

SOLVED EXAMPLES

Ex. 1 Find

(a) The coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

(b) The coefficient of x^{-7} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$

Also, find the relation between a and b , so that these coefficients are equal.

Sol. (a) In the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$, the general term is :

$$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r = {}^{11}C_r \cdot \frac{a^{11-r}}{b^r} \cdot x^{22-3r}$$

putting $22 - 3r = 7$

$$\therefore 3r = 15 \Rightarrow r = 5$$

$$\therefore T_6 = {}^{11}C_5 \frac{a^6}{b^5} \cdot x^7$$

Hence the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is ${}^{11}C_5 a^6 b^{-5}$.

Note that binomial coefficient of sixth term is ${}^{11}C_5$.

(b) In the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$, general term is :

$$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(\frac{-1}{bx^2}\right)^r = (-1)^r {}^{11}C_r \frac{a^{11-r}}{b^r} \cdot x^{11-3r}$$

putting $11 - 3r = -7$

$$\therefore 3r = 18 \Rightarrow r = 6$$

$$\therefore T_7 = (-1)^6 \cdot {}^{11}C_6 \frac{a^5}{b^6} \cdot x^{-7}$$

Hence the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6 a^5 b^{-6}$.

Also given :

$$\text{Coefficient of } x^7 \text{ in } \left(ax^2 + \frac{1}{bx}\right)^{11} = \text{coefficient of } x^{-7} \text{ in } \left(ax - \frac{1}{bx^2}\right)^{11}$$

$$\Rightarrow {}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6}$$

$$\Rightarrow ab = 1 \quad (\rightarrow {}^{11}C_5 = {}^{11}C_6)$$

which is the required relation between a and b .

Ex. 2 Find the numerically greatest term in the expansion of $(3 - 5x)^{15}$ when $x = \frac{1}{5}$.

Sol. Let r^{th} and $(r + 1)^{\text{th}}$ be two consecutive terms in the expansion of $(3 - 5x)^{15}$

$$T_{r+1} \geq T_r$$

$${}^{15}C_r 3^{15-r} (-5x)^r \geq {}^{15}C_{r-1} 3^{15-(r-1)} (-5x)^{r-1}$$

$$\frac{(15)!}{(15-r)!r!} |-5x| \geq \frac{3 \cdot (15)!}{(16-r)!(r-1)!}$$

$$5 \cdot \frac{1}{5} (16-r) \geq 3r$$

$$16-r \geq 3r$$

$$4r \leq 16$$

$$r \leq 4$$

Ex. 3 Given T_3 in the expansion of $(1 - 3x)^6$ has maximum numerical value. Find the range of 'x'.

Sol. Using $\frac{n+1}{1+\left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1+\left|\frac{a}{b}\right|}$

$$\frac{6+1}{1+\left|\frac{1}{-3x}\right|} - 1 \leq 2 \leq \frac{7}{1+\left|\frac{1}{-3x}\right|}$$

Let $|x| = t$

$$\frac{21t}{3t+1} - 1 \leq 2 \leq \frac{21t}{3t+1}$$

$$\begin{cases} \frac{21t}{3t+1} \leq 3 \\ \frac{21t}{3t+1} \geq 2 \end{cases} \Rightarrow \begin{cases} \frac{4t-1}{3t+1} \leq 0 \Rightarrow t \in \left[-\frac{1}{3}, \frac{1}{4}\right] \\ \frac{15t-2}{3t+1} \geq 0 \Rightarrow t \in \left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{2}{15}, \infty\right) \end{cases}$$

Common solution $t \in \left[\frac{2}{15}, \frac{1}{4}\right] \Rightarrow x \in \left[-\frac{1}{4}, -\frac{2}{15}\right] \cup \left[\frac{2}{15}, \frac{1}{4}\right]$

Ex. 4 Find the last two digits of the number $(17)^{10}$.

Sol. $(17)^{10} = (289)^5 = (290 - 1)^5$
 $= {}^5C_0 (290)^5 - {}^5C_1 (290)^4 + \dots + {}^5C_4 (290)^1 - {}^5C_5 (290)^0$
 $= {}^5C_0 (290)^5 - {}^5C_1 \cdot (290)^4 + \dots + {}^5C_3 (290)^2 + 5 \times 290 - 1$
 $= \text{A multiple of } 1000 + 1449$

Hence, last two digits are 49

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Ex. 5 Find the number of rational terms in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$.

Sol. The general term in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$ is

$$T_{r+1} = {}^{1000}C_r \left(9^{1/4}\right)^{1000-r} \left(8^{1/6}\right)^r = {}^{1000}C_r 3^{\frac{1000-r}{2}} 2^{\frac{r}{2}}$$

The above term will be rational if exponents of 3 and 2 are integers

It means $\frac{1000-r}{2}$ and $\frac{r}{2}$ must be integers

The possible set of values of r is {0, 2, 4,, 1000}

Hence, number of rational terms is 501

Ex. 6 Show that the integer just above $(\sqrt{3} + 1)^{2n}$ is divisible by 2^{n+1} for all $n \in \mathbb{N}$.

Sol. Let $(\sqrt{3} + 1)^{2n} = (4 + 2\sqrt{3})^n = 2^n (2 + \sqrt{3})^n = I + f$ (i)

where I and f are its integral & fractional parts respectively
 $0 < f < 1$.

Now $0 < \sqrt{3} - 1 < 1$

$0 < (\sqrt{3} - 1)^{2n} < 1$

Let $(\sqrt{3} - 1)^{2n} = (4 - 2\sqrt{3})^n = 2^n (2 - \sqrt{3})^n = f'$ (ii)

$0 < f' < 1$

adding (i) and (ii)

$$I + f + f' = (\sqrt{3} + 1)^{2n} + (\sqrt{3} - 1)^{2n}$$

$$= 2^n [(2 + \sqrt{3})^n + (2 - \sqrt{3})^n] = 2 \cdot 2^n [{}^nC_0 2^n + {}^nC_2 2^{n-2} (\sqrt{3})^2 + \dots]$$

$$I + f + f' = 2^{n+1} k \quad (\text{where } k \text{ is a positive integer})$$

$$0 < f + f' < 2 \Rightarrow f + f' = 1$$

$$I + 1 = 2^{n+1} k.$$

$I + 1$ is the integer just above $(\sqrt{3} + 1)^{2n}$ and which is divisible by 2^{n+1} .

Ex. 7 If $(1+x)^n = \sum_{r=0}^n {}^nC_r x^r$, then prove that $C_1^2 + 2.C_2^2 + 3.C_3^2 + \dots + n.C_n^2 = \frac{(2n-1)!}{((n-1)!)^2}$

Sol. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots + C_n x^n$ (i)

Differentiating both the sides, w.r.t. x, we get

$$n(1+x)^{n-1} = C_1 + 2C_2 x + 3C_3 x^2 + \dots + n.C_n x^{n-1} \quad \dots(ii)$$

also, we have

$$(x+1)^n = C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + \dots + C_n \quad \dots(iii)$$

Multiplying (ii) & (iii), we get

$$(C_1 + 2C_2 x + 3C_3 x^2 + \dots + n.C_n x^{n-1})(C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + \dots + C_n) = n(1+x)^{2n-1}$$

Equating the coefficients of x^{n-1} , we get

$$C_1^2 + 2C_2^2 + 3C_3^2 + \dots + n.C_n^2 = n \cdot 2^{n-1} C_{n-1} = \frac{(2n-1)!}{((n-1)!)^2}$$

Ex. 8: If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + c_n x^n$, then show that

(i) $C_0 + 3C_1 + 3^2C_2 + \dots + 3^n C_n = 4^n$.

(ii) $C_0 + 2C_1 + 3 \cdot C_2 + \dots + (n+1) C_n = 2^{n-1} (n+2)$.

(iii) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$.

