SOLVED EXAMPLES

Ex.1 A parallel-plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of are A/2 parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.

Sol. Suppose the charge on the capacitor at time t is Q.The electric field between the plates of the capacitor is $E = \frac{Q}{\varepsilon_0 A}$.

The flux through the area considered is

$$\Phi_E = \frac{Q}{\varepsilon_0 A} \cdot \frac{A}{2} = \frac{Q}{2\varepsilon_0}$$

The displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \left(\frac{1}{2\varepsilon_0}\right)\frac{dQ}{dt} = \frac{i}{2}$$

Ex.2 A plane electromagnetic wave propagating in the x-direction hass a wavelength of 5.0 mm. The electric field is in the y-direction and its maximum magnitude is 30 V m^{-1}. Write suitable equations for the electric and magnetic fields as a function of x and t.

Sol. The equation for the electric and the magnetic fields in the wave may be written as

$$E = E_0 \sin \omega\left(t - \frac{x}{c}\right)$$

$$B = B_0 \sin \omega\left(t - \frac{x}{c}\right)$$

We have,

$$\omega = 2\pi v = \frac{2\pi}{\lambda} c$$

Thus, $E = E_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$

$$= (30\ \text{V m}^{-1}) \sin\left[\frac{2\pi}{5.0\ \text{mm}}(ct - x)\right]$$

The maximum magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{30\ \text{V m}^{-1}}{3 \times 10^8\ \text{ms}^{-1}} = 10^{-7}\ \text{T}$$

So, $B = B_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$

$$= (10^{-7}\ \text{T}) \sin\left[\frac{2\pi}{5.0\ \text{mm}}(ct - x)\right].$$

The magnetic field is along the z-axis.

Ex.3 A light beam travelling in the x-direction is described by the electric field $E_y = (300\ V\ m^{-1}) \sin \omega(t - x/c)$. An electron is constrained to move along the y-direction with a speed of $2.0 \times 10^7\ m\ s^{-1}$. Find the maximum electric force and the maximum magnetic force on the electron.

Sol. The maximum electric field is $E_0 = 300\ V\ m^{-1}$. The maximum magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{300 Vm^{-1}}{3 \times 10^8 ms^{-1}} = 10^{-6} T$$

along the z-direction.

The maximum electric force on the electron is

$$F_e = qE_0 = (1.6 \times 10^{-19}\ C) \times (300\ V\ m^{-1})$$
$$= 4.8 \times 10^{-17}\ N.$$

The maximum magnetic force on the electron is

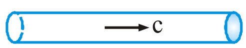
$$F_b = \left|q\vec{v} \times \vec{B}\right|_{max} = qvB_0$$

$= (1.6 \times 10^{-19}\ C) \times (2.0 \times 10^7\ ms^{-1}) \times (10^{-6}\ T)$

$= 3.2 \times 10^{-18}\ N.$

Ex.4 Find the energy stored in a 60 cm length of a laser beam operating at 4mW .

Sol. :



60 cm

The time taken by the electromagnetic wave to move through a distance of 60 cm is $t = \frac{60 cm}{c} = 2 \times 10^{-9} s$. The energy contained in the 60 cm length passes through a cross-section of the beam in 2×10^{-9} s. But the energy passing through any cross section in 2×10^{-9} s is

$$U = (4\ mW) \times (2 \times 10^{-9}\ s)$$
$$= (4 \times 10^{-3}\ Js^{-1}) \times (2 \times 10^{-9}\ s)$$
$$= 8 \times 10_{-12}\ J.$$

Ex.5 Find the amplitude of the electric field in a parallel beam of light of intensity $2.0\ Wm^{-2}$.

