

 \Rightarrow v' = -v

Total mechanical energy released

$$=\frac{1}{2}(m+m+2m)v^2=2mv^2$$

17. COLM :

Along horizontal

$$0 = m(u\cos\theta - v) - 4mv$$

$$\Rightarrow v = \frac{u \cos \theta}{5}$$

: velocity of shell along horizontal w.r.t ground

$$= u \cos \theta - \frac{u \cos \theta}{5} = \frac{4}{5} (u \cos \theta)$$

Time of flight T= $\frac{2 u \sin \theta}{g}$

$$\therefore$$
 x = horizontal displacement

$$= \left(\frac{4}{5}u\cos\theta\right) \left(\frac{2u\sin\theta}{g}\right) = \frac{4u^2\sin2\theta}{5g}$$

18. The ball & the earth forms a system and no external force acts on it. Hence total momentum remains constant.

19. COME :
$$mv \cos \theta = \frac{m}{2} (-v \cos \theta) + \frac{m}{2} v$$

$$\Rightarrow \frac{3 \operatorname{mv} \cos \theta}{2} = \frac{\mathrm{m}}{2} \mathrm{v}' \Rightarrow \mathrm{v}' = 3 \mathrm{v} \cos \theta$$

$$\bigvee \cos \theta$$

$$\bigvee \cos \theta$$

$$\frac{\mathrm{m}}{2} \frac{\mathrm{m}}{2} \frac{\mathrm{m}}{2}$$

20. COME :
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1 \times \mathbf{u} + \mathbf{0} = 1 \times \frac{\mathbf{u}}{4} + \mathbf{m}\mathbf{v}_2$$

$$\frac{3}{4}$$
 u = mv₂

$$\mathbf{e} = -\left(\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}\right) = -\left(\frac{\mathbf{v}_2 - \mathbf{u}/4}{0 - \mathbf{u}}\right) = \frac{\mathbf{v}_2 - \mathbf{u}/4}{\mathbf{u}}$$

....(i)

$$\Rightarrow v_2 = \frac{u}{4} + u = \frac{5u}{4} \qquad \dots (ii)$$

(i) & (ii)
$$\frac{3}{4}u = m\left(\frac{5}{4}u\right) \implies m = \frac{3}{5} = 0.6 \text{ kg}$$

For collision between B and C :

$$v_{\rm B} = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) u_1 + \frac{m_2(1 + e)}{(m_1 + m_2)} u_2$$
$$= \left(\frac{m - 4m}{5m}\right) v + 0 = -3/5 v$$

$$\mathbf{v}_{\rm C} = \frac{\mathbf{m}_1(\mathbf{l} + \mathbf{e})}{(\mathbf{m}_1 + \mathbf{m}_2)}\mathbf{u}_1 + \left(\frac{\mathbf{m}_2 - \mathbf{e}\mathbf{m}_1}{\mathbf{m}_1 + \mathbf{m}_2}\right)\mathbf{v}_2 = \frac{\mathbf{m}(\mathbf{l} + 1)}{4\mathbf{m}}\mathbf{v} + \mathbf{0} = \frac{\mathbf{v}}{2}$$

For collision between A and B :

$$v_A = 0 + \frac{m(l+1)}{5m} \times \left(\frac{3v}{5}\right) = \frac{6}{25}v$$

$$\mathbf{v}_{\mathrm{B}}' = \mathbf{0} + \left(\frac{\mathrm{m} - 4\,\mathrm{m}}{5\,\mathrm{m}}\right) \left(-\frac{3\,\mathrm{v}}{5}\right) = -\frac{9\,\mathrm{v}}{2\,5}$$
$$\mathbf{v}_{\mathrm{B}}' < \mathbf{v}_{\mathrm{C}}$$

B will not collide with C.

Therefore there will be only two collisions.

...(i)

22. Along tangent

$$u\cos\theta = v\sin\theta$$

u sin θ

Along normal

$$e = \frac{v \cos \theta}{v \cos \theta} = \cot^2 \theta = \cot^2 60 = \cot^$$

23. At the lowest position
COME :
$$M\sqrt{2}gL = (M + m)v$$
(i)
COME : $\frac{1}{2}(M+m)v^2 = (M+m)gh$ (ii)
 $\Rightarrow v = \sqrt{2}gh = \frac{M\sqrt{2}gL}{(M+m)} \Rightarrow h = \left(\frac{M}{m+M}\right)^2 L$

24. After 1s,

 $v_A = 20-10 \times 1 = 10$ m/s and $v_B = 0 + 10 \times 1 = 10$ m/s At the time of collision , $V_A = V_B = 5$ m/s after collision, velocity gets interchanged.

25. COLM
$$\Rightarrow$$
 2mu + 0= 2mv + mu \Rightarrow v = $\frac{u}{2}$

$$\mathbf{e} = -\left(\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}\right) = -\left(\frac{\mathbf{u} - \mathbf{u}/2}{\mathbf{0} - \mathbf{u}}\right) = \frac{1}{2} \begin{array}{c} 2\mathsf{m} \\ \mathsf{O} \\ \mathsf{u} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\ \mathsf{O} \\ \mathsf{V}_{(\text{Let})} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\ \mathsf{O} \\ \mathsf{v} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\ \mathsf{O} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\ \mathsf{O} \\ \mathsf{v} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\ \mathsf{O} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\\mathsf{O} \end{array} \xrightarrow{\mathsf{C}} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\\mathsf{m} \\\mathsf{O} \end{array} \xrightarrow{\mathsf{C}} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{C}} \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\\mathsf{m} \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\ \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m} \\ \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \end{array} \xrightarrow{\mathsf{m} } \begin{array}{c} \mathsf{m$$



$$\mathbf{26.} \quad \mathbf{e} = -\left(\frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{u}_2 - \mathbf{u}_1}\right)$$

$$\Rightarrow 1 = -\frac{5 - v_1}{5 - (-10)} = \frac{v_1 - 5}{15} \Rightarrow v_1 = 20 \text{ m/s}$$

:. Impulse on ball= $m(v - u) = 1 \times [20 - (10)] = 30$ N-s

27. For second object

 $2v_1 = 0 + at; t = \frac{2v_1}{a} ...(i)$ $u = 0 + at; t = \frac{2v_1}{a} ...(i)$ $s_1 = s_2 + d; \ \frac{1}{2}at^2 = ut + d$ $\Rightarrow \frac{1}{2}a \times \left(\frac{2v_1}{a}\right)^2 = v_1\left(\frac{2v_1}{a}\right) + d \Rightarrow d = \frac{2v_1^2}{a} - \frac{2v_1^2}{a} = 0$ 28. **†**⊻₁ $\text{COLM} \implies 2\text{mv}_0 = 3\text{mv}_1 \implies \text{v}_1 = \frac{2}{3}\text{v}_0$ COME $\Rightarrow 2 \times \frac{1}{2} m v_0^2 = 2 \times \frac{1}{2} m (v_1^2 + v_2^2) + \frac{1}{2} m v_1^2$ $\Rightarrow 2mv_0^2 = 2m\left(\frac{2}{3}v_0\right)^2 + 2mv_2^2 + m\left(\frac{2}{3}v_0\right)^2$ $\Rightarrow 2v_2^2 = 2v_0^2 - 3\left(\frac{2}{3}v_0\right)^2 \Rightarrow v_2 = \frac{v_0}{\sqrt{3}}$

Velocity of particle =
$$\sqrt{v_1^2 + v_2^2} = \sqrt{\frac{4v_0^2}{9} + \frac{v_0^2}{3}} = \frac{\sqrt{7}}{3}v$$

29. Average power

$$\frac{\Delta W}{\Delta t} = \left(\frac{\Delta K + \Delta U}{\Delta t}\right) = \frac{1}{2} \frac{(\lambda \Delta x)v^2}{\Delta t} + \frac{\lambda \Delta xg(\Delta x/2)}{\Delta t}$$
$$< P > = \frac{1}{2} \lambda v^3 + \frac{\lambda l}{2} vg$$

EXERCISE - 2
Part # 1 : Multiple Choice
1. Momentum of the coin perpendicular to the common normal remains constant.

$$\therefore v_{y} = -3 (constant) v_{y}t = -6$$

$$\Rightarrow t = 2 \sec \& v_{x}t = -4 v_{x} = -2 m/s$$
Which is given by striker.
So initial velocity of striker = 2ms⁻¹
Final velocity of the striker = 0.
2. In the absence of external forces, the linear momentum of the system remains constant.
3. $F_{ext} = m \frac{dv}{dt} + v_{rel} \frac{dm}{dt}$

$$\Rightarrow 0 = m \frac{dv}{dt} + 2 \frac{dm}{dt} \Rightarrow - \int_{+m}^{m/2} \frac{2 dm}{m} = \int_{0}^{v} dv$$

$$\Rightarrow v = 2 \bullet n2$$
4. Force exerted by one leg on the ground
 $N = \frac{1}{4} [wt + rate of change of momentum]$

$$= \frac{1}{4} [Mg + n (mv \cos 60^{\circ}) \times 2] = 1N$$

In the ground frame

5.

6.

$$: m_A \Delta x_A + m_B \Delta x_B + m_p \Delta x_p = 0$$

$$\Rightarrow 40 \times 60 + 0 + 40 \times \Delta x = 0$$

$$\Rightarrow \Delta x_{a} = -60$$
 (to the left)

Hence A & B meet at the right end.

