of these
- 14.** $\frac{1-2i}{2+i} + \frac{4-i}{3+2i} =$
 (A) $\frac{24}{13} + \frac{10}{13}i$ (B) $\frac{24}{13} - \frac{10}{13}i$
 (C) $\frac{10}{13} + \frac{24}{13}i$ (D) $\frac{10}{13} - \frac{24}{13}i$
- 15.** $a + ib > c + id$ can be explained only when
 (A) $b = 0, c = 0$ (B) $b = 0, d = 0$
 (C) $a = 0, c = 0$ (D) $a = 0, d = 0$

Complex Number

- 16.** If $x, y \in \mathbb{R}$ and $(x + iy)(3 + 2i) = 1 + i$, then (x, y) is
 (A) $\left(1, \frac{1}{5}\right)$ (B) $\left(\frac{1}{13}, \frac{1}{13}\right)$
 (C) $\left(\frac{5}{13}, \frac{1}{13}\right)$ (D) $\left(\frac{1}{5}, \frac{1}{5}\right)$
- 17.** If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
 (A) $a = 2, b = -1$ (B) $a = 1, b = 0$
 (C) $a = 0, b = 1$ (D) $a = -1, b = 2$
- 18.** If $z_1 = (4, 5)$ and $z_2 = (-3, 2)$ then $\frac{z_1}{z_2}$ equals
 (A) $\left(\frac{-23}{12}, \frac{-2}{13}\right)$ (B) $\left(\frac{2}{13}, \frac{-23}{13}\right)$
 (C) $\left(\frac{-2}{13}, \frac{-23}{13}\right)$ (D) $\left(\frac{-2}{13}, \frac{23}{13}\right)$
- 19.** If $z = 1 + i$, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$)
 (A) $2i$ (B) $1 - i$
 (C) $-i/2$ (D) $i/2$
- 20.** If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then (x, y) is
 (A) $(3, 1)$ (B) $(1, 3)$
 (C) $(0, 3)$ (D) $(0, 0)$
- 23.** If the conjugate of $(x + iy)(1 - 2i)$ be $1 + i$, then
 (A) $x = \frac{1}{5}$ (B) $y = \frac{3}{5}$
 (C) $x + iy = \frac{1-i}{1-2i}$ (D) $x - iy = \frac{1-i}{1+2i}$
- 24.** The conjugate of $\frac{(2+i)^2}{3+i}$, in the form of $a + ib$, is
 (A) $\frac{13}{2} + i\left(\frac{15}{2}\right)$ (B) $\frac{13}{10} + i\left(\frac{-15}{2}\right)$
 (C) $\frac{13}{10} + i\left(\frac{-9}{10}\right)$ (D) $\frac{13}{10} + i\left(\frac{9}{10}\right)$
- 25.** If $z = 3 + 5i$, then $z^3 + \bar{z} + 198 =$
 (A) $-3 - 5i$ (B) $-3 + 5i$
 (C) $3 + 5i$ (D) $3 - 5i$
- 26.** If z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1 + z_2)}{(z_1 - z_2)}$ may be
 (A) Purely imaginary
 (B) Real and positive
 (C) Real and negative
 (D) None of these
- 27.** The moduli of two complex numbers are less than unity, then the modulus of the sum of these complex numbers
 (A) Less than unity
 (B) Greater than unity
 (C) Equal to unity
 (D) Any
- 28.** The product of two complex numbers each of unit modulus is also a complex number, of
 (A) Unit modulus
 (B) Less than unit modulus
 (C) Greater than unit modulus
 (D) None of these

Conjugate, Modulus and Argument of complex numbers

- 21.** If $\frac{z-i}{z+i}$ ($z \neq -i$) is a purely imaginary number, then $z\bar{z}$ is equal to
 (A) 0 (B) 1
 (C) 2 (D) None of these
- 22.** If $\frac{c+i}{c-i} = a + ib$, where a, b, c are real, then
 $a^2 + b^2 =$
 (A) 1 (B) -1
 (C) c^2 (D) $-c^2$