Sol. (i) $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_n x^n$

put $x = 3$

$$C_0 + 3 \cdot C_1 + 3^2 \cdot C_2 + \dots + 3^n \cdot C_n = 4^n$$

(ii) **I Method : By Summation**

$$\text{L.H.S.} = {}^nC_0 + 2 \cdot {}^nC_1 + 3 \cdot {}^nC_2 + \dots + (n+1) \cdot {}^nC_n$$

$$= \sum_{r=0}^n (r+1) \cdot {}^nC_r = \sum_{r=0}^n r \cdot {}^nC_r + \sum_{r=0}^n {}^nC_r$$

$$= n \sum_{r=0}^n {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r = n \cdot 2^{n-1} + 2^n = 2^{n-1} (n+2). \quad \text{RHS}$$

II Method : By Differentiation

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_n x^n$$

Multiplying both sides by x ,

$$x(1+x)^n = C_0x + C_1x^2 + C_2x^3 + \dots + C_n x^{n+1}$$

Differentiating both sides

$$(1+x)^n + x n (1+x)^{n-1} = C_0 + 2 \cdot C_1x + 3 \cdot C_2x^2 + \dots + (n+1)C_n x^n$$

putting $x = 1$, we get

$$C_0 + 2 \cdot C_1 + 3 \cdot C_2 + \dots + (n+1) C_n = 2^n + n \cdot 2^{n-1}$$

$$C_0 + 2 \cdot C_1 + 3 \cdot C_2 + \dots + (n+1) C_n = 2^{n-1} (n+2) \quad \text{Proved}$$

(iii) **I Method : By Summation**

$$\text{L.H.S.} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + (-1)^n \frac{C_n}{n+1}$$

$$= \sum_{r=0}^n (-1)^r \cdot \frac{{}^nC_r}{r+1} = \frac{1}{n+1} \sum_{r=0}^n (-1)^r \cdot {}^{n+1}C_{r+1} \quad \left\{ \frac{n+1}{r+1} \cdot {}^nC_r = {}^{n+1}C_{r+1} \right\}$$

$$= \frac{1}{n+1} [{}^{n+1}C_1 - {}^{n+1}C_2 + {}^{n+1}C_3 - \dots + (-1)^n \cdot {}^{n+1}C_{n+1}]$$

$$= \frac{1}{n+1} [-{}^{n+1}C_0 + {}^{n+1}C_1 - {}^{n+1}C_2 + \dots + (-1)^n \cdot {}^{n+1}C_{n+1} + {}^{n+1}C_0]$$

$$= \frac{1}{n+1} = \text{R.H.S.}, \quad \text{since } \{ -{}^{n+1}C_0 + {}^{n+1}C_1 - {}^{n+1}C_2 + \dots + (-1)^n \cdot {}^{n+1}C_{n+1} = 0 \}$$

II Method : By Integration

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n.$$

Integrating both sides, within the limits -1 to 0 .

$$\left[\frac{(1+x)^{n+1}}{n+1} \right]_{-1}^0 = \left[C_0x + C_1 \frac{x^2}{2} + C_2 \frac{x^3}{3} + \dots + C_n \frac{x^{n+1}}{n+1} \right]_{-1}^0$$

$$\frac{1}{n+1} - 0 = 0 - \left[-C_0 + \frac{C_1}{2} - \frac{C_2}{3} + \dots + (-1)^{n+1} \frac{C_n}{n+1} \right]$$

$$C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \dots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1} \quad \text{Proved}$$

Ex. 9 Prove that $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n \cdot {}^{2n}C_n$

Sol. $(1-x)^{2n} = {}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 - \dots + (-1)^n {}^{2n}C_{2n}x^{2n}$...**(i)**

and $(x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + \dots + {}^{2n}C_{2n}$...**(ii)**

Multiplying **(i)** and **(ii)**, we get

$$(x^2-1)^{2n} = ({}^{2n}C_0 - {}^{2n}C_1x + \dots + (-1)^n {}^{2n}C_{2n}x^{2n}) \times ({}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + \dots + {}^{2n}C_{2n})$$
 ...**(iii)**

Now, coefficient of x^{2n} in R.H.S.

$$= ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2$$

→ General term in L.H.S., $T_{r+1} = {}^{2n}C_r(x^2)^{2n-r}(-1)^r$

Putting $2(2n-r) = 2n$

∴ $r = n$

∴ $T_{n+1} = {}^{2n}C_n x^{2n} (-1)^n$

Hence coefficient of x^{2n} in L.H.S. = $(-1)^n {}^{2n}C_n$

But **(iii)** is an identity, therefore coefficient of x^{2n} in R.H.S. = coefficient of x^{2n} in L.H.S.

⇒ $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n \cdot {}^{2n}C_n$

Ex. 10 In the expansion of $\left(1+x+\frac{7}{x}\right)^{11}$, find the term independent of x .

Sol. $\left(1+x+\frac{7}{x}\right)^{11} = \sum_{r_1+r_2+r_3=11} \frac{(11)!}{r_1!r_2!r_3!} (1)^{r_1} (x)^{r_2} \left(\frac{7}{x}\right)^{r_3}$

The exponent 11 is to be divided among the base variables 1 , x and $\frac{7}{x}$ in such a way so that we get x^0 .

Therefore, possible set of values of (r_1, r_2, r_3) are $(11, 0, 0)$, $(9, 1, 1)$, $(7, 2, 2)$, $(5, 3, 3)$, $(3, 4, 4)$, $(1, 5, 5)$

Hence the required term is

$$\begin{aligned} & \frac{(11)!}{(11)!} (7^0) + \frac{(11)!}{9!1!1!} 7^1 + \frac{(11)!}{7!2!2!} 7^2 + \frac{(11)!}{5!3!3!} 7^3 + \frac{(11)!}{3!4!4!} 7^4 + \frac{(11)!}{1!5!5!} 7^5 \\ &= 1 + \frac{(11)!}{9!2!} \cdot \frac{2!}{1!1!} 7^1 + \frac{(11)!}{7!4!} \cdot \frac{4!}{2!2!} 7^2 + \frac{(11)!}{5!6!} \cdot \frac{6!}{3!3!} 7^3 + \frac{(11)!}{3!8!} \cdot \frac{8!}{4!4!} 7^4 + \frac{(11)!}{1!10!} \cdot \frac{(10)!}{5!5!} 7^5 \\ &= 1 + {}^{11}C_2 \cdot {}^2C_1 \cdot 7^1 + {}^{11}C_4 \cdot {}^4C_2 \cdot 7^2 + {}^{11}C_6 \cdot {}^6C_3 \cdot 7^3 + {}^{11}C_8 \cdot {}^8C_4 \cdot 7^4 + {}^{11}C_{10} \cdot {}^{10}C_5 \cdot 7^5 \\ &= 1 + \sum_{r=1}^5 {}^{11}C_{2r} \cdot {}^{2r}C_r \cdot 7^r \end{aligned}$$

Ex. 11 If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$ then show that the sum of the products of the C_i 's taken two at a time represented by : $\sum_{0 \leq i < j \leq n} C_i C_j$ is equal to $2^{2n-1} - \frac{2n!}{2 \cdot n! \cdot n!}$

Sol. Since $(C_0 + C_1 + C_2 + \dots + C_{n-1} + C_n)^2$
 $= C_0^2 + C_1^2 + C_2^2 + \dots + C_{n-1}^2 + C_n^2 + 2(C_0 C_1 + C_0 C_2 + C_0 C_3 + \dots + C_0 C_n + C_1 C_2 + C_1 C_3 + \dots + C_1 C_n + C_2 C_3 + C_2 C_4 + \dots + C_2 C_n + \dots + C_{n-1} C_n)$
 $(2^n)^2 = 2^n C_n + 2 \sum_{0 \leq i < j \leq n} C_i C_j$

Hence $\sum_{0 \leq i < j \leq n} C_i C_j = 2^{2n-1} - \frac{2n!}{2 \cdot n! \cdot n!}$

Ex. 12 If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then prove that (a) $a_r = a_{2n-r}$ (b) $\sum_{r=0}^{n-1} a_r = \frac{1}{2}(3^n - a_n)$

Sol. (a) We have

$$(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r \quad \dots(A)$$

Replace x by $\frac{1}{x}$

$$\therefore \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^n = \sum_{r=0}^{2n} a_r \left(\frac{1}{x}\right)^r$$

$$\Rightarrow (x^2 + x + 1)^n = \sum_{r=0}^{2n} a_r x^{2n-r}$$

$$\Rightarrow \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{2n} a_r x^{2n-r} \quad \{ \text{Using (A)} \}$$

Equating the coefficient of x^{2n-r} on both sides, we get

$$a_{2n-r} = a_r \text{ for } 0 \leq r \leq 2n.$$

Hence $a_r = a_{2n-r}$.

(b) Putting $x=1$ in given series, then

$$a_0 + a_1 + a_2 + \dots + a_{2n} = (1+1+1)^n$$

$$a_0 + a_1 + a_2 + \dots + a_{2n} = 3^n$$

....(1)

But $a_r = a_{2n-r}$ for $0 \leq r \leq 2n$

$\therefore$ series (1) reduces to

$$2(a_0 + a_1 + a_2 + \dots + a_{n-1}) + a_n = 3^n.$$

$$\therefore a_0 + a_1 + a_2 + \dots + a_{n-1} = \frac{1}{2}(3^n - a_n)$$

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Ex. 13 Find the value of $e^{-1/5}$ correct to four places of decimal.