6.

$$T_{\Delta t} = m(v - u); \quad N\Delta t - T\Delta t = m(v - 0)$$

$$T\Delta t = 3m(v - 0)$$

$$\Rightarrow \quad v = u/5$$

$$\therefore \quad \text{Impulsive tension } T\Delta t = \frac{3mu}{5}$$
7.
$$COLM : m_{R}(0.8) + m_{S}(0) = m_{R}(0.2) + m_{S}(1.0)$$

$$\Rightarrow 0.6 m_{R} = m_{S} \implies m_{R} > m_{S}$$



8.
$$\sqrt[h]{t_{(h-d)}} \sqrt{2g(h-d)} = e\sqrt{2gh} \Rightarrow \frac{h}{d} = \frac{1}{1-e^2}$$

9.
$$\overline{\mathbf{x}} = \frac{\mathbf{M}_1 \mathbf{x}_1 + \mathbf{M}_2 \mathbf{x}_2}{\mathbf{M}_1 + \mathbf{M}_2} = \frac{\frac{\mathbf{M}_2 \left(-\frac{\mathbf{a}}{2}\right) + \frac{\mathbf{M}_2 \left(\frac{\mathbf{a}}{3}\right)}{2}}{\mathbf{M}} = -\frac{\mathbf{a}}{12}$$

 $v_{f} = \frac{mvN}{M + Nm}$

10.
$$\text{Nmv} = (M + \text{Nm})\text{v}_{\text{f}} \implies$$

11.

$$\overline{\mathbf{x}} = \frac{(\mathbf{M} \times \mathbf{0}) + \left(-\frac{\mathbf{M}}{4} \times \frac{-\mathbf{L}}{3}\right) + \left(\frac{\mathbf{M}}{4} \times \frac{4\mathbf{L}}{6}\right)}{\mathbf{M}} = \frac{\mathbf{L}}{4}$$

12.
$$\Delta \overline{\mathbf{x}} = \frac{3 \,\mathrm{M.x} + \mathrm{M}(\mathrm{x}+2)}{4 \,\mathrm{M}} = 0 \implies x = -\frac{1}{2}$$

13. Let x = displacement of ring to the left. $\Rightarrow \Delta x_{cm}$

$$=\frac{2\mathrm{mx}+\mathrm{m}(\mathrm{x}+\mathrm{L}-\mathrm{L}\cos\theta)}{3\mathrm{m}}=0 \implies x=-\frac{L}{3}(1-\cos\theta)$$

 $P(x=y) \qquad \frac{x}{2} - \frac{y}{6} = 1$

14.

Co-ordinate of P = (3,3)

 \therefore Speed of 3rd particle = $3\sqrt{2}$ m/s

15. COLM
$$\Rightarrow$$
 m(v₁ cos 45° + v₂) + mv₂ = 0

 $\Rightarrow v_1 = -2\sqrt{2}.v_2$ COME $\Rightarrow K_i + U_i = K_f + U_f$

 $\sqrt{3\sqrt{2}}$

$$0 + \frac{mgR}{\sqrt{2}} = \frac{1}{2}mv_{2}^{2} + \frac{1}{2}m\left[\frac{v_{1}^{2}}{2} + \left(\frac{v_{1}}{\sqrt{2}} - \frac{v_{1}}{2\sqrt{2}}\right)^{2}\right]$$

$$16. Mv = 0 + \frac{M}{10}v_2 \Longrightarrow v_2 = 10v_2$$

- Initially when the shell is empty the C.M. lies at its geometric centre. Also when the shell is filled with sand CM lies at its geometric centre.
- **18.** $\Delta p = \text{change in momentum} = 2\text{mv}$

$$\Delta t$$
 = time between two collision = $\frac{2(L-d)}{v}$
 \therefore Force exerted on wall = $\frac{\Delta p}{\Delta t} = \frac{mv^2}{(L-d)}$

19.
$$a_{em} = \frac{F_{net}}{Totalmass} = \frac{(0.2)(3)(10)}{1+2} = 2 \text{ ms}^{-2}$$

Acceleration of 1kg w.r.t. ground

$$=(0.1)(10)=1$$
 ms⁻²

a _{cn}

Acceleration of 2 kg w.r.t. ground

$$= \frac{(0.2)(3)(10) - (0.1)(10)}{2} = \frac{5}{2} \text{ms}^{-2}$$

$$= \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{(1)(1) + (2)(5/2)}{1+2} = 2 \text{ms}^{-2}$$

20. In elastic head on collision if the masses of the colliding bodies are equal, the velocities after collision are interchanged.

For Ist bead,
$$Fd = \frac{1}{2}mu^2 \implies u = \sqrt{\frac{2Fd}{m}}$$

21.
$$t_1 = \frac{L}{v}$$
 (time for Ist collision)
 $t_2 = \frac{2L}{v}$ (time for IInd collision)
 $t_3 = \frac{3L}{v}$ (time for 3rd collision)

 $1 \setminus I$

$$t_{(n-1)} = \frac{L}{v} (n-1) (time \text{ for } (n^{th}) \text{ collision})$$

22.
$$\overrightarrow{P}_{A} + \overrightarrow{P}_{B} = \overrightarrow{A} + \overrightarrow{J} \quad J \rightarrow \overrightarrow{B}$$

For A : P - J = mv₁(i) for B : J = mv₂(ii)

$$\therefore e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left[\frac{\frac{J}{m} - \left(\frac{P - J}{m}\right)}{0 - \frac{P}{m}}\right] = \frac{2J}{p} - 1$$



23.
$$\Delta \overline{x} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3}{m_1 + m_2 + m_3}$$

$$\Delta \overline{x} = \frac{(80 \times 2) - (50 \times 2) + (70 \times 0)}{80 + 50 + 70}$$

$$= 30 \text{ cm towards right}$$
24. Cart frame :
$$44 + 44$$
Ground frame :
$$44 + 44$$
The velocity of rebound = $4\sqrt{5}$ m/s
25. $mu - I_1 = -mu$
 $\therefore I_1 = 2mu \& mu - I_2 = 0$
 $\Rightarrow I_2 = mu (\text{for II}^{nd} \text{ ball}) \therefore I_2 = I_1/2$
26. COLM \Rightarrow
 $mv^{\$} + mv^{\$} + m\left(\frac{-v}{\sqrt{2}}\$ - \frac{v}{\sqrt{2}}\$\right) + m\overline{v}_4 = 0$
 $\overline{v}_4 = -v\left(1 - \frac{1}{\sqrt{2}}\right)\$ - v\left(1 - \frac{1}{\sqrt{2}}\right)\$$
Total energy released
 $= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m\left[v^2\left(1 - \frac{1}{\sqrt{2}}\right)^2 \times 2\right]$
 $= mv^2(3 - \sqrt{2})$
27. COME : $-MV + m(v\cos 60 - V) = 0 \Rightarrow v = 10 \text{ m/s}$
28. $F_{ext} = \Delta mv$ (for first body)
 $\Rightarrow F_{ext} = \frac{10 \times (15 - 0)}{3} = 50 \text{ N}$

COME:
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

 $\Rightarrow 10 \times 15 + 25 \times u_2 = (10 + 25)5$
 $\Rightarrow u_2 = 1 \text{ m/s}$

29.
$$x_{cm} = \frac{m \times 0 + m \times R}{m + m} = \frac{R}{2}$$

 $x_{cG} = \frac{W_1(0) + W_2(R)}{W_1 + W_2} = \frac{mgR}{mg} = R$
 $\therefore x_{cG} - x_{cM} = R - \frac{R}{2} = \frac{R}{2}$
30. $\Delta \overline{x} = \frac{m\Delta x_1 + M\Delta x_2}{m + M} = 0$
 $\Rightarrow \frac{1(l \sin 30^\circ + l \sin 30^\circ - x) + 4x}{1 + 4} = 0$
 $\Rightarrow Displacement of bar x = 0.2$
31. $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$
 $v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2$

For C:
$$v_c = \frac{2mu}{3m} = \frac{2}{3}u$$

32. Velocity before strike
$$u = \sqrt{2 \text{ gh}}$$

Impulse = $F\Delta t = m(v-u)$
 $\Rightarrow F = \frac{m(v-u)}{t} = \frac{w(0 - \sqrt{2 \text{ gh}})}{g \times 0.15} = 5.21 \text{ W}$

33. COLM
$$\Rightarrow$$
 $m_1u_1 + m_2u_2 = (m_1 + m_2)v$
 $5 \times 10^3 \times 1.2 + 0 = (5+1) \times 10^3 \times v \Rightarrow v = 1 \text{ m/s}$

A

34. $2r\sin\theta = \frac{r}{2}$; $\sin\theta = \frac{1}{4}$ r/2 r/

35. From COLM
$$Mv_{x_A} + 2Mv_{x_B} = 0 \Rightarrow v_{x_A} = -2v_{x_F}$$

So $r_A = -2v_x + vk$

36. Component of velocity of A along common normal is $v \cos 60^{\circ}$ and this velocity of A after collision with B is interchanged. Hence A moves along $v \sin 60^{\circ}$ which is normal to common normal.





37. $m \times 1 = mv_1 + (2mv_2 \times \cos 60^\circ)$

$$\Rightarrow \mathbf{v}_{2} = \frac{1}{2} & \mathbf{v}_{1} = \frac{1}{2}$$

$$(KE)_{initial} = \frac{1}{2} \times 1 \times 1^{2} = 0.5 \text{ J}$$

$$(KE)_{final} = \frac{1}{2} (1) \times \left(\frac{1}{2}\right)^{2} + \left\{\frac{1}{2} \times 1 + \times \left(\frac{1}{2}\right)^{2}\right\} \times 2$$

$$= 0.25 + 0.125 = 0.3755$$

$$\Delta KE = 0.5 - 0.375 = 0.125 \text{ J}$$

38. At the time of maximum compression, COLM : $mu = 2mv (for A \& B) \Rightarrow v = u/2$

COME:
$$\frac{1}{2}mu^2 = \frac{1}{2}2mv^2 + \frac{1}{2}kx^2 \implies x = v\sqrt{\frac{m}{2k}}$$

39. At the time of collision both particles have common velocity and hence the system has minimum kinetic energy.
 COME : mu + 0 = 3mv ⇒ v = u/3

$$KE_{initial} = \frac{1}{2} mu^{2} = 3J$$

$$KE_{collision} = \frac{1}{2} (3m)v^{2} = \frac{1}{2} (3m)\frac{u^{2}}{9} = 1J$$

$$PE_{collision} = (3-1) = 2J$$

Total energy remains constant and hence KE of system First decreases & then increases.

