HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. $I_{rms} = \left[\dfrac{\int_0^T i^2\,dt}{T}\right]^{\frac{1}{2}}$

$= \left[\int_0^T \dfrac{[3+4\sin(\omega t+\pi/3)]^2}{T}dt\right]^{1/2} = \sqrt{17}$.

3. $V = 100\sin 100\pi t\cos 100\pi t$

$V = 50\sin 200\pi t$

here $V_0 = 50$

& $\omega = 200\pi$

$f = 100$ Hz

4. If net area of E – t curve is zero for given interval then average value will be zero.

9. $P_{av} = v_{rms}\, I_{rms}\cos\phi$

Here $\phi = 90^\circ$ So $P_{av} = 0$

12. $<P> = I^2_{rms} R = \left(\dfrac{I_P}{\sqrt{2}}\right)^2 R = \dfrac{I_P^2 R}{2}$

14. $I^2R = 100$

$R = \dfrac{100}{I^2} = \dfrac{100}{(2)^2} = 25.$

16. $I_0 = \dfrac{V_0}{\omega L} = \dfrac{10}{100\times 5\times 10^{-3}}$

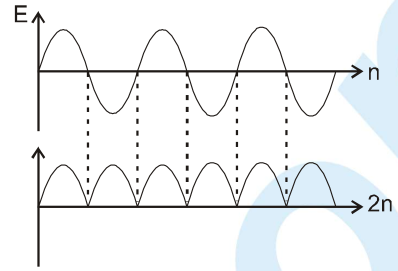
17. $\cos\phi = \dfrac{R}{z}$

% change $= \dfrac{z'-z}{z}\times 100 = 100\%$.

18. $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{V_{rms}}{\sqrt{R^2+(\omega L)^2}} = 2A$

$\tan\phi = \dfrac{\omega L}{R} = \dfrac{66}{88} = \dfrac{3}{4}$.

19.



20. $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{100}{\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}}$

P.d. across resistance $= R\, I_{rms} = 100$ volt.

23. $R = \dfrac{V_0}{I_0} = \dfrac{200}{5} = 40\,\Omega$ (For circuit x)

$X_L = \dfrac{V_0}{I_0} = 40\,\Omega$ (For circuit y)

If x & y are in series

$I = \dfrac{200}{40\times\sqrt{2}} = \dfrac{5}{\sqrt{2}}$ Amp.

$\Rightarrow I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{5}{2}$ amp.

25. $\tan\phi = \tan 45^\circ = \dfrac{\omega L}{R}$

$X_L = \omega L = R.$

26. $I = \dfrac{200\sqrt{2}}{(X_C)\times\sqrt{2}} = 200\times\omega C = 20$ mA.

27. Voltage of source is always less than $(V_1+V_2+V_3)$, $V_{net} = \sqrt{V_1^2+V_2^2+V_3^2}$

31. Current lags behind voltage.

If $X_L > X_C \Rightarrow 2\pi\nu L > \dfrac{1}{2\pi\nu C} \Rightarrow \nu > \dfrac{1}{2\pi\sqrt{LC}}$

But as $\nu_r = \dfrac{1}{2\pi\sqrt{LC}}$

therefore, $\nu > \nu_r$

32. $I_{rms} = \frac{60}{120} = \frac{1}{2}$ Amp.

$V_L = I_{rms} \times (\omega L)$

$40 = \frac{1}{2} \times (40 \times 10^3) \times L$

$L = 20$ mH

At resonance $V_C = I_{rms}\left(\frac{1}{\omega c}\right) = V_L$

$C = \frac{1}{2} \times \frac{1}{4 \times 10^3} \times \frac{1}{40}$

$C = \frac{25}{8}$ μF.

34. $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{8}{1}$

$V_2 = 8 \times 120 = 960$ volt

$I = \frac{960}{10^4} = 96$ mA.

36. $I_1 E_1 = I_2 E_2$

$I_2 = \frac{I_1 E_1}{E_2} = \frac{5 \times 220}{22000} = .05$ A

EXERCISE - 2

Part # I : Multiple Choice

1. ABC

2. ABCD

3. $\cos\phi = \frac{R}{z} = \frac{R}{\sqrt{R^2 + (x_L - x_C)^2}} = 1$

Because $x_L = x_C$

4. BC

5. Resonance frequency $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = 500$ Hz

At resonance

$Z = R$

& $I = \frac{V}{z} = \frac{V}{R}$

L & C are in out of phase.

6. AB

7. AB

8. $P_{avr} = I_{rms} V_{rms} \cos\phi$

$\cos\phi$ can not be more than 1 so power can not be more than 1000.

9. $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(100)^2 + (100 - 200)^2}$

$= 100\sqrt{2}$

$I_{rms} = \frac{V_{rms}}{Z}$

$P_R = I_{rms}{}^2 R$

$P_L = 0$

$P_C = 0$

10. AB

11. AC

12. $I_0 = \frac{V_0}{\omega L} = \frac{10}{\omega \times 5 \times 10^{-3}}$

13. BD

14. AC

Part # II : Assertion & Reason

1. A

2. Statement 1 is false because the given relation is true if all voltages are instantaneous.

3. D

4. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

EXERCISE - 3

Part # I : Matrix Match Type

1. (A) → (r), (B) → (q), (C) → (p), (D) → (q)

2. **(A)** Inductance of a coil depends on its shape and magnetic properties of its core (medium inserted)

(B) Capacitance of capacitor depends on its shape and dielectric properties of medium inserted.

