# HINTS & SOLUTIONS

## EXERCISE - 1

### Single Choice

**1.** Slope $\frac{dy}{dx} = 3x^2 - 6x - 9$

if tangent is parallel to the x–axis then $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0 \qquad \Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0 \qquad \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = 3 \text{ or } x = -1 \qquad \Rightarrow y = -20 \text{ or } y = 12$

**2.** Enclosed area : $A = \pi r^2$

So $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Here $r = 8$ cm, $\frac{dr}{dt} = 5$ cm/s

$\Rightarrow \frac{dA}{dt} = (2\pi)(8)(5) = 80\pi \text{ cm}^2/\text{s}$

**3.** $\because p = t\ell nt$

$\therefore F = \frac{dp}{dt} = \frac{d}{dt}(t\ell nt) = (1)\ell nt + (t)\left(\frac{1}{t}\right) = 1 + \ell nt$

$F = 0 \Rightarrow 1 + \ell nt = 0 \Rightarrow \ell nt = -1 \Rightarrow t = e^{-1} = \frac{1}{e}$

**4.** Check $\vec{A}.\vec{B} = 0$

**5.** Let side of cube be x then $\frac{dx}{dt} = 3$ cm/s

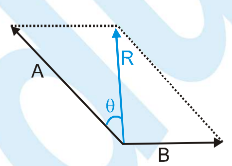
$\because V = x^3 \therefore \frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3 \times 10^2 \times 3 = 900 \text{ cm}^3/\text{s}$

**6.** Resultant $= \sqrt{3^2 + 4^2 + 12^2} = \sqrt{5^2 + 12^2} = 13N$

**7.** $\sqrt{(0.5)^2 + (-0.8)^2 + c^2} = 1$

$\Rightarrow 0.25 + 0.64 + c^2 = 1 \Rightarrow c^2 = 0.11 \Rightarrow c = \pm\sqrt{0.11}$

**8.** Let forces be A and B and B < A then A + B = 16




$A\cos\theta = R = 8$ and $A\sin\theta = B$

$\Rightarrow A^2 = 8^2 + B^2 \qquad \Rightarrow A^2 - B^2 = 64$

$\Rightarrow (A-B)(A+B) = 64 \qquad \Rightarrow A - B = 4$

$\Rightarrow A = 10N \;\&\; B = 6N$

**9.** Required unit vector

$$= \frac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+2^2}} = \frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$$

**12.** For zero resultant, sum of any two forces $\geq$ remaining force

**13.** $\vec{a} = \vec{c} + \overrightarrow{RP}$ and $\vec{b} = \vec{c} + \overrightarrow{RQ}$ but $\overrightarrow{RP} = -\overrightarrow{RQ}$

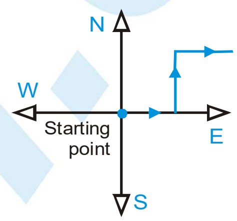
$\Rightarrow \vec{a} + \vec{b} = 2\vec{c} + \overrightarrow{RP} + \overrightarrow{RQ} \Rightarrow \vec{a} + \vec{b} = 2\vec{c}$

**14.** $\vec{R} = \vec{P} + \vec{Q},\; \vec{R}' = \vec{P} + 2\vec{Q}$

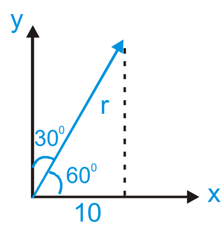
$\because \vec{R}' \cdot \vec{P} = 0 \therefore \left(\vec{P} + 2\vec{Q}\right)\cdot\vec{P} = 0 \Rightarrow P^2 + 2\vec{Q}.\vec{P} = 0$

$R^2 = P^2 + Q^2 + 2\vec{P}.\vec{Q} = P^2 + Q^2 - P^2 = Q^2 \Rightarrow R = Q$

**15.**




**16.** $\cos 60^0 = \frac{10}{r} \Rightarrow r = \frac{10}{1/2} = 20$ units




**17.** $$\hat{v} = \frac{(4-1)\hat{i} + (2+2)\hat{j} + (3-3)\hat{k}}{\sqrt{(4-1)^2 + (2+2)^2 + (3-3)^2}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$\vec{v} = (10)\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right) = 6\hat{i} + 8\hat{j}$

**18.** Use $R^2 = A^2 + B^2 + 2AB\cos\theta$ or see options

**20.** Required angle $= \frac{2\pi}{12} = \frac{360}{12} = 30^0$

**21.** Displacement $= \sqrt{12^2 + 5^2 + 6^2}$

$= \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31$ m

**22.** $\because |\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}\cdot\vec{B} \therefore AB\sin\theta = \sqrt{3}AB\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos 60°}$

$= \sqrt{A^2+B^2+2AB\left(\frac{1}{2}\right)} = \sqrt{A^2+B^2+AB}$

**24.** $\because \vec{P}+\vec{Q}=\vec{R} \quad \therefore \vec{Q}=\vec{R}-\vec{P}$

$\Rightarrow Q^2 = R^2+P^2-2RP\cos\theta_1$

$\Rightarrow \cos\theta_1 = \frac{1}{2} \Rightarrow \theta_1 = \frac{\pi}{3}$

Now $\because \vec{P}+\vec{Q}+\vec{R}=\vec{0} \therefore \vec{P}+\vec{R}=-\vec{Q}$

$\Rightarrow P^2+R^2+2PR\cos\theta_2 = Q^2$

$\Rightarrow \cos\theta_2 = -\frac{1}{2} \Rightarrow \theta_2 = \frac{2\pi}{3}$

**25.** $\because \vec{A}.\vec{B} = AB\cos\theta$

$\therefore$ Projection of $\vec{A}$ on $\vec{B} = A\cos\theta = \frac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

**26.** Resultant $= \sqrt{(x^2+y^2)}$

$= \sqrt{(x+y)^2+(x-y)^2+2(x+y)(x-y)\cos\theta}$

$\Rightarrow x^2+y^2 = 2(x^2+y^2)+2(x^2-y^2)\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}\left(\frac{x^2+y^2}{y^2-x^2}\right)$

**27.** Projection on x–y plane $= \sqrt{3^2+1^2} = \sqrt{10}$

**30.** Velocity of one ball $\vec{v}_1 = \hat{i}+\sqrt{3}\hat{j}$

Velocity of second ball $\vec{v}_2 = 2\hat{i}+2\hat{j}$

Angle between their path :

$\cos\theta = \frac{\vec{v}_1.\vec{v}_2}{v_1 v_2} = \frac{2+2\sqrt{3}}{(2)(2\sqrt{2})} = \frac{1+\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 15°$

**31.** In a clockwise system $\hat{k}\times\hat{j} = \hat{i}$

**32.** $|\vec{e}_1-\vec{e}_2| = \sqrt{1^2+1^2-2(1)(1)\cos\theta} = 2\sin\frac{\theta}{2}$

**34.** $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 0 & 4 & -3\end{vmatrix}$

$= \hat{i}(6-8)-\hat{j}(-3)+\hat{k}(4) = -2\hat{i}+3\hat{j}+4\hat{k}$

$|\vec{v}| = \sqrt{2^2+3^2+4^2} = \sqrt{4+9+16} = \sqrt{29}$

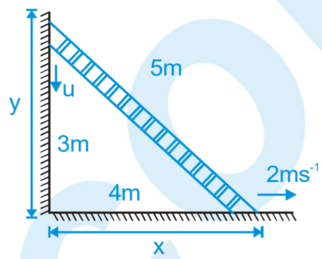
## EXERCISE - 2

### Part # I : Multiple Choice

**1.** $x^2+4=y \Rightarrow 2xdx = dy$ but $dy = 2dx$

So $2xdx = 2dx \Rightarrow 2x = 2 \Rightarrow x=1 \Rightarrow y = 1^2+4 = 5$

**2.** At any instant $x^2+y^2 = 5^2$




Differentiating w.r.t. time $2x\frac{dx}{dt}+2y\frac{dy}{dt} = 0$

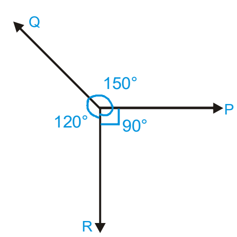
Here $\frac{dx}{dt} = 2, \frac{dy}{dt} = u \Rightarrow u = \frac{8}{3}$ m/s