40. COLM : mu +0 = (m + M)v
$$\Rightarrow v = \left(\frac{m}{M+m}\right)u$$

KE after collision $= \frac{1}{2}(m + M) \times \left(\frac{m}{m + M}\right)^2 u^2$
 $= \frac{m^2 u^2}{2(m + M)}$
41. PE of solid sphere = mgR = mg $\frac{D}{2} = \rho g D^4 \left(\frac{\pi}{12}\right)$
PE of solid cube = mg $\frac{D}{2} = \rho g D^4 \left(\frac{1}{2}\right)$
PE of solid cone = mg $\frac{D}{4} = \rho g D^4 \left(\frac{\pi}{48}\right)$
PE of solid cylinder = mg $\frac{D}{2} = \rho g D^4 \left(\frac{\pi}{8}\right)$
42. $p_i = -mv$, $p_e = m(v + 2u)$ $\therefore \Delta p = 2m(v + u)$

$$\therefore \quad \text{Force} = \frac{\Delta p}{\Delta t} = \frac{2m(v+u)}{\Delta t}$$

KE_{initial} =
$$\frac{1}{2}$$
mu², KE_{final} = $\frac{1}{2}$ m(2 v + u)²
∴ ΔKE = $\frac{1}{2}$ m[4v² + u² + 4uv - u²]
= $\frac{1}{2}$ m[4v(v + u)] = 2mv(u + v)
Part # II : Assertion & Reason
1. A 2. A 3. A 4. A 5. A 6. C
7. C 8. B 9. B
EXERCISE - 3
Part # I : Matrix Match Type

1. Impulse = change in momentum

(A) For body M :
$$\mathbf{p} = \begin{vmatrix} \mathbf{r} & \mathbf{p} \\ p_f - p_1 \end{vmatrix} \implies p_f = \mathbf{p}$$

(B) For body 2M : $\mathbf{p} = \begin{vmatrix} \mathbf{r} & \mathbf{p} \\ p_f - p_1 \end{vmatrix} \implies p_f = 2\mathbf{p}$

(C)
$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow \frac{\frac{p}{M} - \frac{p}{M}}{\frac{2p}{M} - \frac{p}{2M}} = 0 \Rightarrow e = 0$$

By applying conservation of momentum Before collision After collision $\underset{p=\sqrt{2mK}}{\overset{K=P^2}{\longleftarrow}} \underset{p=\sqrt{2mK}}{\overset{V/3}{\longleftarrow}} \underset{q}{\overset{V/3}{\longleftarrow}} \underset{q}{\overset{2w/3}{\longleftarrow}}$ $m_1u_1 + 2m(0) = mv_1 + 2mv_2$(i) Also $\mathbf{u} = \mathbf{v}_2 - \mathbf{v}_1$(ii) $v_2 = \frac{2v}{3}$ and $v_1 = -\frac{v}{3}$; $p_2 = \frac{4mv}{3} = \frac{4p}{3}$, $p_1 = -\frac{mv}{3} = -\frac{p}{3}; K_2 = \frac{8K}{9}; K_1 = \frac{K}{9}$ 3. For 1kg $v_1 = (2t)$ \$ = 4 \$; $a_1 = 2$ \$ = 2 \$ For 2kg $v_2 = t^2$ = 4; $a_2 = 2t$ = 4(A) Acceleration of centre of mass = $\frac{m_1a_1 + m_2a_2}{m_1 + m_2}$ $a_{cm}^{r} = \frac{2}{3} \$ + \frac{8}{3} \$ \implies a_{cm}^{r} = \sqrt{\frac{4}{9} + \frac{64}{9}} \implies \frac{\sqrt{68}}{3} m/s^{2}$ $f = ma_{cm} = \sqrt{68} N$ (B) Velocity of centre of mass ${r \atop v_{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \left(\frac{4}{3} \$ + \frac{8}{3} \$\right) m/s$ $\Rightarrow |v_{cm}| = \sqrt{\frac{16}{9} + \frac{64}{9}} \Rightarrow \frac{\sqrt{80}}{3}$



(C)
$$r_{cm} = \frac{1(2t)\hat{i} + 2(t^2\hat{j})}{1+2} = \frac{2}{3}t\hat{i} + \frac{2}{3}t^2$$

Displacement =
$$\int_{0}^{2} \int_{0}^{r} v_{cm} dt = \left[\frac{3}{2}\left(\frac{t^{2}}{2}\right)^{2} \hat{i} + \frac{2}{3}\left(\frac{t^{3}}{3}\right)\hat{j}\right]_{0}^{2}$$
$$= \frac{4}{3}\hat{i} + \frac{16}{9}\hat{j}$$
$$\Rightarrow |Dispalcement| = \sqrt{\left(\frac{4}{3}\right)^{2} + \left(\frac{16}{9}\right)^{2}} = \frac{20}{9}units$$

- 4. (A) Net force on block m acts in downward direction
 - :. Acceleration of centre of mass is in downward direction.
 - (B) Net force acts in downward as well as in horizontal direction.



- \therefore a_{cm} moves both in horizontal & vertical direction.
- (C) As the mass of monkey & block is same both moves upward.
- .: Centre of mass moves upward
- (D) Centre of mass of the system does not moves
- 5. As no external force acts on the system velocity of centre of mass remain same

$$v_{cm} = \frac{2m(u)}{3m} = \frac{2u}{3}$$

In frame of centre of mass velocity of B is 2u/3 and It

oscillates from $\left(-\frac{2u}{3},\frac{2u}{3}\right)$

In frame of centre of mass velocity of A is u/3 and It

oscillates from $\left(-\frac{u}{3}, \frac{u}{3}\right)$. In ground frame velocity of B $\left[0, \frac{4u}{3}\right]$. In ground frame velocity of A $\left[\frac{u}{3}, u\right]$ and by conservation of energy $\Delta K.E. = \Delta U_s$

Part # II : Comprehension

Comprehension #1

By applying conservation of momentum

$$2(6) + 1(4) = 1v_2 + 2v_1; 16 = v_2 + 2v_1$$

By applying newton law of collision

$$1 = \frac{v_2 - v_1}{2} \implies v_2 - v_1 = 2 \dots (ii) \implies v_2 = \frac{20}{3}, v_1 = \frac{14}{3}$$

1. Impulse = change in momentum = $\frac{20}{3} - 4 = \frac{8}{3}$ N-s

2. To change the direction of a block impulse should be greater than 12 N-s

Comprehension #2

2.

3.

1. By applying conservation of momentum

$$mv_1 + Mv_2 = 0 \implies v_1 = -4v_2$$

By applying conservation of energy

$$\frac{1}{2} \operatorname{mv}_{1}^{2} + \frac{1}{2} \operatorname{Mv}_{2}^{2} = \operatorname{mgh} \implies \frac{\operatorname{v}_{1}^{2}}{2} + 2 \operatorname{v}_{2}^{2} = 20 \quad \dots (ii)$$

$$v_{2} = \sqrt{2} \operatorname{m} / \operatorname{s}; v_{1} = 4 \sqrt{2} \operatorname{m/s}$$

....(i)

- When 'm' leaves the wedge 'M' then wedge moves distance 'x' in left side
 - \therefore m(4-x) = Mx \Rightarrow x = 0.8 m
 - :. Co-ordinate where block will leave wedge x = 4 0.8 = 3.2

Time for m will strike the ground is = $\sqrt{\frac{2 \times 2}{10}}$

:.
$$x_f = 3.2 + 4\sqrt{2} \times \frac{2}{\sqrt{10}} = 6.8 \text{ m}$$

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$
(1) a cos 60 = 4a

$$a_M = \frac{5\sqrt{3}}{8} \text{ m/s}^2$$
N cos 30 = 4ma_M
N $\frac{\sqrt{3}}{2} = 4(1)\frac{5\sqrt{3}}{8}$

⇒ N = 5 Newton



CENTRE OF MASS & COLLISIONS

4.
$$N_g = N \sin 30 + 40 \implies N_g = 42.5$$



Comprehension #3

As horizontal velocity (i.e. velocity along the surface)

remains constant so required time = $\frac{30}{5} = 6$ s

Comprehension #4

1.
$$\mathbf{r}_{cm} = \frac{\mathbf{m}_{1}\mathbf{r}_{10} + \mathbf{m}_{2}\mathbf{r}_{20}}{\mathbf{m}_{1} + \mathbf{m}_{2}} = \frac{1(3\hat{i}) + 2(9\hat{j})}{1 + 2} = (\hat{i} + 6\hat{j})\mathbf{m}$$
$$\mathbf{r}_{cm} = \frac{\mathbf{m}_{1}\mathbf{v}_{1} + \mathbf{m}_{2}\mathbf{v}_{2}}{\mathbf{m}_{1} + \mathbf{m}_{2}} = \frac{1(3\hat{i}) + 2(6\hat{j})}{1 + 2}(\hat{i} + 4\hat{j})\mathbf{m} / \mathbf{s}$$
$$\mathbf{Now} \ \Delta \mathbf{r}_{cm}^{\mathbf{r}} = \mathbf{v}_{cm}t \implies \mathbf{r}_{cm} - \mathbf{r}_{cm_{0}} = \mathbf{v}_{cm}t$$
$$\implies (x - 1)\hat{i} + (y - 6)\hat{j} = (\hat{i} + 4\hat{j})t$$
$$\implies x = 1 + t \text{ and } y = 6 + 4t \implies y = 4x + 2$$

