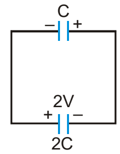
# HINTS & SOLUTIONS

## EXERCISE - 1

### Single Choice

**1.** $V_0(C + CV) = CV + (2C)(2V)$

$V_0 = V$ (Final pot. diff.)



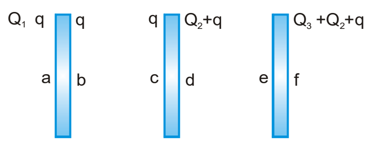

$\therefore U_{final} = \frac{1}{2}(C+2C)V_2 = \frac{3CV^2}{2}$

**2.** $P = \frac{\Delta U}{\Delta t} = \frac{\frac{1}{2}CV^2}{\Delta t} = \frac{\frac{1}{2}\times 40\times 10^{-6}\times 9\times 10^{6}}{2\times 10^{-3}} = 90\text{kW}$

**3.** $k = \frac{f}{x} = \frac{5000}{0.2} = 2{,}5000 \text{ N/m}$

$\frac{U_{SPR}}{U_{CAP}} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}CV^2} = \frac{25000\times 0.2\times 0.2}{10\times 10^{-6}\times 10^{8}} = 1$

**4.**




Here $Q_1 - q = Q_2 + Q_3 + q \Rightarrow q = \frac{Q_1 - (Q_2 + Q_3)}{2}$

Charge on a= Charge on f

$\Rightarrow Q_1 - q = \frac{\Sigma Q}{2} = \frac{Q_1 + Q_2 + Q_3}{2}$

**5.** $C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} \left(t = \frac{d}{2}, K = \infty\right)$

$= \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\epsilon_0 A}{d} = 2C_0$

**6.** Before sharing $U_i = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

After sharing $U_f = \frac{(Q_1 + Q_2)^2}{2(C_1 + C_2)}$

$\Delta U = U_f - U_i = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)} - \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

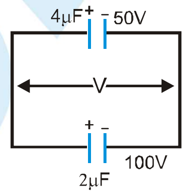
$= -\frac{(Q_1C_2 - Q_2C_1)^2}{2C_1C_2(C_1 + C_2)}$

–ve sign indicates there is decrease in energy

But $Q_1C_2 - Q_2C_1 \neq 0 \quad \Rightarrow \quad Q_1C_2 \neq Q_2C_1$

$\Rightarrow Q_1 4\pi\varepsilon_0 R_2 \neq Q_2 4\pi\varepsilon_0 R_1 \quad \Rightarrow \quad Q_1R_2 \neq Q_2R_1$

**7.** $(4+2)V = (4\times 50) + (2\times 100)$




$V = \frac{400}{6} = \frac{200}{3}V$

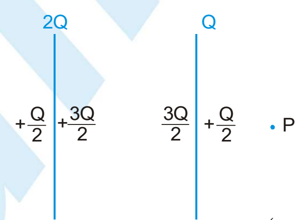
$U_{initial} = \left(\frac{1}{2}\times 4\times (50)^2 + \frac{1}{2}\times 2\times (100)^2\right)\times 10^{-6}$

$= (5000 + 10000)\times 10^{-6} = 1.5\times 10^{-2}\text{ J}$

$U_{final} = \frac{1}{2}(4+2)\times 10^{-6}\times \frac{200}{3}\times \frac{200}{3}$

$= 1.33\times 10^{-2}\text{ J}$

**8.**




Force on either plate $= \frac{(3Q/2)^2}{2A\epsilon_0} = \frac{9Q^2}{8A\epsilon_0}$

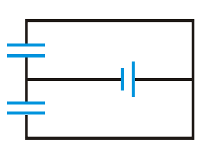
Force on point 'P' due to capacitor = 0

Potential diff. between the plates $= \frac{3Q}{2C}$

Energy stored in electric field between the plates

$= \frac{1}{2}C\times\left(\frac{3Q}{2C}\right)^2 = \frac{9Q^2}{8C}$

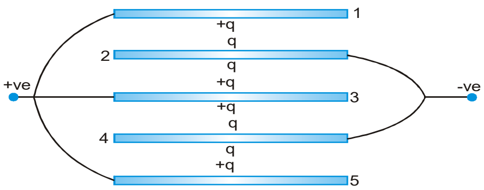
**9.**



$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$

$= \frac{1}{2}\left(\frac{8.85\times 10^{-12}\times 0.1}{0.885\times 10^{-3}}\times 2\right)\times 10^2 = 10^{-1}\mu\text{J}$

**10.**

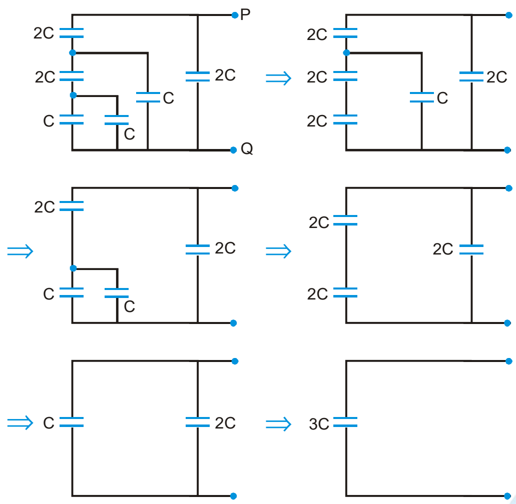



Therefore

$q_2 = -2q,\ q_3 = +2q,$

$q_4 = -2q$ and $q_5 = +q$

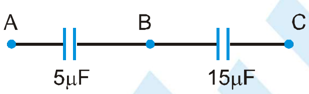
**11.**




**12.** Each capacitor has potential difference 'V' and energy $\frac{1}{2}CV^2$. After reconnecting total energy remains constant and total voltage becomes NV.

**13.** For 'n' plates; effective C will be (n–1)C.

**14.**




$5(V_A - V_B) = 15(V_B - V_C)$

$\Rightarrow 5(2000 - V_B) = 15(V_B - 0)$

$\Rightarrow 2000 - V_B = 3V_B$

$\Rightarrow V_B = 500V$

**15.** $C_{eff} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + ....$

$$= \frac{C}{1 - 1/2} = 2C = 2\mu F$$

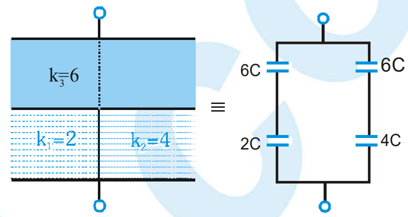
**16.** $C = \frac{\epsilon_0 A}{d} = 9pF$

$$C' = \frac{\epsilon_0 A}{d - t_1 + \frac{t_1}{K_2} - t_2 + \frac{t_2}{K_2}} = \frac{\epsilon_0 A}{d - \frac{d}{3} + \frac{d}{9} - \frac{2d}{3} + \frac{d}{9}}$$

$$= \frac{9}{2}\frac{\epsilon_0 A}{d} = \frac{81}{2} pF = 40.5pF$$

**17.** $CV + 2CV = KCV' + 2CV' \Rightarrow V' = \frac{3V}{K+2}$

**18.**

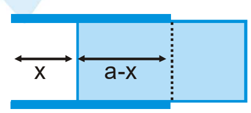



where $C = \frac{\epsilon_0 A}{d}$

$$C_{eq} = \frac{6C \times 2C}{8C} + \frac{6C \times 4C}{10C} = 3.9\ C$$

**19.** $\frac{1}{2}CV^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

**20.**

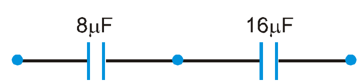



$$C = \frac{\epsilon_0\, ax}{d} + \frac{K \epsilon_0 (a - x)a}{d}$$

$$C = \frac{K \epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)}{d} x \quad \text{where } x = vt$$

$\therefore$ C– t graph is linear with negative slope.

**21.**



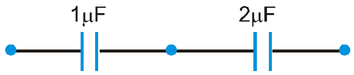

| | | |
|---|---|---|
| Breaking voltage | 20 V | 80 V |
| Safe Voltage | 20 V | 10 V |

$\therefore$ Charge on each capacitor = 20 × 8 = 160 μC

**22.** $C = 4\pi \epsilon_0 a$

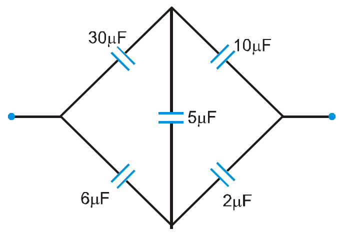
$$C' = \frac{4\pi \epsilon_0 ab}{b - a} = \frac{4\pi \epsilon_0 a}{1 - \frac{a}{b}} = \frac{4\pi \epsilon_0 a}{1 - \left(\frac{n-1}{n}\right)} = n(4\pi \epsilon_0 a)C$$

**23.** Capacitance between 1 and 3 and between 2 and 4 are symmetrical.

**24.** 

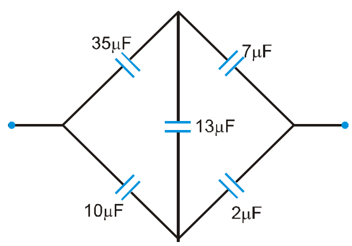

| | | |
|---|---|---|
| Breaking voltage | 6 kV | 4 kV |
| Safe Voltage | 6 kV | 3 kV |

$\therefore$ Total voltage $= 9$ kV

**25.** 


The system is a balanced Wheatstone bridge.

$$\therefore C_{eff} = \left(\frac{10 \times 30}{10 + 30} + \frac{6 \times 2}{6 + 2}\right) = 9\,\mu F$$

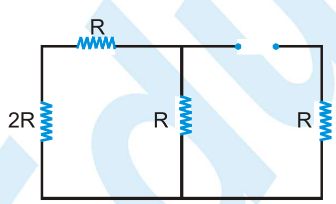
**26.** 


The system is a balanced Wheatstone bridge.

$$\therefore C_{eff} = \left(\frac{35 \times 7}{35 + 7} + \frac{10 \times 2}{10 + 2}\right) = \frac{15}{2}\,\mu F$$

**27.** There is no closed path for flow of current. Hence no current flows. Hence heat developed is zero.

**28.** $VA = 3\left(\frac{q}{C}\right) = 3 \times 2.5 = 7.5$ volt

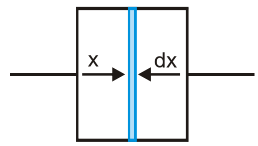
**29.** 


To find the time constant of a RC circuit, Short circuit the battery

$$R_{eff} = \frac{7R}{4} \qquad \therefore \tau = \frac{7RC}{4}$$

## EXERCISE - 2

### Part # I : Multiple Choice

**1.** 


$$\int \frac{1}{dC} = \int \frac{dy}{K \in_0 A} = \int_0^d \frac{dy}{\lambda \in_0 A \sec\left(\frac{\pi y}{2d}\right)}$$

$$\Rightarrow \quad C = \frac{\lambda \in_0 A\pi}{2d}$$

**2.** Both A and B are always in parallel.

**3.** $E = \frac{V_0}{d} \Rightarrow E_F < E_D$ Also $\sigma_A > \sigma_B$

**4.** $q = q_0 e^{-t/\tau} \quad \therefore \quad i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = i_0 e^{-t/\tau}$

$\therefore \quad q_0 = i_0 \tau$

Initial stored energy $= \frac{1}{2} CV^2 = \frac{1}{2}(CV)V$

$$= \frac{1}{2}(i_0\tau)(i_0R) = \frac{1}{2} i_0^2 R\tau$$

**5.** $V = V_0.e^{-t/RC}$

$$\left|\frac{dV}{dt}\right| = \frac{V_0}{RC} e^{-t/RC} = \text{slope}\ (<ky)$$

At $t = 0$, for $R = R_A$;slope is least in curve–3.

**6.** $q = q_0 e^{-t/\tau}$

$$\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau} = \frac{CV_0}{RC} e^{-t/\tau} = \frac{V_0}{R} e^{-t/\tau}$$

At t=0; $i_1 = \frac{V_0}{R_1}$; $\quad i_2 = \frac{V_0}{R_2}$

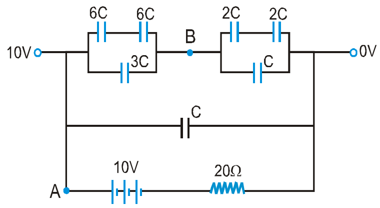
$\because R_1 = R_2 \qquad \therefore i_1 = i_2$

As $\tau$ is less for $C_1$ and hence it looses charges faster than $C_2$.