**3.** $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2 = \frac{8}{15}\pi\rho R^5$

$\frac{dI}{dt} = \left(\frac{8}{15}\pi\rho\right)(5R^4)\frac{dR}{dt} = \left(\frac{8\pi}{15}\right)\left(\frac{M}{4/3\ \pi R^3}\right)(5R^4)$

$\frac{dR}{dt} = 2MR\left(\frac{dR}{dt}\right) = (2)(1)(1)(2) = 4\ kg\ m^2s^{-1}$

**4.**




$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°}$

$\Rightarrow \frac{P}{\sqrt{3}/2} = \frac{Q}{1} = \frac{R}{1/2}$

$\Rightarrow \frac{2P}{\sqrt{3}} = \frac{Q}{1} = \frac{2R}{1} = k$ (constant)

$\Rightarrow P:Q:R = \frac{\sqrt{3}k}{2} : k : \frac{k}{2} = \sqrt{3}:2:1$

**5.** Angle between $\vec{a}$ and $\vec{b}$,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{ab} = \frac{-x+2+x+1}{\sqrt{1+4+1}\sqrt{x^2+1^2+(x+1)^2}}$$

$$= \frac{3}{\sqrt{6\left[x^2+1+(x+1)^2\right]}} > 0$$

**6.** ∵ $|\hat{a}+\hat{b}| = 1$ ∴ $2\cos\frac{\theta}{2} = 1$

$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$

$|\hat{a}-\hat{b}| = 2\sin\frac{\theta}{2} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

**7.** ∵ $|\hat{a}+\hat{b}+\hat{c}| = 1$

∴ $|\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) = 1$

$\Rightarrow 1+1+1+2(\cos\theta_1+\cos\theta_2+\cos\theta_3) = 1$

$\Rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3 = -1$

**8.** $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$

∵ $\vec{a}\cdot(\vec{b}+\vec{c}) = 0$, $\vec{b}\cdot(\vec{c}+\vec{a}) = 0$ & $\vec{c}\cdot(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = 0$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2} = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$

**9** $a_x = 2a_y$, $\cos\gamma = \frac{a_z}{a} = \cos 135° = -\frac{1}{\sqrt{2}}$

$\Rightarrow a_z = -\frac{a}{\sqrt{2}} = -\frac{5\sqrt{2}}{\sqrt{2}} = -5$

Now $a_x^2+a_y^2+a_z^2 = 50 \Rightarrow 4a_y^2+a_y^2+25 = 50$

$\Rightarrow a_y^2 = 5 \Rightarrow a_y = \pm\sqrt{5} \Rightarrow a_x = \pm2\sqrt{5}$

**12.** ∵ $\vec{C} = \vec{A}+\vec{B}$ ∴ $C^2 = A^2+B^2+2AB\cos\theta$

If $C^2 < A^2+B^2$ then $\cos\theta < 0$.

Therefore $\theta > 90^0$

**13.** $\vec{F}_1+\vec{F}_2+\vec{F}_3+\vec{F}_4$

$= (4\hat{i}-5\hat{j}+5\hat{k})+(-5\hat{i}+8\hat{j}+6\hat{k})+(-3\hat{i}+4\hat{j}-7\hat{k})$

$+(12\hat{i}-3\hat{j}-2\hat{k}) = 4\hat{j}+2\hat{k}$

$\Rightarrow$ motion will be in y–z plane

**14.** Area of triangle $= \frac{1}{2}(\vec{a}\times\vec{b}) = \frac{1}{2}(\vec{b}\times\vec{c}) = \frac{1}{2}(\vec{c}\times\vec{a})$

**15.** $\vec{r} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$

velocity $= \vec{v} = \frac{d\vec{r}}{dt} = -a\omega\sin\omega t\hat{i} + a\omega\cos\omega t\hat{j}$

Acceleration $= \frac{d^2\vec{r}}{dt^2} = -a\omega^2\cos\omega t\,\hat{i} - a\omega^2\sin\omega t\hat{j} = -\omega^2\vec{r}$

**16.** $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 3 & 1\\ -3 & 1 & 5\end{vmatrix} = 14\hat{i}-38\hat{j}+16\hat{k}$

**17.** $|\vec{A}\cdot\vec{B}| = AB|\cos\theta| = 8$, $|\vec{A}\times\vec{B}| = AB|\sin\theta| = 8\sqrt{3}$

$\Rightarrow |\tan\theta| = \frac{8\sqrt{3}}{8} = \sqrt{3} \Rightarrow \theta = 60°, 120°$

**18.** Here $\alpha = 45°$ so inclination of AC with x–axis is 45°. So unit vector along AC

$= \cos45°\hat{i} + \sin45°\hat{j} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

**19.** Displacement $d\vec{r} = dx\hat{i} + dy\hat{j}$

but $3y + kx = 5$ so $3dy + kdx = 0$

$\Rightarrow d\vec{r} = dx\hat{i} - \frac{k}{3}dx\hat{j} = \left(\hat{i}-\frac{k}{3}\hat{j}\right)dx$

Work done is zero if $\vec{F}.d\vec{r} = 0$

$(2\hat{i}+3\hat{j})\cdot\left(\hat{i}-\frac{k}{3}\hat{j}\right)dx = 0 \Rightarrow (2-k)dx = 0 \Rightarrow k=2$

**20.** For triangle ABC : $\vec{AB}+\vec{BC}+\vec{CA} = \vec{0}$

Now $\vec{AB}+\vec{BC}+2\vec{CA}$

$= \vec{AB}+\vec{BC}+\vec{CA}+\vec{CA} = \vec{0}+\vec{CA} = \vec{CA}$

**21.** $(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b}) = 0$

$\Rightarrow 7a^2 - 15b^2 + 16\,\vec{a}\cdot\vec{b} = 0$ ...(i)

and $(\vec{a}-4\vec{b})\cdot(7\vec{a}-2\vec{b}) = 0$

$\Rightarrow 7a^2 + 8b^2 - 30\,\vec{a}\cdot\vec{b} = 0$ ...(ii)

By adding (i) and (ii)

$\Rightarrow -23b^2 + 46\,\vec{a}\cdot\vec{b} = 0 \Rightarrow 2\vec{a}\cdot\vec{b} = b^2$

So $7a^2 - 15b^2 + 8b^2 = 0 \Rightarrow a^2 = b^2$

$\Rightarrow 2ab\cos\theta = b^2 \Rightarrow 2\cos\theta = 1$

$\Rightarrow \theta = \cos^{-1}(1/2) = 60°$

## Part # II : Assertion & Reason

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | A | **2.** | C | **3.** | C | **4.** | D | **5.** | B | **6.** | A |
| **7.** | C | **8.** | B | **9.** | D | **10.** | A | **11.** | A | **12.** | A |

## EXERCISE - 3

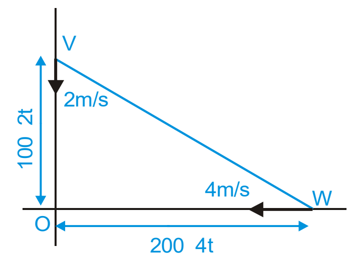
### Part # I : Matrix Match Type

1. (A) – q, (B)–r, (C)–s, (D)–s
2. (A) $\to$ r, (B) $\to$ p, (C) $\to$ q, (D) $\to$ s
3. (A) $\to$ q, (B) $\to$ r, (C) $\to$ p, (D) $\to$ s

### Part # II : Comprehension

**Comprehension 1**

1. $l = \sqrt{(100-2t)^2 + (200-4t)^2}$




2. For shortest distance $\frac{dl}{dt} = 0 \Rightarrow t = 50$ sec
3. $l_{min} = \sqrt{(100-2\times 50)^2 + (200-4\times 50)^2} = 0$

**Comprehension 2**

1. $x = at,\ y = -bt^2 \Rightarrow a^2y + bx^2 = 0$
2. $\frac{d\vec{r}}{dt} = a\hat{i} - 2bt\hat{j}$ at $t = 0$, $\frac{d\vec{r}}{dt} = a\hat{i}$
3. $\frac{d^2\vec{r}}{dt^2} = -2b\hat{j}$