2.
$$\mathbf{a}_{cm}^{r} = \frac{\mathbf{m}_{1}\hat{\mathbf{a}}_{1} + \mathbf{m}_{2}\hat{\mathbf{a}}_{2}}{\mathbf{m}_{1} + \mathbf{m}_{2}} = \frac{1(-2\hat{\mathbf{i}}) + 2(-2\hat{\mathbf{j}})}{1 + 2}$$

$$= -\frac{2}{3}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\mathbf{m} / \mathbf{s}^{2}$$
By using $\mathbf{v}_{cm}^{r} = \mathbf{u}_{cm}^{r} + \mathbf{a}_{cm}^{r}\mathbf{t}$; we get t =3s

3. By using $s = ut + \frac{1}{2} at^2$ for individual particles For Ist particle :sx = (3) (1.5) + $\frac{1}{2}$ (2) (1.5)² = 2.25 m For IInd particle = s_y =(6) (3) + $\frac{1}{2}$ (-2)(3)² = 9m Therefore $x_{cm} = \frac{2.25}{3} + 1 = 1.75$ m and $y_{cm} = \frac{2(9)}{3} + 6 = 12m$

Comprehension #5

1.
$$f = v \frac{dm}{dt} = 2(20) = 40 \implies f_r = \mu mg$$

 $m = 40 = M_0 - 2(t) \implies t = 5 \text{ sec}$
so (0.1) (50-2t) = 40 \implies t = 5 sec

2.
$$v = v \bullet n \left(\frac{m}{m_0}\right) - gt = 20 \bullet n \left[\frac{4}{3}\right] - gt = 20 (0.28) - 5$$

= 5.6 - 5 = 0.6m/s

Comprehension #6

1. ByCOLM

$$0 = m_{A} v_{A} + m_{B} v_{B}; v_{B} = -\frac{m_{A}}{m_{B}} v_{A}$$

Both velocity are opposite in direction

- 2. If $m_A = m_B \implies v_B = -v_A \Rightarrow \text{Graph II}$; $m_A > m_B \implies v_B > v_A \Rightarrow \tan\theta_B > \tan\theta_A \Rightarrow \text{Graph IV}$ If $m_A < m_B \implies v_A > v_B \Rightarrow \text{Graph IV}$
- 3. v_{cm} is not zero in graph (I), (III) and (VI)

Comprehension #7

1. As
$$\operatorname{ma}_{cm} = f \implies \operatorname{a}_{cm} = \frac{f}{2m}$$
 so $\operatorname{s}_{cm} = \frac{1}{2}\operatorname{at}^2 = \left(\frac{f}{4m}\right) t^2$

$$\mathbf{x}_{cm} = \frac{\mathbf{m}_1 \mathbf{x}_1 + \mathbf{m}_2 \mathbf{x}_2}{2\mathbf{m}} \implies \mathbf{x}_1 + \mathbf{x}_2 = \frac{\mathbf{f}}{2\mathbf{m}} \mathbf{t}^2 \implies \mathbf{x}_1 - \mathbf{x}_2 = \mathbf{x}_0$$

Therefore $\mathbf{x}_1 = \frac{\mathbf{f}}{4\mathbf{m}} \mathbf{t}^2 + \frac{\mathbf{x}_0}{2}$

From above equations
$$x_2 = \frac{\pi}{4m} - \frac{x_0}{2}$$

Comprehension #8

 As no external force acts on the two blocks friction acts like an internal forces and the two blocks will move with common velocity. By applying conservation of momentum

$$(1 \text{ kg}) (15 \text{ m/s}) = 1 \text{ v} = 2 \text{ v} \implies \text{ v} = 5 \text{ m/s}$$

P₁ = 5 N-s ; P₂ = 10 N-s

 $2. \qquad \frac{dp}{dt} = f_{ext}$

For block of 1 kg friction $f_r = \mu mg = 0.4 \times 1 \times 10 = 4 N$

3. $v = u + at \Rightarrow 5 = 15 - (4) t \Rightarrow t = 2.5 sec$



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8.
$$x_{cm} = \frac{\left(\rho 2 r^2\right) \frac{r}{2} - \left(\rho \frac{\pi r^2}{2}\right) \left(\frac{4 r}{3 \pi}\right)}{\left(\rho 2 r^2\right) - \left(\rho \frac{\pi r^2}{2}\right)} = \frac{2 r}{3 (4 - \pi)^2}$$

9.
$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int_{0}^{a} x \rho y dx}{\int_{0}^{a} \rho y dx} = \frac{3}{4}a$$

10. COLM
$$\Rightarrow$$
 3mv = 30 $\sqrt{2}$ m \Rightarrow v = 10 $\sqrt{2}$ m/s



- 11. (i) CM does not shift
 - (ii) Plank moves towards right.

(iii)
$$\Delta x_{CM} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2 + M \Delta x}{m_1 + m_2 + M} = 0$$

$$\Rightarrow 0 = \frac{50(x+2) + 70(x-2) + 80x}{50 + 70 + 80}$$
$$\Rightarrow x = 0.2m \text{ (right)}$$

(iv) $\Delta x_{m_1} = x + 2 = 2.2 \text{ m} (\text{right})$

(v)
$$\Delta x_{m_2} = x - 2 = -1.8m$$
 (left)

12. For
$$0 \le t < 1$$
 from $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

13. From work energy theorem W_F + W_g = ΔKE ⇒ F_{avg}(h₂ - h₁) - mg(h₃-h₁)=0 ⇒ F_{avg} = $\frac{mg(h_3 - h_1)}{(h_2 - h_1)}$

14. Velocity of mass-1 when string is in normal length.

$$v_{A} = \sqrt{6 g l - 2 g l} = 2 \sqrt{g l}$$

Now impulsive tension acts on both bodies to come to common velocity $v_{common} = \frac{v_A}{2} = \sqrt{gl}$

Displacement of C.M. when string becomes taut.

$$\Delta y_{1\,cm}=\frac{m\times l+m\times 0}{m+m}=\frac{l}{2}$$

Displacement of CM when masses reach the max. height

$$\Delta y_{2m} = \frac{v^2}{2g} = \frac{1}{2}$$
 : $\Delta y_{cm} = \frac{1}{2} + \frac{1}{2} = 1$

15. Let v = velocity of wedge and u = velocity of particle relative to wedge



- \Rightarrow m (v + u cos θ) + 4 mv = 0
- \Rightarrow $u \cos \theta = -5v; \ \omega = \frac{u}{R} = \frac{5v}{R\cos\theta}$
- 16. In the presence of gravity, the CM shifts along vertically downward direction.

For point P : $y = r \sin \theta$

$$\mathbf{x} = (\mathbf{\Phi}/2 - \mathbf{r}) \cos\theta$$

$$\mathbf{1} = \left(\frac{\mathbf{x}}{\frac{1}{2} - \mathbf{r}}\right)^2 + \left(\frac{\mathbf{y}}{\mathbf{r}}\right)^2$$

$$\mathbf{y}_{\text{cm}} = \mathbf{p}_{\text{cm}} \mathbf{r}$$

7.
$$\left(\frac{1}{2}gt^{-1}\right) + \left(\frac{49t - \frac{1}{2}gt^{-1}}{2}\right) = 98$$

 $\Rightarrow t = 2s$ $u_1 = gt(s) = -19.6 sm/s$
 $u_2 = (u - gt)s = +29.4 sm/s$
 $v_f = \frac{u_1 + u_2}{2} = 4.9 m/s$
 $\frac{98m}{78.4 m}$
 $\frac{98m}{49 m/s}$

h = 4.9 × 2
$$-\frac{1}{2}$$
 × 9.8 × 2 × 2 = 78.4 m

For the combined mass $x = ut + \frac{1}{2} \times 9.8 t^2$, t = 4.53 \therefore Total time of height = 2 + 4.53 = 6.53 sec.

18.

$$\begin{array}{c}
\mathbf{m} \\ \mathbf{C} \\
\mathbf{V}_{0} \\
\mathbf{A} \\
\mathbf{A} \\
\mathbf{C} \\
\mathbf{C} \\
\mathbf{A} \\
\mathbf{C} \\$$

For IInd collision

$$\frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} \sqrt{2 \operatorname{gl}} = \sqrt{2 \operatorname{gl} (1 - \cos \theta)}$$

For nth collision

$$\left(\frac{2}{\sqrt{5}}\right)^n = \sqrt{1 - \cos\theta} \implies \left(\frac{4}{5}\right)^n = 1 - \cos\theta$$

$$\cos\theta = 1 - \left(\frac{4}{5}\right)^n$$
 Put n=0,1,2,3 and get answer

21.
$$\frac{u}{o}$$
 o $=$ $\frac{v}{o}$
 m_1 m_2 (m_1+m_2)

COLM \Rightarrow m₁u +0 = (m₁ + m₂)v(i) Energy equation

$$\left(\frac{1}{2}mu^2\right) \times \frac{2}{3} = \frac{1}{2}(m_1 + m_2)v^2$$
(ii)

$$\Rightarrow \frac{m_1}{m_2} = 2:1$$

22. In this elastic collision velocity of masses are exchanged. So $v_A = 0 \implies A$ does not rise



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m=100gm

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23.
$$x_{un} = \frac{4ML + M(L + 5R)}{5M} = \frac{4MX + M(X - 5R)}{5M}$$

 $\Rightarrow x = L + 2R$
 $(x_{1}) = (x_{1}) = (x_{2}) = \frac{4MX + M(X - 5R)}{5M}$
 $\Rightarrow x = L + 2R$
 $(x_{1}) = (x_{1}) = (x_{2}) = (x$

+91-9350679141

CENTRE OF MASS & COLLISIONS

3 v g

▶

34.
$$u_{s} = \sqrt{2g(1 - \cos 60^{\circ})} = \sqrt{2g \times \frac{1}{2}} = 3.13 \text{ m/s}$$

COLM $\Rightarrow 5u_{n} = 4 \times u_{n} \Rightarrow u_{n} = \frac{4}{5}\sqrt{g} = 2.53 \text{ m/s}$
Linergy equation
 $\frac{1}{2} \ln(\frac{4}{5}\sqrt{g})^{2} = \mu mg \times 0.8 \Rightarrow \mu = 0.4$
 $e = -\left(\frac{v_{y} - v_{1}}{u_{x} - u_{y}}\right) = 0.8$
35. $Av_{em} = 0 = \frac{mx_{e} + m_{1}(x_{e} - h \cot \alpha) + m_{2}(x_{e} - h)}{m + m_{1} + m_{2}}$
 $\Rightarrow x_{e} = \frac{h(m_{1} + m_{1} \cot \alpha)}{(m + m_{1} + m_{1} + m_{2})}$
36. (a) $mu = TAt = mv$...(i)
 $TAt = mv$...(ii)
 $rat = \frac{mu\sqrt{3}}{4}$...(ii)
 $rat = \frac{mu\sqrt{3}}{4}$...(ii)
37. $2mg = T = 2ma$...(i)
 $T = mg = ma$...(ii)
 $Tat = mv$...(ii)
 $Tat = \frac{mu\sqrt{3}}{4}$...(ii)
 $Tat = 2mk$...(ii)
 $Tat = \frac{g}{3}$...(ii)
 $Tat = \frac{g}{3}$...(ii)
 $Tat = \frac{g}{3}$...(ii)
 $Tat = \frac{g}{3}$...(ii)
 $Tat = 2mk$...(ii)
 $Tat = 7kk$..