- 29.** Let z be a complex number, then the equation $z^4 + z + 2 = 0$ cannot have a root, such that
 (A) $|z| < 1$ (B) $|z| = 1$
 (C) $|z| > 1$ (D) None of these
- 30.** If $|z_1| = |z_2| = \dots = |z_n| = 1$, then the value of $|z_1 + z_2 + z_3 + \dots + z_n| =$
 (A) 1
 (B) $|z_1| + |z_2| + \dots + |z_n|$
 (C) $\left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right|$
 (D) None of these
- 31.** If $|z| = 1$, ($z \neq -1$) and $z = x + iy$, then $\left(\frac{z-1}{z+1} \right)$ is
 (A) Purely real (B) Purely imaginary
 (C) Zero (D) Undefined
- 32.** The minimum value of $|2z-1| + |3z-2|$ is
 (A) 0 (B) $1/2$
 (C) $1/3$ (D) $2/3$
- 33.** If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then $\operatorname{Re}(\omega)$ is
 (A) 0 (B) $-\frac{1}{|z+1|^2}$
 (C) $\left| \frac{z}{z+1} \right| \cdot \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$
- 34.** A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix} \right) = \alpha - i\beta$ (α, β real), if
 (A) $\alpha^2 - \beta^2 = -1$ (B) $\alpha^2 - \beta^2 = 1$
 (C) $\alpha^2 + \beta^2 = 1$ (D) $\alpha^2 - \beta^2 = 2$
- 35.** Let z_1 be a complex number with $|z_1| = 1$ and z_2 be any complex number, then $\left| \frac{z_1 - z_2}{1 - z_1 \bar{z}_2} \right| =$
 (A) 0 (B) 1
 (C) -1 (D) 2
- 36.** $\arg \left(\frac{3+i}{2-i} + \frac{3-i}{2+i} \right)$ is equal to
 (A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$
 (C) 0 (D) $\frac{\pi}{4}$
- 37.** If $z_1 z_2 \dots z_n = z$, then
 $\arg z_1 + \arg z_2 + \dots + \arg z_n$ and $\arg z$ differ by a
 (A) Multiple of π (B) Multiple of $\frac{\pi}{2}$
 (C) Greater than π (D) Less than π
- 38.** Let z be a purely imaginary number such that $\operatorname{Im}(z) > 0$. Then $\arg(z)$ is equal to
 (A) π (B) $\frac{\pi}{2}$
 (C) 0 (D) $-\frac{\pi}{2}$
- 39.** Let z be a purely imaginary number such that $\operatorname{Im}(z) < 0$. Then $\arg(z)$ is equal to
 (A) π (B) $\frac{\pi}{2}$
 (C) 0 (D) $-\frac{\pi}{2}$
- 40.** If z is a purely real number such that $\operatorname{Re}(z) < 0$, then $\arg(z)$ is equal to
 (A) π (B) $\frac{\pi}{2}$
 (C) 0 (D) $-\frac{\pi}{2}$