Sol. The intensity of a plane electromagnetic wave is

$$I = u_{av} c = \frac{1}{2} \varepsilon_0 E_0^2 c$$

or, $$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

$$= \sqrt{\frac{2 \times (2.0\ W\ m^{-2})}{(8.85 \times 10^{-12} C^2 N^{-1} m^{-2}) \times (3 \times 10^8 ms^{-1})}}$$

$= 38.8\ N\ C^{-1}.$

Exercise # 1 [Single Correct Choice Type Questions]

1. The fundamental source of e. m. waves
(A) is varying magnetic field **(B)** constant magnetic and electric fields
(C) are continous oscillations of electric charge **(D)** is planets

2. The displacement current was first postulated by
(A) Ampere **(B)** Maxwell **(C)** Hertz **(D)** Marconi

3. An accelerated electric charge emits
(A) β – rays **(B)** γ – rays **(C)** e.m. waves **(D)** none of the above

4. The speed of e. m. waves is given by the relation
(A) $\mu_0 \varepsilon_0$ **(B)** $\sqrt{\mu_0 \varepsilon_0}$ **(C)** $1/\mu_0 \varepsilon_0$ **(D)** $1/\sqrt{(\mu_0 \varepsilon_0)}$

5. Electromagnetic waves in nature are
(A) longitudinal **(B)** longitudinal stationary **(C)** transverse **(D)** transverse – stationary

6. An accelerated charge
(A) emits an electromagnetic wave **(B)** does not emits electromagnetic wave
(C) produces a gravitational field **(D)** none of the above

7. Electromagnetic waves
(A) are longitudinal waves
(B) travel in free space at the speed of light
(C) are produced by charges moving with uniform velocity
(D) travel with the same speed in all media

8. Choose the only wrong statement from the following about electromagnetic waves
(A) are transverse **(B)** travels free space at the speed of light
(C) are produced by accelerating charges **(D)** travel with the same speed in all media

9. In an electromagnetic wave, electric field E and magnetic field B are
(A) mutually perpendicular to each other **(B)** all parallel
(C) at 30^0 to each other **(D)** at 60^0 to each other

10. Electromagnetic wave obey the principle of
(A) superposition **(B)** interference **(C)** 1 and 2 both **(D)** none of the above

11. If E and B be the electric and magnetic fields of electromagnetic waves, then the direction of propagation of e. m. wave is along the direction of
(A) E **(B)** B **(C)** $E \times B$ **(D)** None of the above

12. Which of the following pairs of space and time varying E and B fields would generate a plane electromagnetic wave travelling in the Z– direction
(A) E_x, B_y **(B)** E_y, B_x **(C)** E_x, B_z **(D)** E_z, B_x

13. Choose the correct statement about electromagnetic waves
(A) they are supersonic waves **(B)** they are the electric charged particles
(C) they travel with the speed of light **(D)** they can only be produced in laboratory.

14. Hertz produced electromagnetic wave by using
(A) L C R circuit **(B)** C R circuit **(C)** L C circuit **(D)** None of the above

15. The shortest wavelength is for
(A) γ – rays **(B)** x – rays **(C)** ultraviolet rays **(D)** microwaves

16. Visible range of wavelength in cm is
(A) 3×10^{-6} to 10^{-10} **(B)** 7×10^{-5} to 4×10^{-5}
(C) 4×10^{-5} to 3×10^{-6} **(D)** 6×10^{4} to 1.5×10^{3}

17. Which has the longest wavelength `
(A) infrared light **(B)** ultraviolet light **(C)** radio–waves **(D)** X–rays

18. The earth's atmosphere is richer in
(A) infra - red radiation **(B)** ultraviolet radiations **(C)** visible radiations **(D)** blue colour radiations

19. Microwaves are used for
(A) television **(B)** radio transmission **(C)** radar system **(D)** all the above

20. Ionosphere is mainly composed of
(A) electrons and positive ions **(B)** ozone gas
(C) nitrogen and oxygen **(D)** b and c both

21. The upper atmosphere layer is known as
(A) troposphere **(B)** mesosphere **(C)** ionospere **(D)** chromosphere

22. Greenhouse effect keeps the earth surface
(A) cold in night **(B)** dusty and cold **(C)** warm in night **(D)** moist in night

23. Greenhouse effect is due to
(A) visible radiations **(B)** red colour radiation **(C)** violet colour radiation **(D)** infra - red radiation

24. Ozone layer in atmosphere exist at the height of
(A) 20 km **(B)** 50 km **(C)** 120 km **(D)** 150 km

25. Ozone layer exist in
(A) ionosphere **(B)** mesosphere **(C)** troposphere **(D)** stratosphere

26. Ozone layer protectes the living cells from
(A) ultraviolet radiations **(B)** unfra –red radiations
(C) X–rays **(D)** all the radiations