**7.** As B is in parallel with C and the potential develops slowly. Hence during charging more heat is produced in A than in B. In steady state, same current passes through A and B.

$$\therefore \text{Vcapacitor} = \frac{E}{2} \quad \therefore E_{capacitor} = \frac{1}{2}C\left(\frac{E}{2}\right)^2 = \frac{CE^2}{8}$$

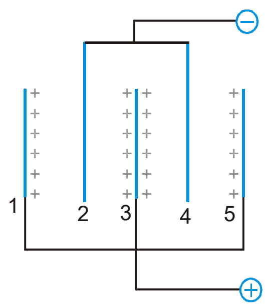
**8.**




$(V_A - V_B)\,6C = (V_B - 0)\,2C \Rightarrow V_B = 7.5\text{ V}$

$\therefore V_A - V_B = 10 - 7.5 = 2.5\text{ V}$

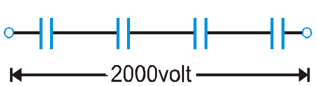
**9.**




Charge on plate ≠ 1 $= \dfrac{\epsilon_0 AV}{d}$

Charge on plate ≠ 4 $= -\dfrac{2\epsilon_0 AV}{d}$

**10.**




$C_{eff} = 1/4\ \mu F$

$\therefore$ Total no. of rows of capacitor $= \dfrac{C_{net}}{C_{eff}} = \dfrac{3}{1/4} = 12$

$\therefore$ Total no. of capacitors needed $= 12 \times 4 = 48$

**11.** Force on plate

$= \dfrac{\sigma^2 A}{2\epsilon_0} = \dfrac{Q^2}{2A\epsilon_0} = Kx = mg$

$\therefore Q = \sqrt{2mgA\epsilon_0}$

**12.** $i = 10e^{-t/RC} \Rightarrow 2.5 = 10\,e^{-2/RC}$

$\Rightarrow RC = \tau = \dfrac{1}{\ln 2}$ & $C = \dfrac{1}{10\ln 2}$

For capacitor

$\dfrac{V_0}{R} = 10 \Rightarrow V_0 = 10R = 100$ volt

Total heat developed = Total initial energy stored in capacitor. $= \dfrac{1}{2}CV^2 = \dfrac{500}{\ln 2}$

Thermal power in resistor $P = i^2R = 100\,Re^{-2t/RC}$

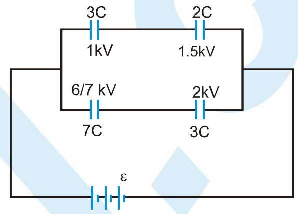
$\therefore$ Time–constant $= \dfrac{RC}{2} = \dfrac{1}{2\ln 2}$

**13.** $C_{eff} = C_{EF} = \dfrac{\epsilon_0 A}{d}$ $\quad\therefore E_{net} = \dfrac{1}{2}CV^2 = \dfrac{\epsilon_0 AV^2}{2d}$

**14.** Time constant

$= CR_{eff} = (100 \times 10^{-6})\left(\dfrac{10^3}{2}\right)s = 50\text{ m/s}$

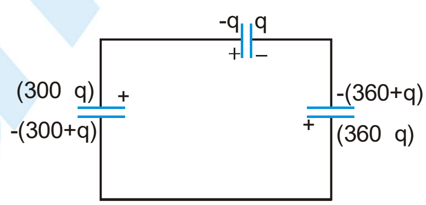
**15.**




Safe voltages in each arm are mentioned.

$\because (1+1.5) < (6/7+2)$ $\quad\therefore E_{safe} = 1 + 1.5 = 2.5\text{ kV}$

**16.**




$\Rightarrow \dfrac{300-q}{2} - \dfrac{q}{1.5} + \dfrac{360-q}{3} = 0 \Rightarrow q = 180$

$\therefore q_{1.5\mu F} = 180\ \mu C,\ q_{3\mu F} = 540\ \mu C,\ q_{2\mu F} = 480\mu C$

**17.** $i_1 = \dfrac{V}{R}e^{-t/RC_1},\ i_2 = \dfrac{V}{R}e^{-t/RC_2}$

$\therefore \dfrac{i_1}{i_2} = e^{t/R\left(\frac{1}{C_2} - \frac{1}{C_1}\right)} = e^{+\frac{t}{2RC_2}}$

$\Rightarrow i_1/i_2$ increases with time, t.

**18.** At t=0, $V_C = 0 \Rightarrow i_{R_3} = 0$

$$Q_{max} = C\left[\dfrac{\varepsilon}{\dfrac{R_1R_2}{R_1+R_2} + R_3}\right] = \dfrac{10C}{1+1} = 5 \times 1 = 5\mu C$$

$\therefore \left(I_{R_3}\right)_{max} = \frac{V_C}{R_3} = \frac{5}{1} = 5\,A$

Since $R_1$ and $R_2$ are in parallel hence current ratio of $R_1$ and $R_2$ will remain same.

**19.** $i = \frac{i_0}{2} = i_0 e^{-t/RC} \Rightarrow \frac{1}{2} = e^{-\ln 4/RC}$

$\Rightarrow RC = 2 \Rightarrow (2+r)\frac{1}{2} = 2 \Rightarrow r = 2\Omega$

**18.** At t=0, $V_C = 0 \Rightarrow i_{R_3} = 0$

$$Q_{max} = C\left[\frac{\varepsilon}{\frac{R_1R_2}{R_1+R_2}+R_3}\right] = \frac{10C}{1+1} = 5\times 1 = 5\,\mu C$$

$\therefore \left(I_{R_3}\right)_{max} = \frac{V_C}{R_3} = \frac{5}{1} = 5\,A$

Since R1 and R2 are in parallel hence current ratio of R1 and R2 will remain same.

**20.** $q = q_0 e^{-t/RC} \Rightarrow I = \frac{q_0}{RC}e^{-t/RC}$

$$\Rightarrow \ln I = \ln\left(\frac{q_0}{RC}\right) - \frac{t}{RC} = \ln\left(\frac{V_0}{R}\right) - \frac{t}{RC}$$

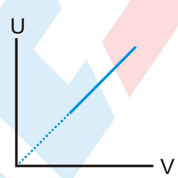
As Imax does not change $\therefore$ R = constant

$$\left|\frac{d(\ln I)}{dt}\right| = \left|0 - \frac{1}{RC}\right| \Rightarrow \left[\frac{d(\ln I)}{dt}\right]_1 > \left[\frac{d(\ln I)}{dt}\right]_2$$

$\therefore C_2 > C_1 \Rightarrow$ C is increased

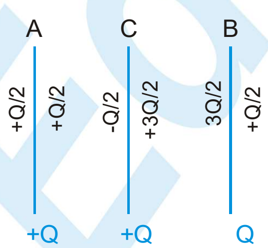
**21.** $C = \frac{\epsilon_0 ax}{d} + \frac{K\epsilon_0 (a-x)a}{d}$

$= \frac{K\epsilon_0 a^2}{d} - \frac{\epsilon_0 a(K-1)vt}{d}$




$V = \frac{Q}{C}$ and $U = \frac{QV}{2}$ $\quad\therefore \frac{U}{V} = \frac{Q}{2}$

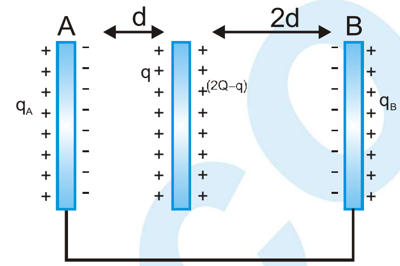
**22.**




Initial $V'_{AB} = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}$

$$\text{Final } V_{AB} = \frac{Q/2}{\left(\frac{2\epsilon_0 A}{d}\right)} + \frac{(3Q/2)}{\left(\frac{2\epsilon_0 A}{d}\right)} = \frac{Qd}{\epsilon_0 A} = V_{AB}$$

**23.**




$\Delta V = \frac{qd}{\epsilon_0 A} = \frac{(2Q-q)(2d)}{\epsilon_0 A} \Rightarrow q = \frac{4Q}{3}$

Total charge on inner faces of A and B = –2Q

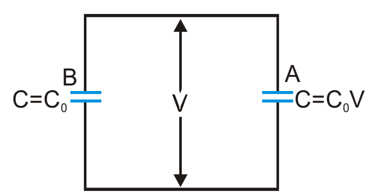
Rest charge will equally appear on their outer faces

$= \frac{Q-(-2Q)}{2} = \frac{3Q}{2}$

Final charge on plate A $= \frac{3Q}{2} - \frac{4Q}{3} = \frac{Q}{6}$

$\therefore$ Charge flown through wire $= Q - \frac{Q}{6} = \frac{5Q}{6}$

**24.**




$(C_0 + C_0V)V = 30C_0$

$\Rightarrow V^2 + V - 30 = 0 \quad\Rightarrow\quad V = 5$ volt

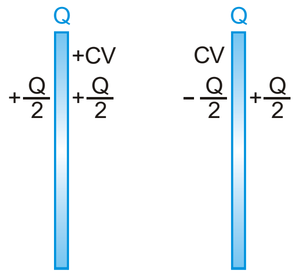
$\therefore V_A = V_B = 5$ volt

$Q_A = 52C_0 = 25C_0;\ Q_B = 5C_0$

**25.** $C_{eq} = \frac{KC}{K+1},\ C'_{eq} = \frac{C}{2}$

$\Rightarrow \frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

**26.** Final charge distribution




Therefore potential difference across the capacitor

$$= \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

**27.** $Q = \frac{C}{2}E$

$$Q' = \frac{KCC}{KC + C}E = \frac{KC}{K+1}E$$

$$\therefore Q' - Q = \frac{KCE}{K+1} - \frac{CE}{2} = \frac{(K-1)CE}{2(K+1)}$$

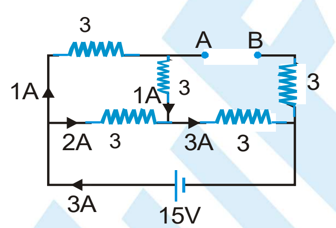
This charge is supplied by battery.

**28.** At t=0, $V_{capacitors} = 0$

$$\Rightarrow I_2 = I_3 = 0 \text{ and } I_1 = \frac{6}{2} = 3A$$

$$\text{At } t \to \infty,\ I_1 = I_3 = \frac{6}{2+8} = 0.6\,A,\ I_2 = 0$$

**29.** At t= ∞, capacitor gets open circuited




$$\therefore I = \frac{15}{5} = 3\,A \Rightarrow V_A - 3 \times 1 - 3 \times 3 = V_B$$

$$\Rightarrow V_A - V_B = 12\ V$$

**30.** In steady state

$$I_{upper\ arm} = I_{lower\ arm} = \frac{120}{6} = 20A$$

For the right most loop $3I - 3I + \frac{q}{C_2} = 0 \Rightarrow q = 0$

For the left most loop

$$20 \times 1 + \frac{q}{C_1} - 20 \times 2 = 0$$

$$\Rightarrow q = (40-20)\,C_1 = 20C_1 = 40\ \mu C$$

**31.** $\varepsilon = \frac{Q_0}{C_1}$ $\quad \therefore Q_1 = Q_0;\quad Q_2 = \left(\frac{Q_0}{C_1}\right)C_2$

$$V_1 = V_2 = \varepsilon = \frac{Q_0}{C_1};\quad U_1 = \frac{1}{2}C_1\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2}{2C_1}$$

$$U_2 = \frac{1}{2}C_2\left(\frac{Q_0}{C_1}\right)^2 = \frac{Q_0^2 C_2}{2C_1^2}$$

**32.** Energy $= \frac{Q^2}{2C} = \frac{Q^2 d}{2\,\epsilon_0\,A}$

As d decreases, E decreases

**33.** Charge on 3μF capacitor

$= 6 \times 7 = 42\ \mu C$

$$\therefore\ V_{3\mu F} = \frac{42}{3} = 14 \text{ volt}$$

$\therefore\ V_{3.9\,\mu F} = 14 + 6 = 20$ volt

Charge on 3.9 μF capacitor $= 20 \times 3.9 = 78\ \mu C$

$\therefore$ Total charge $= 78 + 42 = 120\ \mu C$

$$\therefore\ V_{12\mu F} = \frac{120}{12} = 10V$$

$$\therefore\ \varepsilon = 20 + 10 = 30\ V$$

**34.** $Q = CV = \frac{\epsilon_0\,AV}{d}$

$$E = \frac{V'}{d} = \frac{V/K}{d} = \frac{V}{Kd}$$

$$W = \frac{1}{2}Q^2\left(\frac{1}{C} - \frac{1}{C'}\right) = \frac{CV^2}{2}\left(1 - \frac{1}{K}\right)$$

**35.** $U_{initial} = \frac{1}{2}CV^2;\qquad U_{final} = \frac{1}{2}CV^2 \quad \therefore \Delta U = 0$

$\therefore$ Heat = work done by battery

$= [CV-(-CV)]V = 2CV^2$

**36.** S–open ; $V_{inner} = V_{outer}$

S –closed ; $V_{inner} = 0$

$\Rightarrow \frac{KQ}{3R} + \frac{Kq}{R} = 0 \quad \Rightarrow \quad q = -Q/3$

$C_{initial} = 4\pi \in_0 (3R)$

$C_{final} = 4\pi \in_0 (3R) + \frac{4\pi \in_0 (3R)(R)}{(3R - R)}$

$\therefore C_{final} > C_{initial}$

**37.** $W_{ext} = -\Delta U = U_i - U_f$

$= \frac{1}{2} \times 2\mu F \times 400 - \frac{1}{2} \times 1\mu F \times 400 = 200\mu J$