(C) Impedance of coil $\sqrt{R^2+\omega^2L^2}$ depends on resistivity (due to R), shape (for L), magnetic properties of core inserted and also depends on angular frequency ω of external voltage source.

(D) Reactance of capacitor = $\frac{1}{\omega C}$ depends on shape (for C), nature of dielectric medium (for C) and external voltage source (due to ω).

Part # II : Comprehension

Comprehension#1

1. C

2. C_{eq} decreases thereby increasing resonant frequency.

3. Average energy stored $= \frac{1}{2}\ Li^2_{rms}$

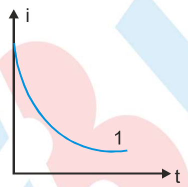
$$= \frac{1}{2}\ (2.4\times10^{-3}\ H)\ .\ (1\ A)^2 = 1.2\ mJ$$

4. B **5.** D

Comprehension#2

1. As current is leading the source voltage, so circuit should be capacitive in nature and as phase difference is not $\frac{\pi}{2}$, it must contain resistor also.

2. A



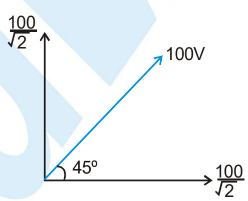
3. For DC circuit

$i = i_0\, e^{-\frac{t}{RC}}$ and RC = 0.01 sec.

Comprehension#3

1. C **2.** B

3. When switch is closed



$V_{rms\ (applied)} = 100$ volts

$V_{peak\ (applied)} = 100\sqrt{2}$

When switch is open

$$f = \frac{1}{2\pi\sqrt{\frac{1}{25\pi}\times\frac{1}{100\pi}}} = \frac{50}{2} = 25\ Hz$$

Resistance $R = X_L = X_C = 2\pi fL = 2\Omega$

Average power consumption

$$= \left(\frac{100}{2}\right)^2 2 = \frac{10000}{2} = 5000\ W.$$

EXERCISE - 4
Subjective Type

1. $\frac{I_o}{e}\sqrt{(e^2-1)/2}$

2. $V_O = \sqrt{2}\ V_{rms} = 220\sqrt{2}$

$$\Delta V = \frac{V_O}{\sqrt{2}}$$

$$\Rightarrow \frac{V_O}{\sqrt{2}} = V_O \sin\omega t$$

$$\Rightarrow 2\pi f\times t = \frac{\pi}{4} \Rightarrow t = 2.5\ ms$$

3. $$I_{rms} = \left[\frac{\int_o^T (a+b\sin\omega t)^2\ dt}{T}\right]^{\frac{1}{2}} = I_{eff} = \left[a^2+\frac{1}{2}b^2\right]^{1/2}$$

4. $V_O = V_{rms}\times\sqrt{2} = 12\sqrt{2}$

5. 0

6. $V_O = 3\times10^6\times\sqrt{2}\times10^{-3} \Rightarrow V_{rms} = \frac{V_O}{\sqrt{2}} = 3\ kV$

7. $P = i\,\varepsilon = [2\sin 250\pi t]\ [10\sin(250\pi t+\pi/3)]$ at

$$t = \frac{2}{3}\times10^{-3} \Rightarrow P = 10\ watt$$

$$\langle P\rangle = i_{rms}\,\varepsilon_{rms}\cos\phi = \frac{2}{\sqrt{2}}\times\frac{10}{\sqrt{2}}\times\cos\pi/3$$

8. 0.72 W

9. $\frac{2.2\sqrt{3}}{\pi}$

10. **(i)** $\left[\frac{1}{\omega C}\times I_{rms}\right]^2 + [20]^2 = [200]^2$

where $I_{rms} = \frac{5}{20}$ & $\omega = 2\pi\times\frac{50}{\pi}\sqrt{11}$

or $[(\omega L)\ I_{rms}]^2 + (20)^2 = 200^2$

(ii) $I_{rms}\times R + 20 = 200$

(iii) does not loss in C and L.

11. **(a)** $I_{rms} = \frac{E_{rms}}{z}$

where $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$

(b) $V_{R\,rms} = I_{rms} R = 30, \quad V_{L\,rms} = I_{rms}(\omega L) = 10$

$V_{C\,rms} = I_{rms}\left(\frac{1}{\omega C}\right) = 50$

12. 25 mJ, 5mJ

13. $I_{rms}^2 \times R = 100$ .... **(1)**

$(1)^2 \times R = 100 \Rightarrow \quad R = 100\,\Omega$

$V_{rms} = I_{rms} Z \Rightarrow \quad 110 = I_{rms} Z$

$110 = 1 \times Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow L = \sqrt{\frac{21}{22}}$ H

14. $Z = 50\sqrt{2}$ ohm, $V_C = 500\sqrt{2}$ volt

and $V_L = 600\sqrt{2}$ volt, $\frac{1}{\sqrt{2}}$

15. Here phase difference $\phi = 360°$ & $\omega = 5000$

at Resonance $C = \frac{1}{\omega^2 L} = \frac{1}{(5000)^2} \times \frac{1}{0.01}$

$R = \frac{V_0}{I_0} = \frac{141.4}{5}$

16. $125\,\Omega$, 288 J

17. **(a)** $\frac{250}{3\pi}$ Hz **(b)** 2 mA

18. 1×10^{-8} henry

19. $R = \frac{24}{6} = 4\,\Omega \Rightarrow I = \frac{12}{4+4} = 1.5$ Amp.