3



44. Let v = velocity of the ball after collision along the normal

.....(i)

J = impulse on ball

$$= v - (-2 \cos 30^\circ) = v + \sqrt{3}$$

Impulse on wedge

$$J \sin 30^\circ = mv_1 = 2v_1$$

 $\Rightarrow v = 4v_1 - \sqrt{3}$

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) \implies \frac{1}{2} = \frac{\left(v + \frac{v_1}{2}\right)}{2\cos 30^{\circ}}$$
$$\implies v = \frac{\sqrt{3}}{2} - \frac{v_1}{2} \qquad \dots \dots (ii)$$

Solving we get $v_1 = \frac{1}{\sqrt{3}}$ m/s

For the ball velocity along incline remains constant. \therefore v' = 2 sin 30° = 1 m/s

:. Final velocity of ball =
$$\sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}$$
 m/s

45. For first collision with plate A, final velocity of ball

$$v_1 = ev_0 = e\sqrt{2 gh_0}$$
(i)

For second collision mv = 4mv'

$$\Rightarrow \mathbf{v}' = \frac{\mathbf{v}}{4} \Rightarrow \mathbf{e}\sqrt{\frac{2\,\mathbf{gh}_0}{4}} = \sqrt{2\,\mathbf{gh}_2} \Rightarrow \mathbf{e} = \frac{2}{3}$$

Height attained after first collision

:.
$$h_1 = e^2 h_0 = \frac{2}{3} \times \frac{2}{3} \times 9 = 4m$$



 \mathbf{m}_{2}

EXERCISE - 5 Part # I : AIEEE/JEE-MAIN

- 1. $v_{cm} = \frac{m(2v) + m(-v)}{m+m} = \frac{v}{2}$
- 2. Linear momentum is a vector quantity whereas kinetic energy is a scalar quantity.



For this system, position of centre of mass remains same

$$\left[Q \ F_{system} = 0 \right]$$

$$\frac{M(0) + 5M(12R)}{M + 5M} = \frac{M(d) + 5M(d + 3R)}{M + 5M} \Longrightarrow d = 7.5R$$

4. In order to shift centre of mass, the system must experience an external force, as there is no external force responsible for explosion, hence centre of mass does not shift.

2

after collision

5. Let maximum momentum be p then

$$\frac{p^2}{2M} = \frac{1}{2}kL^2 \implies p = L\sqrt{Mk}$$

$$\stackrel{\text{m}}{\longrightarrow} v \qquad \stackrel{\text{m}}{(2)} \qquad \stackrel{\stackrel{\text{v}}{\longrightarrow}}{(1)}$$

before collision

6.

From COLM

$$\mathrm{mv}_{2} = \sqrt{(\mathrm{mv})^{2} + \left(\mathrm{m}\frac{\mathrm{v}}{\sqrt{3}}\right)^{2}} \Rightarrow \mathrm{v}_{2} = \frac{2\mathrm{v}}{\sqrt{3}}$$

7. The object will have translation motion without rotation, when $\frac{1}{F}$ is applied at CM of the system.



$$m(21-x) = 2m(x-1) \implies x = \frac{4}{3}$$

8. On applying law of conservation of linear momentum

$${}^{1}P_{i} = {}^{1}P_{f} \implies 16 \times \overset{1}{0} = 4 \overset{r}{\nu_{4}} + 12 \times 4 \hat{i} \implies \overset{r}{\mathbf{v}_{4}} = 12 \left(-\hat{i}\right)$$

The 4 kg block will move in a direction opposite to 12 kg block with a speed of 12 m/s. The corresponding kinetic energy of 4 kg block

m,

$$=\frac{1}{2} \times 4 \times (12)^2 = 288 \text{ J}$$

0. Since mass
$$\infty$$
 area

Here $m_1 d = m_2 x$

9.

1

 \therefore mass of the smaller disc = M

 \therefore mass of the remaining portion = 4M – M = 3M

Now put the cut disc at its place again, centre of mass of the whole disc will be at centre O.

Centre of mass of the smaller disc is at its centre that is at B.

Suppose CM of the remaining portion is at A and AO is X. Let O as origin

$$3M(x)=RM \implies x=\frac{R}{3}$$

This suggests that centre of mass of remaining disc will shift from the centre of original disc by a distance of (1/3)R towards left.

$$\alpha = \frac{1}{3}$$

11. Energy loss
$$= \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} (u_1 - u_2)^2$$

$$=\frac{(0.5)(1)}{2[0.5+1]}(2-0)^2 = \frac{2}{3}J$$

12.
$$X_{cm} = \frac{\int x dm}{\int dm} = \frac{\int_{0}^{L} k \left(\frac{x}{L}\right)^{n} x dx}{\int_{0}^{L} k \left(\frac{x}{L}\right)^{n} dx} = \left(\frac{n+1}{n+2}\right) L$$

For n=0, $x_{cm} = \frac{L}{2}$ and for $n \to \theta$, $x_{cm} = L$
15. 1



16.
$$z_0 = \frac{3h}{4}$$

(From class theory)
17. $1 = \frac{Mx^2}{6}$
 $cge length : (x)$
 $2R = \sqrt{3} x \Rightarrow x = \frac{2R}{\sqrt{3}}$
Now, mass of cube :
 $m = \frac{M}{\left(\frac{4}{3}\pi R^3\right)} \left(\frac{2R^3}{3}\right)^3$
 $m = \frac{M}{\left(\frac{4}{3}\pi R^3\right)} \left(\frac{2R^3}{3}\right)^3$
 $1 = \frac{1}{3} \left(\frac{2M}{\sqrt{3}\pi}\right) \left(\frac{4R^2}{3}\right)^2$
 $1 = \frac{1}{3} \left(\frac{2M}{\sqrt{3}\pi}\right) \left(\frac{4R^2}{3}\right)^2 = \frac{4MR^2}{\sqrt{3}\pi}$
 $1 = \frac{1}{3} \left(\frac{2M}{\sqrt{3}\pi}\right) \left(\frac{4R^2}{3}\right) = \frac{4MR^2}{9\sqrt{3}\pi}$
 $1 = \frac{1}{3} \left(\frac{2M}{\sqrt{3}\pi}\right) \left(\frac{4R}{3}\right) = \frac{4MR^2}{9\sqrt{3}\pi}$
 $1 = \frac{1}{3} \left(\frac{4M}{\sqrt{3}\pi}\right) \left(\frac{4M}{\sqrt{3}\pi}\right) = \frac{4MR^2}{\sqrt{3}\pi}$
 $1 = \frac{1}{3} \left(\frac{4M}{\sqrt{3}\pi}\right) \left(\frac{4M}{\sqrt{3}\pi}\right) = \frac$



CENTRE OF MASS & COLLISIONS

$$\Rightarrow \mathbf{v}_{f} = \frac{\mathbf{v}_{0} \mathbf{cos} \alpha}{2} (\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

 \Rightarrow Angle with horizontal immediately after the collision

 $=\frac{\pi}{4}$

Multiple Choice Questions

1. As $p_1 + p_2 = 0$ so $p_1 + p_2 = 0$ For (A) $\hat{p}_1' + \hat{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$ For (B) $\dot{p}_1' + \dot{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j}$ For (C) $\vec{p}_1' + \vec{p}_2' = (c_1 + c_2)\hat{k}$ For (D) $\dot{p}_1' + \dot{p}_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j}$ But $a_1, b_1, c_1, a_2, b_2, c_2 \neq 0$ Therefore (A) & (D) is not possible to get $r_{p_1} + r_{p_2} = 0$

Comprehension type questions



$$\frac{h_1}{\sqrt{3}} = \tan 60^\circ \Rightarrow h_1 = 3m$$
$$\frac{h_2 - h_1}{\sqrt{2}} = \tan 30^\circ \Rightarrow h_2 - h_1 = 3 \Rightarrow h_2 = 6m$$

$$3\sqrt{3}$$

Velocity of block just before collision at B

$$= \sqrt{2 \text{ gh}} = \sqrt{2 \times 10 \times 3} = \sqrt{60} \text{ ms}^{-1}$$

For totally inelastic collision velocity of block along normal to BC becomes zero and since there is no impulse along BC so momentum (velocity) along BC remains unchanged

Speed of block just after collision

$$v_{\rm B} = \sqrt{60} \cos 30^{\circ} = \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{45} \,{\rm ms}^{-1}$$