**38.** V decreases continuously from left to right except in conductor where it is constant.

**39.** $eV = \frac{1}{2} m \left(v_2^2 - v_1^2\right)$

$\Rightarrow 1.6 \times 10^{-19} \times 20 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (v^2 - 0)$

$\Rightarrow v = 2.65 \times 10^6$ m/s

**40.** $\Delta Q = 2CV - (-CV) = 3CV$

$WB = \Delta Q(2V) = 6CV^2$

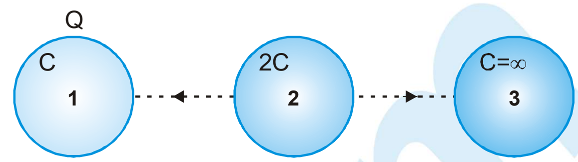
$\Delta U = U_f - U_i = \frac{1}{2} C(2V)^2 - \frac{1}{2} CV^2 = \frac{3CV^2}{2}$

$\therefore$ Heat $= W_B - \Delta U = \frac{9CV^2}{2}$

$U_f = \frac{1}{2} C(2V)^2 = 2CV^2$

$\therefore \frac{Heat}{U_f} = \frac{9}{4} = 2.25$

**41.**




Initial charge on 1 = Q when $C_1$ & $C_2$ touches

$\Rightarrow \frac{Q_1}{Q_2} = \frac{C}{2C} = \frac{1}{2} \Rightarrow Q_1 = \frac{Q}{3}, Q_2 = \frac{2Q}{3}$

Now when $Q_2$ & $Q_3$ is touched

$\Rightarrow \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{2C}{\infty} = 0 \Rightarrow Q_2 = 0$

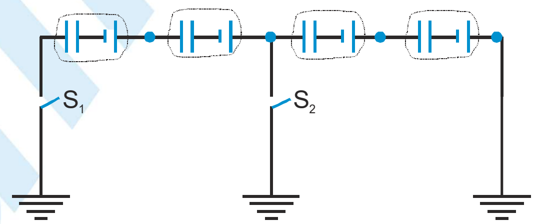
Again when $Q_1$ & $Q_2$ is touched

$Q_2 = 2\frac{(Q/3)}{3} \Rightarrow Q_1 = \frac{(Q/3)}{3} = \frac{Q}{9}$

Similarly we can say after N times it becomes

$Q_1 = \frac{Q}{3^N}$

**42.**




Potential difference across each capacitor and cell combination is zero.

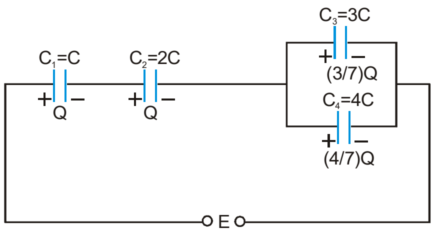
## Part # II : Assertion & Reason

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (D) | **2.** | (C) | **3.** | (A) |
| **4.** | (A) | **5.** | (B) | **6.** | (B) |

## EXERCISE - 3

### Part # I : Matrix Match Type

**1.**




At $C_1 = V_1 = \frac{Q}{C}$ and $U_1 = \frac{Q^2}{2C}$

At $C_2 = V_2 = \frac{Q}{2C}$ and $U_2 = \frac{Q^2}{4C}$

At $C_3 = V_3 = \frac{Q}{7C}$ and $U_3 = \frac{3Q^2}{98C}$

At $C_4 = V_4 = \frac{Q}{7C}$ and $U_4 = \frac{4Q^2}{98C}$

Therefore $V_{max} = V_1$ and $V_{min} = V_3 = V_4$

and $U_{max} = U_1$ and $U_{min} = U_3$

**2.** Initial charge $q_1 = \frac{CE}{2}$

Final charge $q_2 = CE$

Initial stored energy

$$U_1 = \frac{1}{2}C(E/2)^2 + \frac{1}{2}C(E/2)^2 = \frac{CE^2}{4}$$

Final stored shergy$U_2 = \frac{CE^2}{2}$

Charge supplied by battery

$$\Delta Q = q_2 - q_1 = CE - \frac{CE}{2} = \frac{CE}{2}$$

Work done by battery $W_B = \Delta QE = \frac{CE^2}{2}$

Heat developed in the system

$$H = W_B - \Delta U = \frac{CE^2}{2} - \left(\frac{CE^2}{2} - \frac{CE^2}{4}\right) = \frac{CE^2}{4}$$

### Part # II : Comprehension

**Comprehension-1**

**1.** Time Constant

$\tau = R_1C = 8 \times 6 = 48\ \mu s$

**2.** $V_t = 2\tau = V_0(1 - e^{-t/\tau}) = 12(1 - e^{-2\tau/\tau})$

$$= 12\left(1 - \frac{1}{7.4}\right) = 10.4\ V$$

**3.** $(V_{R_1})t = 2\tau = V0 - V_{capacitor} = 12 - 10.4 = 1.6\ V$

**4.** $V_{R_2} = V_0 = 12V$

**Comperehension -2**

**1.** In steady state

$$Icircuit = \frac{V}{R_1 + R_2} = \frac{18}{3+6} = 2A$$

$V_{R_2} = V_{C_2} = IR_2 = 2 \times 6 = 12\ V$

$Q_{C_2} = C_2V_{C_2} = 12 \times 4 = 48\ \mu C$

**2.** $Q_{initial} = Q_{C_1} + Q_{C_2} = IR_1C_1 + IR_2C_2$

$= 3 \times 2 \times 2 + 3 \times 4 \times 4 = 12 + 48 = 60\mu C$

$Q_{final} = V(C_1 + C_2) = 18(2+4) = 108\ \mu C$

$\therefore \Delta Q = 108 - 60 = 48\ \mu C$ (through $S_1$)

**3.** $U_{initial} = \frac{1}{2}C_1V_{12} + \frac{1}{2}C_2V_2^2$

$$= \frac{1}{2} \times 2 \times 6^2 + \frac{1}{2} \times 4 \times 12^2 = 324\ \mu J$$

$$U_{final} = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2+4)182 = 972\mu J$$

$\Delta Q = Q_f - Q_I = 48\ \mu C$

$W_{Battery} = \Delta Q.V = 48 \times 18 = 864\ \mu J$

$\therefore$ Heat $= W_B - \Delta U = 864 - (972 - 324) = 216\ \mu J$

**Comprehension-3**

**1.** $q_{1max} = q_{2max}$

$C_1$ and $C_2$ may be different and hence $E_1$ and $E_2$ may be different.

**2.** $\tau_2 > \tau_1 \Rightarrow R_2C_2 > R_1C_1 \Rightarrow \frac{R_1}{R_2} < \frac{C_2}{C_1}$

## Comprehension-4

**1.** $\frac{C_A}{C_B} = \frac{\epsilon_0 A/d}{K\epsilon_0 A/d} = 1:K$

**2.** $\frac{V_A}{V_B} = \frac{Q/C_A}{Q/C_B} = \frac{C_B}{C_A} = K:1$

**3.** $(V_A)_{initial} = \frac{V}{2};\ \ (V_A)_{final} = \frac{E}{C}\frac{(KC)}{(K+1)} = \frac{KE}{K+1}$

$\therefore \frac{(V_A)_{Initial}}{(V_A)_{Final}} = \frac{K+1}{2K}$

**4.** $(V_B)_{Initial} = \frac{V}{2};(V_B)_{Final} = \frac{Q}{C_B} = \frac{E(KC)}{(K+1)} \times \frac{1}{KC} = \frac{E}{K+1}$

$\therefore \frac{(V_B)_{Initial}}{(V_B)_{Final}} = (K+1):2$

**5.** $(U_A)_{final} = \frac{Q^2}{2C_A};(U_B)_{final} = \frac{Q^2}{2C_B}$

$\therefore \left(\frac{U_A}{U_B}\right)_{Final} = K:1$

## Comprehension -5

**1.** $V_b = \varepsilon_0(1 - e^{-t/RC})$

$\Rightarrow 110 = 120(1 - e^{-t/RC})$

$\Rightarrow e^{-t/RC} = 1/12$

$\Rightarrow t/RC = \bullet n/12 = 2.5$

$\Rightarrow t = RC \times 2.5 = 10^6 \times 10^{-6} \times 2.5 = 5/2$ sec

**2.** $\tau_0 = 10^{-6} \times 10 = 10\ \mu s$

**3.** Flash duration$= 3\tau_0 = 30\ \mu s$

**4.** Energy in flash

$= \frac{1}{2}CV^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 110 \times 110 = 6.1$ mJ

## EXERCISE - 4

### Subjective Type

**1.** Equivalent capacity between A and B

$C = \frac{9}{3} + 3 = 6\mu F$

**(i)** Stored charge

$Q = CV = 6 \times 10^{-6} \times 4 = 24\ \mu C$

**(ii)** Stored energy

$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 16 = 48\ \mu J$

**2.** $CV = \frac{qt}{t} \Rightarrow 400 \times 10–6 \times 100 = 100\,t$

$\Rightarrow t = 400$ s

**3.** Electric field

$E = \frac{V_A - V_B}{d} = \frac{(10{,}000 - 0)}{(2 \times 10^{-3})} = 5 \times 106$ V/m

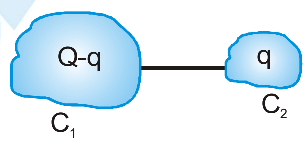
**4.** Common potential

$V_{cm} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2 \times 200 + 3 \times 400}{2+3} = 320V$

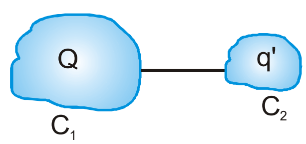
Charge on $C_1Q_1 = C_1\ V_{cm} = 2 \times 320\mu C = 640\mu C$

Charge on $C_2Q_2 = C_2\ V_{cm} = 3 \times 320\mu C = 960\mu C$

**5.**




$\frac{Q-q}{C_1} = \frac{q}{C_2}$ ...(i)




$\frac{Q}{C_1} = \frac{q'}{C_2}$ ...(ii)

Eq. (i) ÷ (ii) : $q' = \frac{Qq}{Q-q}$

**6.** By using KCL

$C_1(V_A - V_0) + C_2(V_B - V_0) + C_3(V_C - V_0) = 0$

$\Rightarrow V_0 = \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$

**7.** ✈ $C = \frac{\epsilon_0 A}{d}$ ; $q = \left(\frac{\epsilon_0 A}{d}\right)V$

Slope $= \frac{\epsilon_0 A}{d}$ $\quad\therefore C_2 > C_1 > C_3$

**8. (i)** On connecting with the second capacitor the charge distributes equally

$$\therefore \quad V_{CM} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{0.1 \times 10}{0.1 + 0.1} = 5V$$

Total stored energy

$$U_f = \frac{1}{2}C_1V_{CM}^2 + \frac{1}{2}C_2V_{CM}^2$$

$$= \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2 + \frac{1}{2} \times 0.1 \times 10^{-6} \times (5)^2$$

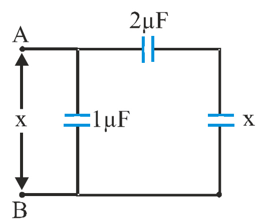
$= 2.5\ \mu J$

**(ii)** Initial stored energy in first capacitor

$$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 0.1 \times 10^{-6} \times 10^2 = 5.0\ \mu J$$

$$\Rightarrow \frac{U_f}{U_i} = \frac{2.5}{5.0} = \frac{1}{2}$$

**9.** $x = \frac{2x}{2+x} + 1$




(Let $C_{eq} = x$)

$$x = \frac{2x + 2 + x}{2 + x}$$

$\Rightarrow x(2 + x) = 3x + 2$

$\Rightarrow 2x + x^2 = 3x + 2 \Rightarrow x^2 - x - 2 = 0$

Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = 2$$

and $-1$

$x = 2$, $C_{eq} = 2\mu F$

**10.** $C_X = \frac{\varepsilon_0 A}{d}$, $C_Y = \frac{5\varepsilon_0 A}{d} \Rightarrow CY = 5CX$

**(i)** $C_X$ and $C_Y$ are in series, so charge on each

$$q = C_X V_X = C_Y V_Y \Rightarrow \frac{V_X}{V_Y} = 5$$

$\because V_X + V_Y = 12 \quad \therefore 6V_Y = 12$

$$\Rightarrow V_Y = \frac{12}{6} = 2 \text{ volt} \quad \text{and} \quad V_X = 10 \text{ volt}$$