20. Transformer does not work on D.C.

21. zero

22. Give that

$L = 0.1$ H

$C = 500 \times 10^{-6}$ F

$V_{rms} = 230$ volt, $\quad f = \frac{100}{\pi}$ Hz

(a) $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{Z}$

where $Z = \omega L - \frac{1}{\omega c} = 2\pi f L - \frac{1}{2\pi f c}$

$= 20 - 10 = 10$

& $I_O = \sqrt{2} \times I_{rms} = 23\sqrt{2}$

(b) $V_L = I_{rms}(\omega L) \quad V_C = I_{rms}\left(\frac{1}{\omega c}\right)$

(c) $\langle P_L \rangle = I_{rms} V_{rms} \cos\phi$

here $\phi = 90°$ So $\langle P \rangle = 0$

(d) $\langle P_C \rangle = 0$ (e) $\langle P_{Net} \rangle = 0$

23. **(a)** $23\sqrt{2}$ A, 23 A **(b)** 460 volt, 230 volt **(c)** zero **(d)** zero **(e)** zero

24. $f = \frac{50}{100} \times f_r \Rightarrow f = \frac{1}{2} \times \frac{1}{2\pi\sqrt{LC}} \Rightarrow \omega = \frac{1}{2\sqrt{LC}}$

(a) $X = \left|\omega L - \frac{1}{\omega C}\right| = 150$ **(b)** $I_0 = \frac{V_0}{Z} = \frac{150\sqrt{2}}{\sqrt{R^2 + x^2}}$

$P_{CW} = I_{rms} V_{rms} \cos\phi = \frac{I_0 V_0}{2} \times \frac{R}{Z}$

25. **(a)** $150\,\Omega$ **(b)** 1 amp, 75 watt.

26. general equation of V

$V = \frac{V_0}{T/2} t - V_0 = \frac{2V_0}{T} t - V_0$

$$V_{rms} = \left[\frac{\int_0^T V^2 dt}{T}\right]^{\frac{1}{2}} = \left[\frac{\int_0^T \left(\frac{2V_0}{T} t - V_0\right)^2 dt}{T}\right]^{\frac{1}{2}} = \frac{V_0}{\sqrt{3}}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

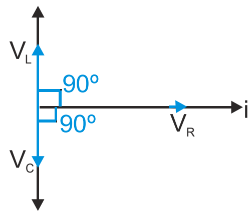
1. $\frac{q^2}{2C} + \frac{1}{2} L I^2 = \frac{Q^2}{2C}$

but $\frac{1}{2} L I^2 = \frac{q^2}{2C}$

So $2\left(\frac{q^2}{2C}\right) = \frac{Q^2}{2C} \Rightarrow q = \frac{Q}{\sqrt{2}}$

2. The core of transformer is laminated to reduce energy loss due to eddy currents.

3. In an LCR series a.c. circuit, the voltage across inductor L leads the current by 90º and the voltage across capacitor C lags behind the current by 90º



Hence, the voltage across LC combination will be zero.

4. The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an a.c. cycle, the net value of current average out to zero. While for the half cycle, the value of current is different at different points. Hence, the alternating current cannot be measured by D.C. ammeter

5. In the condition of resonance

$X_L = X_C$

or $\omega L = \dfrac{1}{\omega C}$ ....(i)

Since, resonant frequency remains unchanged,

so, $\sqrt{LC} =$ constant

or $LC =$ constant

$\therefore\ L_1C_1 = L_2C_2$

$\Rightarrow\ L \times C = L_2 \times 2C \Rightarrow L_2 = \dfrac{L}{2}$

6. Power factor

$= \cos\phi = \dfrac{R}{Z} = \dfrac{12}{15} = \dfrac{4}{5} = 0.8$

7. $\tan\phi = \dfrac{X}{R} = \infty = \dfrac{1}{0} \Rightarrow \quad R = 0$

8. $\tan 30^\circ = \dfrac{X_L}{R} \Rightarrow X_L = \dfrac{R}{\sqrt{3}} = \dfrac{200}{\sqrt{3}}$

$\tan 30^\circ = \dfrac{X_C}{R} \Rightarrow X_c = \dfrac{200}{\sqrt{3}}$

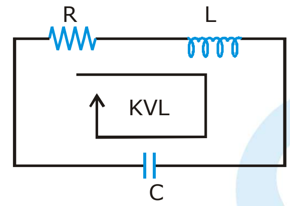
$Z = \sqrt{R + (X_L - X_C)^2} = 200\ \Omega$

$i_{rms} = \dfrac{220}{200} = 1.1$

$P = (i_{rms})^2 \times R = (1.1)^2 \times 200$

$P = 242$ W

9.



$IR + L\dfrac{dI}{dt} - \dfrac{q}{C} = 0$

$L\dfrac{d^2q}{dt^2} = -R\dfrac{dq}{dt} + \dfrac{q}{C}$

comparing with equation of damped oscillation

$d\dfrac{d^2y}{dt^2} = -\gamma\dfrac{dy}{dt} - ky$

The equation of amplitude is $y = Ae^{-bt}$

where $b = \dfrac{\gamma}{2m} = \dfrac{R}{2L}$

$\therefore\ q_{max} = q_0 e^{-\frac{Rt}{2L}}$

$\therefore\ q^2_{max} = q^2_0 e^{-\frac{Rt}{L}}$

$\therefore$ time constant $\tau = \dfrac{R}{L}$

since $L_1 > L_2$

$\tau_1 < \tau_2$

Hence correct graph is 3.

Alternative solution

The value of Q_{max} reduces because of energy dissipation in resistor. As the value of inductor increases the time taken for capacity to discharge or charge increases therefore heat dissipation time decreases. Hence correct graph is 3.