**Comprehension 3**

1. $x = t^3 - 3t^2 + 12t + 20$

$$v = \frac{dx}{dt} = 3t^2 - 6t + 12$$

$t = 0 \quad \Rightarrow \quad v = 12$ m/s

2. $a = \frac{dv}{dt} = 6t - 6$

$t = 0 \quad \Rightarrow \quad A = -6\ m/s^2$

3. $a = 0 \quad \Rightarrow \quad t = 1$ sec

$v = 3t^2 - 6t + 12 = 9$ m/s

**Comprehension 4**

1. (A) 2. (B) 3. (C)

## EXERCISE - 4

### Subjective Type

1. (i) Let the angle between $\vec{A}$ and $\vec{B}$ is $\theta$, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(2\hat{i}-2\hat{j}-\hat{k})\cdot(\hat{i}+\hat{j})}{|2\hat{i}+2\hat{j}-\hat{k}|\cdot|\hat{i}+\hat{j}|} = \frac{0}{3\sqrt{2}} = 0$$

$\Rightarrow \theta = 90°$

(ii) Resultant

$$\vec{R} = \vec{A}+\vec{B} = (2\hat{i}-2\hat{j}-\hat{k}) + (\hat{i}+\hat{j}) = 3\hat{i}-\hat{j}-\hat{k}$$

Projection of resultant on x–axis = 3

(iii) Required vector

$$= \hat{j} - \vec{A} = \hat{j} - (2\hat{i}-2\hat{j}-\hat{k}) = -2\hat{i}+3\hat{j}+\hat{k}$$

2. $\because$ $A_x = 4$, $A_y = 6$ so $A_x + B_x = 10$ and $A_y + B_y = 9$

(i) $B_x = 10 - 4 = 6$m and $B_y = 9 - 6 = 3$m

(ii) length $= \sqrt{B_x^2 + B_y^2} = \sqrt{36+9} = \sqrt{45}$ m

(iii) $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{3}{6}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

3. (i) Component of $\vec{A}$ along $\vec{B}$ = $\left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\frac{\vec{B}}{B} = \left[\frac{(3\hat{i}+\hat{j})\cdot(\hat{j}+2\hat{k})}{\sqrt{5}}\right]\frac{(\hat{j}+2\hat{k})}{\sqrt{5}} = \frac{1}{5}(\hat{j}+2\hat{k})$$

Component of $\vec{A} \perp \vec{B}$

$$= \vec{A} - \left[\frac{\vec{A}\cdot\vec{B}}{B}\right]\hat{B} = 3\hat{i}+\hat{j} - \left[\frac{1}{5}(\hat{j}+2\hat{k})\right]$$

(ii) Area of the parallelogram

$$= |\vec{A}\times\vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = |2\hat{i} - 6\hat{j} + 3\hat{k}|$$

$$= \sqrt{2^2 + (-6)^2 + 3^2} = 7 \text{ units}$$

(iii) Unit vector perpendicular to both $\vec{A}$ & $\vec{B}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{2\hat{i}-6\hat{j}+3\hat{k}}{7} = \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$$

5. Component along the vector $\hat{i}+\hat{j}$

$$= (A\cos\theta)\hat{B} = \frac{(\vec{A}\cdot\vec{B})}{B^2}\vec{B} = \frac{(3\hat{i}+4\hat{j})\cdot(\hat{i}+\hat{j})}{(\sqrt{2})^2}(\hat{i}+\hat{j})$$

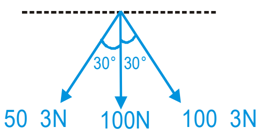
$$= \frac{3+4}{2}(\hat{i}+\hat{j}) = \frac{7}{2}(\hat{i}+\hat{j})$$

Component along the vector $\hat{i}-\hat{j}$

$$=(A\cos\theta)\,\hat{B}=\frac{(\vec{A}.\vec{B})}{B^2}\vec{B}=\frac{(3\hat{i}+4\hat{j}).(\hat{i}-\hat{j})(\hat{i}-\hat{j})}{(\sqrt{2})^2}$$

$$=\frac{(3-4)}{2}(\hat{i}-\hat{j})=-\frac{1}{2}(\hat{i}-\hat{j})$$

**6.** Resultant force in vertical direction




$$=50\sqrt{3}\cos 30^\circ+100+100\sqrt{3}\cos 30^\circ$$

$$=50\times\frac{3}{2}+100+100\times\frac{3}{2}=325\text{ N}$$

Resultant force in horizontal direction

$$=100\sqrt{3}\sin 30^\circ-50\sqrt{3}\sin 30^\circ$$

$$=100\frac{\sqrt{3}}{2}-\frac{50\sqrt{3}}{2}=25\sqrt{3}\text{N}$$

So resultant pull $=\sqrt{(325)^2+(25\sqrt{3})^2}=327.9\text{N}$

**7.** $|\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2AB\cos\theta}$

$$=\sqrt{(10)^2+(6)^2-2(10)(6)\cos 60^\circ}=2\sqrt{19}$$

$$\tan\alpha=\frac{6\sin 60^\circ}{10-6\cos 60^\circ}=\frac{6\times\sqrt{3}/2}{10-3}=\frac{3\sqrt{3}}{7}$$

$$\Rightarrow\quad \alpha=\tan^{-1}\left(\frac{3\sqrt{3}}{7}\right)$$

**8.** Let two forces are A and B then

$$A+B=P,\ A-B=Q\Rightarrow A=\frac{P+Q}{2},\ B=\frac{P-Q}{2}$$

Resultant $k=\sqrt{A^2+B^2+2AB\cos\theta}$

$$=\sqrt{\left(\frac{P+Q}{2}\right)^2+\left(\frac{P-Q}{2}\right)^2+2\left(\frac{P+Q}{2}\right)\left(\frac{P-Q}{2}\right)\cos 2\alpha}$$

$$=\sqrt{\frac{P^2}{2}+\frac{Q^2}{2}+\frac{1}{2}(P^2-Q^2)\cos 2\alpha}$$

$$=\sqrt{\frac{P^2}{2}(1+\cos 2\alpha)+\frac{Q^2}{2}(1-\cos 2\alpha)}$$

$$=\sqrt{P^2\cos^2\alpha+Q^2\sin^2\alpha}$$

**9.** $x=at,\ y=-bt^2=-b\left(\frac{x}{a}\right)^2$

**10.** Let $\vec{c}$ is $=c_x\hat{i}+c_y\hat{j}$

then according to question $=\sqrt{c_x^2+c_y^2}=5$

$\Rightarrow\quad c_x^{\,2}+c_y^{\,2}=25$ ....(i)

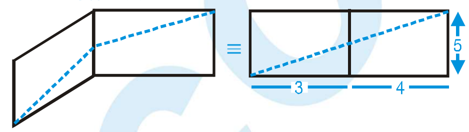
and $\vec{a}.\vec{c}=0\ \Rightarrow 3c_x+4c_y=0-$ ...(ii)

from equation (i) and (ii) $c_x=\pm 4,\ c_y=\mp 3$

**12. (i)**

$$|\text{displacement}|=\sqrt{(3)^2+(4)^2+(5)^2}=\sqrt{50}\,\text{m}$$

Distance




**(ii)** $L=\sqrt{(7)^2+(5)^2}=\sqrt{74}\,\text{m}$

**13.** $\vec{v}=\frac{d\vec{r}}{dt}=(6t-6)\,\hat{i}+(-12t^2)\,\hat{j}$ m/s

$$\vec{a}=\frac{d\vec{v}}{dt}=(6\hat{i}-24t\hat{j})\,\text{m/s}^2$$

**(i)** $\vec{F}=m\vec{a}=6(6\hat{i}-24t\hat{j})=(36\hat{i}-144t\hat{j})\text{N}$

**(ii)** $\vec{\tau}=\vec{r}\times\vec{F}=\left[(3t^2-6t)\hat{i}+(-4t^3)\hat{j}\right]\times\left[36\hat{i}-144t\hat{j}\right]$

$$=[(-144\times 3t^2)+(144\times 6t^2)+144\,t^3]\,\hat{k}$$

$$=(-288\,t^3+864\,t^2)\,\hat{k}$$

**(iii)** $\vec{p}=m\vec{v}=6[(6t-6)\hat{i}+(-12t^2)\hat{j}]$

$$=[36(t-1)\hat{i}-72t^2\hat{j}]$$

**(iv)** $\vec{L}=\vec{r}\times\vec{p}=\left[(3t^2-6t)\hat{i}+(-4t^3)\hat{j}\right]$

$$\times\left[36(t-1)\hat{i}-72t^2\hat{j}\right]=\left[-72t^4+288t^3\right]\hat{k}$$