**(ii)** Energy stored in capacitor

$$U = \frac{q^2}{2C} \Rightarrow \frac{U_X}{U_Y} = \left(\frac{q^2}{2C_X}\right)\left(\frac{2C_Y}{q^2}\right) = \frac{C_Y}{C_X} = 5$$

**11.**
$$\frac{C_A}{C_B} = \frac{\left(\frac{K_1 \in_0 A}{d/4}\right)}{\left(\frac{K_2 \in_0 A}{3d/4}\right)} = \frac{3K_1}{K_2} = 3 \times 3 = 9$$

Net capacity

$$C = \frac{C_A C_B}{C_A + C_B} = \frac{(9C_B)(C_B)}{9C_B + C_B} = \frac{9}{10}C_B$$

$$= \frac{9}{10}\left[\frac{K_2 \in_0 A}{(3d/4)}\right] = \frac{6K_2 \in_0 A}{5d} = \frac{1.2K_2 \in_0 A}{d}$$

**12.** $\because E = \frac{V}{d}$

$$\therefore d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}\ m$$

Now $C = \frac{\in_0 \in_r A}{d}$

$$\Rightarrow A = \frac{Cd}{\in_0 \in_r} = \frac{88.5 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 10} = 10^{-3}\ m^2$$

**13.** When $S_{W1}$ is closed and $S_{W2}$ is open then capacitor B is charged upto 10V.

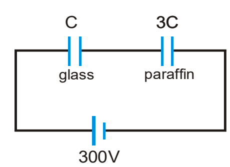
Now $S_{W1}$ is open and $S_{W2}$ is closed then

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{3 \times 10 + 2 \times 0}{3 + 2} = 6V$$

$Q_A = 2 \times 10^{-6}\ V_{cm} = 12\ \mu C$

$Q_B = 3 \times 10^{-6}\ V_{cm} = 18\ \mu C$

**14.** $CV_1 = 3CV_2$ ....(i)




$V_1 + V_2 = 300$ ....(ii)

$\Rightarrow V_1 = 75V;\ V_2 = 225\ V$

**(i)** $\therefore E_1 = \frac{V_1}{d_1} = \frac{75 \times 100}{0.5} = 1.5 \times 10^4\ V/m$

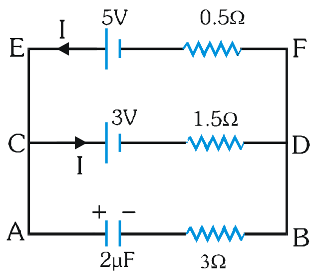
$$E_2 = \frac{V_2}{d_2} = \frac{225 \times 100}{0.5} = 4.5 \times 10^4\ V/m$$

**(ii)** $V_1 = 75\ V;\quad V_2 = 225\ V$

**(iii)** $Q = \left(\frac{C_1C_2}{C_1 + C_2}\right)V = \frac{3}{4}C_1V = \frac{3}{4}\left(\frac{2 \in_0 A}{d}\right)300$

$$\Rightarrow \frac{Q}{A} = \frac{6 \times 300 \times 8.89 \times 10^{-12}}{4 \times 0.5 \times 10^{-2}} = 8 \times 10^{-7}\ C/m^2$$

**15.** (a) In steady state no current in capacitor's branch.




So current $I = \dfrac{2}{0.5 + 1.5} = 1A$

voltage across capacitor

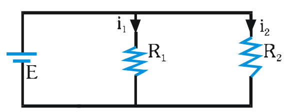
$V_C = 3 + 1.5 \times 1 = 4.5$ V

$\Rightarrow Q = CV_C = 2 \times 10^{-6} \times 4.5$

$= 9 \times 10^{-6}$ C

**16. (i)** At t = 0, capacitor has zero resistance, i.e., $R_1$ and $R_2$ are in parallel.

The simple circuit is shown in figure




$i1 = \dfrac{E}{R_1}$ and $i2 = \dfrac{E}{R_2}$

**(ii)** At steady state $(t = \infty)$, capacitor has infinite resistance.

Hence, $i_1 = \dfrac{E}{R_1}, \; i_2 = 0$

**(iii)** Final potential difference across capacitor is E.

$\therefore$ Final energy stored

$$U = \frac{1}{2}CE^2$$

**(iv)** When switch is opened, capacitor will discharge through two resistance as $R_1$ and $R_2$ (both in series).

Hence, $\tau_c = C(R_1 + R_2)$

**(v)** When switch is closed, capacitor will charged through resistance $R_2$.

So $\tau = R_2C$

**17.** For the circuit ACDA and the cell :

$6 - I_1(5) - 6 = 0 \quad \Rightarrow \quad I_1 = 0, \; \therefore \; I = 0$

For the loop BCD : $V_{2\mu F} = 6V$

For the loop ABD : $V_{7\mu F} = 6V$

$\therefore Q_{7\mu F} = 6 \times 7 = 42\ \mu C$

**18.** $R_{eff} = \dfrac{2 \times 3}{3 + 2} + 2.8 = 4\Omega$

$I = \dfrac{V}{R_{eff}} = \dfrac{6}{4} = 1.5$ A

$\therefore I_{2\Omega} = I\left(\dfrac{3}{2+3}\right) = \dfrac{1.5 \times 3}{5} = 0.9$ A

**19.** Total heat dissipated

$H = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 5 \times 10^{-6} \times 200 \times 200 = 0.1$ J

$H_1$ = Heat developed across $R_1 = \int I^2 R_1 dt$

$H_2$ = Heat developed across $R_2 = \int I^2 R_2 dt$

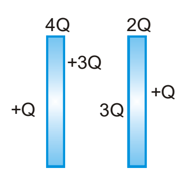
$\Rightarrow H_1 = \dfrac{(H_1 + H_2)R_1}{(R_1 + R_2)} = \dfrac{H\ R_1}{(R_1 + R_2)}$

$= \dfrac{0.1 \times 500}{(500 + 330)} = 60$ mJ

**20.** $\dfrac{1}{C_{arm}} = \dfrac{1}{C}\left(1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\right) = \dfrac{1}{C\left(1 - \dfrac{1}{2}\right)} = \dfrac{2}{C}$

$\therefore C_{effective} = 2C_{arm} = \dfrac{2C}{2} = C$

**21.**

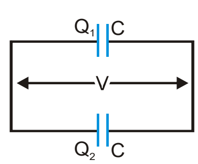



Initial effective charge = 3Q

$CV + CV = Q_1 + Q_2$

$= 3Q + 0 = 3Q$

$\therefore \quad V = \dfrac{3Q}{2C}$



**22.** $V_{2\ initial} = \frac{20}{2} = 10V$

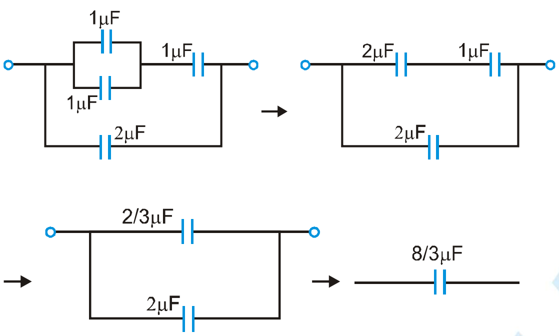
$V_{5\ initial} = \frac{50}{5} = 10V$

There is no potential difference.
Hence no charge flows.
Heat produce is zero.

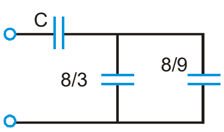
**23.** $E_{final} = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2 d}{2 \in_0 A}$ ; $E_{initial} = 0$

$\therefore$ Heat $= -(E_{initial} - E_{final}) = \frac{Q^2 d}{2 \in_0 A}$

**24.**

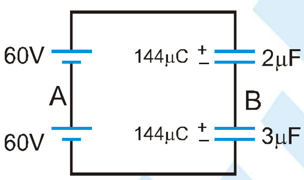



**25.**



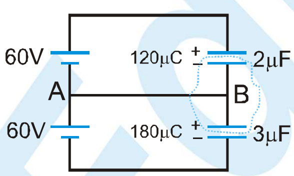

$\Rightarrow \frac{1}{C} + \frac{9}{32} = 1 \Rightarrow C = \frac{32}{23}\mu F$

**26.**



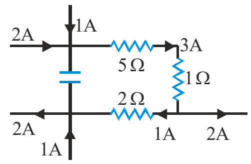

$C_{eff} = \frac{2\times 3}{2+3} = 1.2\mu F$

$\therefore Q_2 = Q_3 = +144\mu C$




$Q_{total}$ on the middle plates
$= +180 + (-120) = +60\ \mu C$
This charge flows from A to B.

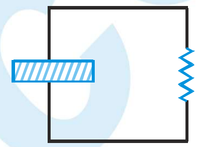
**27.**




$V_C = (5+1)\times 3 + 2\times 1 = 20V$

$U_{cap} = \frac{1}{2}CV^2 = \frac{1}{2}\times 4\times 20^2 = 0.8\ mJ$

**28.** $\frac{q}{C} - iR = 0 \Rightarrow \frac{q}{C} + \frac{dq}{dt}R = 0 \Rightarrow q = q_0 e^{-t/RC}$



equivalent circuit $\Rightarrow i = \frac{dq}{dt} = \frac{q_0}{RC}e^{-t/RC}$

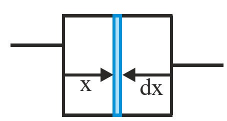
Where $R = \frac{L}{SA}$, $C = \frac{k \in_0 A}{4}$

$\therefore RC = \frac{k \in_0}{S} = \frac{5\times 8.85\times 10^{-12}}{7.4\times 10^{-12}} = \frac{5\times 8.85}{7.4}$

$\therefore i = \frac{q_0}{R_C}e^{-t/RC} = \frac{8.85\times 10^{-3}}{\left(\frac{5\times 8.85}{7.4}\right)}e^{-12/6}$

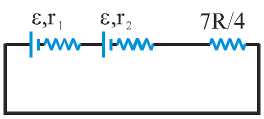
$= \frac{7.4}{5}\times\frac{1}{7.4}mA = 0.2mA$

**29.**




$\int\frac{1}{dC} = \int\frac{dx}{KS \in_0} = \int_0^d \frac{dx}{KS \in_0\left(1+\sin\frac{\pi x}{d}\right)}$

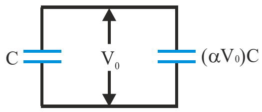
$\Rightarrow C = \frac{K_1 S \in_0 \pi}{2d}\left[\int_0^d \frac{dx}{\left(1+\sin\frac{\pi x}{d}\right)} = \frac{2d}{\pi}\right]$

**30.** 


$$I = \frac{2\varepsilon}{r_1 + r_2 + \frac{7R}{4}}$$

Pot. diff. across $(\varepsilon, r_1)$ cell : $\varepsilon - Ir_1 = 0$

$$\Rightarrow \varepsilon = Ir_1 \Rightarrow \varepsilon = \frac{2\varepsilon r_1}{r_1 + r_2 + \frac{7R}{4}} \Rightarrow \frac{4(r_1 - r_2)}{7} = R$$

**31.** 

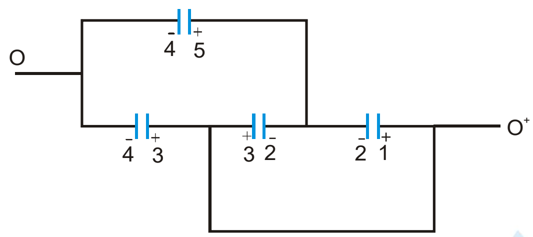

Total charge remains constant

$156C = (\alpha V_0)CV_0 + CV_0$

$\Rightarrow V_0^2 + V_0 - 156 = 0 \; (\alpha = 1)$

$\Rightarrow (V_0 + 13)(V_0 - 12) = 0 \Rightarrow V_0 = 12$ volt

**32.** $C_{eq} = \frac{2C}{3} + C = \frac{5C}{3}$




$$Q_3 = \frac{4}{3}\epsilon_0 \frac{AV_0}{d} \;\&\; Q_5 = \frac{2}{3}\frac{\epsilon_0 AV_0}{d}$$

**33.** $Q_{total} = C_1 V = \left[C_1 + \frac{C_2C_3}{C_2 + C_3}\right]V_0$

$$\Rightarrow V_0 = \frac{C_1(C_2 + C_3)V}{C_1C_2 + C_2C_3 + C_3C_1}$$

$\therefore$ Charge on $C_1$,

$$q_1 = C_1V_0 = \frac{C_1^2V(C_2 + C_3)}{C_1C_2 + C_2C_3 + C_3C_1}$$

Charge on $C_2$ and $C_3$

$$q_2 = q_3 = \left(\frac{C_2C_3}{C_2 + C_3}\right)V_0 = \frac{C_1C_2C_3V}{C_1C_2 + C_2C_3 + C_3C_1}$$