Part # II : IIT-JEE ADVANCED

1. **(i)** here current lead the voltage

$\tan\phi = \dfrac{R}{1/\omega c}$

$\tan\dfrac{\pi}{4} = R\omega C$

$RC = \dfrac{1}{\omega} = \dfrac{1}{100}$

2. Inductive reactance

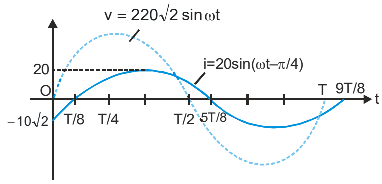
$$X_L = \omega L = (50)(2\pi)(35\times10^{-3}) \approx 11\Omega$$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2+(11)^2} = 11\sqrt{2}\ \Omega$

Given $v_{rms} = 220$ V

Hence, amplitude of voltage

$v_0 = \sqrt{2}\ v_{rms} = 220\sqrt{2}$ V



$\therefore$ Amplitude of current $i_0 = \dfrac{v_0}{Z} = \dfrac{220\sqrt{2}}{11\sqrt{2}}$ or $i_0 = 20$ A

Phase difference $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right) = \dfrac{\pi}{4}$

In L–R circuit voltage leads the currents, Hence, instantaneous current in the circuit is,

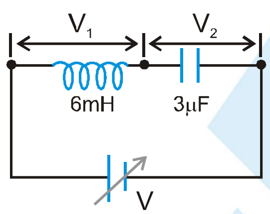
$i = (20A)\sin(\omega t - \pi/4)$

Corresponding i–t graph in shown in figure.

3. $i_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{\omega^2 c^2}}}$

when ω increases, i_{rms} increases so the bulb glows brighter

4. **(p)**

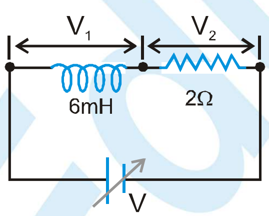


As I is steady state current

$V_1 = 0$; $I = 0$

Hence, $V_2 = V$ So, answer of P $\Rightarrow$ C

(q)



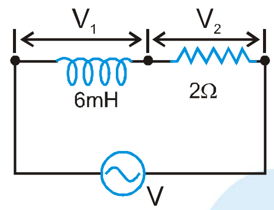
In the steady state ;

$V_1 = 0$ as $\dfrac{dI}{dt} = 0$ $\therefore$ $V_2 = V = IR$

or $V_2 \propto I$ and $V_2 > V_1$

So, answer of q $\Rightarrow$ B, C, D

(r)



Inductive reactance $X_L = \omega L$

$X_L = 6\pi\times10^{-1}\ \Omega$

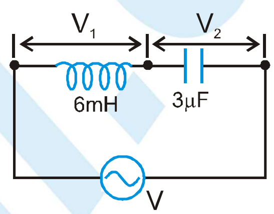
and resistance $= R = 2\Omega$

So, $V_1 = IX_L$

and $V_2 = IR$ Hence, $V_2 > V_1$

So, Answer of r $\Rightarrow$ A,B,D

(s)



Here, $V_1 = IX_L$, where, $X_L = 6\pi\times10^{-1}\ \Omega$

Also, $V_2 = IX_C$, where, $X_C = \dfrac{10^4}{3\pi}$

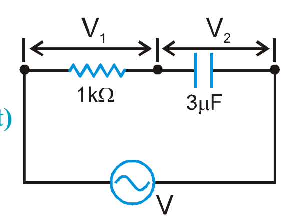
So, $V_2 > V_1$

$V_1 \propto I$

$V_2 \propto I$

So, answer of s $\Rightarrow$ A,B,D

(t)



Here, $V_1 = IR$, where, $R = 1000\ \Omega$, $X_C = \dfrac{10^4}{3\pi}\ \Omega$

$V_2 = IX_C$, where, $X_C = \dfrac{10^4}{3\pi}\ \Omega$

So, $V_2 > V_1$

and $V_1 \propto I$

$V_2 \propto I$

So, answer of t $\Rightarrow$ A,B,D

Ans. (A) – r,s,t ; **(B)** – q,r,s,t ; **(C)** – p,q ; **(D)** – q,r,s,t

Note : For circuit 'p' :

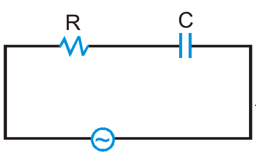
$$V - \frac{Ldi}{dt} - \frac{q}{C} = 0 \quad \text{or} \quad CV = CL\frac{di}{dt} + q \quad \text{or}$$

$$0 = LC\frac{d^2i}{dt^2} + \frac{dq}{dt} \quad \text{or} \quad \frac{d^2i}{dt^2} = -\frac{1}{LC}\frac{dq}{dt}$$

So, $i = i_0 \sin\left(\frac{1}{\sqrt{LC}}t + \varphi_0\right)$

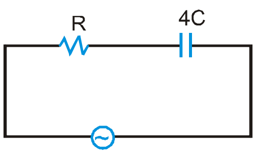
As per given conditions, there will be no steady state in circuit 'p'. So it should not be considered in options

5. Case I



$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Case II



$I_R^A = \frac{V}{Z} \Rightarrow Z' < Z$

$I_R^B = \frac{V}{Z'} \Rightarrow I_R^A < I_R^B \Rightarrow V_R^A < V_R^B$

So. $V_C^A > V_C^B \quad \rightarrow V_R^2 + V_C^2 = V_0^2$

6.