Sol. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \text{to } \infty$ (i)

putting $x = \left(-\frac{1}{5}\right)$ in (1), we get

$$e^{-1/5} = 1 - \frac{1}{5} + \frac{1}{2!} \left(-\frac{1}{5}\right)^2 + \frac{1}{3!} \left(-\frac{1}{5}\right)^3 + \frac{1}{4!} \left(-\frac{1}{5}\right)^4 + \dots \text{to } \infty$$

$$\Rightarrow e^{-1/5} = 1 - \frac{2}{10} + \frac{2^2}{2!} \cdot \frac{1}{10^2} - \frac{2^3}{3!} \cdot \frac{1}{10^3} + \frac{2^4}{4!} \cdot \frac{1}{10^4} + \dots$$

$$\Rightarrow e^{-1/5} = 1 - 0.200000 + 0.0200000 - 0.001333 + 0.000066$$

$$\Rightarrow e^{-1/5} = 0.8187 \text{ (correct to 4 decimal places)}$$

Ex. 14 If $(6\sqrt{6} + 14)^{2n+1} = [N] + F$ and $F = N - [N]$; where $[.]$ denotes greatest integer function, then find value of NF .

Sol. Since $(6\sqrt{6} + 14)^{2n+1} = [N] + F$

Let us assume that $f = (6\sqrt{6} - 14)^{2n+1}$; where $0 \leq f < 1$.

Now, $[N] + F - f = (6\sqrt{6} + 14)^{2n+1} - (6\sqrt{6} - 14)^{2n+1}$

$$= 2 \left[{}^{2n+1}C_1 (6\sqrt{6})^{2n} (14) + {}^{2n+1}C_3 (6\sqrt{6})^{2n-2} (14)^3 + \dots \right]$$

$$\Rightarrow [N] + F - f = \text{even integer.}$$

Now $0 < F < 1$ and $0 < f < 1$

so $-1 < F - f < 1$ and $F - f$ is an integer so it can only be zero

Thus $NF = (6\sqrt{6} + 14)^{2n+1} (6\sqrt{6} - 14)^{2n+1} = 20^{2n+1}$.

Ex. 15 If $y > 0$, then prove that

$$\log_e y = 2 \left[\left(\frac{y-1}{y+1} \right) + \frac{1}{3} \left(\frac{y-1}{y+1} \right)^3 + \frac{1}{5} \left(\frac{y-1}{y+1} \right)^5 + \dots \right] \text{ and calculate } \log_e 2 \text{ to three places of decimal.}$$

Sol. We know $\log_e \left(\frac{1+x}{1-x} \right) = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \text{to } \infty \right)$

Putting $x = \frac{y-1}{y+1}$, we get $\log_e \left(\frac{1 + \frac{y-1}{y+1}}{1 - \frac{y-1}{y+1}} \right) = 2 \left[\frac{y-1}{y+1} + \frac{1}{3} \left(\frac{y-1}{y+1} \right)^3 + \frac{1}{5} \left(\frac{y-1}{y+1} \right)^5 + \dots \right]$

$$\log_e y = 2 \left[\frac{y-1}{y+1} + \frac{1}{3} \left(\frac{y-1}{y+1} \right)^3 + \frac{1}{5} \left(\frac{y-1}{y+1} \right)^5 + \dots \right]$$

putting $y = 2$, we get $\log_e 2 = 2 \left[\frac{1}{3} + \frac{1}{3} \left(\frac{1}{3} \right)^3 + \frac{1}{5} \left(\frac{1}{3} \right)^5 + \dots \right] = \frac{2}{3} \left[1 + \left(\frac{1}{3} \right)^3 + \frac{1}{5} \left(\frac{1}{3} \right)^4 + \dots \right] = 0.693$



Exercise # 1

[Single Correct Choice Type Questions]

1. The sum of the co-efficients in the expansion of $(1 - 2x + 5x^2)^n$ is 'a' and the sum of the co-efficients in the expansion of $(1 + x)^{2n}$ is b. Then -
 (A) $a = b$ (B) $a = b^2$ (C) $a^2 = b$ (D) $ab = 1$
2. If the coefficients of x^7 & x^8 in the expansion of $\left[2 + \frac{x}{3}\right]^n$ are equal, then the value of n is -
 (A) 15 (B) 45 (C) 55 (D) 56
3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$ is equal to:
 (A) ${}^{47}C_5$ (B) ${}^{52}C_5$ (C) ${}^{52}C_4$ (D) ${}^{49}C_4$
4. The last two digits of the number 3^{400} are -
 (A) 81 (B) 43 (C) 29 (D) 01
5. The sum of the binomial coefficients of $\left[2x + \frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is -
 (A) 1120 (B) 2110 (C) 1210 (D) none
6. If $|x| < 1$, then the co-efficient of x^n in the expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is
 (A) n (B) $n - 1$ (C) $n + 2$ (D) $n + 1$
7. Number of rational terms in the expansion of $(\sqrt{2} + \sqrt[4]{3})^{100}$ is -
 (A) 25 (B) 26 (C) 27 (D) 28
8. Sum of the infinite series $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots$ to ∞
 (A) $\frac{e}{3}$ (B) e (C) $\frac{e}{2}$ (D) none of these
9. If 'a' be the sum of the odd terms & 'b' be the sum of the even terms in the expansion of $(1+x)^n$, then $(1-x^2)^n$ is equal to -
 (A) $a^2 - b^2$ (B) $a^2 + b^2$ (C) $b^2 - a^2$ (D) none
10. The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is
 (A) 196 (B) 197 (C) 198 (D) 199
11. If $a_n = \sum_{r=0}^n \frac{1}{{}^nC_r}$, then $\sum_{r=0}^n \frac{r}{{}^nC_r}$ equals -
 (A) $(n-1)a_n$ (B) na_n (C) $na_n/2$ (D) none of these
12. $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} =$
 (A) $\frac{n}{2}$ (B) $\frac{n+1}{2}$ (C) $(n+1)\frac{n}{2}$ (D) $\frac{n(n-1)}{2(n+1)}$

13. Let the co-efficients of x^n in $(1+x)^{2n}$ & $(1+x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{Q}\right)^5 =$
 (A) 9 (B) 27 (C) 81 (D) none of these
14. $\binom{47}{4} + \sum_{j=1}^5 \binom{52-j}{3} = \binom{x}{y}$, then $\frac{x}{y} =$
 (A) 11 (B) 12 (C) 13 (D) 14
15. Find the sum of the series $\frac{1}{2} - \frac{1}{2 \times 2^2} + \frac{1}{3 \times 2^3} - \frac{1}{4 \times 2^4} + \dots$
 (A) $\log_e \left(\frac{3}{2}\right)$ (B) $\log_e \left(\frac{2}{3}\right)$ (C) $\log_e \left(\frac{4}{7}\right)$ (D) none of these
16. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_{10}}{11}$ is equal to (here $C_r = {}^{10}C_r$)
 (A) $\frac{2^{11}}{11}$ (B) $\frac{2^{11}-1}{11}$ (C) $\frac{3^{11}}{11}$ (D) $\frac{3^{11}-1}{11}$
17. If $(1+x)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{10}x^{10}$, then value of $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is
 (A) 2^{10} (B) 2 (C) 2^{20} (D) None of these
18. The expression $\frac{1}{\sqrt{4x+1}} \left[\left[\frac{1+\sqrt{4x+1}}{2} \right]^7 - \left[\frac{1-\sqrt{4x+1}}{2} \right]^7 \right]$ is a polynomial in x of degree -
 (A) 7 (B) 5 (C) 4 (D) 3
19. Find numerically greatest term in the expansion of $(2+3x)^9$, when $x = 3/2$.
 (A) ${}^9C_6 \cdot 2^9 \cdot (3/2)^{12}$ (B) ${}^9C_3 \cdot 2^9 \cdot (3/2)^6$ (C) ${}^9C_5 \cdot 2^9 \cdot (3/2)^{10}$ (D) ${}^9C_4 \cdot 2^9 \cdot (3/2)^8$
20. The value of $\sum_{r=0}^{10} \binom{10}{r} \binom{15}{14-r}$ is equal to -
 (A) ${}^{25}C_{12}$ (B) ${}^{25}C_{15}$ (C) ${}^{25}C_{10}$ (D) ${}^{25}C_{11}$
21. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r \right) \left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k} \right)$ is :
 (A) 2^{10} (B) 2^{20} (C) 1 (D) 2^5
22. If $k \in \mathbb{R}$ and the middle term of $\left(\frac{k}{2} + 2 \right)^8$ is 1120, then value of k is:
 (A) 3 (B) 2 (C) -3 (D) -4
23. Let $R = (5\sqrt{5} + 11)^{31} = I + f$, where I is an integer and f is the fractional part of R , then $R \cdot f$ is equal to -
 (A) 2^{31} (B) 3^{31} (C) 2^{62} (D) 1