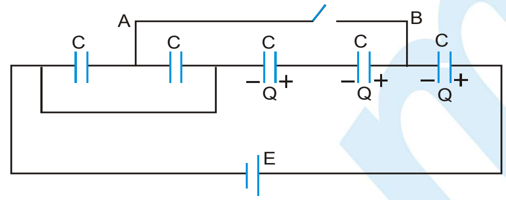
**34.** Extra weight needed

$$= \left(\frac{6}{\epsilon_0}\right)^2 \times \frac{\epsilon_0 A}{2} = E_2 \times \frac{\epsilon_0 A}{2} = \left(\frac{V}{d}\right)^2 \frac{\epsilon_0 A}{2}$$

$$\Rightarrow mg = \left(\frac{5000}{5\times 10^{-3}}\right)^2 \times \frac{8.85\times 10^{-12}\times 100}{2\times 100\times 100}$$

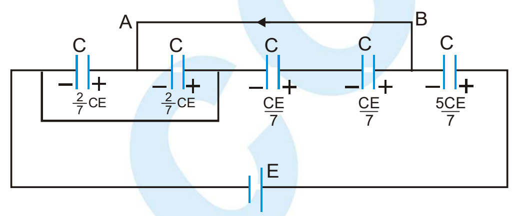
$\Rightarrow m = 4.52 \times 10^{-3}$ kg

**35.** Initial condition



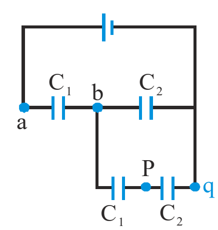

$$Q = \frac{CE}{3}$$

Final condition




Charge flown from B to A $= \frac{4}{7}CE$

**36.** $Q_{pq} = 2C_2 = 6C_1 = Q_{bp}$




$$\therefore V_{bp} = \frac{6C_1}{C_1} = 6\,V$$

$\therefore V_{bq} = 6 + 2 = 8V$

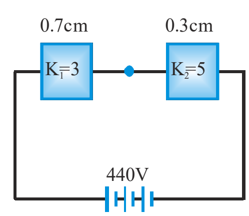
Total charge flown into right loop

$= C_2V_{bq} + C_1V_{bp}$

$= 3C_1 \times 8 + C_1 \times 6 = 30C_1$

$$\therefore V_{ab} = \frac{Q_{total}}{C_{ab}} = \frac{30C_1}{C_1} = 30 \text{ volt}$$

**38.** $V_1C_1 = V_2C_2$ and $V_1 + V_2 = 440$




$$\Rightarrow V_2 = \frac{V_1C_1}{C_2} \Rightarrow V_1 + \frac{V_1C_1}{C_2} = 440$$

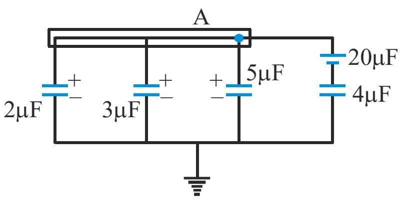
$$\Rightarrow V_1 = \frac{440C_2}{C_1 + C_2} = \frac{440}{\frac{C_1}{C_2} + 1}$$

$$\therefore E_1 = \frac{V_1}{d} = \frac{350\times 100}{0.7} = 5\times 10^4 \text{ V/m}$$

$$E_2 = \frac{V_2}{d} = \frac{90}{0.3} \times 100 = 3 \times 10^4 \text{ V/m}$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V_1^2}{\frac{1}{2}C_2V_1^2} = \frac{35}{9}$$

**39.** Applying junction law at A :

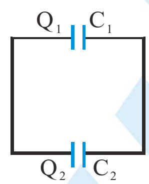



$$2(V_A-5)+3(V_A-20)+5(V_A-10)+4(V_A-20)=0$$

$$\Rightarrow V_A = \frac{100}{7} = 14.28 \text{ volt}$$

$\therefore Q_{2\mu F} = 28.56\ \mu C$, $Q_{3\mu F} = 42.84\ \mu C$,
$Q_{5\mu F} = 71.40\ \mu C$, $Q_{4\mu F} = 22.88\ \mu C$

**40.** $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ....(i)




$Q_1 + Q_2 = 2Q$ ....(ii)

$C_1 = \frac{\epsilon_0 A}{d-x}$ and $C_2 = \frac{\epsilon_0 A}{d+x}$

$\Rightarrow Q_2 = \frac{Q(d-x)}{d}$ and $Q_1 = \frac{Q(d+x)}{d}$

$\Rightarrow \frac{dQ_2}{dt} = -\frac{Q}{2d}\left(\frac{dx}{dt}\right)$ & $\frac{dQ_1}{dt} = \frac{Q}{2d}\left(\frac{dx}{dt}\right)$

$$\therefore I = \frac{dQ_1}{dt} - \frac{dQ_2}{dt} = \frac{Q}{d}\left(\frac{dx}{dt}\right) = \frac{200}{0.1} \times 0.001 = 2\mu A$$

**41.** Work done by battery $= \Delta QV = (3CV)V = 3CV^2$

Energy stored in capacitors $= \frac{1}{2}(3C)V^2$

**(i)** $\therefore$ Heat developed $= W_B - \Delta U = \frac{1}{2}(3C)V^2$

**(ii)** Work done by external agent = – (K–1)

**(iii)** Final voltage after 'dielectric is removed = V'

$$3CV' = (K+2)CV \Rightarrow V' = V\left(\frac{K+2}{3}\right)$$

$$W_{agent} = U_i - U_f$$

$$= \frac{1}{2}(3C)V^2\left(\frac{K+2}{3}\right)^2 - \frac{1}{2}(K+2)CV^2$$

$$= \frac{(K+2)(K-1)CV^2}{6}$$

**42.** $C_{initial} = \frac{2C \times C}{2C+C} = \frac{2C}{3}$ ; $C_{final} = C$

**(i)** $\therefore \Delta Q = \Delta C \times V$

$$= \left(C - \frac{2C}{3}\right)V = \frac{CV}{3} = \frac{2 \times 30}{3} = 20\mu C$$

**(ii)** $H = W_B - \Delta U = \Delta QV - \left(\frac{1}{2}CV^2 - \frac{1}{2}\frac{2CV^2}{3}\right)$

$= 600 - (900 - 600) = 300\ \mu J = 0.3$ mJ

**(iii)** Energy supplied by the battery

$= \Delta QV = 600\ \mu J = 0.6$ mJ

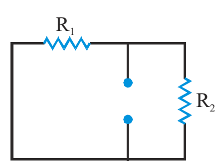
**(iv)** Initial charge on each capacitor

$$= \frac{2C}{3}V = 40\mu C$$

Final charge on right capacitor = 60 μC
Final charge on left capacitors = 0
$\therefore$ Total charge from through switch, S = 60μC

**43.** $U_1 = \frac{Q^2}{2C_1}$, $C_1 = 4\pi\epsilon_0\left[\frac{ab}{b-a} + b\right]$

$U_2 = \frac{Q^2}{2C_2}$, $C_2 = 4\pi\epsilon_0 b$ $\quad \therefore \Delta U = U_1 - U_2 = 9$ J




Here, $R_{net}$ is equivalent resistance across capacitor after short circuiting the battery.

$$R_{net} = \frac{R_1R_2}{R_1 + R_2}$$

(As $R_1$ and $R_2$ are in parallel)

$$\alpha = \frac{1}{C\left(\frac{R_1R_2}{R_1+R_2}\right)} = \frac{R_1+R_2}{C\ R_1R_2}$$

## EXERCISE - 5

### Part # I : AIEEE/JEE-MAIN

**1.** $C_{eff_{(Parallel)}} = nC$

If connected across V volts then energy stored

$$= \frac{1}{2}(nC)V^2$$

**2.** Capacitance of an isolated sphere is

$C = (4pe_0)(\text{Radius})$

$$C = \frac{1}{9\times10^9}\times1 = 0.11\times10^{-9} = 1.1\times10^{-10}\,F$$

**3.** (2)

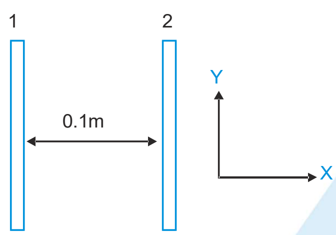
**4.** Work done $= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\left(8\times10^{-18}\right)^2}{100\times10^{-6}}$

$$= \frac{1}{2}\times\frac{64\times10^{-36}}{10^{-4}} = 32\times10^{-32}\,J$$

**5.** $\frac{1}{2}cv^2 = ms\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

**6.** Two plates stacked together form a single capacitor of capacitance C. n plates stacked together form (n–1) number of capacitors of effective capacitance (n–1)C.

**7.**




Applying law of conservation of energy

We get $\frac{1}{2}mv^2 = eV$

[Here, v = speed of electron, $V = V_2 - V_1$ = potential difference]

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9\times10^{-31}}}$$

On solving, we get $v = 2.65\times10^6$ m/s

**8.** Energy stored in a capacitor when it is charged by a potential difference of $V_0$ volt $= \frac{1}{2}QV_0$

Total work done by battery in sending a charge of Q through emf $V_0 = QV_0$

hence $\frac{\text{energy stored in capacitor}}{\text{work done by battery}} = \frac{\frac{1}{2}QV_0}{QV_0} = \frac{1}{2}$

**9.** Net work done by the system in the process is zero, as in removing the dielectric, work done is equal and opposite to the work done is re-inserting the dielectric.

**10.** $C = \frac{\delta A}{d} = 9\,Pf$; $C_{eq} = \frac{C_1C_2}{C_1+C_2}$

$$\Rightarrow \frac{\left(\frac{3\varepsilon_0AK_1}{d}\right)\left(\frac{3\varepsilon_0AK_2}{2d}\right)}{\frac{3\varepsilon_0AK_1}{d}+\frac{3\varepsilon_0AK_1}{2d}} \Rightarrow \frac{dl}{2}F = 40.5\,pF$$

**11.** $U = \frac{1}{2}CV^2$ ; $\frac{U_0}{2} = \frac{1}{2}CV_0^2e^{-2t_1/RC}$

$\frac{1}{2} = e^{-2t_1/RC}$ $(U0 = \frac{1}{2}CV_0^2)$

$$\frac{2t_1}{RC} = \ln 2$$

$t_1 = \frac{RC\ln 2}{2}$ ...(i) and $\frac{q_0}{4} = q_0e^{-t_2/RC}$

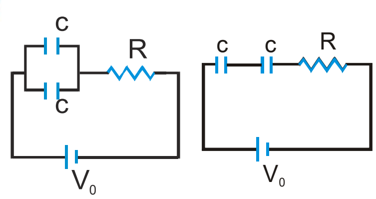
$\frac{t_2}{RC} = 2\ln 2$ ; t2 = 2RC ●n2...(ii)

from equation (i) and (ii) $\frac{t_1}{t_2} = \frac{1}{4}$

**12.** $V = V_0\left(1-e^{-t/RC}\right) \Rightarrow 120 = 200\left(1-e^{-\frac{5}{RC}}\right)$

$\Rightarrow R = 2.7\times106\ \Omega$

**13.** Parallel Series




$\frac{V_0}{2} = v_0\left(1-e^{-\frac{t_p}{R\times2C}}\right)$ ...(i)

$\frac{V_0}{2} = v_0\left(1-e^{-\frac{t_s}{R\times\frac{C}{2}}}\right)$ ...(ii)

from (i) and (ii) $e^{-\frac{t_p}{2Rc}} = e^{-\frac{2t_s}{Rc}}$

$$t_s = \frac{t_p}{4} = \frac{10}{4} = 2.5\text{ sec}$$

**14.** $t = 0.37\%$ of $V_0$

$= 0.37\times25 = 9.25$ volt

where is in between 100 and 150 sec.

**15.** Common voltage = $\dfrac{C_1V_1 - C_2V_2}{C_1 + C_2}$

(positive plate of one capacitor is connected with negative plate of second capacitor)

$\Rightarrow 120\ C_1 = 200\ C_2 \Rightarrow 3C_1 = 5C_2$

**16.** 3

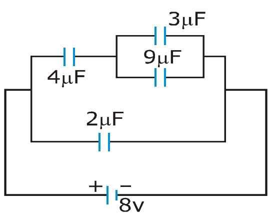
**17.** $q = \left(\dfrac{3C}{C+3}\right)E \Rightarrow q = CV \Rightarrow q \propto C$

$q_2 = \left(\dfrac{3C}{C+3}\right)E\left(\dfrac{2}{3}\right) \Rightarrow q_2 = \left(\dfrac{2C}{C+3}\right)E$

$q_2 = \left(\dfrac{2C}{1+\dfrac{3}{C}}\right)E \quad q = CV \Rightarrow C\uparrow q_2\uparrow$

If $C \to \infty$, q = constant value.