W = 500 rad/s

$$Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2(1.25)$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2 + \frac{R^2}{4}$$

$$\Rightarrow \frac{1}{\omega C} = \frac{R}{2} \quad \Rightarrow CR = \frac{2}{\omega} = \frac{2}{500} \text{ sec.}$$

$$= \frac{2}{500} \times 10^3 \text{ ms} = \frac{2 \times 1000}{500} \text{ ms} = 4 \text{ ms}$$

7. $C = 100\,\mu F,\ \frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})}$

$X_C = 100\,\Omega,\quad X_L = \omega L = (100)(.5) = 50\,\Omega$

$Z_1 = \sqrt{x_C^2 + 100^2} = 100\sqrt{2}\Omega$

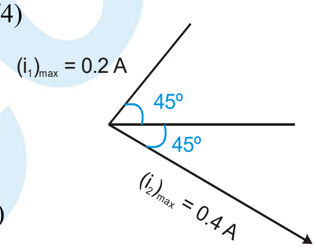
$Z_2 = \sqrt{x_L^2 + 50^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}$

$\varepsilon = 20\sqrt{2}\sin\omega t$

$i_1 = \frac{20\sqrt{2}}{100\sqrt{2}}\sin(\omega t + \pi/4)$

$i_1 = \frac{1}{5}\sin(\omega t + \pi/4)$

$I_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin(\omega t - \pi/4)$



$I = \sqrt{(.2)^2 + (.4)^2} = (.2)\sqrt{1+4} = \frac{1}{5}\sqrt{5} = \frac{1}{\sqrt{5}}$

$(I)_{rms} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \approx 0.3\text{ A}$

$(V_{100\Omega})_{rms} = (I_1)_{rms}) \times 100$

$= \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\text{ V}$

$V_{50\Omega})_{rms} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\text{ V}$

Since $I_{rms} \approx 0.3$ A so A may or may not be correct.

8. CD

MOCK TEST

1. From Kirchoff's current law,

$i_3 = i_1 + i_2 = 3\sin\omega t + 4\sin(\omega t + 90°)$

$= \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90°}\ \sin(\omega t + \phi)$

where $\tan\phi = \frac{4\sin 90°}{3 + 4\cos 90°} = \frac{4}{3}$

$\therefore i_3 = 5\sin(\omega t + 53°)$

2. $\varepsilon = \varepsilon_0 \sin\omega t$

If $i = i_m \sin(\omega t - \phi)$

then $v_C = \left(\frac{1}{\omega C}\right) i_m \sin(\omega t - \phi - \pi/2)$

and $v_L = (\omega L)\, i_m \sin(\omega t - \phi + \pi/2)$.

So $v_C + v_L + v_R = \varepsilon_0 \sin \omega t$

$\Rightarrow 0 + v_R = \varepsilon_0 \sin \omega t \Rightarrow v_R = \varepsilon_0 \sin \omega t$

Also $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R} = 0$, so $i = i_m \sin \omega t$

Hence answer is (B)

$$z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = R$$

3. When capacitance is removed

$\tan\theta = \dfrac{\omega L}{R}$ or $\omega L = 100 \tan 60^\circ$ ...(1)

when inductance is removed

$\tan\phi = \dfrac{1}{(\omega C)(R)}$ or $\dfrac{1}{\omega C} = 100 \tan 60^\circ$ ...(2)

From equation (1) & (2) $\omega L = \dfrac{1}{\omega C}$

So it is condition of resonance.

so $z = R = 100\ \Omega$

$I = v/R = 200/100 = 2A$

Power $P = I^2 R = 4 \times 100 = 400$ W

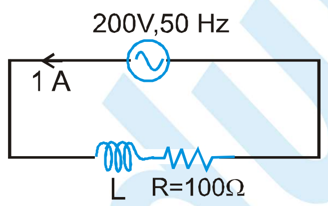
4. Current leads emf so circuit is R – C.

$\tan\phi = x_C/R$, $\phi = 45^\circ$, $R = 1000\ \Omega$, $\omega = 100$

since $\tan 45^\circ = \dfrac{1}{\omega CR}$ So $C = 10\ \mu F$

5. From the rating of the bulb , the resistance of the bulb can be calculated.

$$R = \frac{V_{rms}^{\ 2}}{P} = 100\ \Omega$$



For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $I_{rms} = \dfrac{V_{rms}}{Z}$

$\therefore\ 1 = \dfrac{200}{\sqrt{100^2 + (2\pi 50 L)^2}}$ $L = \dfrac{\sqrt{3}}{\pi}$ H

6 According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \quad ...(1)$$

and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \quad ...(2)$$

Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}.$$

i.e., $$\frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

7. $V_{rms} = \sqrt{16^2 + 20^2} = 25.6$ V

8. $i = 3 \sin \omega t + 4 \cos \omega t$

$= 5\left[\frac{3}{5}\sin \omega t + \frac{4}{5}\cos \omega t\right]$

$= 5\,[\sin(\omega t + \delta)]$ ....(1)

$\Rightarrow$ rms value $= \dfrac{5}{\sqrt{2}}$ $\Rightarrow$ mean value $= \dfrac{\int_{T_1}^{T_2} i\,dt}{\int_{T_1}^{T_2} dt}$

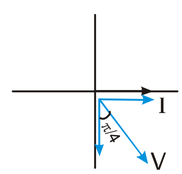
$\therefore$ Initial value of time is not given hence the mean value will be different for various time intervals.

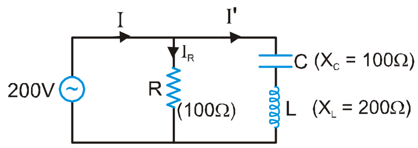
If voltage applied is $V = V_m \sin\omega t$ then i given by equation (1) indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R & C.