**16.** $\vec{F}_1=5P\hat{j},\ \vec{F}_2=4P\hat{i},\ \vec{F}_3=10P\left(\frac{3\hat{i}+4\hat{j}}{5}\right)=(6\hat{i}+8\hat{j})P$

$$\vec{F}_4=15P\frac{((-\hat{i}-3\hat{i})+(\hat{j}-4\hat{j}))}{5}=-12P\hat{i}-9P\hat{j}$$

$$\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3+\vec{F}_4=5P\hat{j}+4P\hat{i}+6P\hat{i}+8P\hat{j}$$

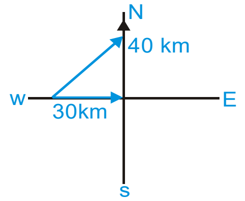
$$-12P\hat{i}-9P\hat{j}=-2P\hat{i}+4P\hat{j}$$

$$|\vec{F}|=P\sqrt{(-2)^2+(4)^2}=\sqrt{20}P$$

$$\tan\alpha=\frac{4}{-2}\Rightarrow\alpha=\tan^{-1}(-2)$$

**17.** Displacement

$$= \sqrt{(30)^2 + (40)^2} = 50\,\text{km}$$




$$\tan\alpha = \frac{40}{30} \Rightarrow \alpha = 53^\circ \text{ N to E}$$

**18.** Speed $= |\vec v| = \sqrt{9+25+16} = 5\sqrt{2}\,\text{m/s}$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\times 200\times 10^{-3}\times 50 \text{ J=5J}$$

**20.** **(i)** $\vec v = v_0\hat i + a_0 b_0 e^{b_0 t}\hat k$

**(ii)** $|\vec v| = \sqrt{v_0^2 + a_0^2 b_0^2 e^{2b_0 t}}$

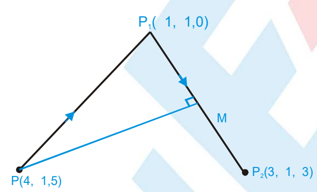
**(iii)** $\vec a = a_0 b_0^2 e^{b_0 t}\hat k$

**21.** From graph $\frac{dv}{dx} = \frac{90-50}{40-20} = \frac{40}{20}\ \frac{dv}{dx} = 2$

$v\,(\text{at } x = 20) = 50$ m/s

$$a = v\frac{dv}{dx} \Rightarrow a = 50\times 2 = 100\ \text{m/s}^2$$

**22.** Vector $\overrightarrow{PP_1} = -5\hat i - 5\hat k$ and $\overrightarrow{P_1P_2} = 4\hat i - 3\hat k$




Let angle between these vectors be $\theta$ then

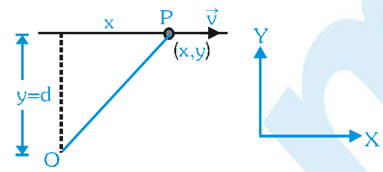
$$\cos\theta = \frac{(-5\hat i - 5\hat k)\cdot(4\hat i - 3\hat k)}{(5\sqrt{2})(5)} = -\frac{1}{5\sqrt{2}}$$

As PM = $PP_1\sin\theta$

$$\text{so } PM = (5\sqrt{2})\left(\frac{7}{5\sqrt{2}}\right) = 7\ \text{m}$$

$$\text{Therefore } t = \frac{7\ \text{m}}{2\ \text{m/s}} = 3.5\ \text{s}$$

**23.** Let $\vec v = v\hat i$ & $\overrightarrow{OP} = x\hat i + y\hat j = x\hat i + d\hat j$




so $\overrightarrow{OP}\times\vec v = (x\hat i + d\hat j)\times v\hat i = -dv\hat k$

(d = is constant)

which is independent of position.

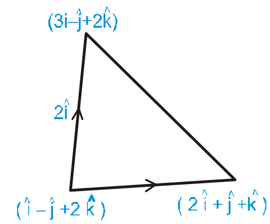
**24.** $\vec v = \frac{d\vec r}{dt} = 1.2\hat i + 1.8t\hat j - 1.8t^2\hat k$

At t= 4s, $\vec v = 1.2\hat i + 7.2\hat j - 28.8\hat k$

$$P = \vec F\cdot\vec v = (60\hat i - 25\hat j - 40\hat k)\cdot(1.2\hat i + 7.2\hat j - 28.8\hat k)$$

$= 1044$W

**25.** Area of triangle




$$= \frac{1}{2}|\vec A\times\vec B| = \frac{1}{2}(4\hat k + 2\hat j)$$

$\vec A = (\hat j + 2\hat k)$

$|A| = \sqrt{5}\,\text{m}^2$

**26.** $\vec v = A\hat i + (3Bt^2 - 2)\hat j + (2ct - 4)\hat k$

At t=2, $A\hat i + (12B - 2)\hat j + (4c - 4)\hat k = 3\hat i + 22\hat j$

Thus, A =3, B=2, C=1

$\therefore\ \vec v = 3\hat i + (6t^2 - 2)\hat j + (2t - 4)\hat k$

At t=4, $\vec v = 3\hat i + (96 - 2)\hat j + (8 - 4)\hat k = 3\hat i + 94\hat j + 4\hat k$

**27.** $\vec r = t\hat i + \frac{t^2}{2}\hat j + t\hat k$

**(i)** $\vec v = \frac{d\vec r}{dt} = \hat i + t\hat j + \hat k$ **(ii)** speed $|\vec v| = \sqrt{t^2 + 2}$

**(iii)** $\vec a = \frac{d\vec v}{dt} = \hat j$ **(iv)** $|\vec a| = 1$

**(v)** $$\vec a_T = (\vec a\cdot\hat v)\hat v = \left(\left[\hat j\cdot\frac{(\hat i + t\hat j + \hat k)}{\sqrt{t^2+2}}\right]\right)\frac{(\hat i + t\hat j + \hat k)}{\sqrt{t^2+2}}$$

$$\vec a_T = \left(\frac{t}{\sqrt{t^2+2}}\right)\hat v = \frac{t(\hat i + t\hat j + \hat k)}{(t^2+2)}\ ;\ |\vec a_T| = \frac{t}{\sqrt{t^2+2}}$$

As $a_N^2 + a_T^2 = a^2$

so $a_N = \sqrt{a^2 - a_T^2} = \dfrac{\sqrt{2}}{\sqrt{t^2+2}}$

**28.** $\vec{a} = 5\cos t\hat{i} - 3\sin t\hat{j}$

$\Rightarrow \int d\vec{v} = \int 5\cos t\, dt\hat{i} - \int 3\sin t dt\hat{j}$

Therefore $\int_{-3}^{v} dv_x = \int_0^t 5\cos t dt \Rightarrow v_x = 5\sin t - 3$

$\dfrac{dx}{dt} = (5\sin t - 3) \quad \Rightarrow \int_{-3}^{x} dx = \int_0^t (5\sin t - 3)dt$

$x+3 = 5 - 5\cos t - 3t \Rightarrow x = 2 - 5\cos t - 3t$

Similarly,

$\int_2^v dv_y = -\int_0^t 3\sin t dt$

$\Rightarrow v_y - 2 = 3(\cos t - 1) \Rightarrow v_y = 3\cos t - 1$

$\Rightarrow \int_2^y dy = \int_0^t (3\cos t - 1)dt$

$\Rightarrow y-2 = 3\sin t - t \Rightarrow y = 2 + 3\sin t - t$

Thus, $\vec{v} = (5\sin t - 3)\hat{i}| (3\cos t - 1)\hat{j}$

and $\vec{s} = (2 - 5\cos t - 3t)\hat{i} + (2 + 3\sin t - t)\hat{j}$

## EXERCISE - 5

### Part # I : AIEEE/JEE-MAIN

**1.** $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

This is only possible if the value of both vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ is zero. This occurs when the angle between $\vec{A}$ and $\vec{B}$ is $\pi$.