**18.**




Potential at 4μF = 6 volt

$\therefore$ charge $q_1 = 24\mu C$

Potential at 9μF = 2 volt

$\therefore$ charge $q_2 = 18\mu C$

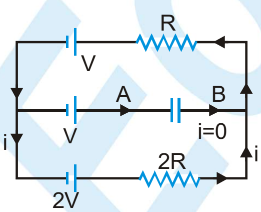
total q = 42 μC

$E = \dfrac{kq}{r^2} = \dfrac{9\times10^9 \times 42\times10^{-6}}{900} = 420\ N/C$

## Part # II : IIT-JEE ADVANCED

### Straight Objective type question

**1.** In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,




$i = \dfrac{2V - V}{2R + R} = \dfrac{V}{3R}$

Potential difference between A and B :

$V_A - V + V + iR = V_B$

$\therefore V_B - V_A = iR = \left(\dfrac{V}{3R}\right)R = \dfrac{V}{3}$

**2.** Charging current $I = \dfrac{E}{R}e^{-\frac{t}{RC}}$

Taking log both sides $\log I = \log\left(\dfrac{E}{R}\right) - \dfrac{t}{RC}$

When R is doubled, slope of curve increase. Also at t=0, the current will be less. Graph Q represents the best.

**3.** Given : $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$\therefore V_C = \dfrac{3}{4}V \Rightarrow V(1–e–t/RC) = \dfrac{3}{4}V \quad \therefore e–t/RC = \dfrac{1}{4}$

Here $\tau_c = cR = 10s$

Substituting this value of τc in equation and solving

We get : t = 13.86 s

**4.** τ=CR

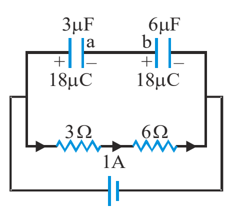
$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\ \mu s$

$\tau_2 = \left(\dfrac{C_1C_2}{C_1 + C_2}\right)\left(\dfrac{R_1R_2}{R_1 + R_2}\right) = \dfrac{8}{6}\times\dfrac{2}{3} = \dfrac{8}{9}\mu s$

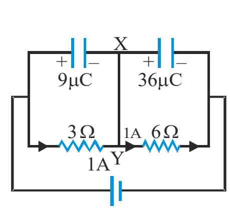
$\tau_3 = (C_1 + C_2)\left(\dfrac{R_1R_2}{R_1 + R_2}\right) = (6)\left(\dfrac{2}{3}\right) = 4\mu s$

**5.** From Y to X charge flows to plates a and b.

$(q_a + q_b)_i = 0, (q_a + q_b)_f = 27\mu C$




Initial Figure
(when switch was open)




Final Figure
(when switch is closed)

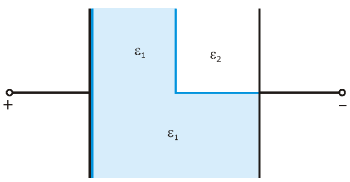
$\therefore$ 27μC charge flows from Y to X.

**6.** Time constant

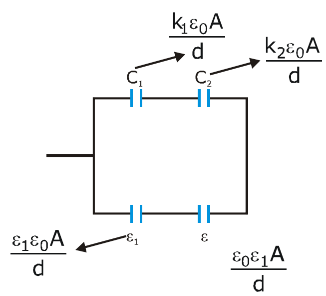
Where $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\left(\frac{2d}{3} + Vt\right)}{\epsilon_0} + \frac{\left(\frac{d}{3} - Vt\right)}{2\epsilon_0}$

$\Rightarrow C = \frac{6\epsilon_0}{5d + 3Vt}$

**9.**



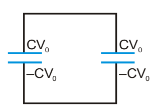

$C_1 = \frac{\varepsilon_0 A}{d} = C$



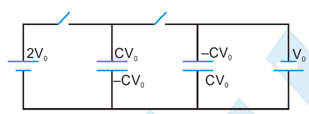

$\frac{4}{3} + 1 = \frac{7}{3} \Rightarrow C_2 = \frac{7C}{3}$

### Multiple Correct type question

**1.** Before $S_3$ is pressed




After $S_3$ is pressed

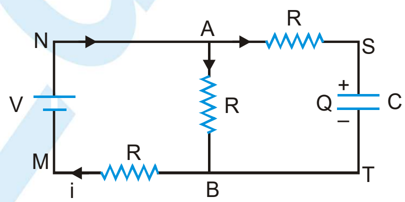



**2.** A,D

### Subjective

**1.** Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t.

$\therefore i_1 = \frac{dQ}{dt}$




**(i)** Applying Kirchhoff's second law in loop MNABM

$V = (i - i_1)R + iR \Rightarrow V = 2iR - i_1R$ ...**(i)**

Simillarly, applying Kirchhoff's second law in loop MNSTM

we have $V = i_1R + \frac{Q}{C} + iR$ ....**(ii)**

Eliminating i from equations **(i)** and **(ii)**, we get

$$V = 3i_1R + \frac{2Q}{C} \Rightarrow 3i_1R = V - \frac{2Q}{C}$$

$$\Rightarrow i_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right) \Rightarrow \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R} \Rightarrow \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}(1 - e^{-2t/3RC})$$

**(ii)** $i_1 = \frac{dQ}{dt} = \frac{V}{3R}e^{-2t/3RC}$

From equation **(i)**

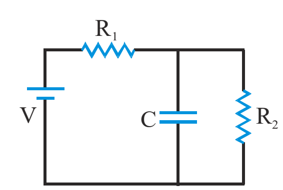
$$i = \frac{V + i_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

$\therefore$ Current through AB

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC} \Rightarrow i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

**2.** $Q_0$ is the steady state charge stored in the capacitor.




$Q_0 = C$ [PD across capacitor in steady state]

$= C$[ steady state current through $R_2$] $(R_2)$

$= C\left(\frac{V}{R_1 + R_2}\right)R_2$

$$\therefore Q_0 = \frac{CV R_2}{R_1 + R_2} \quad \alpha \text{ is } \frac{1}{\tau_C} \Rightarrow \frac{1}{C R_{net}}$$

## MOCK TEST

**1. (i)** $E = \dfrac{2Q}{2A\epsilon_0} + \dfrac{Q}{2A\epsilon_0}$

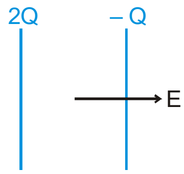
$\Rightarrow \quad E = \dfrac{3Q}{2A\epsilon_0}$

$E = \dfrac{3}{2}\dfrac{Q}{Cd} \Rightarrow \quad E_d = \dfrac{3Q}{2C} = V$

**(ii)** $F = EQ/2$

$F = \left(\dfrac{2Q}{2A\epsilon_0}\right) \times \dfrac{(-Q)}{1} = -\dfrac{Q^2}{A\epsilon_0}$

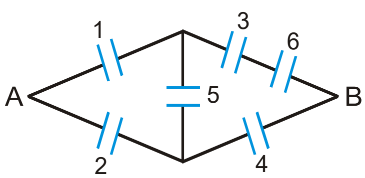
$F = \dfrac{Q^2}{A\epsilon_0}$



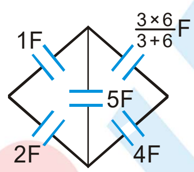

**(iii)** Energy $= \dfrac{1}{2}\epsilon_0 E_2 Ad$

$= \dfrac{1}{2}\epsilon_0\left(\dfrac{3Q}{2Cd}\right)^2 Ad = \dfrac{9}{8}\dfrac{Q^2}{C}$

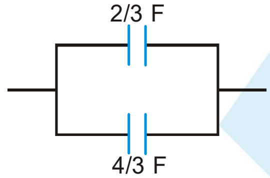
**2.** Equivalent circuit is




$\Rightarrow$

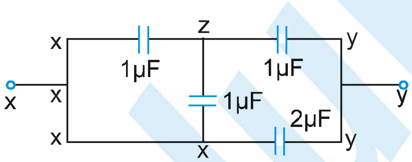



$\Rightarrow$

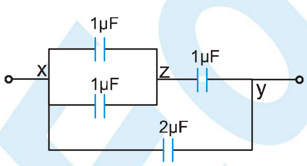



$= 2$ F

**3.**




Rearrange the circuit

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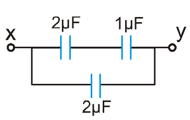


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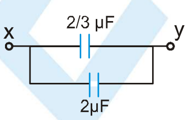
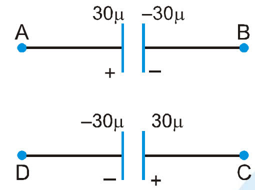
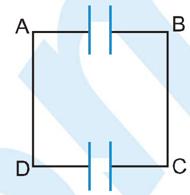



$\Rightarrow \quad C_{eq} = \dfrac{8}{3}\,\mu F$

**4.** $V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = 0$



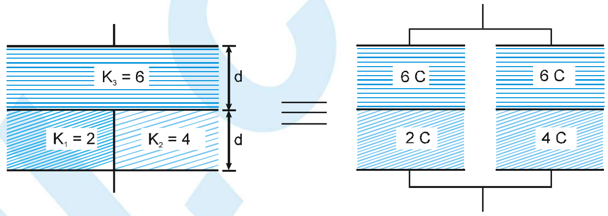




Final potential difference = zero

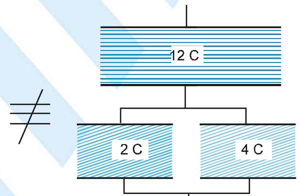
Final charge = Zero

Charge flow 30 μc from A to D

**5.**




(right)




as potential difference across dielectrics of dielectric constant 2 and 4 is not same.

The equivalent capacitance $C_{eq} = \dfrac{2C \times 6C}{2C + 6C} + \dfrac{6C \times 4C}{6C + 4C} = 3.9\,C$

**6.** Let the capacitance before insertion of dielectric be C and the resistance be R.

$\therefore \quad q = q_0\, e^{-\frac{t}{RC}}$ and $i = \dfrac{q/C}{R} = \dfrac{q}{RC}$

✈ Just after insertion of dielectric the capacitance increases.

$\therefore$ The charge just after insertion of dielectric remains same, but the current decreases.

$\Rightarrow$ **(A)** and **(B)** are false

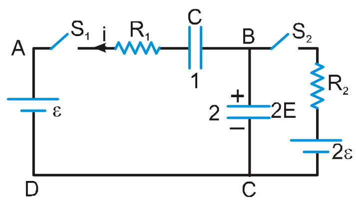
The energy stored in capacitor is $\dfrac{q^2}{2C}$, hence energy decreases

$\Rightarrow$ **(C)** is false

The time constant is RC and hence increases.

$\Rightarrow$ **(D)** is true

**7.**



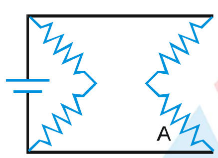

just before $S_1$ is closed the potential difference across capacitor 2 is 2E.

just after $S_1$ is closed the potential difference across capacitors 1 and 2 are 0 and 2E respectively. Applying KVL to loop ABCD immediately after $S_1$ is closed.

$$E = -iR_1 + 0 + 2E$$

or $i = \dfrac{E}{R_1}$ towards left

**8.** This is a DC circuit because the battery is the only source of voltage. Hence the capacitors behave like open circuits. An equivalent circuit is then two parallel sets of two identical series resistors, see figure. The voltage drop across each parallel branch must be the battery voltage of 3V. Since the resistors are identical there is an equal voltage drop of 1.5V across each resistor. In particular there is a drop of 1.5 V across resistor A.




**9.** Rate of change of energy = V.I.

Initially V = 0 hence VI = 0

finally I = 0 hence VI = 0

∴ first increases then decreases

**10.** As the key is connected to 1 and 2 frequently and at equal intervals, the two emf's $E_1$ & $E_2$ behave as d.c. sources in continuous contact.

The potential due to the two cells is :

$$V = \left(\frac{E_1R_2 + E_2R_1}{R_1 + R_2}\right)$$

Hence the charge on the capacitor is q = CV

$$= \frac{(E_1R_2 + E_2R_1)C}{(R_1 + R_2)}$$

**11.** $U_i = \dfrac{1}{2}C\varepsilon_1^2$

$U_f = \dfrac{1}{2}C\varepsilon_2^2 \qquad \Delta U = \dfrac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2)$

$Q_{in} = +C\varepsilon_1 \quad ; \quad Q_{final} = -C\varepsilon_2$

$\Delta Q = c(\varepsilon_1 + \varepsilon_2)$

work done by battery $W_b = E_2 \,.\, \Delta Q$

$= C(\varepsilon_2 + \varepsilon_1)\varepsilon_2$

Heat generated $= W_b - \Delta U$

$= c\,\varepsilon_2^2 + c\,\varepsilon_1\varepsilon_2 - \dfrac{1}{2}C(\varepsilon_2^2 - \varepsilon_1^2)$

$= \dfrac{1}{2}C(\varepsilon_2^2 + \varepsilon_1^2 + 2\varepsilon_1\varepsilon_2)$

$= \dfrac{1}{2}C(\varepsilon_1 + \varepsilon_2)^2$

**12.** The equivalent circuit is as shown in figure (b).

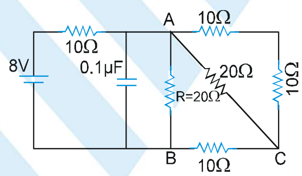



Fig. (a)

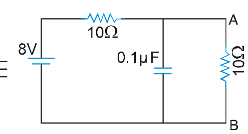



Fig. (b)

In the steady state the potential difference across AB is 4 volts.