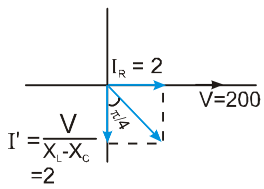
Hence (C).

9. $v = v_0 \sin(\omega t + \pi/4) = v_0 \cos(\omega t - \pi/4)$

Since V lags current, an inductor can bring it in phase with current.



10. 



$$I_R = \frac{V}{R} = \frac{200}{100} = 2A \quad I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{{I_R}^2 + I'^2} = 2\sqrt{2} \text{ Amp.}$$

11. Resultant voltage = 200 volt

Since V_1 and V_3 are out of phase 180º, the resultant voltage is equal to V_2

$\therefore \quad V_2 = 200$ volt

12. The circuit will have inductive nature if $\omega > \frac{1}{\sqrt{LC}} \left(\omega L > \frac{1}{\omega C}\right)$.

Hence A is false. Also if circuit has inductive nature the current will lag behind voltage. Hence D is also false.

If $\omega = \frac{1}{\sqrt{LC}} \left(\omega L = \frac{1}{\omega C}\right)$ the circuit will have resistance nature. Hence B is false

$$\text{Power factor } \cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1 \text{ it } \omega L$$

$= \frac{1}{\omega C}$. Hence C is true.

13. $X_L = X_C$ at resonance

$\therefore \quad \frac{X_L}{X_C} = 1.$ for both circuits

14. Since, $\quad \cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also $\cos\theta$ can never be greater than 1)

Hence (C) is wrong.

Also, $\quad Ix_C > Ix_L \quad \Rightarrow \quad x_C > x_L$.

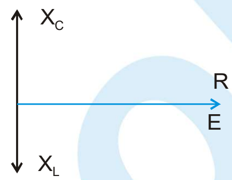
$\therefore$ Current will be leading.

In a LCR circuit

$$v = \sqrt{(v_L - v_C)^2 + {v_R}^2} = \sqrt{(6-12)^2 + 8^2}$$

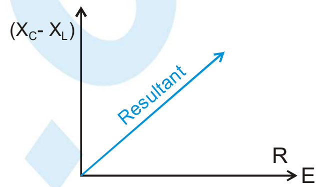
v = 10 ; which is less than voltage drop across capacitor.

15. If we have all R,L and C then I vs. E will be :



To obtain a leading phase difference of π/4 :

if $X_L < X_C$ and we use all R, L and C in the circuit, then the resultant graph will be :



which can give a leading phase difference of π/4 :

Similarly, if we have only resistance and capacitor then we can obtain a phase difference of π/4 (leading) for suitable values of I , X_C and R. But we cannot obtain a leading phase difference of π/4 if we use only capacitor (phase difference of π/2), or only (inductor and resistor) (phase difference of π/2), or only resistor (phase difference of 0).

16. (i) $V = \frac{V_0}{T/4} t \qquad V = \frac{4V_0}{T} t$

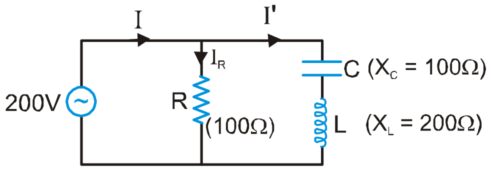
$$\Rightarrow V_{rms} = \sqrt{\langle V^2 \rangle} = \frac{4V_0}{T} \sqrt{\langle t \rangle}$$

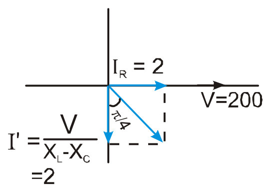
$$= \frac{4V_0}{T} \left\{ \frac{\int_0^{T/4} t^2\, dt}{\int_0^{T/4} dt} \right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

17. $\langle V \rangle = \frac{\int_0^T v\,dt}{\int_0^T dt} = 0.$

18. $z = \sqrt{3^2 + 4^2} = 5$

19.





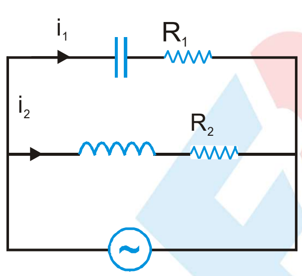
$$I_R = \frac{V}{R} = \frac{200}{100} = 2A$$

$$\Rightarrow \quad I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{{I_R}^2 + I'^2} = 2\sqrt{2} \text{ Amp.}$$

20. $$i_{1_{rms}} = \frac{E_{rms}}{\sqrt{X_C^2 + R_1^2}} = \frac{130}{13} = 10\text{ A}$$

$$i_{2_{rms}} = \frac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13\text{ A}$$



Power dissipated = $i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6$

$= i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6$

= power delivered by battery

$= 500 + 169 \times 6 = 1514$ watt

21. $P = VI$

For secondary :

$$V_2 = \frac{P_2}{I_2} = \frac{500}{12.5} = 40 \text{ volts}$$

For an ideal transformer (100% efficient)

$$P_{input} = P_{output}$$

$$\Rightarrow \quad V_1 I_1 = V_2 I_2$$

$$\Rightarrow \quad I_1 = \frac{V_2 I_2}{V_1} = \frac{40(12.5)}{40 \times 5} = 2.5\text{ A}$$

$$\left[\because \frac{n_1}{n_2} = \frac{V_1}{V_2} \Rightarrow \frac{5}{1} = \frac{V_1}{40}\right].$$

22. It is apparent from the graph that emf attains its maximum value before the current does, therefore current lags behind emf in the circuit. Nature of the circuit is inductive.