24. Sum of the infinite series $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots$ to ∞
- (A) $\frac{e}{3}$ (B) e (C) $\frac{e}{2}$ (D) none of these
25. The term independent of x in the product $(4+x+7x^2)\left(x-\frac{3}{x}\right)^{11}$ is -
- (A) $7 \cdot {}^{11}C_6$ (B) $3^6 \cdot {}^{11}C_6$ (C) $3^5 \cdot {}^{11}C_5$ (D) $-12 \cdot 2^{11}$
26. If the second term of the expansion $\left[a^{1/13} + \frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is:
- (A) 4 (B) 3 (C) 12 (D) 6
27. If $n \in \mathbb{N}$ & n is even, then $\frac{1}{1 \cdot (n-1)!} + \frac{1}{3! \cdot (n-3)!} + \frac{1}{5! \cdot (n-5)!} + \dots + \frac{1}{(n-1)! \cdot 1!} =$
- (A) 2^n (B) $\frac{2^{n-1}}{n!}$ (C) $2^n n!$ (D) none of these
28. If the 6th term in the expansion of $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x\right]^8$ is 5600, then $x =$
- (A) 10 (B) 8 (C) 11 (D) 9
29. The greatest terms of the expansion $(2x+5y)^{13}$ when $x=10, y=2$ is -
- (A) ${}^{13}C_5 \cdot 20^8 \cdot 10^5$ (B) ${}^{13}C_6 \cdot 20^7 \cdot 10^4$ (C) ${}^{13}C_4 \cdot 20^9 \cdot 10^4$ (D) none of these
30. The value of, $\frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 81 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64}$ is :
- (A) 1 (B) 2 (C) 3 (D) none

Exercise # 2

Part # I

[Multiple Correct Choice Type Questions]

- Let $(1+x^2)^2(1+x)^n = A_0 + A_1x + A_2x^2 + \dots$. If A_0, A_1, A_2 are in A.P. then the value of n is -
 (A) 2 (B) 3 (C) 5 (D) 7
- If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio of 1 : 7 : 42, then n is divisible by -
 (A) 9 (B) 5 (C) 3 (D) 11
- The sum of the co-efficients in the expansion of $(1-2x+5x^2)^n$ is a and the sum of the co-efficients in the expansion of $(1+x)^{2n}$ is b . then:
 (A) $a = b$ (B) $(x-a)^2 + (x-b)^2 = 0$, has real roots
 (C) $\sin^2 a + \cos^2 b = 1$ (D) $ab = 1$
- If $(1+2x+3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$, then :
 (A) $a_1 = 20$ (B) $a_2 = 210$ (C) $a_4 = 8085$ (D) $a_{20} = 2^2 \cdot 3^7 \cdot 7$
- Let $n \in \mathbb{N}$. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ and $a_{n-3}, a_{n-2}, a_{n-1}$ are in AP, then -
 (A) a_1, a_2, a_3 are in AP (B) a_1, a_2, a_3 are in HP (C) $n = 7$ (D) $n = 14$
- In the expansion of $(x+y+z)^{25}$
 (A) every term is of the form ${}^{25}C_r \cdot {}^rC_k \cdot x^{25-r} \cdot y^{r-k} \cdot z^k$
 (B) the coefficient of $x^8y^9z^9$ is 0
 (C) the number of terms is 325
 (D) none of these
- In the expansion of $\left(x^{2/3} - \frac{1}{\sqrt{x}}\right)^{30}$, a term containing the power x^{13} -
 (A) does not exist (B) exists and the co-efficient is divisible by 29
 (C) exists and the co-efficient is divisible by 63 (D) exists and the co-efficient is divisible by 65
- The coefficient of x^4 in $\left(\frac{1+x}{1-x}\right)^2, |x| < 1$, is
 (A) 4 (B) -4 (C) $10 + {}^4C_2$ (D) 16
- The value r for which $\binom{30}{r}\binom{15}{r} + \binom{30}{r-1}\binom{15}{1} + \dots + \binom{30}{0}\binom{15}{r}$ is maximum is/are -
 (A) 21 (B) 22 (C) 23 (D) 24
- In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$
 (A) the number of irrational terms is 19 (B) middle term is irrational
 (C) the number of rational terms is 2 (D) 9th term is rational

11. The number $101^{100} - 1$ is divisible by -
 (A) 100 (B) 1000 (C) 10000 (D) 100000
12. If the expansion of $(3x + 2)^{-1/2}$ is valid in ascending powers of x , then x lies in the interval.
 (A) $(0, 2/3)$ (B) $(-3/2, 3/2)$ (C) $(-2/3, 2/3)$ (D) $(-\infty, -3/2) \cup (3/2, \infty)$
13. $7^9 + 9^7$ is divisible by :
 (A) 16 (B) 24 (C) 64 (D) 72
14. In the expansion of $\left(x^3 + 3 \cdot 2^{-\log_{\sqrt{2}} \sqrt{x^3}}\right)^{11}$ -
 (A) there appears a term with the power x^2 (B) there does not appear a term with the power x^2
 (C) there appears a term with the power x^{-3} (D) the ratio of the co-efficient of x^3 to that of x^{-3} is $\frac{1}{3}$
15. If $(9 + \sqrt{80})^n = I + f$, where I, n are integers and $0 < f < 1$, then :
 (A) I is an odd integer (B) I is an even integer
 (C) $(I + f)(1 - f) = 1$ (D) $1 - f = (9 - \sqrt{80})^n$
16. If the co-efficients of r th, $(r + 1)$ th, and $(r + 2)$ th terms in the expansion of $(1 + x)^{14}$ are in A.P. then r is/are
 (A) 5 (B) 12 (C) 10 (D) 9
17. If recursion polynomials $P_k(x)$ are defined as $P_1(x) = (x - 2)^2$, $P_2(x) = ((x - 2)^2 - 2)^2$
 $P_3(x) = (((x - 2)^2 - 2)^2 - 2)^2$ (In general $P_k(x) = (P_{k-1}(x) - 2)^2$, then the constant term in $P_k(x)$ is
 (A) 4 (B) 2 (C) 16 (D) a perfect square
18. If $(1 + x + x^2 + x^3)^{100} = a_0 + a_1x + a_2x^2 + \dots + a_{300}x^{300}$, then -
 (A) $a_0 + a_1 + a_2 + a_3 + \dots + a_{300}$ is divisible by 1024
 (B) $a_0 + a_2 + a_4 + \dots + a_{300} = a_1 + a_3 + \dots + a_{299}$
 (C) coefficients equidistant from beginning and end are equal
 (D) $a_1 = 100$
19. If the 6th term in the expansion of $\left(\frac{3}{2} + \frac{x}{3}\right)^n$ when $x = 3$ is numerically greatest then the possible integral value(s) of n can be -
 (A) 11 (B) 12 (C) 13 (D) 14
20. The sum of the coefficient in the expansion of $(1 + ax - 2x^2)^n$ is
 (A) positive, when $a < 1$ and $n = 2k, k \in \mathbb{N}$ (B) negative, when $a < 1$ and $n = 2k + 1, k \in \mathbb{N}$
 (C) positive, when $a > 1$ and $n \in \mathbb{N}$ (D) zero, when $a = 1$
21. The co-efficient of the middle term in the expansion of $(1 + x)^{2n}$ is -
 (A) $\frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n - 1)}{n!} 2^n$ (B) ${}^{2n}C_n$
 (C) $\frac{(n+1)(n+2)(n+3) \dots (2n-1)(2n)}{1 \cdot 2 \cdot 3 \dots (n-1) \cdot n}$ (D) $\frac{2 \cdot 6 \cdot 10 \cdot 14 \dots (4n-6)(4n-2)}{1 \cdot 2 \cdot 3 \cdot 4 \dots (n-1) \cdot n}$

These questions contains, Statement I (assertion) and Statement II (reason).

- (a) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
 (b) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
 (c) Statement-I is true, Statement-II is false.
 (d) Statement-I is false, Statement-II is true.

- Statement-I :** The greatest value of ${}^{40}C_0 \cdot {}^{60}C_r + {}^{40}C_1 \cdot {}^{60}C_{r-1} + \dots + {}^{40}C_{40} \cdot {}^{60}C_{r-40}$ is ${}^{100}C_{50}$
Statement-II : The greatest value of ${}^{2n}C_r$ (where r is constant) occurs at $r = n$.
- Statement - I :** If n is even, then ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + \dots + {}^{2n}C_{n-1} = 2^{2n-1}$.
Statement - II : ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + \dots + {}^{2n}C_{2n-1} = 2^{2n-1}$.
- Statement-I :** Coefficient of $ab^8c^3d^2$ in the expansion of $(a+b+c+d)^{14}$ is 180180
Statement-II : General term in the expansion of $(a_1 + a_2 + a_3 + \dots + a_m)^n = \sum \frac{n!}{n_1!n_2!n_3!\dots n_m!} a_1^{n_1} a_2^{n_2} \dots a_m^{n_m}$,
 where $n_1 + n_2 + n_3 + \dots + n_m = n$.
- Statement - I :** Any positive integral power of $(\sqrt{2}-1)$ can be expressed as $\sqrt{N} - \sqrt{N-1}$ for some natural number $N > 1$.
Statement - II : Any positive integral power of $\sqrt{2} - 1$ can be expressed as $A + B\sqrt{2}$ where A and B are integers.
- Statement-I :** If $x = {}^nC_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + \dots + {}^{2n}C_{n-1}$, then $\frac{x+1}{2n+1}$ is integer.
Because
Statement-II : ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ and nC_r is divisible by n if n and r are co-prime.
- Statement - I :** The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(2m)!}{(m!)^2}$.
Statement - II : The coefficient of x^b in the expansion of $(1+x)^n$ is nC_b .
- Statement-I :** If $q = \frac{1}{3}$ and $p+q=1$, then $\sum_{r=0}^{15} r \cdot {}^{15}C_r p^r q^{15-r} = 15 \times \frac{1}{3} = 5$
Statement-II : If $p+q=1$, $0 < p < 1$, then $\sum_{r=0}^n r \cdot {}^nC_r p^r q^{n-r} = np$
- Statement - I :** If n is an odd prime then $\left[(\sqrt{5}+2)^n\right] - 2^{n+1}$ is divisible by 20 n, where $[.]$ denotes greatest integer function.
Statement - II : If n is prime then ${}^nC_1, {}^nC_2, \dots, {}^nC_{n-1}$ must be divisible by n.