∴ Charge on capacitor in steady state is

$q = CV = 0.4\ \mu C$

Current through resistor R is $I = \dfrac{V}{R} = \dfrac{4}{20} = 0.2\ A$

**13.** $U = \dfrac{1}{2}C_{eq}V^2 \Rightarrow C_1 = \dfrac{k\epsilon_0 A}{d/2} = \dfrac{2\epsilon_0 A}{(d/2)}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} \quad \Rightarrow C_2 = \dfrac{\epsilon_0 A}{d/2};$

$$C_{eq} = \frac{\left(2\dfrac{\epsilon_0 A}{d/2}\right) \times \dfrac{\epsilon_0 A}{d/2}}{3\dfrac{\epsilon_0 A}{d/2}} = \frac{4}{3}\frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\left(\frac{4}{3}\frac{\epsilon_0 A}{d}\right)V^2 = \frac{2}{3}\left(\frac{\epsilon_0 A}{d}\right)V^2$$

**14. Method I**

Force between plates

$$F = \frac{Q^2}{2A\epsilon_0} = \frac{\left(\frac{\epsilon_0 A}{x} V\right)^2}{2A\epsilon_0} = \frac{\epsilon_0 AV^2}{2x^2}$$

where x is separation between plates

dW = F dx

$$W = \int_d^{2d} \frac{\epsilon_0 AV^2}{2x^2} dx = \left[\frac{\epsilon_0 AV^2}{4x}\right]_d^{2d} = \frac{CV^2}{4} = 200\ \mu J$$

**Method II**

$U_{\bullet} + W_B + W_{ext} = U_f + \text{loss}$

Process is slow so energy loss is zero work done by battery = $W_B = Q_E$

$Q = Q_f - Q_i = 20 - 40 = -20$

$W_B = -20 \times 20$

$$\frac{1}{2} 2 \times 202 - 20 \times 20 + W_{ext} = \frac{1}{2} 1 \times 202 + 0$$

$W_{ext} = 200\ \mu J$

**15.** Force on metal plate S due to electrostatic attraction by plate T is $F = \dfrac{Q^2}{2A\epsilon_0}$

Force exerted on plate S by spring is = mg

In equilibrium $\Rightarrow \dfrac{Q^2}{2A\epsilon_0} = mg$ or

$Q = \sqrt{2mgA\epsilon_0}$

**16.** Ceff $= \dfrac{\varepsilon_0 A}{d}$ since effective capacitance between plates A and E is zero.

$\therefore U = \dfrac{1}{2} CV^2 = \dfrac{\varepsilon_0 A}{2d} V^2$

**17.** When switch $S_2$ is closed, due to symmetry no charge will flow through $S_2$.

Alternate solution :

After closing and before closing the switch there is no change in potential of any point.

**18.** $I_{max} = \dfrac{2\varepsilon}{R}$ at $t = 0$

$I = \dfrac{\varepsilon}{R}$ at $t = \infty$

so charge on the capacitor is $C\varepsilon$, when current is 50% of maximum current.

**19.** In the given cross-section which lies inside the capacitor plates , no charge flows. hence the required charge is 0.

**20.** Energy density= $\dfrac{1}{2} k\epsilon_0 E^2$

Since the cell remains connected, V remains unchanged (and therefore E remains unchanged)

$\Rightarrow$ Energy density will increase k times.

**21.** Charge on outer surface of C = – charge on inner surface of C

Hence potential at B due to charge on conductor C = 0

charge on outer surface of dielectric = – charge on inner surface of dielectric

$\therefore$ Potential at B due to charge on dielectric = 0

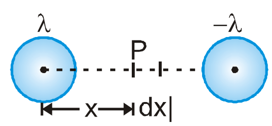
Potential at B due to charge on A $= \dfrac{Q}{4\pi\epsilon_0 b}$

$\therefore$ net potential at B $= \dfrac{Q}{4\pi\epsilon_0 b}$.

**22.** Let σ be the charge density of conducting plate and V be the volume of either dielectric

$$\therefore \frac{U_1}{U_2} = \frac{\left(\frac{1}{2} K_1 \epsilon_0 E_1^2\right) V}{\left(\frac{1}{2} K_2 \epsilon_0 E_2^2\right) V} = \frac{K_1}{K_2} \frac{\left(\frac{\sigma}{K_1 \epsilon_0}\right)^2}{\left(\frac{\sigma}{K_2 \epsilon_0}\right)^2} = \frac{K_2}{K_1}$$

**23.** (C) Let us give equal and opposite charges to two wires so that they would have linear charge density as + λ and – λ.




Electric field at point P,

$$E = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0 (\eta a - x)}$$

$$\int dV = -\int E\,dx = -\int_a^{\eta a - a} E\,dx$$

where a is radius of wire $\Rightarrow C = \dfrac{\lambda}{|V|} = \dfrac{\pi\epsilon_0}{\ln\eta}$

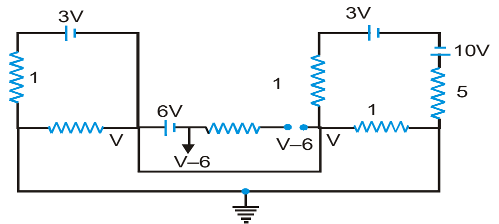
**24.** The resultant force acting per unit area of each plate can be written as $F = F_0 - F'$.
where $F_0$ is the force acting on unit area of plate due to other plate and F' is the force acting on unit area of plate from the dielectric.

Now $F = \dfrac{\frac{q^2}{2\varepsilon_0 \varepsilon A}}{A} = \dfrac{\left(\frac{\varepsilon_0 \varepsilon A}{d} V\right)^2}{2\varepsilon_0 \varepsilon A} \times \dfrac{1}{A} \Rightarrow F = \dfrac{\varepsilon_0 \varepsilon V^2}{2d^2}$

Also $F_0 = F \times \varepsilon$

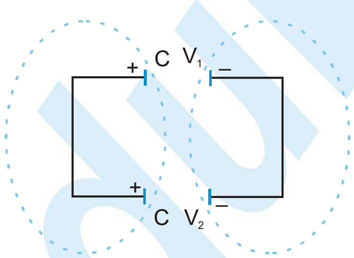
So $F' = F_0 - \dfrac{F_0}{\varepsilon} = F_0\left(1 - \dfrac{1}{\varepsilon}\right) = \varepsilon F\left(1 - \dfrac{1}{\varepsilon}\right) = \dfrac{\varepsilon(\varepsilon - 1)\varepsilon_0 V^2}{2d^2}$

**25.** In steady state the capacitor is fully charged and is treated as open circuit, so no current flows through branch containing capacitor in steady state. So the circuit can be redrawn as :




Potential difference across the capacitor in steady state
$= V - 6 - V = -6V$
(–ve sign signifies that left hand plate is of negative polarity)
Charge $= CV = 1 \times 6 = 6\ \mu C$

**26.** As the charge of isolated system remains conserved, so the sum of charges of plates having-ve polarity remains constant. As potential of two capacitors are different so some charge flows into the circuit till both acquire the same potential.




As charge flows, $\Delta H \neq 0$, and hence $\sum u_i \neq \sum U_f$
Let final common potential be V, then

$V = \dfrac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \dfrac{V_1 + V_2}{2}$ [as $C_1 = C_2 = C$]

**27.** $V_0 = I_0 R = 10 \times 10 = 100$ volts(since, $I_0 = 10$amp from figure) Hence (A) is correct
Also $I = I_0 e - t/RC$

Taking log $\lambda n\left(\dfrac{I_0}{I}\right) = \dfrac{t}{RC}$

$\Rightarrow C = \dfrac{t}{R \ln(I_0 / I)}$

At ; t = 2 sec, $I = 2.5\,A$

$C = \dfrac{2}{10 \ln\left(\dfrac{10}{2.5}\right)}$

$C = \dfrac{2}{10 \ln 4} = \dfrac{2}{10 \times 2 \ln 2} = \dfrac{1}{10 \ln 2}$ F

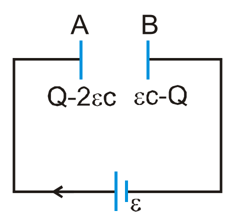
Hence (B) is correct.

Heat produced $= \dfrac{1}{2} CV^2 = \dfrac{1}{2}\left(\dfrac{1}{10 \ln 2}\right)(1002)$

$= \dfrac{500}{\ln 2}$ joules.

Hence (C) is correct
Thermal power in the resistor will decrease with a time constant $\dfrac{1}{2 \ln 2}$ second. Hence (D) is correct

**28.** Suppose charge flown through the battery is Q, then charge distribution will be as :




The electric field in the region between A and B is =

$\dfrac{Q - 2\varepsilon C}{2A \in_0} - \dfrac{\varepsilon C - Q}{2A \in_0} = \dfrac{2Q - 3\varepsilon C}{2A \in_0}$

$\therefore$ Potential difference between the plates,

$\dfrac{2Q - 3\varepsilon C}{2A \in_0}.d = \varepsilon \Rightarrow \dfrac{2Q - 3\varepsilon C}{2} \dfrac{1}{C} = \varepsilon \Rightarrow 2Q = 5\,\varepsilon C$

$\Rightarrow Q = \dfrac{5\varepsilon C}{2}$

$\therefore$ work done by battery $= \varepsilon Q = \dfrac{5\varepsilon^2 C}{2}$

**29.** **(A)** $E = \frac{1}{2}CV^2$

As potential difference source between the plates is connected, p.d. remains constant. But capacitance C becomes KC hence energy stored is increased by factor K.

**(B)** Electric field $\frac{V}{d}$ is not changed.

**(C)** Charge on each plate is increased by factor K hence force between them increases by factor $K^2$. For effect of the medium, they must completely lie in the medium.

**(D)** $Q = CV$

Hence charge becomes KQ as C becomes KC and V remain unchanged.

**30.** Charge on capacitor before insertion of dielectric slab = 100 μC

Charge on capacitor after insertion of dielectric slab = 300 μC

Increase in charge on the capacitor = 300 – 100 = 200 μC

Heat produced = 0

Energy supplied by the cell = increase in stored potential energy + work done on the person who filling the dielectric slab + heat produced.

**31.** The instantaneous charge on the capacitor is

$q(t) = q_0 [1 - e–t/RC] = CV [1 - e–t/RC]$

The instantaneous current

$$i = \frac{dq}{dt} = CV\left(\frac{1}{RC}\right) e–t/RC$$

$$\therefore\ i = \frac{V}{R} e–t/RC \quad \text{or} \quad i_0 = \frac{V}{R} \quad (\because t = 0)$$

Given that V and R are same for both capacitors, so the initial current in both condensers is same moreover this is not zero.

During discharge, the instantaneous charge q is

$q(t) = q_0 e–t/RC$

Let $q' = q_0/2$ at $t = t$, then $\frac{q_0}{2} = q_0 e–1t/RC$

or $t = RC \log_e 2$

If $t_1$ and $t_2$ be the times in which the two capacitors lose 50% of their charge, then

$$\frac{t_1}{t_2} = \frac{RC_1 \log_e 2}{RC_2 \log_e 2} = \frac{C_1}{C_2} = \frac{1}{2}$$

$t_1 = t_2/2$

This shows that $C_1$ loses 50% charge sooner than $C_2$ because it takes time $t_1$ which is half of $t_2$]

**32.** If potential difference across an isolated charged capacitor is doubled by doubling separation between plates, the energy stored is capacitor from $U = \frac{Q^2}{2C}$ becomes double of previous value. Hence statement–1 is false.

**33.** Let the electric field in region I and II be $E_1$ and $E_2$. The potential difference across left half capacitor and right half capacitor is same.

Therefore $E_1 d = E_2 d$ where d = inter planar gap.

$\therefore\ E_1 = E_2$

Hence statement – 1 is false, statement – 2 is correct by definition.

**34.** The electrostatic force on metal of capacitor is

$= \text{pressure} \times \text{area of plate} = \frac{\sigma^2}{2\epsilon_0}A$

σ = charge per unit area on plate.

Since charge on metal plate of an "isolated" capacitor does not change, force on metal plate remains same. Electric field decreases due to induced charges in dielectric, but this does not effect the charge distribution on isolated metal plate.

**35.** The battery energizes the circuit and maintains the flow of electron from positive plate of capacitor to positive terminal of battery through wires and from wires to negative plate on other side.

No transfer of charge takes place within the plates in spite of having the electric field in between the plates.