2.
$$v_c^2 = v_B^2 + 2g(h_2 - h_1)$$

 $\Rightarrow v_c = \sqrt{45^2 + 2 \times 10 \times 3} = \sqrt{105} \text{ ms}^{-1}$
3. $\sqrt{15}$ $\sqrt{45}$ $\sqrt{5}$ \sqrt

$$= -10 [0.05 + 0.25 \times 0.99] = -2.99 \text{ m/s}^2$$

$$v^2 - u^2 = 2as \implies v = \sqrt{100 + 2 - (-2.99) \times 6} = 8 \text{ m/s}$$



After collision

=

Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision.Body of mass M moving from B to C and coming to rest.

$$\begin{split} &u = v_2; \quad v = 0, \qquad a = - \ 2.99 \ \text{m/s}^2 \\ &\text{and } s = 0.5 \quad v^2 - u^2 = 2 as \\ & \Rightarrow \quad (0)^2 - v_2^2 = 2(-2.99) \times 0.5 \implies v_2^2 = 1.73 \ \text{m/s} \end{split}$$

Body of mass m moving from B to A after collision

$$u = v_1; v = +1 \text{ m/s}$$

(K.E. + P.E.)_{initial} = (K.E. + P.E.)_{final} + W_{frication}

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$$\frac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \frac{1}{2}(1)^2 + 0.25 \times 10 \times 6$$
$$v_1 = -5m/s$$

$$\begin{array}{c} & & \\ & & \\ & & \\ A \end{array} \end{array} \begin{array}{c} & & \\$$

: Coefficient of restitution

$$e = \left| \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} \right| = \left| \frac{-5 - 1.73}{8 - 0} \right| = 0.84$$



2. Consider the vertical motion of the cannon ball

→ S = ut +
$$\frac{1}{2}$$
 at²

∴ -120 = 50t₀ - 5t₀²

⇒ 5t₀² - 50t₀ - 120 = 0 ⇒ t₀² - 10 t₀ - 24 = 0

∴ t₀ = - $\frac{(-10) \pm \left[\sqrt{100} - 4(1)(-24)\right]}{2}$ = 12 or -2

The horizontal velocity of the cannon ball remains the same

:.
$$v_x = 100 \cos 30^\circ + 5\sqrt{3} = 55\sqrt{3}$$
 m/s
+ P u = $5\sqrt{3}$ m/s
- $\sqrt{30^\circ}v = 5\sqrt{3}$ m/s
120m
A B u_r=0

 \therefore Apply conservation of linear momentum to the cannon ball-trolley system in horizontal direction. If m is the mass of cannon ball and M is the mass of the trolley then

$$mv_x + M \times 0 = (m + M) V_x$$
 : $V_x = \frac{mv_x}{m + M}$

where v_x is the velocity of the (cannon ball- trolley)

system
$$V_x = \frac{1 \times 55\sqrt{3}}{1+9} = 5.5\sqrt{3} \text{ m/s}$$

The second ball was projected after 12 second.

Horizontal distance covered by the car

 $P = 12 \times 5\sqrt{3} = 60\sqrt{3}m$

Since the second ball also struck the trolley

:. In time 12 seconds the trolley covers a distance of $60\sqrt{3}$

For trolley in 12 sec

From

$$s = \left(\frac{u+v}{2}\right) 60\sqrt{3} = \left(\frac{5.5\sqrt{3}+v}{2}\right) (12) \implies v = 7.8 \text{ m/s}$$

To find the final velocity of the carriage after the second impact we again apply conservation of linear momentum in the horizontal direction

$$m v_x + (M + m)7.8 = (M + 2m) v_f$$

$$v_{\rm f} = 1 \times 55 \sqrt{3} + (9+1)7.8 = (9+2)v_{\rm f} \implies v_{\rm f} = 15.75 \text{ m/s}$$

$$\begin{split} \stackrel{r}{v_2} &= (-v_2 \sin \omega t \hat{i} + v_2 \cos \omega t \hat{j}) \text{ and } \stackrel{r}{v_1} = v_1 \hat{j} \\ \stackrel{r}{v_{21}} &= \stackrel{r}{v_2} - \stackrel{r}{v_1} = -v_2 \sin \omega t \hat{i} + (v_2 \cos \omega t - v_1) \hat{j} \\ \stackrel{r}{p_{21}} &= \stackrel{r}{mv_{21}} = -mv_2 \sin \omega t \hat{i} + m(v_2 \cos \omega t - v_1) \hat{j} \\ \text{where } \omega = \frac{v_2}{R} \end{split}$$

The string snaps and the spring force comes into play. The spring force being an internal force for the two mass-spring system will not be able to change the velocity of centre of mass. This means the location of centre of mass at time t will be v_0 t

$$\begin{aligned} \mathbf{x}_{cm} &= \frac{\mathbf{m}_{1}\mathbf{x}_{1} + \mathbf{m}_{2}\mathbf{x}_{2}}{\mathbf{m}_{1} + \mathbf{m}_{2}} = \mathbf{v}_{0}\mathbf{t} \\ \Rightarrow & \mathbf{m}_{1}[\mathbf{v}_{0}\mathbf{t} - \mathbf{A}(1 - \cos \omega \mathbf{t})] + \mathbf{m}_{2}\mathbf{x}_{2} = \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{1} + \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{2} \\ \Rightarrow & \mathbf{m}_{2}\mathbf{x}_{2} = \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{1} + \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{2} - \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{1} + \mathbf{m}_{1}\mathbf{A} (1 - \cos \omega \mathbf{t}) \\ \Rightarrow & \mathbf{m}_{2}\mathbf{x}_{2} = \mathbf{v}_{0}\mathbf{t}\mathbf{m}_{2} + \mathbf{m}_{1}\mathbf{A}(1 - \cos \omega \mathbf{t}) \\ \Rightarrow & \mathbf{m}_{1} \end{aligned}$$

$$\Rightarrow x_2 = v_0 t + \frac{m_1}{m_2} A(1 - \cos \omega t)$$

(b) Given that $x_1 = v_0 t - A(1 - \cos \omega t)$

4.

$$\frac{dx_1}{dt} = v_0 - A\omega \sin \omega t \implies \therefore \frac{d^2 x_1}{dt^2} = -A\omega^2 \cos \omega t \quad \dots (i)$$

This is the acceleration of mass m_1 . When the spring comes to its natural length instantaneously then

$$\frac{d^2 x_1}{dt^2} = 0 \text{ and } x_2 - x_1 = \Phi_0$$

$$\left[v_0 t + \frac{m_1}{m_2} A(1 - \cos \omega t) \right] - \left[v_0 t - A(1 - \cos \omega t) \right] = \Phi_0$$

$$\left(\frac{m_1}{m_2} + 1 \right) A(1 - \cos \omega t) = \Phi_0$$
Also when $\frac{d^2 x_1}{dt^2} = 0$; $\cos \omega t = 0$ from (1)
$$\therefore \quad \Phi_0 = \left(\frac{m_1}{m_2} + 1 \right) A$$

5. Since the plate is held horizontal therefore net torque acting on the plate is zero.





3.

2MV

$$F = n \frac{dp}{dt} \times (Area) = n \times (2mv) \times a \times \frac{b}{2} \qquad \dots (ii)$$

From (i) and (ii) Mg × $\frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$
 $\Rightarrow 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$
 $\Rightarrow v = 10 \text{ m/s}$

6. For collision between A & B

$$v_{A} = \frac{(m-2m)}{(m+2m)}(9) = -3ms^{-1} \Rightarrow v_{B} = \frac{2(m)}{(m+2m)}(9) = +6ms^{-1}$$

For collision between B and C

$$\mathbf{v}_{\rm C} = \left(\frac{2\,\mathrm{m}}{2\,\mathrm{m}+\mathrm{m}}\right)(6) = 4\,\mathrm{m}\mathrm{s}^{-1}$$

MOCK TEST

- 1. Since $\Sigma \stackrel{P}{\mathsf{F}_{\mathsf{ext}}} = 0^{\mathsf{P}}$
 - :. Momentum of system will remain conserved, equal to zero.
- 2. The line of impact for duration of collision is parallel to x-axis.

The situation of striker and coin just before the collision is given as



Because masses of coin and striker are same, their components of velocities along line of impact shall exchange. Hence the striker comes to rest and the x-y component of velocities of coin are u and 3 m/s as shown in figure.



For coin to enter hole, its velocity must be along PO

$$\tan \theta = \frac{6}{4} = \frac{3}{u}$$
 or $u = 2 \text{ m/s}$

3. If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle.

Velocity of centre of mass is :

$$V_{CM} = R_{CM} \cdot \omega = \frac{2R}{\pi} \cdot \omega = \frac{2R}{\pi} \cdot \left(\frac{V}{R}\right) (\Theta \ \omega = \frac{V}{R})$$
$$\Rightarrow V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$$

As the linear momentum of any system = MV_{CM}

- ... The linear momentum of the train
- 4. As shown in figure the component of momentum of one shell along initial direction and perpendicular to initial

direction are
$$P_{1X} = 10\sqrt{3}$$
 Ns and $P_{1y} = 10$ Ns.

For momentum of the system to be zero in y-direction P_{2y} must be 10 Ns. 2nd part of shell may or may not have momentum in x-direction

5. Let h be the height of water surface, finally

$$a^{2}h = a \cdot \frac{a}{2} \cdot \frac{a}{2} ; h = \frac{a}{4}$$

$$\therefore C.M. \text{ gets lowered by}$$

$$a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$$

$$\therefore$$
 Work done by gravity = mg $\frac{5a}{8}$

6. Neglecting gravity,

$$v = u \bullet n \left(\frac{m_0}{m_t} \right);$$

u = ejection velocity w.r.t. balloon .

 $m_0 = initial mass$

$$m_t = mass at any time t.$$

$$= 2 \bullet n \left(\frac{m_0}{m_0 / 2} \right) = 2 \bullet n2.$$

7. Let the tube displaced by x towards left, then

5

$$mx = m(R - x) \Longrightarrow x = \frac{R}{2}$$

Taking the origin at the centre of the plank. 8.