### Part # II : IIT-JEE ADVANCED

**5. (4)**

Let $\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}\sqrt{4 - \dfrac{1}{3\sqrt{2}}...}}} = y$

So, $4 - \dfrac{1}{3\sqrt{2}}y = y^2 \qquad (y > 0)$

$\Rightarrow y^2 + \dfrac{1}{3\sqrt{2}}y - 4 = 0 \quad \Rightarrow y = \dfrac{8}{3\sqrt{2}}$

so, the required value is $6 + \log_{3/2}\left(\dfrac{1}{3\sqrt{2}} \times \dfrac{8}{3\sqrt{2}}\right)$

$= 6 + \log_{3/2}\dfrac{4}{9} = 6 - 2 = 4.$

**6. (ABC)**

$\log_2 3^x = (x-1)\log_2 4 = 2(x-1)$

$\Rightarrow x = \log_2 3 = 2x - 2$

$\Rightarrow x = \dfrac{2}{2 - \log_2 3}$

Rearranging, we get

$x = \dfrac{2}{2 - \dfrac{1}{\log_3 2}} = \dfrac{2\log_3 2}{2\log_3 2 - 1}$

Rearranging again,

$x = \dfrac{\log_3 4}{\log_3 4 - 1} = \dfrac{\dfrac{1}{\log_4 3}}{\dfrac{1}{\log_4 3} - 1} = \dfrac{1}{1 - \log_4 3}$

## MOCK TEST : BASIC MATHS

**1.** Since x = 0 is one of the solution so the product will be zero.

**2.** $\log(-2x) = 2\log(x+1)$

$-2x > 0 \Rightarrow x < 0$ .....(i)

$x + 1 > 0 \Rightarrow x > -1$ .....(ii)

from (i) & (ii), we get $x \in (-1, 0)$

$\therefore\ -2x = (x+1)^2 \Rightarrow x^2 + 4x + 1 = 0$

$\Rightarrow x = \dfrac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$

so $x = -2 + \sqrt{3}$ only one solution lies in $(-1, 0)$

**3.** $\log_2 15 \log_{1/6} 2 \log_3 1/6 = \dfrac{\log 15}{\log 2} \times \dfrac{\log 2}{\log 1/6} \times \dfrac{\log 1/6}{\log 3}$

$= \dfrac{\log(3 \times 5)}{\log 3} = 1 + \log_3 5 > 2$ (but $< 3$)

**4.** **Case I**

$[x] - 2x = 4$ ... (i)

$\Rightarrow [x] - 2([x] + \{x\}) = 4$

$\Rightarrow [x] + 2\{x\} + 4 = 0$ ... (ii)

$\because\ 0 \le 2\{x\} < 2$

$\therefore\ 0 \le -[x] - 4 < 2 \Rightarrow -6 < [x] \le -4$

$\Rightarrow [x] = -4, -5$

$\therefore$ from (i) we get $x = -4, \dfrac{-9}{2}$

**Case II**

$[x] - 2x = -4$ ...(iii)

$\Rightarrow [x] = 2x - 4$

$\Rightarrow [x] = 2([x] + \{x\}) - 4$

$\Rightarrow 2\{x\} = 4 - [x]$ ... (iv)

$\therefore\ 0 \le 2\{x\} < 2$

$\Rightarrow 0 \le 4 - [x] < 2$

$\Rightarrow 2 < [x] \le 4 \quad \therefore [x] = 3, 4$

$\therefore$ from (iii) we get $x = 4, \dfrac{7}{2}$

$\therefore$ Number of solutions of $|[x] - 2x| = 4$ are 4.

**5.** (i) $\log_{\frac{1}{3}}(x^2 + x + 1) > -1 \Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x+2)(x-1) < 0$

$\Rightarrow x \in (-2, 1)$ ........(1)

and (ii) $x^2 + x + 1 > 0 \Rightarrow x \in R$ ........(2)

by (1) & (2) $x \in (-2, 1)$

**6** $5\{x\} = x + [x]$ .... (i)

$[x] - \{x\} = \dfrac{1}{2}$ .... (ii)

$\because\ 0 \le \{x\} < 1$

$\Rightarrow 0 \le [x] - \dfrac{1}{2} < 1$ (by (ii)

$\Rightarrow [x] = 1 \quad \therefore \{x\} = \dfrac{1}{2}$

$\therefore$ from (i) we get $\dfrac{5}{2} = x + 1$

$\therefore\ x = \dfrac{3}{2}$, (one value)

**7.** $|x^2 - 9| + |x^2 - 4| = 5$

$|x^2 - 9| + |x^2 - 4| = |(x^2 - 9) - (x^2 - 4)|$

$\Rightarrow (x^2 - 9)(x^2 - 4) \le 0$ $\{\because\ |a| + |b| = |a - b| \Leftrightarrow a \cdot b \le 0\}$

$\Rightarrow x \in [-3, -2] \cup [2, 3]$

**8.** Here $x \ne 0$

**Case I** when $x \ge -2$

$\dfrac{|x+2| - x}{x} < 2 \Rightarrow \dfrac{2}{x} < 2$

$\Rightarrow \dfrac{1}{x} < 1 \Rightarrow (x - 1)/x > 0$

$x \in [-2, 0) \cup (1, \infty)$ ...(i)

**Case II** when $x < -2$

$\dfrac{|x+2| - x}{x} < 2 \Rightarrow \dfrac{-2 - 2x}{x} < 2 \Rightarrow \dfrac{1 + x}{x} + 1 > 0$

$\Rightarrow (1 + 2x)/x > 0 \Rightarrow x \in (-\infty, -2)$ ...(ii)

$\therefore$ from (i) and (ii) we get $x \in (-\infty, 0) \cup (1, \infty)$

**9.** $0 \le \log_e[2x] \le 1$

$1 \le [2x] \le e \Rightarrow [2x] = 1, 2 \Rightarrow 1 \le 2x < 3$

$\therefore\ \dfrac{1}{2} \le x < \dfrac{3}{2}$

**10.** $|a| + |b| = |a - b| \Rightarrow a.b \le 0$

$(x^2 - 5x + 7)(x^2 - 5x - 14) \le 0$

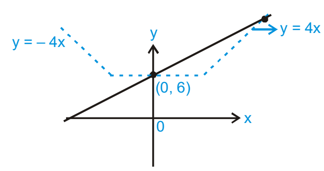
$(x - 7)(x + 2) \le 0 \Rightarrow x \in [-2, 7]$

**11.** When (i) P = 0 then it has infinte slution

(ii) if $-4 < P < 0$ or $0 < P < 4$

then it intersect at 2 points

(iii) $P \ge 4$ or $P \le -4$ then it has only one solution



**12.** use $A^{\log_A B} = B$

**Basic (E)**

$e^{\ln(\ln 3)} = \ln 3$

$\therefore \quad e^{e^{\ln(\ln 3)}} = e^{\ln 3} = 3$

**13.** $N = \dfrac{(3^4)^{\log_9 5} + 3^{3\log_3 \sqrt{6}}}{409}\left[7^{\log_7 25} - (5^3)^{\log_{5^2} 6}\right] \Rightarrow N$

$= \dfrac{3^{\log_3 25} + 3^{\log_3 \sqrt{6}^3}}{409}[25 - 6\sqrt{6}\,]$

$N = \dfrac{(25+6\sqrt{6})(25-6\sqrt{6})}{409}$

$N = 1$

$\log_2 N = \log_2 1 = 0$

**14. S1 :**

$3^{\sqrt{\log_3 7}} = 7^{\sqrt{\log_7 3}}$

taking log with base 3 both sides

we get $\sqrt{\log_3 7} = (\log_3 7)\sqrt{(\log_7 3)}$

$1 = \sqrt{\log_3 7}\,.\,\sqrt{\log_3 7}$

**S2 :** $x^{\log_{10} 2x} = 5$ taking log both sides

$\Rightarrow (\log_{10} x)(\log_{10} x + \log_{10} 2) = \log_{10} 5$

$\Rightarrow t(t + \log_{10} 2) = 1 - \log_{10} 2 \quad \{\text{where } t = \log_{10} x\}$