27. The ionosphere does not allow to pass the waves which are termed as
(A) microwaves **(B)** visible light waves
(C) 1 and 2 both **(D)** amplitude modulated waves

28. Practically ozone layer absorbs the radiation of wavelength
(A) less than 3×10^{-7} m **(B)** greater than 3×10^{-7} m
(C) equal to 3×10^{-7} m **(D)** all the above

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. The waves which can travel directly along surface of the earth are known as
(A) ground waves (B) X–rays
(C) α–rays (D) sky waves

2. The ionosphere bends the e. m. waves having the frequencies
(A) less than 40 MHz (B) beyond 40 MHz
(C) nothing is certain (D) depends on the moisture present

3. The S.I unit of displacement current is
(A) H (B) A (C) Fm^{-1} (D) C

4. Transmission of T. V. signals from the surface of the moon can be received on earth. But transmitted T. V. Signals from Delhi can not be received beyond 110 km distance. The reason is
(A) there is no atmosphere on the moon
(B) strong gravitational effect on T. V. signals
(C) T. V. signals travel along a straight line, they do not follow the curvature of earth
(D) there is atmosphere around the earth

5. The number of radio frequency carrier waves transmitted by a television transmitter is
(A) three (B) two (C) one (D) four

6. The speed of electromagnetic waves is independent of
(A) wavelength (B) frequency (C) intensity (D) medium, in which it travels

7. An electromagnetic radiation of frequency ν, wavelength λ, travelling with velocity c in air, enters a glass slab of refractive index μ. The frequency, wavelength and velocity of light in the glass slab will be respectively :

(A) $\frac{\nu}{\mu}, \frac{\lambda}{\mu}$ and $\frac{c}{\mu}$ (B) $\nu, \frac{\lambda}{\mu}$ and $\frac{c}{\mu}$

(C) $\nu, 2\lambda$ and $\frac{c}{\mu}$ (D) $\frac{2\nu}{\mu}, \frac{\lambda}{\mu}$ and c

8. If ε_0 and μ_0 are the electric permittivity and magnetic permeability in free space, ε and μ are the corresponding quantities in a medium, then index of refraction of the medium is

(A) $\sqrt{\frac{\varepsilon_0\mu}{\varepsilon\mu_0}}$ (B) $\sqrt{\frac{\varepsilon}{\varepsilon_0}}$ (C) $\sqrt{\frac{\varepsilon_0\mu_0}{\varepsilon\mu}}$ (D) $\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$

9. Dimension of $\varepsilon_0\mu_0$ is :
(A) LT^{-1} (B) $L^{-1}T$ (C) L^2T^{-2} (D) $L^{-2}T^2$

10. For television transmission, the frequency employed is normally in the range
(A) 30–300 MHz (B) 30–300 GHz (C) 300–300 kHz (D) 30–300 Hz

11. Red light differs from blue light in its
(A) speed. **(B)** frequency **(C)** intensity **(D)** amplitude

12. If an electromagnetic wave propagating through vacuum is described by
$E = E_0 \sin(kx - \omega t)$; $B = B_0 \sin(kx - \omega t)$,
(A) $E_0 k = B_0 \omega$ **(B)** $E_0 B_0 = \omega k$ **(C)** $E_0 \omega = B_0 k$ **(D)** $E_0 B_0 = \omega / k$

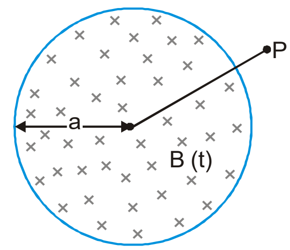
13. The electromagnetic waves used in the telecommunication are
(A) ultraviolet **(B)** infra-red **(C)** visible **(D)** microwaves.