Exercise # 3

Part # I

[Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1.

column-I	column-II
(A) $(2n+1)(2n+3)(2n+5) \dots (4n-1)$ is equal to	(p) $\frac{(n+1)^n}{n!}$
(B) $\frac{C_1}{C_0} + \frac{2 \cdot C_2}{C_1} + \frac{3 \cdot C_3}{C_2} + \dots + \frac{n \cdot C_n}{C_{n-1}}$ is equal to here C_r stand for nC_r .	(q) $n \cdot 2^n \cdot (2^n - 1)$
(C) If $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \dots (C_{n-1} + C_n)$ $= m \cdot C_1 C_2 C_3 \dots C_{n-1}$, then m is equal to	(r) $\frac{(4n)! n!}{2^n \cdot (2n)! (2n)!}$
(D) If C_r are the binomial co-efficients in the expansion of $(1+x)^n$, the value of $\sum_{i=1}^n \sum_{j=1}^n (i+j) C_i C_j$ is	(s) $\frac{n(n+1)}{2}$

2.

Column - I	Column - II
(A) If $(r+1)^{\text{th}}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r (where $ x < 1$) is	(p) divisible by 2
(B) The coefficient of y in the expansion of $(y^2 + 1/y)^5$ is	(q) divisible by 5
(C) nC_r is divisible by n , ($1 < r < n$) if n is	(r) divisible by 10
(D) The coefficient of x^4 in the expression $(1+2x+3x^2+4x^3+\dots \text{up to } \infty)^{1/2}$ is c , ($c \in \mathbb{N}$), then $c+1$ (where $ x < 1$) is	(s) a prime number

3.

Column - I	Column - II
(A) If $x = (7 + 4\sqrt{3})^{2n} = [x] + f$, then $x(1-f) =$	(p) 6
(B) If second, third and fourth terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then n is equal to	(q) 1
(C) value of ${}^4C_0 {}^4C_4 - {}^4C_1 {}^4C_3 + {}^4C_2 {}^4C_2 - {}^4C_3 {}^4C_1 + {}^4C_4 {}^4C_0$ is	(r) 2
(D) If x is very large as compare to y , then the value of k in $\sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} = 1 + \frac{y^2}{kx^2}$	(s) 5

Comprehension # 1

If n is positive integer and if $(1 + 4x + 4x^2)^n = \sum_{r=0}^{2n} a_r x^r$, where a_i 's are ($i = 0, 1, 2, 3, \dots, 2n$) real numbers.

On the basis of above information, answer the following questions :

- The value of $2 \sum_{r=0}^n a_{2r}$ is -
 (A) $9^n - 1$ (B) $9^n + 1$ (C) $9^n - 2$ (D) $9^n + 2$
- The value of $2 \sum_{r=1}^n a_{2r-1}$ is -
 (A) $9^n - 1$ (B) $9^n + 1$ (C) $9^n - 2$ (D) $9^n + 2$
- The value of a_{2n-1} is -
 (A) 2^{2n} (B) $(n-1) \cdot 2^{2n}$ (C) $n \cdot 2^{2n}$ (D) $(n+1) \cdot 2^{2n}$
- The value of a_2 is -
 (A) $8n$ (B) $8n^2 - 4$ (C) $8n^2 - 4n$ (D) $8n - 4$

Comprehension # 2

Let P be a product given by $P = (x + a_1)(x + a_2) \dots (x + a_n)$

and Let $S_1 = a_1 + a_2 + \dots + a_n = \sum_{i=1}^n a_i$, $S_2 = \sum_{i < j} a_i \cdot a_j$, $S_3 = \sum_{i < j < k} a_i \cdot a_j \cdot a_k$ and so on,

then it can be shown that

$$P = x^n + S_1 x^{n-1} + S_2 x^{n-2} + \dots + S_n.$$

- The coefficient of x^8 in the expression $(2+x)^2(3+x)^3(4+x)^4$ must be
 (A) 26 (B) 27 (C) 28 (D) 29
- The coefficient of x^{203} in the expression $(x-1)(x^2-2)(x^3-3) \dots (x^{20}-20)$ must be
 (A) 11 (B) 12 (C) 13 (D) 15
- The coefficient of x^{98} in the expression of $(x-1)(x-2) \dots (x-100)$ must be
 (A) $1^2 + 2^2 + 3^2 + \dots + 100^2$
 (B) $(1+2+3+\dots+100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)$
 (C) $\frac{1}{2} [(1+2+3+\dots+100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)]$
 (D) None of these

Comprehension # 3

Consider, sum of the series $\sum_{0 \leq i < j \leq n} f(i) f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^n \sum_{j=1}^n f(i) f(j) = \sum_{i=1}^n \left(f(i) \left(\sum_{j=1}^n f(j) \right) \right)$ i and j are independent. In this summation, three types of

terms occur, those when $i < j$, $i > j$ and $i = j$.

Also, sum of terms when $i < j$ is equal to the sum of the terms when $i > j$ if $f(i)$ and $f(j)$ are symmetrical.

So, in that case

$$\sum_{i=1}^n \sum_{j=1}^n f(i) f(j) = \sum_{0 \leq i < j \leq n} f(i) f(j) + \sum_{0 \leq i > j \leq n} f(i) f(j) + \sum_{i=j} f(i) f(j) = 2 \sum_{0 \leq i < j \leq n} f(i) f(j) + \sum_{i=j} f(i) f(j)$$

$$\Rightarrow \sum_{0 \leq i < j \leq n} f(i) f(j) = \frac{\sum_{i=1}^n \sum_{j=1}^n f(i) f(j) - \sum_{i=j} f(i) f(j)}{2}$$

When $f(i)$ and $f(j)$ are not symmetrical, we find the sum by listing all the terms.

1. $\sum_{0 \leq i < j \leq n} {}^nC_i \cdot {}^nC_j$ is equal to

(A) $\frac{2^{2n} - {}^nC_n}{2}$

(B) $\frac{2^{2n} + {}^nC_n}{2}$

(C) $\frac{2^{2n} - {}^nC_n}{2}$

(D) $\frac{2^{2n} + {}^nC_n}{2}$

2. $\sum_{m=0}^n \sum_{p=0}^m {}^nC_m \cdot {}^mC_p$ is equal to

(A) 2^{n-1}

(B) 3^n

(C) 3^{n-1}

(D) 2^n

3. $\sum_{0 \leq i \leq j \leq n} ({}^nC_i + {}^nC_j)$

(A) $n2^n$

(B) $(n+1)2^n$

(C) $(n-1)2^n$

(D) $(n+1)2^{n-1}$

Exercise # 4

[Subjective Type Questions]

1. If the coefficients of the r^{th} , $(r+1)^{\text{th}}$ & $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{14}$ are in AP, find r .
2. In the binomial expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of the 7th term from the beginning to the 7th term from the end is $1:6$; find n .
3. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ & $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$.
4. Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.
5. Prove that : ${}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \dots + {}^rC_r = {}^nC_{r+1}$.
6. Prove that the co-efficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the co-efficients of middle terms in the expansion of $(1+x)^{2n-1}$.
7. Prove that : $({}^{2n}C_1)^2 + 2.({}^{2n}C_2)^2 + 3.({}^{2n}C_3)^2 + \dots + 2n.({}^{2n}C_{2n})^2 = \frac{(4n-1)!}{[(2n-1)!]^2}$
8. If ${}^{40}C_1 \cdot x(1-x)^{39} + 2 \cdot {}^{40}C_2 x^2(1-x)^{38} + 3 \cdot {}^{40}C_3 x^3(1-x)^{37} + \dots + 40 \cdot {}^{40}C_{40} x^{40} = ax + b$, then find a & b .
9. Find the index 'n' of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient ($n \in \mathbb{N}$).
10. Prove that : $\frac{C_0}{2} + \frac{C_1}{3} + \frac{C_2}{4} + \frac{C_3}{5} + \dots + \frac{C_n}{n+2} = \frac{1+n.2^{n+1}}{(n+1)(n+2)}$
11. If $(1+x)^n = \sum_{r=0}^n C_r \cdot x^r$ then prove that ; $\frac{2^2 \cdot C_0}{1 \cdot 2} + \frac{2^3 \cdot C_1}{2 \cdot 3} + \frac{2^4 \cdot C_2}{3 \cdot 4} + \dots + \frac{2^{n+2} \cdot C_n}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$
12. $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4) \dots (C_{n-1} + C_n) = \frac{C_0 C_1 C_2 \dots C_{n-1} (n+1)^n}{n!}$.
13. Prove that $\sum_{k=0}^n {}^nC_k \sin Kx \cdot \cos(n-K)x = 2^{n-1} \sin nx$.
14. $C_0 - 2C_1 + 3C_2 - 4C_3 + \dots + (-1)^n (n+1) C_n = 0$

15. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$, prove that

(i) $C_0 C_3 + C_1 C_4 + \dots + C_{n-3} C_n = \frac{(2n)!}{(n+3)!(n-3)!}$

(ii) $C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n = \frac{(2n)!}{(n+r)!(n-r)!}$

(iii) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = 0$ or $(-1)^{n/2} C_{n/2}$ according as n is odd or even.