$\Rightarrow t^2 + t(\log_{10} 2) + (\log_{10} 2 - 1) = 0$

$\Rightarrow t = -1, -\log_{10} 2 + 1 \Rightarrow x = 5, \dfrac{1}{10}$

$\therefore$ there are two values of x

**S3:** $\left(\dfrac{1}{3}\right)^{\log_{1/9}\left(x^2 - \frac{10}{3}x + 1\right)} \le 1 \Rightarrow \sqrt{\left(x^2 - \dfrac{10x}{3} + 1\right)} \le 1$

$\Rightarrow 0 < x^2 - \dfrac{10x}{3} + 1 \le 1 \quad \Rightarrow x \in \left[0, \dfrac{1}{3}\right) \cup \left(3, \dfrac{10}{3}\right]$

**S4 :** Solution set of $\dfrac{(x-2)}{(x-4)} \le 0 \Rightarrow x \in [2, 4)$

**15. S1 :** $x^2 + |x| + 1$ is always positive for all x

**S2 :** $x^2 - 5|x| + 6 = 0$

$|x|^2 - 5|x| + 6 = 0$

$(|x| - 3)(|x| - 2) = 0$

$|x| = 2$ or $|x| = 3$

$x = \pm 2 \qquad x = \pm 3$

Number of solution is 4

**S3 :** $|x|^2 - |x| - 2 = 0$

$(|x| - 2)(|x| + 1) = 0$

$|x| + 1 = 0 \qquad |x| - 2 = 0$

$|x| = -1$ not possible $|x| = 2$

$x = \pm 2$

Two solution

**S4 :** $x^2 - |x| = 0$

$|x|(|x| - 1) = 0$

$x = 0$ or $x = \pm 1$

**16. S1 :** $e^{y\ln 7 - x\ln 11} = e^{\ln\frac{7^y}{11^x}} = \dfrac{7^y}{11^x}$

$= \dfrac{7^{\sqrt{\log_7 11}}}{11^{\sqrt{\log_{11} 7}}} = \dfrac{7^{\frac{\log_7 11}{\sqrt{\log_7 11}}}}{11^{\sqrt{\log_{11} 7}}} = \dfrac{11^{\sqrt{\log_{11} 7}}}{11^{\sqrt{\log_{11} 7}}} = 1$

**S2 :** $\log_x 3 > \log_x 2 \Rightarrow x > 1$

**S3 :** $|x - 2| = [-\pi]$

$|x - 2| = -4$ no solution

**S4 :** $\log_{25}(2 + \tan^2\theta) = 0.5$

$\Rightarrow \dfrac{1}{2}\log_5(2 + \tan^2\theta) = \dfrac{1}{2}$

$\Rightarrow 2 + \tan^2\theta = 5 \Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3}$

$\Rightarrow \theta$ may take values $\dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$

**17.** $\ln(x+z) + \ln(x-2y+z) = 2\ln(x-z)$

$\ln(x+z)(x-2y+z) = \ln(x-z)^2$

$x^2 - 2xy + 2zx - 2yz + z^2 = x^2 + z^2 - 2zx$

$\Rightarrow y = \dfrac{2xz}{z+x}$ or $\dfrac{x}{z} = \dfrac{x-y}{y-z}$

**18.** $\log_{10} 5 \,.\, \log_{10} 20 + (\log_{10} 2)^2 = \log_{10} 5\,(1 + \log_{10} 2) + (\log_{10} 2)^2 = \log_{10} 5 + \log_{10} 2\,[\log_{10} 5 + \log_{10} 2] = 1$

**19. (A)** $\log_3 \log_{27} \log_4 64 = \log_3 \log_{27} 3 = \log_3\left(\dfrac{1}{3}\right) = -1$

**(B)** $2\log_{18}(\sqrt{2} + \sqrt{8}) = 2\,.\,\log_{(3\sqrt{2})^2}\left(\sqrt{2} + 2\sqrt{2}\right)$

$= 2\,.\,\log_{(3\sqrt{2})^2}\left(3\sqrt{2}\right) = \dfrac{2}{2} = 1$

**(C)** $\log_2\left(\sqrt{10} \times \dfrac{2}{\sqrt{5}}\right) = \log_2 2\sqrt{2} = \dfrac{3}{2}$

**(D)** $-\log_{\sqrt{2}-1}(\sqrt{2} + 1) = \log_{\sqrt{2}-1}\left(\sqrt{2} + 1\right)^{-1}$

$= \log_{\sqrt{2}-1}(\sqrt{2} - 1) = 1$

**20.** ✈ AM ≥ GM

$$\frac{x+y}{2} \geq \sqrt{xy} \quad \Rightarrow \quad \left(\frac{2-z}{2}\right) \geq \sqrt{xy}$$

$$\frac{2-z}{2} \geq \sqrt{\frac{z^2+4}{2}} \Rightarrow \frac{4+z^2-4z}{4} \geq \frac{4+z^2}{2} \ (z+2)^2 \leq$$

$0 \quad \therefore \quad z = -2$

$x + y = 4$ and $xy = 4$

$x - y = 0$

$\therefore$ $x = 2$, $y = 2$ and $z = -2$

only one real solution.

**21.** Statement 1 is false

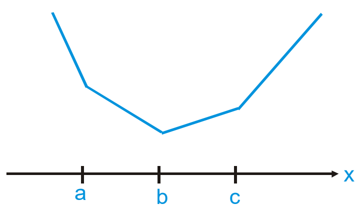
✈ Sum of the length of any two sides of a triangle is greater than length of third side

Statement 2 is true

✈ $a^2 + c^2 - b^2 < 0$

then $\cos B < 0 \Rightarrow$ B is obtuse

**22.** Graph of $y = |x-a| + |x-b| + |x-c|$




We get its minimum value at $x = b$.

So minimum value $|b-a| + |b-c|$

**23.** The result can be easily understood with the help of nature of graph of $y = \log_a x$

**24.** Statement 2 is correct and from statement 1

$\Rightarrow$ $x^2 - 5x + 6 = 0$ (for $x \in z$)

$\Rightarrow$ $x = \{2, 3\}$

Also, $x^2 - 5x + 6 = -1$ (for $x \notin z$)

$\Rightarrow$ $x^2 - 5x + 7 = 0 \Rightarrow$ no real root

$\Rightarrow$ St. 1 is true.

**25.** $\log_a b < 0 \Rightarrow$ either $\{0 < a < 1$ and $b > 1\}$

or $\{a > 1$ and $b < 1\}$

$\Rightarrow$ 1 lies between the roots

$\therefore$ $a + \alpha + \beta < 0$ and so $\alpha + \beta < 0$

$\therefore$ both the statements are true. Statement-2 is a correct explanation for Statement-1.

**26.** $|x^3 - x| + |2 - x| = (x^3 - x) - (2 - x)$

$\therefore$ $x^3 - x \geq 0$ and $2 - x \leq 0$

$x^3 - x \geq 0$ and $x \geq 2$

$x(x^2 - 1) \geq 0$ and $x \geq 2$

$\therefore$ $x \in [2, \infty)$

**27.** $(x^2 - x)(x + 3) \leq 0$

$x(x-1)(x+3) \leq 0$


$x \in (-\infty, -3] \cup [0, 1]$

**28.** $|f(x) - g(x)| = |f(x)| + |g(x)|$

obviously $f(x) \cdot g(x) \leq 0$

**29.** Since $2m - n = 3$ has the solution $m = 4$

and $a_5 - (a_1 + a_2 + a_3 + a_4) = 9 - (1 + 3 + 4 + 7) = -6 < 5$

$\therefore$ there are 2 solutions

**30.** Since $2m - n = 2$ is not possible

but $2m - n + 1 = 2$ has the solution $m = 3$ and $2 < 5$

and $10 - (1 + 3 + 4) = 2 > 1$

$\therefore$ there is no solution

**31.** Since $2m - n = 2$ has no solution

$2m - n + 1 = 2$ has a solution $m = 3$ and $2 < 5$

and $7 - (1 + 2 + 4) = 0 < 10$

$\therefore$ there are two solutions.