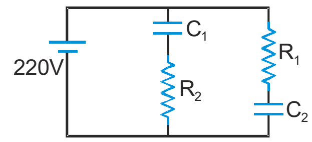
**36.** $i = 2 \times 10^{-2}$ A

$P_{R_1} = i^2 R_1 = (2 \times 10^{-2})^2 \times 4 \times 10^3 = 1.6$ W

**37.** $Q_{C_1} = V_{R_1} \times C_1 = 80 \times 3 \times 10^{-6} = 240\ \mu C$

$Q_{C_2} = V_{R_2} \times C_2 = 140 \times 6 \times 10^{-6} = 840\ \mu C$

**38.** $Q_{C_1} = EC_1 = 220 \times 3 \times 10^{-6} = 660\ \mu C$




**39.** (C) **40.** (D) **41.** (C)

**39. to 41.**

For t = 0 to to = RC seconds, the circuit is of charging type. The charging equation for this time is

$$q = CE(1 - e^{-\frac{t}{RC}})$$

Therefore the charge on capacitor at time t0 = RC is

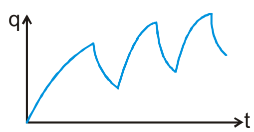
$$q_o = CE(1 - \frac{1}{e})$$

For t = RC to t = 2RC seconds, the circuit is of discharging type. The charge and current equation for this time are

$$q = q_o e^{-\frac{t-t_o}{RC}} \quad \text{and} \quad i = \frac{q_o}{RC} e^{-\frac{t-t_o}{RC}}$$

Hence charge at t = 2 RC and current at t = 1.5 RC are

$$q = q_o e^{-\frac{2RC-RC}{RC}} = \frac{q_o}{e} = \frac{1}{e}CE(1 - \frac{1}{e})$$




$$\text{and } i = \frac{q_o}{RC} e^{-\frac{1.5RC-RC}{RC}} = \frac{q_o}{\sqrt{e}RC} = \frac{E}{\sqrt{e}R}(1 - \frac{1}{e})$$

respectively

Since the capacitor gets more charged up from t = 2RC to t= 3RC than in the interval t=0 to t=RC, the graph representing the charge variation is as shown in figure

**42. (A)** By inserting dielectric slab, capacitance of 1 increases there by increasing charge on capacitor 2 as more charge is flown through the battery. Energy stored in capacitor also increases.

**(B)** By increasing separation between the plates, capacitor $C_1$ decreases. Charge on $C_2$ also decreases.

**(C)** By shorting capacitor-1, only capacitor 2 remains in the circuit. Potential difference across $C_2$ increases thereby increasing charge on 2 as well as energy stored.

**(D)** By earthing plate of capacitor 1 potentials will change but there will be no potential difference change, making no overall change in the circuit.

**43.**

**(A)** At constant potential difference, when interplanar separation is increased, the capacitance decreases.

From $U = \frac{1}{2}CV^2$, the potential energy decreases.

Also from $E = \frac{V}{d}$ electric field decreases

**(B)** At constant charge when interplanar separation is increased the capacitance decreases.

From $U = \frac{Q^2}{2C}$, the potential energy increases

Since charge density on plates is constant, electric field remains same.

**(C)** At constant potential difference, when area of plate increases the capacitance increases.

Hence from $U = \frac{1}{2}CV^2$, the potential energy increases

Also from $E = \frac{V}{d}$, the electric field remains same.

**(D)** At constant charge on increase in area of plates

From $U = \frac{1}{2}\frac{Q^2}{C}$, the potential energy decreases and since charge density on plate decreases electric field decreases.

**44.** The initial charge on capacitor = $CV_i$ = 1 × 2 μC = 2 μC

The final charge on capacitor = $CV_f$ = 1 × 4 μC = 4 μC

∴ Net charge crossing the cell of emf 4V is

$q_f - q_i = 4 - 2 = 2$ μC

The magnitude of work done by cell of emf 4V is

$W = (q_f - q_i)\, 4 = 8$ μJ

The gain in potential energy of capacitor is

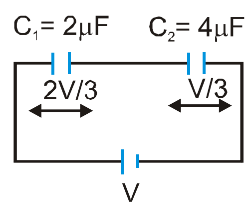
$$\Delta U = \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}\ 1 \times [42 - 22]\ \mu J = 6\ \mu J$$

Net heat produced in circuit is

$\Delta H = W - \Delta U = 8 - 6 = 2$ μJ

**45.** $U_i$ for $C_1 = C_1 \times \frac{4}{9} V_2 \times \frac{1}{2} = \frac{4V^2}{9}$

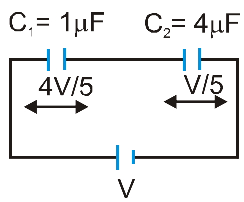
$U_i$ for $C_2 = 4 \times \frac{V^2}{9} \times \frac{1}{2} = \frac{2V^2}{9}$



When separation of plates of $C_1$ gets doubled, its capacity becomes half :

$$U_f \text{ for } C_1 = 1 \times \frac{16V^2}{25} \times \frac{1}{2} = \frac{8V^2}{25}$$

$$U_f \text{ for } C_2 = 4 \times \frac{V^2}{25} \times \frac{1}{2} = \frac{2V^2}{25}$$




**46.** Energy taken from cell $= 20 \times 30\ \mu J$

$= 600\ \mu J$

Energy stored in capacitors $= \frac{1}{2}\ 3.\ 10^2 = 150\ \mu J$

$\therefore$ Heat produced in resistors $= 600 - 150 = 450\ \mu J$

Divide this heat in $2\Omega$ and

(equivalent of $3\Omega$ and $6\Omega$)

i.e., in $2\Omega$ and $2\Omega$

which is $225\ \mu J$, $225\ \mu J$

$\therefore$ Heat produced in $2\Omega = 225\ \mu J$

Further divide $225\ \mu J$ in $3\Omega$ and $6\Omega$ in inverse ratio of resistance $\left(\because P = \frac{V^2}{R}\right)$

$$\text{Heat in } 3\Omega = \frac{225}{9} \times 6 = \frac{225 \times 2}{3} = 75 \times 2 = 150\ \mu J$$

$$\text{Heat in } 6\Omega = \frac{225}{9} \times 3 = 75\ \mu J$$

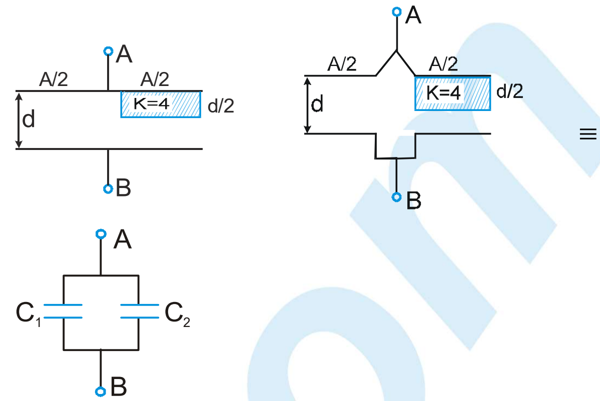
**47.** The charge on the capacitor when current reaches $I_0$

$q_0 = (I_0R).C_1$

When the switch is in position 2, this charge is shared with capacitor $C_2$ and at steady state potential across $C_1$ is equal to that across $C_2$. The energy lost in this process

$$\Delta U = \frac{1}{2}.\frac{C_1C_2}{C_1+C_2} \times \left(\frac{q_0}{C_1}\right)^2 = \frac{1}{2}.\frac{C_1C_2}{C_1+C_2} \times (I_0R)^2 = 4\ J$$

**48.**




$$C_1 = \frac{\varepsilon_0 A/2}{d},\ C_2 = \frac{\varepsilon_0 A/2}{\frac{d/2}{k} + \frac{d}{2}} = \frac{4\varepsilon_0 A}{5d}$$

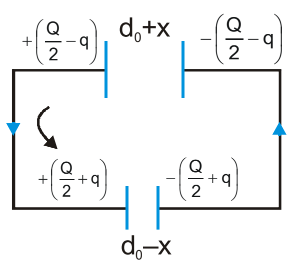
$$\Rightarrow C = C_1 + C_2 = \frac{13}{10}\frac{\varepsilon_0 A}{d} \qquad \text{Ans. } \frac{13}{10}\frac{\varepsilon_0 A}{d}$$

**49.** $E < 10^6 \Rightarrow \frac{10^3}{d} < 10^6$

$$d > 10^{-3}\ m2 \Rightarrow C = \frac{k\varepsilon_0 A}{d} \Rightarrow d = \frac{k\varepsilon_0 A}{C} > 10^{-3}$$

$$A > \frac{10^{-3}\ x\ C}{k\varepsilon_0} \Rightarrow A > \frac{10^{-3}\ x\ 50\ x\ 10^{-12}}{(6\pi)\ x \left(\frac{1}{36\pi} x 10^{-9}\right)} = 300\ mm^2$$

**50.** Let each plate moves a distance 'x' from its initial position.



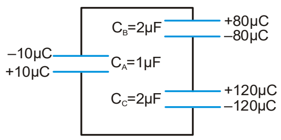

Let q charge flows in the loop. using KVL

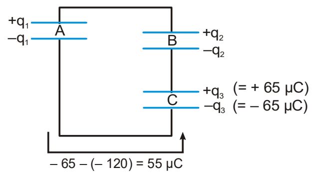
$$\frac{\left(\frac{Q}{2} - q\right)(d+x)}{\epsilon_0 A} - \frac{\left(\frac{Q}{2} + q\right)(d-x)}{\epsilon_0 A} = 0$$

$$\therefore \quad q = \frac{Qx}{2d_0}\ ;\ I = \frac{dq}{dt} = \frac{Q}{2d_0}\left(\frac{dx}{dt}\right);$$

$$\text{Ans, } I = \frac{Qv_0}{2d_0}$$

**51.** Initial state Final state




From conservation of charge

$-q_1 - q_3 = 10 - 120 = -110\ \mu C$ ..............**(1)**

$-q_2 + q_3 = -80 + 120 = +40\ \mu C$ ..............**(2)**

In the final state

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} + \frac{q_3}{C_C}$$

$$\Rightarrow \frac{q_1}{1} = \frac{q_2}{2} + \frac{q_3}{2} \quad \text{at} \quad q_2 + q_3 = 2q_1$$

$$\Rightarrow \frac{q_1}{1} = \frac{q_2}{2} + \frac{q_3}{2} \ ; \ q_2 + q_3 = 2q_1$$

Solving we get a $q_3 = 65\ \mu C$.

The charge on lower plate of capacitor $C_C$ changes from $-120\ \mu C$ to $-65\ \mu C$.

Hence the charge flowing through shown connecting wire is

$(120 - 65) = 55\ \mu C$.

final charges

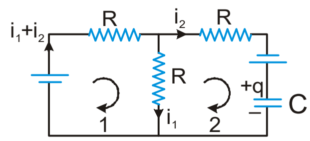
$q_3 = 65\mu C\ ;\ q_2 = 25\mu C\ ;\ q_1 = 45\mu C$

Heat produced $= U_i - U_f$

$$= \left[\frac{(120\mu C)^2}{2\times 2\mu F} + \frac{(80\mu C)^2}{2\times 2\mu F} + \frac{(10\mu C)^2}{2\times 1\mu F}\right] -$$

$$\left[\frac{(65\mu C)^2}{2\times 2\mu F} + \frac{(25\mu C)^2}{2\times 2\mu F} + \frac{(45\mu C)^2}{2\times 1\mu F}\right] = 3025\ \mu J$$

**52.**




Applying Kirchoff's law in Loop1

$\varepsilon - (i_1 + i_2)R - i_1 R = 0$ ...**(1)**

Loop 2

$-i_2 R + \varepsilon - \dfrac{q}{C} + i_1 R = 0$ ....**(2)**

eliminating $i_1$ from **(1)** and **(2)**

$$\varepsilon - \frac{q}{C} - i_2 R + \frac{\varepsilon - i_2 R}{2} = 0 \text{ or } \frac{3\varepsilon}{2} - \frac{q}{C} - \frac{3}{2} i_2 R = 0$$

$$i_2 = \frac{dq}{dt} \Rightarrow \frac{3C\varepsilon - 2q}{2C} = \frac{3}{2} R \frac{dq}{dt}$$

$$\text{or} \int_0^q \frac{dq}{3C\varepsilon - 2q} = \int_0^t \frac{dt}{3RC}$$

$$\text{or } -\frac{1}{2} \ln\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{3RC} \quad \text{or } 1 - \frac{2q}{3C\varepsilon} = e^{-\frac{2t}{3RC}}$$

$$\Rightarrow q = \frac{3C\varepsilon}{2}\left(1 - e^{-\frac{2t}{3RC}}\right) \Rightarrow i_2 = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right) e^{-\frac{2t}{3RC}}$$

$$\text{from (1)},\ i_1 = \frac{\varepsilon - i_2 R}{2R} = \frac{\varepsilon}{2R}\left(1 - e^{-\frac{2t}{3RC}}\right)$$

$$\text{Ans. } i = \frac{\varepsilon}{2R}\left(1 - e^{-\frac{2t}{3RC}}\right)$$