16. Prove the identity $\frac{1}{{}^{2n+1}C_r} + \frac{1}{{}^{2n+1}C_{r+1}} = \frac{2n+2}{2n+1} \frac{1}{{}^{2n}C_r}$.

17. If $y = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \text{to } \infty \right)$ and $|x| < 1$, prove that $x = \left(y + \frac{y^2}{2!} + \frac{y^3}{3!} \dots \text{to } \infty \right)$.

18. Prove that : $\sum_{r=0}^{n-2} \binom{n-1}{r} \binom{n}{r+2} = \binom{2n-1}{n-2}$

19. Find the sum of the following infinite series : $\frac{1}{2} \left(\frac{1}{5} \right)^2 + \frac{2}{3} \left(\frac{1}{5} \right)^3 + \frac{3}{4} \left(\frac{1}{5} \right)^4 + \dots$

20. Prove that : $\sum_{r=1}^n \binom{n-1}{n-r} \binom{n}{r} = \binom{2n-1}{n-1}$

Exercise # 5

Part # I > [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$ is : [AIEEE 2002]
 (A) 21 (B) 25 (C) 26 (D) none of these
2. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[3]{5})^{256}$ is : [AIEEE 2003]
 (A) 32 (B) 33 (C) 34 (D) 35
3. If x is positive, the first negative term in the expansion of $(1 + x)^{\frac{27}{5}}$ is : [AIEEE 2003]
 (A) 7th term (B) 5th term (C) 8th term (D) 6th term.
4. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals : [AIEEE 2004]
 (A) $-\frac{5}{3}$ (B) $\frac{10}{3}$ (C) $-\frac{3}{10}$ (D) $\frac{3}{5}$
5. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is- [AIEEE 2004]
 (A) $(n - 1)$ (B) $(-1)^n(1 - n)$ (C) $(-1)^{n-1}(n - 1)^2$ (D) $(-1)^{n-1}n$
6. If $s_n = \sum_{r=0}^n \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^n \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to- [AIEEE 2004]
 (A) $\frac{n}{2}$ (B) $\frac{n}{2} - 1$ (C) $n - 1$ (D) $\frac{2n - 1}{2}$
7. If the coefficients of r^{th} , $(r + 1)^{\text{th}}$ and $(r + 2)^{\text{th}}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation : [AIEEE 2005]
 (A) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$. (B) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$.
 (C) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$. (D) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$.
8. The value of ${}^{50}C_4 + \sum_{r=1}^6 {}^{56-r}C_3$ is : [AIEEE 2005]
 (A) ${}^{56}C_4$ (B) ${}^{56}C_3$ (C) ${}^{55}C_3$ (D) ${}^{55}C_4$
9. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1 + x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1 - x)^{1/2}}$ may be approximated as [AIEEE 2005]
 (A) $\frac{x}{2} - \frac{3}{8}x^2$ (B) $-\frac{3}{8}x^2$ (C) $3x + \frac{3}{8}x^2$ (D) $1 - \frac{3}{8}x^2$
10. If the expansion in powers of x of the function $\frac{1}{(1 - ax)(1 - bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$, then a_n is : [AIEEE 2006]
 (A) $\frac{a^n - b^n}{b - a}$ (B) $\frac{a^{n+1} - b^{n+1}}{b - a}$ (C) $\frac{b^{n+1} - a^{n+1}}{b - a}$ (D) $\frac{b^n - a^n}{b - a}$
11. For natural numbers m, n if $(1 - y)^m(1 + y)^n = 1 + a_1y + a_2y^2 + \dots$ and $a_1 = a_2 = 10$, then (m, n) is : [AIEEE 2006]
 (A) (35, 20) (B) (45, 35) (C) (35, 45) (D) (20, 45)

12. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots + {}^{20}C_{10}$ is [AIEEE 2007]
 (A) $-{}^{20}C_{10}$ (B) $\frac{1}{2} {}^{20}C_{10}$ (C) 0 (D) ${}^{20}C_{10}$
13. **Statement-I:** $\sum_{r=0}^n (r+1) {}^nC_r = (n+2) 2^{n-1}$ [AIEEE 2008]
Statement-II: $\sum_{r=0}^n (r+1) {}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$
 (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
 (C) Statement-1 is True, Statement-2 is False
 (D) Statement-1 is False, Statement-2 is True
14. Let $S_1 = \sum_{j=1}^{10} j(j-1) {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j {}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2 {}^{10}C_j$. [AIEEE 2009]
Statement -I : $S_3 = 55 \times 2^9$.
Statement -II : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.
 (A) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
 (B) Statement-1 is true, Statement-2 is false.
 (C) Statement -1 is false, Statement -2 is true.
 (D) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.
15. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is : [AIEEE-2011]
 (A) 144 (B) -132 (C) -144 (D) 132
16. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is : [AIEEE-2012]
 (A) an irrational number (B) an odd positive integer
 (C) an even positive integer (D) a rational number other than positive integers
17. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is : [AIEEE -2013]
 (A) 4 (B) 120 (C) 210 (D) 310
18. If the coefficients of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to : [JEE Main 2014]
 (A) $\left(16, \frac{251}{3}\right)$ (B) $\left(14, \frac{251}{3}\right)$ (C) $\left(14, \frac{272}{3}\right)$ (D) $\left(16, \frac{272}{3}\right)$
19. The sum of coefficients of integral powers of x in the binomial expansion of $(1-2\sqrt{x})^{50}$ is [JEE Main 2015]
 (A) $\frac{1}{2}(3^{50}-1)$ (B) $\frac{1}{2}(2^{50}+1)$ (C) $\frac{1}{2}(3^{50}+1)$ (D) $\frac{1}{2}(3^{50})$
20. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : [JEE Main 2016]
 (A) 2187 (B) 243 (C) 729 (D) 64

1. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is
 (A) 6 (B) 9 (C) 12 (D) 24. [IIT-JEE - 1999]
2. For $2 \leq r \leq n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$
 (A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$ [IIT-JEE-2000]
3. For any positive integer m, n (with $n \geq m$), let $\binom{n}{m} = {}^nC_m$. Prove that
 $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots + \binom{m}{m} = \binom{n+1}{m+1}$. Hence or otherwise, prove that
 $\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$. [IIT-JEE 2000]
4. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then a/b equals:
 (A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$ [IIT-JEE-2001]
5. The sum $\sum_{i=0}^m \binom{10}{i} \binom{20}{m-i}$, (where $\binom{p}{q} = 0$, if $p < q$) is maximum when 'm' is :
 (A) 5 (B) 10 (C) 15 (D) 20 [IIT-JEE-2002]
6. Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is :
 (A) ${}^{12}C_6 + 3$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) ${}^{12}C_6 + 2$ [IIT-JEE-2003]
7. Prove that $2^k \binom{n}{0} \binom{n}{k} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n}{2} \binom{n-2}{k-2} - \dots + (-1)^k \binom{n}{k} \binom{n-k}{0} = \binom{n}{k}$.
 [IIT-JEE-2003]
8. If ${}^{(n-1)}C_r = (k^2-3) {}^nC_{r+1}$, then an interval in which k lies is
 (A) $(2, \infty)$ (B) $(-\infty, -2)$ (C) $[-\sqrt{3}, \sqrt{3}]$ (D) $(\sqrt{3}, 2]$ [IIT-JEE-2004]

9. The value of [IIT-JEE-2005]

$$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \dots + \binom{30}{20}\binom{30}{30}$$
 is :
 (A) $\binom{60}{20}$ (B) $\binom{30}{10}$ (C) $\binom{30}{15}$ (D) None of these
10. For $r = 0, 1, \dots, 10$, let A_r , B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to [IIT-JEE 2010]
 (A) $B_{10} - C_{10}$ (B) $A_{10}(B_{10}^2 - C_{10}A_{10})$ (C) 0 (D) $C_{10} - B_{10}$
11. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is [JEE Ad. 2014]
 (A) 1051 (B) 1106 (C) 1113 (D) 1120
12. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is. [JEE Ad. 2015]
13. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{40} + (1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integer n . Then the value of n is [JEE Ad. 2016]