 $m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 = 0 \qquad (\Rightarrow \Delta x_{CM} = 0)$ (Assuming the centres of the two men are exactly at the axis shown.)

60(0) + 40(60) + 40(-x) = 0, x is the displacement of the block.

- \Rightarrow x = 60 cm
- i.e. A & B meet at the right end of the plank. $\begin{array}{ll} m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 = 0 & (\Rightarrow \Delta x_{CM} = 0) \\ 60(0) + 40(60) + 40 (-x) = 0, & \Rightarrow x = 60 \text{ cm} \end{array}$
- 9. For 1st collision

 $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}$

Since e = 1:

B) C Α

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 \implies \mathbf{v}_2 = \mathbf{v} - \mathbf{v}_1$$

By momentum conservation :

$$m_{\rm p} v = -m_{\rm p} v_1 + m_{\rm c} v_2$$

or
$$\mathbf{m}_{\mathrm{B}} \mathbf{v} = -\mathbf{m}_{\mathrm{B}} \mathbf{v}_{1} + 4 \mathbf{m}_{\mathrm{B}} \mathbf{v}_{2}$$
 ($\Theta \mathbf{m}_{\mathrm{C}} = 4 \mathbf{m}_{\mathrm{B}}$)
 $\Rightarrow \mathbf{v}_{2} = \frac{\mathbf{v}_{1} + \mathbf{v}}{4}$ (2)

From (1) to (2):
$$v_1 = \frac{3}{5}v$$
 and $v_2 = \frac{2}{5}$

For second collision :

$$V_3 \leftarrow \rightarrow V_1$$

A) (B)

$$e = 1 \implies v_1 = v_1' + v_3 \implies v_3 = v_1 - v_1' \quad \dots (3)$$

By momentum conservation :
$$-m_B v_1 = m_B v_1' - m_A v_3$$

or
$$-m_B v_1 = m_B v_1' - 4 m_B v_3 \quad (\Theta \ m_A = 4 m_B)$$

$$\Rightarrow v_3 = \frac{v_1 + v_1}{4} \qquad \dots$$

From (3) and (4):

$$v'_1 = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$$

Clearly % $\frac{9}{25} v < \frac{2}{5} v$

Therefore 'B' can not collide with 'C' for the second time.

Hence ; total number of collisions is 2.

10. Force on table due to collision of balls :

$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2 \text{ N}$$

Net force on one leg = $\frac{1}{2} (2 + 0.2 \times 10) = 1 \text{ N}$



- Since, both have positive final velocities, hence, **(i)** both moved in the same direction after collision.
- (ii) at t = 2 sec, both had equal velocities.
- (iii) by conservation of linear momentum, we can say that mass of R was greater than mass of S.
- 12. By conservation of linear momentum along the string,

mu =
$$(m + m + 3m)v$$
 or $v = \frac{u}{5}$
and impulse on the block A = $3m(v - 0) = \frac{3mu}{5}$

13. Let the three mutually perpendicular directions be along x, y and z-axis respectively,

$$\begin{aligned} \beta_1 &= mv_0 \hat{i} \\ \beta_2 &= mv_0 \hat{j} \quad \text{where,} \quad \frac{1}{2}mv_0^2 = E_0 \\ \beta_3 &= mv_0 \hat{k} \quad \text{and} \quad \beta_4 = m\hat{v} \\ \text{By linear momentum conservation,} \\ 0 &= \beta_1 + \beta_2 + \beta_3 + \beta_4 \quad \text{or} \quad \hat{v} = -v_0(\hat{i} + \hat{j} + \hat{k}) \\ \text{or} \quad v &= v_0 \quad \sqrt{1^2 + 1^2 + 1^2} = v_0 \sqrt{3} \\ \text{tal energy} &= 3\left(\frac{1}{2}mv_0^2\right) + \frac{1}{2}mv^2 \quad = 3E_0 + 3E_0 = 6E_0 \end{aligned}$$



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to

11.

.... (1)

(4)

- 14. Let v_1 , v_2 and v_3 be velocities of blocks 1, 2 and 3 after 17. During 1st collision perpendicular component of v_1 , v_{\perp} suffering collision each.
 - $\mathbf{mv} = \mathbf{mv}_1 + \mathbf{Mv}_2$ and $\mathbf{v}_1 \mathbf{v}_2 = -\mathbf{v}$ solving we get $v_1 = \frac{m-M}{m+M} < 0$ $\Box m < M$ $\rightarrow V_1 \qquad M \rightarrow V_2$ m $\therefore |v_1| = \frac{M-m}{m+M} v$ (1) and $v_2 = \frac{2 m v}{m + M}$ Similarly, $v_3 = \frac{2m}{m+M} \times v_2 = \frac{4Mmv}{(m+M)^2}$ (2)

:.
$$\frac{M-m}{M+m}v = \frac{4Mmv}{(M+m)^2}$$
 or $M^2 - m^2 = 4Mm$.

$$\frac{M}{m} = 2 + \sqrt{5} \text{ Ans.}$$

15. After collision by momentum conservation : Along y-axis

$$0 = 0 + mv_2 \sin\theta - mv_3 \sin\theta$$

$$\Rightarrow$$
 v₂ = v₃

Along x-axis

$$mv = 0 + mv_2 \cos\theta + mv_3 \cos\theta$$

 $mv = 2 m v_2 \cos\theta$
 $v_2 = \frac{v}{2} \frac{1}{\cos\theta}$

So
$$v_2 = v_3 > \frac{v}{2} \Rightarrow \cos\theta < 1$$

16. $\int dp = p_f - p_i = \int F dt = Area under the curve.$ = 0

Net Area =
$$16 - 2 - 1 = 13$$
 N-s

$$\frac{p_f}{M} = V_f = \frac{13}{2} = 6.5 \ \hat{j} \ m/s$$

[As momentum is positive, car is moving along positive x axis.]

becomes e times, while IInd component V_{II} remains unchanged and similarly for second collision. The end result is that both v_{\parallel} and v_{\perp} becomes e times their initial value and hence v'' = -ev (the (-) sign indicates the reversal of direction).



18. It can be shown that

 $K_0 = K_{cm} + \frac{1}{2}MV_{cm}^2$ where M is the total mass of the system and V_{cm} is velocity of centre of mass with respect to ground.

Due to internal changes K_{cm} can change but V_{cm} will remain same. Hence only K_{CM} portion of kinetic energy can be transformed to some other form of energy. Thus D is the wrong statement.

19. For first collision
$$v = 10$$
 m/s. $t_1 = \frac{\pi(5)}{10} = \pi/2$ sec.

velocity of sep = e. velocity of opp.

$$v_2 - v_1 = \frac{1}{2} (10) \implies v_2 - v_1 = 5 \text{ m/s}$$

for second collision

:.
$$t_2 = \frac{2\pi(5)}{5} = 2\pi$$

$$\therefore \text{ total time } t = t + t_2 = \pi/2 + 2\pi \implies t = \frac{5}{2}\pi$$

20. Just before the particle transfers to inclined surface, we resolve its velocity along and normal to the plane.



For the trajectory of the particle to sharply change from the horizontal line to the inclined line, the impact of the particle with inclined plane should reduce the $usin\theta$ component of velocity to zero. Hence the particle starts to move up the incline with speed $u \cos\theta$.

Hence as θ increases, the height to which the particle rises shall decrease.



- 21. Friction force between wedge and block is internal i.e. will not change motion of COM. Friction force on the wedge by ground is external and causes COM to move towards right. Gravitational force (mg) on block brings it downward hence COM comes down.
- **22.** $U_Q > U_S > U_P > U_R$
- $\Rightarrow M_Q > M_S > M_P > M_R \text{ and CM of cone is on smallest height}$
- **23.** Since, $F_{ext} = 0$

Hence, momentum will remain conserved equal to mv.

≋ wal

$$mv = (m + M) v' \text{ or } v' = \frac{mv}{m + M}$$

and final kinetic energy is $\frac{1}{2}$ (m + M) v'²

$$=\frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{m^2v^2}{2(m+M)}$$

24. (B), (C)

in an elastic collision

 $v_{sep} = v_{app}$ or v' - u = v + uor v' = v + 2uchange in momentum of ball is $|p_f - p_i|$

= |m(-v') - mv| = m(v' + v) = 2m(u + v)

average force =
$$\frac{\Delta p}{\Delta t} = \frac{2m(u+v)}{\Delta t}$$

change in KE = $K_f - K_i = \frac{1}{2} mv'^2 - \frac{1}{2} mv^2 = 2mu(u+v)$

25. (A, B, C, D)



Impulse $(J) = \Delta P$ = mv sin ϕ - m(-u sin θ) = m(v sin ϕ + u sin θ) = m(V_{sep} + V_{app})

$$= m (eV_{app} + V_{app}) \qquad [e = \frac{V_{sep}}{V_{app}}]$$
$$= m V_{app} (e+1)$$
$$J = m(u \sin \theta) (1+e)$$

In horizontal direction, momentum is conserved :

$$u \cos\theta = v \cos\phi \text{ or } v = \frac{u\cos\theta}{\cos\phi}$$

or
$$e = \frac{V_{sep}}{V_{app}} = \frac{v \sin \phi}{u \sin \theta} = \frac{tan \phi}{tan \theta}$$

or $tan \phi = e tan \theta$
in vertical direction, $e = \frac{v \sin \phi}{u \sin \theta}$
or $v \sin \phi = eu \sin \theta$,
 $v = \sqrt{(eu \sin \theta)^2 + (u \cos \theta)^2} = u \sqrt{e^2 \sin^2 \theta + \cos^2 \theta}$
 $v = u \sqrt{1 - (1 - e^2) \sin^2 \theta}$
final kinetic energy $= \frac{1}{2} mv^2$
initial kinetic energy $= \frac{1}{2} mu^2$
ratio $= \frac{v^2}{u^2} = e^2 \sin^2 \theta + \cos^2 \theta$

26,. Sphere A moving with velocity v has a component v/ 2 along the line joining the centres of the spheres at

the time of collision and another component $v\sqrt{3}/2$ perpendicular to the previous direction. After collision the component along the line will interchange i.e. B will move with v/2 velocity i.e. 4 m/s along the

line joining the centres and A moves with $v\sqrt{3}/2$ velocity at perpendicular direction to B.