**32. (A)** $$\frac{5x + 1 - x^2 - 2x - 1}{(x+1)^2} < 0$$

$-x^2 + 3x < 0,\ x \neq -1$

$x(x-3) > 0,\ x \neq -1$

$\therefore$ $x \in (-\infty, -1) \cup (-1, 0) \cup (3, \infty)$

**(B)** $$|x| + |x-3| = \begin{cases} -2x+3 & : \quad x < 0 \\ 3 & : \quad 0 \leq x \leq 3 \\ 2x-3 & : \quad x > 3 \end{cases}$$

$\therefore$ $x \in (-\infty, 0) \cup (3, \infty)$

**(C)** $$\frac{1}{|x|-3} - \frac{1}{2} < 0 \Rightarrow \frac{2-|x|+3}{2(|x|-3)} < 0$$

$(5 - |x|) / (|x| - 3) < 0 \quad \Rightarrow \quad |x| < 3$ or $|x| > 5$

$\Rightarrow$ $x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$

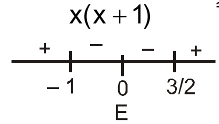
**(D)** $$\frac{x^4}{(x-2)^2} > 0 \Rightarrow x \in (-\infty, 0) \cup (0, 2) \cup (2, \infty)$$

**33. (A)** $\log_{\sin x}(\log_3(\log_{0.2} x)) < 0$

$\Rightarrow$ $0 < x < \frac{1}{125}$ (which also satisfy $0 < \sin x < 1$)

**(B)** $$\frac{(e^x - 1)(2x-3)(x^2+x+2)}{(\sin x - 2)\, x(x+1)} \leq 0$$

$$\frac{(e^x - 1)(x - 3/2)}{x(x+1)} \geq 0$$




$\Rightarrow$ $x \in (-\infty, -1) \cup [3/2, \infty)$

(C) $|2-|[x]-1||\le 2 \Rightarrow ||[x]-1|-2|\le 2$

$\Rightarrow 0\le |[x]-1|\le 4 \Rightarrow -3\le [x]\le 5 \Rightarrow x\in[-3,6)$

(D) $|\sin^{-1}(3x-4x^3)|\le \frac{\pi}{2} \Rightarrow -\frac{\pi}{2}\le \sin^{-1}(3x-4x^3)\le \frac{\pi}{2}$

$\Rightarrow -1\le 3x-4x^3\le 1 \Rightarrow -1\le x\le 1$

**34.** By using definitions of modulus, greatest integer and fractional part function, obviously.

**35.** (A) $(3-x)>3\sqrt{1-x^2}$

Case-I (i) $3-x\ge 0 \Rightarrow x\le 3$

(ii) $\sqrt{1-x^2}\ge 0 \Rightarrow x\in[-1,1]$

(iii) $9+x^2-6x>9-9x^2 \Rightarrow 10x^2-6x>0$

$x(5x-3)>0 \Rightarrow x\in(-\infty,0)\cup\left(\frac{3}{5},\infty\right)$

$\therefore x\in[-1,0)\cup\left(\frac{3}{5},1\right]$

Case-II (i) $3-x<0$

– ve > + ve not possible

$\therefore$ by case-I & II $x\in[-1,0)\cup\left(\frac{3}{5},1\right]$

(B) $-\sqrt{x+2}<-x \Leftrightarrow x<\sqrt{x+2}$

Case-I (i) $x\ge 0$

(ii) $x+2>0$

(iii) $x^2<x+2$

so $x\in[0,2)$

Case-II (i) $x<0$

(ii) $x+2\ge 0$

(iii) –ve < + ve

so $x\in[-2,0)$

by case-I & II $x\in[-2,2)$

(C) $\log_5(x-3)+\frac{1}{2}\log_5 3<\frac{1}{2}\log_5(2x^2-6x+7)$

$3(x-3)^2<(2x^2-6x+7) \Rightarrow x\in(2,10)$

$\Rightarrow x>3$

so $x\in(3,10)$

(D) $7^{x+2}-\frac{1}{7}.7^{x+1}-14.7^{x-1}+2.7^x=48$

Let $7^x=t$

$49t-t-2t+2t=48$

$\therefore t=1$ so $x=0$

**36.** Case – I

$0<x^2<1 \Rightarrow x\in(-1,1)-\{0\}$ ............(i)

$|x-1|<1 \Rightarrow -1<x-1<1 \Rightarrow 0<x<2$ ............(ii)

from (i) and (ii), we get $x\in(0,1)$

Case – II

$x^2>1 \Rightarrow x\in(-\infty,-1)\cup(1,\infty)$ ............(iii)

$|x-1|>1 \Rightarrow x-1>1$ or $x-1<-1$

$\Rightarrow x>2$ or $x<0$ ............(iv)

from (iii) and (iv), we get $x\in(-\infty,-1)\cup(2,\infty)$

$\therefore x\in(-\infty,-1)\cup(0,1)\cup(2,\infty)$

inequality is not defined for

$x=-1,0,1,2$

$\therefore$ sum of their absolute values $=|-1|+|0|+|1|+|2|=4$.

**37.** $|x^2-3x-1|<|3x^2+2x+1|+|2x^2+5x+2|,\ x^2-3x-1\ne 0$

$\Leftrightarrow |(3x^2+2x+1)-(2x^2+5x+2)|<|3x^2+2x+1|+|2x^2+5x+2|,\ x^2-3x-1\ne 0$

The inequality holds if and only if

$(3x^2+2x+1)(2x^2+5x+2)>0$

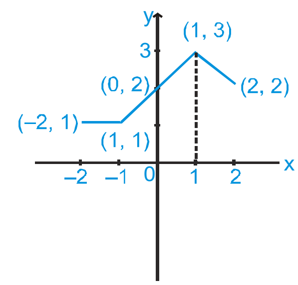
i.e. $2x^2+5x+2>0$

i.e. $(2x+1)(x+2)>0$

i.e. $x\in(-\infty,-2)\cup(-1/2,\infty) \Rightarrow a=2$ and $b=\frac{1}{2}$

$\therefore a+\log ab=2$

**38.** Drawing the graph of $y=f(x)$




Clearly the range of $y=f(x)$ is $[1,3]$

when $-2\le x\le -1$, $\{f(x)\}=0$

when $-1\le x\le 0$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

when $0\le x\le 1$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

when $1\le x\le 2$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

Hence the total number of values of x for which $\{f(x)\}=\frac{1}{2}$ are 3

**39.** $\sqrt{\left[x+\left[\frac{x}{2}\right]\right]}+\left[\sqrt{\{x\}}+\left[\frac{x}{3}\right]\right]=3$

$\Rightarrow \sqrt{[x]+\left[\frac{x}{2}\right]}+\left[\sqrt{\{x\}}\right]+\left[\frac{x}{3}\right]=3 \Rightarrow \sqrt{[x]+\left[\frac{x}{2}\right]}+\left[\frac{x}{3}\right]=3$

**Case-1** $\sqrt{[x]+\left[\frac{x}{2}\right]}=3$ and $\left[\frac{x}{3}\right]=0$

i.e. $[x]+\left[\frac{x}{2}\right]=9$ and $0\le x<3$

$0\le x<3 \Rightarrow [x]=0, 1$ or $2$ ; $\left[\frac{x}{2}\right]=0$ or $1$

$\therefore$ There is no solution in this case.

**Case-2** $\sqrt{[x]+\left[\frac{x}{2}\right]}=2$ and $\left[\frac{x}{3}\right]=1$

i.e. $[x]+\left[\frac{x}{2}\right]=4$ and $3\le x<6$

$3\le x<6 \Rightarrow [x]=3, 4$ or $5$ ; $\left[\frac{x}{2}\right]=1$ or $2$

$\therefore$ $[x]+\left[\frac{x}{2}\right]=4$ has a solution for $[x]=3$ and $\left[\frac{x}{2}\right]=1$ and $\left[\frac{x}{3}\right]=1$.

i.e. $3\le x<4, 2\le x<4$ and $3\le x<6$

$\therefore$ $[3, 4)$ are solutions.

**Case-3** $\sqrt{[x]+\left[\frac{x}{2}\right]}=1$ and $\left[\frac{x}{3}\right]=2$

i.e. $[x]+\left[\frac{x}{2}\right]=1$ and $6\le x<9$

$6\le x<9 \Rightarrow [x]=6, 7$ or $8$ ; $\left[\frac{x}{2}\right]=3$ or $4$

$\therefore$ $[x]+\left[\frac{x}{2}\right]=1$ and $\left[\frac{x}{3}\right]=2$ is not possible.