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

- The expression, $\left(\sqrt{2x^2+1} + \sqrt{2x^2-1}\right)^6 + \left(\frac{2}{\sqrt{2x^2+1} + \sqrt{2x^2-1}}\right)^6$ is a polynomial of degree
 (A) 5 (B) 6 (C) 7 (D) 8
- In the expansion of $\left(\sqrt[3]{\frac{a}{b}} + \sqrt[3]{\frac{b}{a}}\right)^{21}$ the term containing same powers a and b is
 (A) 11th (B) 13th (C) 12th (D) 6th
- Co-efficient of x^{15} in $(1+x+x^3+x^4)^n$ is :
 (A) $\sum_{r=0}^5 {}^nC_{15-3r} {}^nC_r$ (B) $\sum_{r=0}^5 {}^nC_{5r}$ (C) $\sum_{r=0}^5 {}^nC_{3r}$ (D) $\sum_{r=0}^3 {}^nC_{3-r} {}^nC_{5r}$
- The co-efficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3)\dots(x-n)$ is
 (A) $\frac{n(n^2+2)(3n+1)}{24}$ (B) $\frac{n(n^2-1)(3n+2)}{24}$
 (C) $\frac{n(n^2+1)(3n+4)}{24}$ (D) none of these
- The coefficient of x^n in polynomial $(x+{}^{2n+1}C_0)(x+{}^{2n+1}C_1)\dots(x+{}^{2n+1}C_n)$ is -
 (A) $2n+1$ (B) $2^{2n+1}-1$ (C) 2^{2n} (D) none of these
- If $\sum_{r=0}^{2n} a_r(x-1)^r = \sum_{r=0}^{2n} b_r(x-2)^r$ and $b_r = (-1)^{r-n}$ for all $r \geq n$, then $a_n =$
 (A) ${}^{2n+1}C_{n-1}$ (B) $3^n C_n$ (C) $2^{n+1} C_n$ (D) 0
- $\sum_{r=1}^n \left(\sum_{p=0}^{r-1} {}^nC_r {}^rC_p 2^p \right)$ is equal to -
 (A) $4^n - 3^n + 1$ (B) $4^n - 3^n - 1$ (C) $4^n - 3^n + 2$ (D) $4^n - 3^n$
- $2(1+x^3)^{100} = \sum_{k=0}^{100} \left(a_k x^k - \cos \frac{\pi}{2} (x+k) \right)$ then the value of $a_0 + a_2 + a_4 + \dots + a_{100}$
 (A) 2^{99} (B) 2^{100} (C) 2^{101} (D) None of these
- ${}^nC_0 - 2.3 {}^nC_1 + 3.3^2 {}^nC_2 - 4.3^3 {}^nC_3 + \dots + (-1)^n (n+1) {}^nC_n 3^n$ is equal to
 (A) $(-1)^n 2^n \left(\frac{3n}{2} + 1 \right)$ (B) $2^n \left(n + \frac{3}{2} \right)$ (C) $2^n + 5n 2^n$ (D) $(-2)^n$

10. Consider the following statements :

$$S_1 : {}^nC_0 \cdot {}^nC_1 + {}^nC_1 \cdot {}^nC_2 + \dots + {}^nC_{n-1} \cdot {}^nC_n = \frac{(2n)!}{(n-1)!(n+1)!}$$

$$S_2 : C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{(2n)!}{(n!)^2}$$

$$S_3 : {}^{11}C_0 \cdot {}^{11}C_{11} - {}^{11}C_1 \cdot {}^{11}C_{10} + \dots + (-1)^{11} {}^{11}C_{11} \cdot {}^{11}C_0 = {}^{22}C_{11}$$

$$S_4 : 2^n C_0 + \frac{2^{2^n} C_1}{2} + \frac{2^{3^n} C_2}{3} + \dots + \frac{2^{n+1} C_n}{n+1} = \frac{3^{n+1} - 1}{n+1}$$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FTFF

(B) FTTT

(C) FFFT

(D) TTFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $n^n \left(\frac{n+1}{2} \right)^{2n}$ is

(A) Less than $\left(\frac{n+1}{2} \right)^3$

(B) Greater than or equal to $\left(\frac{n+1}{2} \right)^3$

(C) Less than $(n!)^3$

(D) Greater than or equal to $(n!)^3$

12. Let $a_n = \left(1 + \frac{1}{n} \right)^n$. Then for each $n \in \mathbb{N}$

(A) $a_n \geq 2$

(B) $a_n < 3$

(C) $a_n < 4$

(D) $a_n < 2$

13. Let $a_n = \frac{1000^n}{n!}$ for $n \in \mathbb{N}$, then a_n is greatest, when

(A) $n = 997$

(B) $n = 998$

(C) $n = 999$

(D) $n = 1000$

14. If n is even natural and coefficient of x^r in the expansion of $\frac{(1+x)^n}{1-x}$ is 2^n , ($|x| < 1$), then –

(A) $r \leq n/2$

(B) $r \geq (n-2)/2$

(C) $r \leq (n+2)/2$

(D) $r \geq n$

15. If $(4 + \sqrt{15})^n = I + f$, where n is an odd natural number, I is an integer and $f < 1$, then

(A) I is natural number

(B) I is an even integer

(C) $(I+f)(1-f) = 1$

(D) $1-f = (4 - \sqrt{15})^n$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** Coefficient of $a^2b^3c^4$ in the expansion of $(a+b+c)^8$ is $\frac{8!}{2!3!4!}$
- Statement-II :** Coefficient of $a^\alpha b^\beta c^\gamma$, where $\alpha + \beta + \gamma = n$, in the expansion of $(a+b+c)^n$ is $\frac{n!}{\alpha! \beta! \gamma!}$.
- (A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
 (B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
 (C) Statement I is true, statement II is false
 (D) Statement I is false, statement II is true
17. **Statement-I :** If $q = \frac{1}{3}$ and $p + q = 1$, then $\sum_{r=0}^{15} r {}^{15}C_r p^r q^{15-r} = 15 \times \frac{1}{3} = 5$
- Statement-II :** If $p + q = 1$, $0 < p < 1$, then $\sum_{r=0}^n r {}^nC_r p^r q^{n-r} = np$
- (A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
 (B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
 (C) Statement I is true, statement II is false
 (D) Statement I is false, statement II is true
18. **Statement-I :** Coefficient of x^{51} in the expansion of $(x-1)(x^2-2)(x^3-3) \dots (x^{10}-10)$ is -1
- Statement-II :** Coefficient of $x^{\frac{n(n+1)}{2}-4}$, $n \geq 4$, $n \in \mathbb{N}$, in the expansion of $(x-1)(x^2-2)(x^3-3) \dots (x^n-n)$ is $-4 + (-1)(-3) = -1$
- (A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
 (B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
 (C) Statement I is true, statement II is false
 (D) Statement I is false, statement II is true
19. **Statement-I :** If $r \geq 40$, then greatest possible value of ${}^{40}C_0 \cdot {}^{60}C_r + {}^{40}C_1 \cdot {}^{60}C_{r-1} + \dots + {}^{40}C_{40} \cdot {}^{60}C_{r-40}$ is ${}^{100}C_{50}$
- Statement-II :** The greatest value of ${}^{2n}C_r$ occurs at $r = n$.
- (A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
 (B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
 (C) Statement I is true, statement II is false
 (D) Statement I is false, statement II is true
20. **Statement-I :** If $x = {}^nC_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + \dots + {}^{2n}C_{n-1}$, then $\frac{x+1}{2n+1}$ is integer.
- Statement-II :** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ and nC_r is divisible by n if n and r are co-prime.
- (A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
 (B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
 (C) Statement I is true, statement II is false
 (D) Statement I is false, statement II is true

SECTION - IV : MATRIX - MATCH TYPE

21. Match the column

Column – I

(A) ${}^m C_1 {}^n C_m - {}^m C_2 {}^{2n} C_m + {}^m C_3 {}^{3n} C_m - \dots$ is

(B) ${}^n C_m + {}^{n-1} C_m + {}^{n-2} C_m + \dots + {}^m C_m$ is

(C) $C_0 C_n + C_1 C_{n-1} + \dots + C_n C_0$ is

(D) $2^m {}^n C_0 \cdot {}^n C_m - 2^{m-1} {}^n C_1 {}^{n-1} C_{m-1} + \dots + (-1)^m {}^n C_m {}^{n-m} C_0$ is