14. A magnetic field can be produced by
(A) A charge at rest only **(B)** A moving charge only
(B) a changing electric field **(D)** both by (B) and (C)

15. Which of the following is independent of wavelength ?
(A) k **(B)** ω **(C)** ωk **(D)** k / ω

16. Finger prints on a piece of paper may be detected by sprinkling flourescent powder on the paper and then looking it into
(A) dark-light **(B)** sun-light **(C)** Infra-red-light **(D)** ultraviolet light

17. A uniform but time varying magnetic field B (t) exists in a circular region of radius a is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region.



(A) is zero **(B)** decreases as $1/r$
(C) increases as r **(D)** decreases as $1/r^2$

18. If $\in_0$ and μ_0 represent the permittivity and permeability of vacuum and $\in$ and μ represent the permittivity and permeability of medium, then refractive index of the medium is given by **[CBSE PMT 1997, 2000]**

(A) $\sqrt{\dfrac{\mu_0 \in_0}{\mu \in}}$ **(B)** $\sqrt{\dfrac{\mu \in}{\mu_0 \in_0}}$ **(C)** $\sqrt{\dfrac{\in}{\mu_0 \in_0}}$ **(D)** $\sqrt{\dfrac{\mu_0 \in_0}{\mu}}$

Part # II [Assertion & Reason Type Questions]

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it . Of the statements mark the correct answer as

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

1. **Statement-1:** Sound waves are not electromagnetic waves.
Statement-2: Sound waves require a material medium for propagation.

2. **Statement-1 :** Displacement current arises on account of change in electric flux.
Statement-2 : $I_d = \epsilon_0 \left(\frac{d\Phi_E}{dt}\right)$

3. **Statement-1:** A changing electric field produces a magnetic field
Statement-2: A changing magnetic field produces an electric field.

4. **Statement-1:** In an e.m. wave magnitude of magnetic field vector $\vec{B}$ is much smaller than the magnitude of vector $\vec{E}$.
Statement-2: This is because in an e.m. wave $E/B = c = 3 \times 10^8$ m / s.

5. **Statement-1:** Electromagnetic waves exert pressure called radiation pressure.
Statement-2: This is because they carry energy.

6. **Statement-1:** Electromagnetic waves are transverse in nature
Statement-2: The electric and magnetic fields of an e.m. wave are perpendicular to each other and also perpendicular to the direction of wave propagation.

Exercise # 3 [Subjective Type Questions]

1. Show that the dimensions of the displacement current $\varepsilon_0 \dfrac{d\varphi_E}{dt}$ are that of an electric current.

2. A point charge is moving along a straight line with a constant velocity v. Consider a small area A perpendicular to the direction of motion of the charge (figure). Calculate the displacement current through the area when its distance from the charge is x. The value of x is not large so that the electric field at any instant is essentially given by Coulomb's law.

3. A parallel-plate capacitor having plate-area A and plate separation d is joined to a battery of emf $\in$ and internal resistance R at t = 0. Consider a plane surface of area A/2, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time.

4. Consider the situation of the previous problem. Define displacement resistance $R_d = V/i_d$ of the space between the the plates where V is the potential difference between the plates and i_d is the displacement current. Show that R_d varies with time as

$$R_d = R(e^{t/\tau} - 1).$$

5. Using $B = \mu_0 H$ find the ratio E_0/H_0 for a plane electromagnetic wave propagating through vacuum. Show that it has the dimension of electric resistance. This ratio is a universal constant called the impedance of free space.

6. The sunlight reaching the earth has maximum electric field of 810 V m^{-1}. What is the maximum magnetic field in this light?

7. The magnetic field in a plane electromagnetic wave is given by

$$B = (200\ \mu T)\sin[(4.0 \times 10^{15}\ s^{-1})(t - x/c)].$$

Find the maximum electric field and the average energy density corresponding to the electric field.

8. A laser beam has intensity 2.5×10^{14} W m^{-2}. Find the amplitudes of electric and magnetic fields in the beam.

9. The intensity of the sunlight reaching the earth is 1380 W m^{-2}. Assume this light to be a plane, monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.