- 41.** If $\arg z < 0$ then $\arg(-z) - \arg(z)$ is equal to
 (A) π (B) $-\pi$
 (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$
- 42.** The amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}-i}$ is
 (A) 0 (B) $\pi/6$
 (C) $\pi/3$ (D) $\pi/2$
- 43.** If $z = \frac{-2}{1+\sqrt{3}i}$ then the value of $\arg(z)$ is [
 (A) π (B) $\pi/3$
 (C) $2\pi/3$ (D) $\pi/4$
- 44.** If $z = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$ then
 (A) $|z| = 1$, $\arg z = \frac{\pi}{4}$
 (B) $|z| = 1$, $\arg z = \frac{\pi}{6}$
 (C) $|z| = \frac{\sqrt{3}}{2}$, $\arg z = \frac{5\pi}{24}$
 (D) $|z| = \frac{\sqrt{3}}{2}$, $\arg z = \tan^{-1} \frac{1}{\sqrt{2}}$
- 45.** The amplitude of $\sin \frac{\pi}{5} + i \left(1 - \cos \frac{\pi}{5}\right)$
 (A) $\pi/5$ (B) $2\pi/5$
 (C) $\pi/10$ (D) $\pi/15$
- 47.** The number of non-zero integral solutions of the equation $|1-i|^x = 2^x$ is
 (A) Infinite (B) 1
 (C) 2 (D) None of these
- 48.** $\frac{1+7i}{(2-i)^2} =$
 (A) $\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$
 (B) $\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$
 (C) $\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$
 (D) None of these
- 49.** If $z = r e^{i\theta}$, then $|e^{iz}| =$
 (A) $e^{r \sin \theta}$ (B) $e^{-r \sin \theta}$
 (C) $e^{-r \cos \theta}$ (D) $e^{r \cos \theta}$
- 50.** $\frac{1-i}{1+i}$ is equal to
 (A) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$ (B) $\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$
 (C) $\sin \frac{\pi}{2} + i \cos \frac{\pi}{2}$ (D) None of these
- 51.** The amplitude of $e^{e^{-i\theta}}$ is equal to
 (A) $\sin \theta$ (B) $-\sin \theta$
 (C) $e^{\cos \theta}$ (D) $e^{\sin \theta}$
- 52.** If $z = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$, then $(\bar{z})^{100}$ lies in
 (A) I quadrant (B) II quadrant
 (C) III quadrant (D) IV quadrant
- 53.** If $x + \frac{1}{x} = \sqrt{3}$, then $x =$
 (A) $\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ (B) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
 (C) $\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}$ (D) $\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$

Square root, Representation and Logarithm of complex numbers

- 46.** If $\sqrt{a+ib} = x+iy$, then possible value of $\sqrt{a-ib}$ is
 (A) $x^2 + y^2$ (B) $\sqrt{x^2 + y^2}$
 (C) $x+iy$ (D) $x-iy$

Geometry of complex numbers

Complex Number

- 66.** When $\frac{z+i}{z+2}$ is purely imaginary, the locus described by the point z in the Argand diagram is a
 (A) Circle of radius $\frac{\sqrt{5}}{2}$
 (B) Circle of radius $\frac{5}{4}$
 (C) Straight line
 (D) Parabola
- 67.** If $|z+1| = \sqrt{2}|z-1|$, then the locus described by the point z in the Argand diagram is a
 (A) Straight line (B) Circle
 (C) Parabola (D) None of these
- 68.** The region of the complex plane for which $\left|\frac{z-a}{z+a}\right|=1$ [$R(a) \neq 0$] is
 (A) x -axis
 (B) y -axis
 (C) The straight line $x = a$
 (D) None of these
- 69.** The region of Argand plane defined by $|z-1| + |z+1| \leq 4$ is
 (A) Interior of an ellipse
 (B) Exterior of a circle
 (C) Interior and boundary of an ellipse
 (D) None of these
- 70.** The locus of the points z which satisfy the condition $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is
 (A) A straight line (B) A circle
 (C) A parabola (D) None of these
- 71.** Locus of the point z satisfying the equation $|iz-1| + |z-i| = 2$ is
 (A) A straight line
 (B) A circle
 (C) An ellipse
 (D) A pair of straight lines
- 72.** If $z = x + iy$ is a complex number satisfying $\left|z + \frac{i}{2}\right|^2 = \left|z - \frac{i}{2}\right|^2$, then the locus of z is
 (A) $2y = x$ (B) $y = x$
 (C) y -axis (D) x -axis
- 73.** The locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right) = k$, (where k is non zero) is
 (A) Circle with centre on y -axis
 (B) Circle with centre on x -axis
 (C) A straight line parallel to x -axis
 (D) A straight line making an angle 60° with the x -axis
- 74.** If the amplitude of $z - 2 - 3i$ is $\pi/4$, then the locus of $z = x + iy$ is
 (A) $x + y - 1 = 0$ (B) $x - y - 1 = 0$
 (C) $x + y + 1 = 0$ (D) $x - y + 1 = 0$
- 75.** If $|z^2 - 1| = |z|^2 + 1$, then z lies on
 (A) An ellipse
 (B) The imaginary axis
 (C) A circle
 (D) The real axis