Value of power factor cos ϕ increases by either decreasing L increasing C.

23. Since the circuit is at resonance so current in the circuit is in the phase with applied voltage.

Voltage across inductor leads the current by $\pi/2$ and across a capacitor lags by $\pi/2$. So the voltage across resistance is lagging by 90° than the voltage across capacitor.

24. $V_1 = V_2 \Rightarrow x_L = x_C$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = 125 \text{ Hz}$$

$$I_0 = \frac{v_0}{R} = \frac{200}{100} \qquad (\because X = 0 \quad \therefore \; Z = R) \; = 2A$$

$V_1 = V_2 = IX_L = I.(\omega L)$

$= 2 \times 2\pi \times 125 \times 2/\pi = 1000$ volt

25. $\varepsilon = \varepsilon_0 \sin \omega t$

If $i = i_m \sin(\omega t - \phi)$

then $v_C = \left(\frac{1}{\omega C}\right) i_m \sin(\omega t - \phi - \pi/2)$

and $v_L = (\omega L)\, i_m \sin(\omega t - \phi + \pi/2)$.

So $v_C + v_L + v_R = \varepsilon_0 \sin \omega t$

$\Rightarrow \quad 0 + v_R = \varepsilon_0 \sin \omega t \Rightarrow v_R = \varepsilon_0 \sin \omega t$

Also $\tan\phi = \dfrac{\omega L - \frac{1}{\omega C}}{R} = 0,$

so $i = i_m \sin \omega t$

Hence answer is $z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = R$

26. Statement–1 is true but Statement–2 is false

Both A.C. and D.C. produce heat, which is proportional to square of the current. The reversal of direction of current in A.C. is immaterial so far as production of heat is concerned.

27. Statement 1 is false because the given relation is true if all voltages are instantaneous.

28. In resonance condition when energy across capacitor is maximum, energy stored in inductor is zero, vice versa is also true. Hence statement 1 is false.

29. When current through inductor decreases, the magnetic energy stored in inductor decreases and this energy is absorbed by the ac source.

30. **(D)** **31.** **(C)** **32.** **(A)**

33. to 35. : When connected with the DC source

$$R = \frac{12}{4} = 3\ \Omega$$

When connected to ac source $I = \frac{V}{Z}$

$$\therefore 2.4 = \frac{12}{\sqrt{3^2 + \omega^2 L^2}} \Rightarrow L = 0.08\ H$$

Using $P = I_{rms} V_{rms} \cos\phi = \frac{V_{rms}^2}{Z} \cos\phi$

$$= \frac{V_{rms}^2 \ R}{R^2 + (\omega L - \frac{1}{\omega C})^2} = 24\ W$$

33. Let at an instant $v_R = (V_R)_m \sin(\omega t + \theta)$

$v_R = (V_R)_m \sin(\omega t + \theta)$

$\therefore$ $2 = 4\sin(\omega t + \theta)$

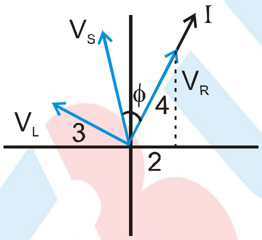
$\sin(\omega t + \theta) = ½$

$\therefore$ $\omega t + \theta = 30°$.

Since V_L is 90° ahead of V_R

$v_L = (V_L)_m \sin(\omega t + \theta + 90)$

$\therefore$ $|(V_L)_m| = 3\cos 30°$



34. From phasor diagram

$$(V_S)_m = \sqrt{(V_R)_m^{\ 2} + (V_L)_m^{\ 2}} = 5\text{ volt.}$$

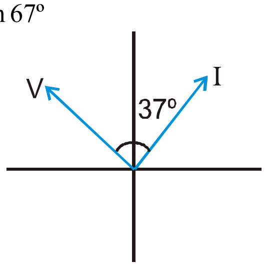
$$(V_S)_m = \sqrt{(V_R)_m^{\ 2} + (V_L)_m^{\ 2}} = 5\text{ volt.}$$

$$\tan\phi = \frac{V_L)_m}{V_R)_m} = \frac{3}{4} \qquad \therefore \phi = 37°$$

$\therefore |v_S| = |(V_S)_m \sin(\omega t + \theta + 37°)|$

$= 5|\sin(30° + 37°)| = 5\sin 67°$

35. From phasor diagram it is clear that instantaneous current will decrease or increases, we cannot say.



36. Current drawn is maximum at resonant angular frequency. $L_{eq} = 4\ mH, C_{eq} = 10\ \mu F$

$L_{eq} = 4\ mH, C_{eq} = 10\ \mu F$

$$\omega = \frac{1}{\sqrt{LC}} = 5000\text{ rad/s}$$

37. C_{eq} decreases thereby increasing resonant frequency.

38. At resonance $i_{rms} = \frac{100}{100} = 1A$

Power supplied $= V_{rms} I_{rms} \cos\phi$

$(\phi = 0$ at resonance $\phi = 0)$ $\quad P = 100\ W$

39. Average energy stored $= \frac{1}{2} Li_{rms}^2$

$$= \frac{1}{2}(2.4 \times 10^{-3}\ H)\ .\ (1\ A)^2 = 1.2\ mJ$$

40. As 1μs time duration is very less than time period T at resonance, thermal energy produced is not possible to calculate without information about start of the given time duration.