Exercise # 4 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. Infrared radiation is detected by **[AIEEE 2002]**

(1) spectrometer **(2)** pyrometer **(3)** nanometer **(4)** photometer

2. Dimensions of $\frac{1}{\mu_0 \in_0}$, where symbols have their usual meanings, are **[AIEEE 2003]**

(1) $[L^{-1} T]$ **(2)** $[L^{-2} T^2]$ **(3)** $[L^2 T^{-2}]$ **(4)** $[L T^{-1}]$

3. Which of the following radiations has the least wavelength ? **[AIEEE 2003]**

(1) γ-rays **(2)** β-rays **(3)** α-rays **(4)** X-rays

4. An electromagnetic wave of frequency n = 3.0 MHz passes vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$, then **[AIEEE 2004]**

(1) wavelength is doubled and the frequency remains unchanged
(2) wavelength is doubled and frequency becomes half
(3) wavelength is halved and frequency remains unchanged
(4) wavelength and frequency both remain unchanged

5. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then **[AIEEE 2012]**

(1) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{B} \times \vec{E}$ **(2)** $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{E} \times \vec{B}$

(3) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{E} \times \vec{B}$ **(4)** $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{B} \times \vec{E}$

6. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is : **[JEE-MAIN 2013]**

(1) 3V/m **(2)** 6V/m **(3)** 9V/m **(4)** 12 V/m

7 Match **List-I** (Electromagnetic wave type) with **List-II** (Its association / application) and select the correct option from the choices given below the lists: **[JEE-Main 2014]**

	List – I	List – II
(a)	Infrared waves	**(i)** To treat muscular strain
(b)	Radio waves	**(ii)** For broadcasting
(c)	X-rays	**(iii)** To detect fracture of bones
(d)	Ultraviolet rays	**(iv)** Absorbed by the ozone layer of the atmosphere

	(a)	**(b)**	**(c)**	**(d)**
(1)	(iii)	(ii)	(i)	(iv)
(2)	(i)	(ii)	(iii)	(iv)
(3)	(iv)	(iii)	(ii)	(i)
(4)	(i)	(ii)	(iv)	(iii)

8. During the propagation of electromagnetic waves in a medium: **[JEE-Main 2014]**

(1) Electric energy density is equal to the magnetic energy density.
(2) Both electric and magnetic energy densities are zero.
(3) Electric energy density is double of the magnetic energy density.
(4) Electric energy density is half of the magnetic energy density.

9. Arrange the following electromagnetic radiations per quantum in the order of increasing energy : **[JEE-Main 2016]**

A : Blue light B : Yellow light C : X–ray D : Radiowave.

(1) A,B,D,C **(2)** C,A,B,D **(3)** B,A,D,C **(4)** D,B,A,C

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is **[IIT - JEE 2000]**

(A) $M L T^{-1}$ (B) $M L^2 T^{-2}$ **(C)** $M L^{-1} T^{-2}$ **(D)** $M L^2 T^{-1}$

ANSWER KEY

EXERCISE - 1

1. C **2.** B **3.** C **4.** D **5.** C **6.** A **7.** B **8.** D **9.** A **10.** C **11.** C **12.** A **13.** C
14. C **15.** A **16.** B **17.** C **18.** A **19.** C **20.** A **21.** C **22.** C **23.** D **24.** B **25.** D **26.** A
27. D **28.** A

EXERCISE - 2 : PART # I

1. A **2.** B **3.** B **4.** C **5.** B **6.** C **7.** B **8.** D **9.** D **10.** A **11.** B **12.** A **13.** D
14. D **15.** D **16.** D **17.** B **18.** B

PART # II

1. A **2.** A **3.** B **4.** A **5.** B **6.** A

EXERCISE - 3 : PART # I

2. $\frac{qAv}{2\pi x^3}$ **3.** $\frac{\varepsilon}{2R}e^{-\frac{td}{\varepsilon AR}}$ **5.** $377\,\Omega$ **6.** $2.7\,\mu T$ **7.** $6\times10^4\ N\,C^{-1},\ 0.016\ J\,m^{-3}$

8. $4.3\times10^8\ N\,C^{-1}, 1.44\ T$ **9.** $1.02\times10^3\ N\,C^{-1}, 3.40\times10^{-6}\ T$

EXERCISE - 4 : PART # I

1. 2 **2.** 3 **3.** 1 **4.** 3 **5.** 2 **6.** 2 **7.** 1 **8.** 1 **9.** 4

PART # II

1. C