- 27. For a system of two isolated sphere having non zero initial kinetic energy, the complete kinetic energy can be converted to other forms of energy if the momentum of system is zero. This is due to the fact that for an isolated system, the net momentum remains conserved. If an isolated system has nonzero momentum, for the momentum to remain constant complete kinetic energy of the system cannot become zero. Hence statement 1 is true while statement 2 is false.
- **28.** Statement-2 contradicts Newton's third law and hence is false.
- **29.** For sum of three non null vectors to be zero, they must be coplanar. Hence Statement-2 is a correct explanation for Statement-1.
- **30.** During collision KE of system is not constant, hence statement-1 is false.
- **31.** (a) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2} a_{COM} t^2 = \frac{Ft^2}{4m} Ans.$$



CENTRE OF MASS & COLLISIONS

32. (A)

 Suppose the displacement of the right block is x₁ and that of the left is x₂. Then,

$$x = \frac{mx_1 + mx_2}{2m} \text{ or, } \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2} \text{ or}$$
$$x_1 + x_2 = \frac{Ft^2}{2m} \qquad \dots (i)$$

Further, the extension of the spring is $x_1 - x_2$. Therefore, $x_1 - x_2 = x_0$...(ii)

From Eqs. (i) and (ii),
$$x_1 = \frac{1}{2} \left(\frac{Ft^2}{2m} + x_0 \right)$$

and
$$x_2 = \frac{1}{2} \left(\frac{Ft^2}{2m} - x_0 \right)$$
 Ans

34. From conservation of momentum $mv = mv' \cos 30^\circ + mv' \cos 30^\circ$

$$\therefore \quad \mathbf{v'} = \frac{\mathbf{v}}{2\cos 30^\circ} = \frac{\mathbf{v}}{\sqrt{3}}$$

35. Loss in kinetic energy

$$= \frac{1}{2}mv^2 - 2 \times \frac{1}{2}m\left(\frac{v}{\sqrt{3}}\right)^2 = \frac{1}{6}mv^2$$

36. Initially B was at rest, therefore line of impact is along final velocity of B.

$$\therefore e = \frac{v' - v' \cos 60^{\circ}}{v \cos 30} = \frac{\frac{1}{2} \frac{v}{\sqrt{3}}}{v \times \frac{\sqrt{3}}{2}} = \frac{1}{3}$$

37. Taking 'O' as the origin ;



$$X_{CM(i)} = \frac{m(0) + 2m(\lambda)}{3m} = \frac{2\lambda}{3}$$

and
$$X_{CM(f)} = \frac{m(x) + 2m(x + \frac{3\lambda}{5})}{m + 2m}$$

As $\Sigma F_x = 0 \implies X_{CM}(i) = X_{CM}(f)$
 $\implies X = \frac{4\lambda}{15}$ Ans.

38. By momentum conservation : (Inhorizontal direction) $mv_1 = 2 mv_2$ & by energy conservation:

$$2 \operatorname{mg} \bullet = \frac{1}{2} \operatorname{mv}_{1}^{2} + \frac{1}{2} \operatorname{2mv}_{2}^{2}$$

$$\Rightarrow 2 \operatorname{g} \bullet = \frac{1}{2} \operatorname{v}_{1}^{2} + \left(\frac{\operatorname{v}_{1}}{2}\right)^{2}$$

$$\Rightarrow 2 \operatorname{g} 1 = \frac{3}{4} \operatorname{v}_{1}^{2} \Rightarrow \operatorname{v}_{1} = \sqrt{\frac{8 \operatorname{g} \lambda}{3}}$$

39. From momentum conservation $0 = m V_r + 2m V_B$ Energy conservation

$$2mg \bullet = \frac{1}{2}mV_{r}^{2} + \frac{1}{2}mV_{B}^{2}$$

Solving above equations, we get $\sqrt{\frac{\log t}{3}}$

$$V_{r} = \sqrt{\frac{8g\lambda}{3}}, V_{B} = \frac{1}{2}\sqrt{\frac{8g\lambda}{3}}$$

Relative velocity of block with
respect to ring is $\frac{3}{2}\sqrt{\frac{8g\lambda}{3}}$.

Applying newton's law equations on the block

$$-2mg = \frac{\left(2m\left(\frac{3}{2}\sqrt{\frac{8g\lambda}{3}}\right)^2}{\lambda} \qquad T = 14 \text{ mg.}$$

- **40.** (A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block $B = \frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.
- (B) Since initial velocity of B is zero, it shall be zero for many other instants of time.
- (C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.



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PHYSICS FOR JEE MAIN & ADVANCED

- (D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.
- **41.** $A \rightarrow P, R, T \Sigma F = 0$

So, linear momentum conservation and centre of mass will not move.

$$B \rightarrow Q, S$$

So, linear momentum will not be conserved and centre of mass will accelerate $W_{ext} = \Delta E$.

 $C \rightarrow P, S, T; D \rightarrow P, S, T$

Line of impact

- (A) Normal force from ground lies along line of impact. Hence (A) is not answer.
- (B) Since no external force act perpendicular to the line of impact. (B) is an answer.
- (C)Horizontal direction is same as direction perpendicular to the line of impact. (C) is an answer.
- (D) Normal impulse from ground lies in vertical direction.

(S) is not an answer.

- (A) The component of normal force from ground lies along the line of impact. Hence not an answer.
- (B) No external force perpendicular to the line of impact for A.
- (C) For the system A + B there is no external force along horizontal direction. Hence an answer.
- (D) For B the normal force from ground is balanced by the impulsive force by A. Initial and final momentum is zero.

Hence an answer.

Line of impact

- (A) The component of tension force of thread lies along the line of impact. Hence not an answer.
- (B) No external force perpendicular to the line of impact for A.
- (C) For system A + B there is no external force along horizontal direction. Hence an answer.
- (D) For B the tension force from thread is balanced by the impulsive force by A. Initial and final momentum is zero.

Hence an answer.



- (A) & (C) are the same direction and there is no external force for the system A + B. Hence answer.
- (B) & (D) are the same direction and there is no net force for the system A + B. Hence answer.

- (A) The component of normal force from ground lies along the line of impact. Hence not an answer.
- (B) Is answer because the normal force from the ground is balanced for A. Hence an answer.
- (C) For the system A + B there is no external force along horizontal direction. Hence an answer.
- (D) For A the normal force from ground is balanced by the impulsive force by B. Initial and final momentum is zero.

Hence an answer.

13. The initial extension in spring is
$$x_0 = \frac{mg}{k}$$

Just after collision of B with A the speed of combined mass is $\frac{V}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (see fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2} 2m\left(\frac{v}{2}\right)^2 + \frac{1}{2} k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$



Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}} \sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2} ;$$

$$v = \sqrt{\frac{2k}{m}} \sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}} = 6 \text{ m/sec}$$

44. For the duration of collision the pendulum does not exert any force on the sphere in the horizontal direction. Hence the horizontal momentum of bullet + sphere is conserved for the duration of collision. Let v' be the velocity of bullet and sphere just after the collision.
from conservation of momentum

$$(m+m) v' = mv$$
 or $v' = \frac{v}{2} = 1m/sec$

45. Force F on plate = force exerted by dust particles

= force on dust particles by the plate

= rate of change of momentum of dust particles = mass of dust particles striking the plate per unit time × change in velocity of dust particles.= A(v+u) $\rho \times (v+u)$ = $A\rho (v+u)^2$

46. Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have, (1 kg)(2 m/s) = (1 kg)V + (1 kg)V or, V = 1 m/s.

Initial kinetic energy = $\frac{1}{2}$ (1 kg) (2 m/s)² = 2 J.

Final kinetic energy

$$= \frac{1}{2} (1 \text{ kg}) (1 \text{ m/s})^2 + \frac{1}{2} (1 \text{ kg}) (1 \text{ m/s})^2 = 1 \text{ J}$$

The kinetic energy lost is stored as the elastic energy in the spring.

Hence,
$$\frac{1}{2} (50 \text{ N/m}) x^2 = 2J - 1J = 1 \text{ J or, } x = 0.2 \text{ m.}$$

47.



after the block B gets off the wedge A and block B at just after the block B gets off the wedge A. Applying conservation of momentum in horizontal direction, we get. mu = mv(1)

Applying conservation of energy between initial and final state as shown in figure (1), we get



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$$mgh = \frac{1}{2} mu^{2} + \frac{1}{2} mv^{2} \qquad(2)$$

solving (1) and (2) we get
 $v = \sqrt{gh} \qquad(3)$

At the instant block B reaches maximum height h' on the wedge C (figure 2), the speed of block B and wedge C are v'. Applying conservation of momentum in horizontal direction, we get



Applying conservation of energy between initial and final state

$$\frac{1}{2} mv^2 = \frac{1}{2} (m+m) v'^2 + mgh' \qquad \dots \dots (5)$$

$$h' = \frac{h}{4} = 25 \text{ cm Ans}$$

48. Since $e = \frac{1}{5}$

:. Final normal component of velocity = $\frac{v \cos 37^{\circ}}{5}$

As the angle of rebound is equal to the angle before impact.

Therefore, both normal & tangential components of velocities must change by the same factor.

Tangential velocity after impact becomes $\frac{v \sin 37^0}{5}$.

Let the time of impact be Δt .

$$N = \frac{m\left(v\cos 37^{0} + \frac{v\cos 37^{0}}{5}\right)}{\Delta t} = \frac{6mv\cos 37^{0}}{5\Lambda t}$$

where N is the normal force imparted on the ball by the wall.