41. **(A)** For sinusoidal curve $i_{rms} = \frac{i_0}{\sqrt{2}}$

(A) sine $i_{rms} = \frac{i_0}{\sqrt{2}}$

(B) $$i_{rms}^2 = \frac{\int_0^T i^2\,dt}{T} = \frac{4\int_0^{T/4} i^2\,dt}{T}$$

$$= \frac{\int_0^{T/4} i^2\,dt}{\frac{T}{4}} = \frac{\int_0^{T/4}\left(\frac{i_0 t}{T/4}\right)^2 dt}{\frac{T}{4}} = \frac{i_0^2}{\left(\frac{T}{4}\right)^3}\int_0^{T/4} t^2\,dt = \frac{i_0^2}{3}$$

$$\Rightarrow i_{rms} = \frac{i_0}{\sqrt{3}}$$

For positive half cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{½(i_0)(T/2)}{(T/2)} = \frac{i_0}{2}$$

Full cycle average current is zero.

(C) For positive half cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0 \quad \Rightarrow \quad i_{rms} = \left[\frac{\int_0^{T/2} i^2\,dt}{T/2}\right]^{1/2} = i_0$$

(D) For full cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0 (T/2) + 0}{T} = \frac{i_0}{2}$$

$$\Rightarrow i_{rms} = \left[\frac{\int_0^{T/2} i^2\,dt}{T/2}\right]^{1/2} = i_0$$

42. (a) $\tan\phi = \frac{1/\omega C}{R} \Rightarrow \phi = \frac{\pi}{4}$, current leads source voltage because reactance is capacitive

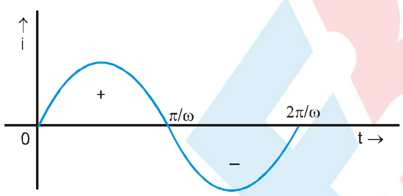
(b) Pure inductive circuit $\phi = \pi/2$, current lags behind source voltage because reactance is inductive

(c) as $R = 0$, $\tan\phi = \infty$

$\phi = \pi/2$, current leads source voltage because reactance is capacitive

(d) $\tan\phi = \frac{\omega L}{R} = 1 \Rightarrow \quad \phi = \frac{\pi}{4}$, current lags behind source voltage because reactance is inductive

43. $$\langle i \rangle = \frac{\int_0^{\frac{2\pi}{\omega}} I_m \sin\omega t\,dt}{\frac{2\pi}{\omega}} = \frac{\frac{I_m}{\omega}\left(1 - \cos\omega\frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$$



It can be seen graphically that the area of $i - t$ graph of one cycle is zero.

$\therefore \quad \langle i \rangle$ in one cycle = 0.

44. $$i_{rms} = \sqrt{\frac{\int_0^{\frac{2\pi}{\omega}} I_m^{\,2} \sin^2\omega t\,dt}{\frac{2\pi}{\omega}}} = \sqrt{\frac{I_m^{\,2}}{2}} = \frac{I_m}{\sqrt{2}}$$

45. Av. electric field energy $= \left(\frac{1}{2}CV_0^2\right) = 25 \times 10^{-3}$ J

$\therefore \quad \frac{1}{2} \times c.I^2_{rms} \times \frac{1}{2\pi^2 v^2 c^2} = 25 \times 10^{-3}$ J $\quad \therefore C = 20\ \mu F$

Av. magnetic energy $\left(\frac{1}{2}LI^2_{rms}\right)$

$\therefore \quad L = \frac{2 \times 5 \times 10^{-3}}{(.10)^2} \Rightarrow L = 1$ henry

$V_R = I_{rms}.R \qquad V_C = I_{rms} X_C \qquad V_L = I_{rms} \times L$

$= (.10).300$

$$= (.10) \times \frac{1}{2\pi\left(\frac{50}{\pi}\right).20 \times 10^{-6}} = (.10) \times 2\pi \times \frac{50}{\pi}\ (1)$$

$= V_R = 30\,V \qquad V_C = 50\,V \qquad V_L = 10\,V$

rsm voltate of source $E_{rms} = \frac{50}{\sqrt{2}}$

$\therefore \quad E_{rms} = 35.36\,V$

Hence $E_{rms} \neq V_R + V_C + V_L \quad E_{rms} < V_R + V_C + V_L$

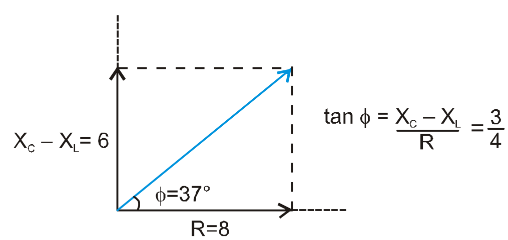
46. (a) impedance of circuit $= \sqrt{R^2 + (X_C - X_L)^2}$

$Z = \sqrt{8^2 + (8-2)^2} = 10\Omega$

(b) The current leads in phase by

$(\because X_C > X_L) \quad \phi = 37°$

$\therefore \quad i = \frac{10\cos(100\pi t + 37°)}{Z} = \cos(100\pi t + 37°)$



The instantaneous potential difference across A B is

$= I_m (X_C - X_L) \cos(100\pi t + 37° - 90°)$

$= 6\cos(100\pi t - 53°)$

The instantaneous potential difference across A B is half of source voltage.

$\Rightarrow \quad 6\cos(100\pi t - 53°) = 5\cos 100\pi t$

solving we get $\cos 100\pi t = \frac{1}{\sqrt{1 + (7/24)^2}} = \frac{24}{25}$

$\therefore \quad$ instantaneous potential difference $= 5 \times \frac{24}{25} = \frac{24}{5}$ volts