De Moivre's theorem and Roots of unity

- 76.** If $\left(\frac{1+\cos\theta+i\sin\theta}{i+\sin\theta+i\cos\theta}\right)^4 = \cos n\theta + i\sin n\theta$, then n is equal to
 (A) 1 (B) 2
 (C) 3 (D) 4
- 77.** The value of expression $\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ $\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)$ to ∞ is
 (A) -1 (B) 1
 (C) 0 (D) 2

- 90.** If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$, then the value of $x^3 + y^3 + z^3$ is equal to
 (A) $a^3 + b^3$ (B) $3(a^3 + b^3)$
 (C) $3(a^2 + b^2)$ (D) None of these
- 91.** The n^{th} roots of unity are in
 (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
- 92.** If $1, \omega, \omega^2$ are the three cube roots of unity, then $(3 + \omega^2 + \omega^4)^6 =$
 (A) 64 (B) 729
 (C) 2 (D) 0
- 93.** $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)$
 to $2n$ factors is
 (A) 2^n (B) 2^{2n}
 (C) 0 (D) 1
- 94.** Let $\Delta = \begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix}$ where ω is the cube root of unity, then
 (A) $\Delta = 0$ (B) $\Delta = 1$
 (C) $\Delta = 2$ (D) $\Delta = 3$
- 95.** If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n = (z+1)^n$, then
 (A) $\operatorname{Re}(z) < 0$ (B) $\operatorname{Re}(z) > 0$
 (C) $\operatorname{Re}(z) = 0$ (D) None of these
- 96.** $\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6$ is equal to
 (A) -2 (B) 0
 (C) 2 (D) 1
- 97.** If ω is an imaginary cube root of unity, $(1 + \omega - \omega^2)^7$ equals
 (A) 128ω (B) -128ω
 (C) $128\omega^2$ (D) $-128\omega^2$
- 98.** $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}} + \frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$ is equal to
 (A) -64 (B) -32
 (C) -16 (D) $\frac{1}{16}$
- 99.** If $\pi/3$ is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \dots\right)}$ is equal to
 (A) -1 (B) 0
 (C) 9 (D) i
- 100.** If cube root of 1 is ω , then the value of $(3 + \omega + 3\omega^2)^4$ is
 (A) 0 (B) 16
 (C) 16ω (D) $16\omega^2$
- 101.** The value of $(8)^{1/3}$ is
 (A) $-1+i\sqrt{3}$ (B) $-1-i\sqrt{3}$
 (C) 2 (D) All of these
- 102.** If ω is a complex cube root of unity, then $225 + (3\omega + 8\omega^2)^2 + (3\omega^2 + 8\omega)^2 =$
 (A) 72 (B) 192
 (C) 200 (D) 248
- 103.** If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} =$$

 (A) 0 (B) 1
 (C) ω (D) ω^2
- 104.** If $\omega = \frac{-1+\sqrt{3}i}{2}$ then $(3 + \omega + 3\omega^2)^4 =$
 (A) 16 (B) -16
 (C) 16ω (D) $16\omega^2$
- 105.** If $1, \omega, \omega^2$ are the roots of unity, then $(1 - 2\omega + \omega^2)^6$ is equal to
 (A) 729 (B) 246
 (C) 243 (D) 81