Column – II

(p) the coefficient of x^m in the expansion of $-(1 - (1+x)^n)^m$

(q) the coefficient of x^m in $\frac{(1+x)^{n+1}}{x}$

(r) the coefficient of x^{n+1} in $(1+x)^{2n}$

(s) the coefficient of x^m in the expansion of $(1+x)^n$

(t) the coefficient of x^n in $(1+x)^{2n}$

22. Match the column

Column – I

(A) If λ denotes the number of terms in the expansion of $(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)^{20}$ & If unit's place & ten's place digits in 3^λ are O and T, then $O + T$ is

(B) The value of $8 \cdot \left\{ \frac{3^{2n}}{8} \right\}$ is (Here $\{ \cdot \}$ denotes fraction part function)

(C) If n be the degree of the polynomial $\sqrt{(3x^2 + 1)} \{ (x + \sqrt{(3x^2 + 1)})^7 - (x - \sqrt{(3x^2 + 1)})^7 \}$ then n is divisible by

(D) The value of $\left(\frac{1}{1} + \frac{1}{2002} \right) \left(\frac{1}{2} + \frac{1}{2001} \right) \left(\frac{1}{3} + \frac{1}{2000} \right) \dots \left(\frac{1}{1001} + \frac{1}{1002} \right)$ is

Column – II

(p) 1

(q) $\frac{(2003)^{1001}}{(2002)!}$

(r) 3

(s) 8

(t) $(2002)^{2001}$

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Consider the identity $(1+x)^{6m} = \sum_{r=0}^{6m} {}^{6m} C_r \cdot x^r$. By putting different values of x on both sides, we can get

summation of several series involving binomial coefficients. For example, by putting $x = \frac{1}{2}$ we get

$$\sum_{r=0}^{6m} {}^{6m} C_r \frac{1}{2^r} = \left(\frac{3}{2} \right)^{6m}.$$

23.

1 The value of $\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ is equal to

- (A) $\frac{3^{6m}}{2}$ (B) $(1 + \sqrt{2})^{3m}$ (C) $(3 + 2\sqrt{2})^{3m}$ (D) None of these

2 The value of $\sum_{r=0}^{3m} (-1)^r {}^{6m}C_{2r}$ is

- (A) 2^{3m} (B) 0 if m is odd (C) -2^{3m} if m is even (D) None of these

3 The value of $\sum_{r=1}^{3m} (-3)^{r-1} {}^{6m}C_{2r-1}$ is

- (A) 0 (B) 1 (C) depends on m (D) None of these

24. Read the following comprehension carefully and answer the questions.

For $k, n \in \mathbb{N}$, we define

$B(k, n) = 1.2.3 \dots k + 2.3.4 \dots (k+1) + \dots + n(n+1) \dots (n+k-1)$, $S_0(n) = n$ and $S_k(n) = 1^k + 2^k + \dots + n^k$.

To obtain value $B(k, n)$, we rewrite $B(k, n)$ as follows

$$B(k, n) = k! \left[{}^k C_k + {}^{k+1} C_k + {}^{k+2} C_k + \dots + {}^{n+k-1} C_k \right] = k! \left({}^{n+k} C_{k+1} \right) \\ = \frac{n(n+1) \dots (n+k)}{k+1} \quad \text{where} \quad {}^n C_k = \frac{n!}{k! (n-k)!}$$

1 $S_2(n) + S_1(n)$ equals

- (A) $B(2, n)$ (B) $\frac{1}{2} B(2, n)$ (C) $\frac{1}{6} B(2, n)$ (D) none of these

2 $S_3(n) + 3S_2(n)$ equals

- (A) $B(3, n)$ (B) $B(3, n) - 2B(2, n)$ (C) $B(3, n) - 2B(1, n)$ (D) $B(3, n) + 2B(1, n)$

3 If $(1+x)^p = 1 + {}^p C_1 x + {}^p C_2 x^2 + \dots + {}^p C_p x^p$, $p \in \mathbb{N}$, then ${}^{k+1} C_1 S_k(n) + {}^{k+1} C_2 S_{k-1}(n) + \dots + {}^{k+1} C_k S_1(n) + {}^{k+1} C_{k+1} S_0(n)$ equals

- (A) $(n+1)^{k+1}$ (B) $(n+1)^{k+1} - 1$ (C) $n^{k+1} - (n-1)^{k+1}$ (D) $(n+1)^{k+1} - (n-1)^{k+1}$.

25. Read the following comprehension carefully and answer the questions.

If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$ (i)

then sum of the series $C_0 + C(b)C_{2k} + \dots$ can be obtained by putting all the roots of the equation $x^k - 1 = 0$ in (i) and then adding vertically.

For Example : Sum of the series $C_0 + C_2 + C_4 + \dots$ can be obtained by putting roots of the equation $x^2 - 1 = 0$

$\Rightarrow x = \pm 1$ in (i)

$$\begin{array}{ll} x = 1 & C_0 + C_1 + C_2 + C_3 + \dots = 2^n \\ x = -1 & C_0 - C_1 + C_2 - C_3 + \dots = 0 \\ \hline \therefore & 2(C_0 + C_2 + C_4 + \dots) = 2^n \\ & C_0 + C_2 + C_4 + \dots = 2^{n-1} \end{array}$$

- 1 Values of x , we should substitute in (i) to get the sum of the series $C_0 + C_3 + C_6 + C_9 + \dots$, are
 (A) $1, -1, \omega$ (B) $\omega, \omega^2, \omega^3$ (C) $\omega, \omega^2, -1$ (D) None of these
- 2 If n is a multiple of 3, then $C_0 + C_3 + C_6 + \dots$ is equal to
 (A) $\frac{2^n + 2}{3}$ (B) $\frac{2^n - 2}{3}$ (C) $\frac{2^n + 2(-1)^n}{3}$ (D) $\frac{2^n - 2(-1)^n}{3}$
- 3 Sum of values of x , which we should substitute in (i) to give the sum of the series $C_0 + C_4 + C_8 + C_{12} + \dots$, is
 (A) 2 (B) $2(1+i)$ (C) $2(1-i)$ (D) 0

SECTION - VI : INTEGER TYPE

26. Let $(1 - x^3)^n = \sum_{r=0}^n a_r x^r (1 - x)^{3n-2r}$, $n > 2$, then find the value of n so that a_1, a_2, a_3 are in A.G.P.
27. If $(1 - x)^{-n} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$, find the value of, $a_0 + a_1 + a_2 + \dots + a_n$.
28. Find the exponent of three in sum of rational coefficients in expansion of $(\sqrt[3]{5x} + \sqrt{3y} + z)^6$
29. Find the remainder when 32^{32} is divided by 7.
30. The value of $\frac{y_1 \cdot y_2 \cdot y_3}{501(y_1 - x_1)(y_2 - x_2)(y_3 - x_3)}$ when $(x_i, y_i), i = 1, 2, 3$ satisfy both $x^3 - 3xy^2 = 2005$ & $y^3 - 3x^2y = 2004$ is

ANSWER KEY

EXERCISE - 1

1. A 2. C 3. C 4. D 5. A 6. D 7. B 8. C 9. A 10. B 11. C 12. A 13. D
14. C 15. A 16. B 17. A 18. D 19. A 20. D 21. C 22. B 23. C 24. C 25. B 26. A
27. B 28. A 29. C 30. A

EXERCISE - 2 : PART # I

1. AB 2. BD 3. ABC 4. ABC 5. AC 6. AB 7. BCD 8. CD 9. BC
10. ABCD 11. ABC 12. AC 13. AC 14. BCD 15. ACD 16. AB 17. AD 18. ABCD
19. BCD 20. ABCD 21. ABCD

PART - II

1. C 2. D 3. C 4. B 5. A 6. A 7. D 8. A

EXERCISE - 3 : PART # I

1. $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$ 2. $A \rightarrow q, s$ $B \rightarrow p, q, r$ $C \rightarrow s$ $D \rightarrow ps$ 3. $A \rightarrow q$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow r$

PART - II

- Comprehension #1: 1. B 2. A 3. C 4. C Comprehension #2: 1. D 2. C 3. C
Comprehension #3: 1. A 2. B 3. A

EXERCISE - 5 : PART # I

1. D 2. B 3. C 4. C 5. B 6. A 7. B 8. A 9. B 10. C 11. C 12. B 13. A
14. B 15. C 16. A 17. C 18. D 19. C 20. C

PART - II

1. C 2. D 4. B 5. C 6. D 8. D 9. B 10. D 11. B 12. 8 13. 5

MOCK TEST

1. B 2. B 3. A 4. B 5. C 6. C 7. D 8. B 9. A
10. B 11. BD 12. ABC 13. CD 14. D 15. ACD 16. D 17. D 18. A
19. A 20. A 21. $A \rightarrow p \quad B \rightarrow q \quad C \rightarrow t \quad D \rightarrow s$ 22. $A \rightarrow r \quad B \rightarrow p \quad C \rightarrow s \quad D \rightarrow q$
23. 1. C 2. B 3. A 24. 1. A 2. C 3. B 25. 1. B 2. C 3. D
26. 7 27. $\frac{(2n)!}{(n!)^2}$ 28. 1 29. 4 30. 2