**Case-4** $\sqrt{[x]+\left[\frac{x}{2}\right]}=0$ and $\left[\frac{x}{3}\right]=3$

i.e. $[x]+\left[\frac{x}{2}\right]=0$ and $9\le x<12$

$9\le x<12 \Rightarrow [x]=9, 10$ or $11$ ; $\left[\frac{x}{2}\right]=4$ or $5$

$\therefore$ $[x]+\left[\frac{x}{2}\right]=0$ has no solution.

Hence the solution set is $[3, 4)$.

## MOCK TEST : VECTOR

**1.** $0=(\vec{a}+\vec{b})\cdot(2\vec{a}+3\vec{b})\times(3\vec{a}-2\vec{b})$

$=(\vec{a}+\vec{b})\cdot(-4\vec{a}\times\vec{b}-9\vec{a}\times\vec{b})=-13\,(\vec{a}+\vec{b})\cdot(\vec{a}\times\vec{b})$

which is true for all values of $\vec{a}$ and $\vec{b}$.

**2.** Volume of the parallelopiped formed by $\vec{a}', \vec{b}', \vec{c}'$ is 4

$\therefore$ Volume of the parallelopiped formed by $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{4}$

$\vec{b}\times\vec{c}=\frac{(\vec{c}'\times\vec{a}')\times\vec{c}}{4}=\frac{1}{4}\vec{a}'$

$\therefore$ $|\vec{b}\times\vec{c}|=\frac{\sqrt{2}}{4}=\frac{1}{2\sqrt{2}}$

$\therefore$ length of altitude $=\frac{1}{4}\times 2\sqrt{2}=\frac{1}{\sqrt{2}}$.

**3.** Let $\vec{a}=\lambda\vec{b}+\mu\vec{c}$

then $\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{\vec{a}\cdot\vec{d}}{ad}$

i.e. $\frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{b}}{b}=\frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{d}}{d}$

i.e. $\frac{[\lambda(2\hat{i}+\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})]\cdot(2\hat{i}+\hat{j})}{\sqrt{5}}$

$=\frac{[\lambda(2\hat{i}+\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})]\cdot(\hat{j}+2\hat{k})}{\sqrt{5}}$

i.e. $\lambda(4+1)+\mu(2-1)=\lambda(1)+\mu(-1+2)$ i.e. $4\lambda=0$

i.e. $\lambda=0$

$\therefore$ $\hat{a}=\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

**4.** $\vec{r}\cdot\vec{a}=0$, $|\vec{r}\times\vec{b}|=|\vec{r}|\,|\vec{b}|$ and $|\vec{r}\times\vec{c}|=|\vec{r}|\,|\vec{c}|$

$\Rightarrow \vec{r}\perp\vec{a}, \vec{b}, \vec{c}$

$\therefore$ $\vec{a}, \vec{b}, \vec{c}$ are coplaner. $\therefore$ $[\vec{a}\ \vec{b}\ \vec{c}]=0$

**5.** Since $\vec{a}_1, \vec{a}_2$ & $\vec{a}_3$ are non-coplanar vectors

$\therefore \quad x+y-3=0 \quad$ ...(i)

$2x-y+2=0 \quad$ ...(ii)

$2x+y+\lambda=0 \quad$ ...(iii)

From (i) & (ii) $x=1/3,\ y=8/3$

$\therefore$ from (iii) $\lambda=-\frac{10}{3}$

**6.** $\vec{a}' = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]} = \frac{\hat{i}+\hat{j}-\hat{k}}{2}$

**7.** $\vec{a}+2\vec{b}+3\vec{c}=\vec{0} \Rightarrow \vec{a}\times\vec{b} + 3\vec{c}\times\vec{b}=\vec{0}$

i.e. $\vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$, $\vec{a}\times\vec{c} + 2\vec{b}\times\vec{c}=\vec{0}$

i.e. $2\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\therefore \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 3\vec{b}\times\vec{c} + \vec{b}\times\vec{c}$

$+2\vec{b}\times\vec{c}=6\vec{b}\times\vec{c}$

**8.** Let $\vec{r} = ●(\vec{b}\times\vec{c})+m(\vec{c}\times\vec{a})+n(\vec{a}\times\vec{b})$

$\vec{r}\cdot\vec{a} = ●\,[\vec{a}\,\vec{b}\,\vec{c}] \Rightarrow ●=1$

similarly $m=2$, $n=3$

$\therefore \vec{r}=(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})+3(\vec{a}\times\vec{b})$

**9.** Since $\vec{a}, \vec{c}, \vec{b}$ form a right-handed system, therefore

$$\vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 1 & 0\\ x & y & z\end{vmatrix} = z\vec{i}-x\vec{k}$$

**10.** **S1 :** Obvious

**S2 :** $(4\hat{i}+7\hat{j}-2\hat{k})-(3\hat{i}-4\hat{j}+7\hat{k}) = \hat{i}+11\hat{j}-9\hat{k}$

$\therefore$ form a triangle.

**S3 :** $\vec{a}\cdot(\vec{a}+\vec{b})\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\cdot(\vec{a}+\vec{b})\times\vec{c}$

$=[\vec{a}\ \vec{b}\ \vec{c}]$

**S4 :** $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c}) \Rightarrow (\vec{b}\cdot\vec{c})\,\vec{a}$

$= (\vec{a}\cdot\vec{b})\,\vec{c} \Rightarrow (\vec{b}\cdot\vec{c})\,\vec{a} - (\vec{a}\cdot\vec{b})\,\vec{c} = \vec{0}$

$\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{b}\cdot\vec{c})\,\vec{a}-(\vec{a}\cdot\vec{c})\,\vec{b}$

$=(\vec{a}\cdot\vec{b})\,\vec{c}-(\vec{a}\cdot\vec{c})\,\vec{b} = (\vec{b}\times\vec{c})\times\vec{a}\neq\vec{0}$

**11.** **S1 :** $\vec{a}$ and $\lambda\vec{a}$ are parallel vectors.

**S2 :** $\vec{a}\cdot\vec{b}$ may take negative values also.

**S3 :** $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = |-(\vec{a}\times\vec{b})+(\vec{b}\times\vec{a})|$

$=2\,|\vec{b}\times\vec{a}|$

**S4 :** $(\vec{a}\times\vec{b})^2 = (\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})$

$=\vec{a}\cdot(\vec{b}\times(\vec{a}\times\vec{b})) = \vec{a}\cdot((\vec{b}\cdot\vec{b})\,\vec{a}-(\vec{a}\cdot\vec{b})\,\vec{b})$

$=(\vec{a}\cdot\vec{a})(\vec{b}\cdot\vec{b})-(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{a})$

**12.** **S1 :** $|\vec{a}|=|\vec{b}|=|\vec{a}-\vec{b}|=1$

$a^2+b^2-2\vec{a}\cdot\vec{b}=1$

$\therefore$ angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$

**S2 :** $\frac{\vec{a}+\vec{b}}{2}$ A vector in the direction of angle bisector of $\vec{a}$ and $\vec{b}$ is $\vec{a}+\vec{b}$

$\therefore$ the given statement is not correct in general

**S3 :** $(\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2 = |\vec{a}|^2=\vec{a}^2$

**S4 :** Any vector in the plane $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ is of the form

$\alpha(\hat{i}+\hat{j}+\hat{k})+\beta(-\hat{i}+\hat{j}+\hat{k}) = (\alpha-\beta)\hat{i}+(\alpha+\beta)\hat{j}+(\alpha+\beta)\hat{k}$

this will be perpendicular with $\hat{i}-\hat{j}-\hat{k}$

if $(\alpha-\beta)-(\alpha+\beta)-(\alpha+\beta)=0 \Rightarrow \alpha=-3\beta$

Hence the required vector is of the form $(-4\hat{i}-2\hat{j}-2\hat{k})$

$\therefore$ statement is false

**13.** **S1 :** Let $\vec{d}=d_1\vec{i}+d_2\vec{j}+d_3\vec{k}$

Then $[\vec{d}\,\vec{j}\,\vec{k}]-d_1$

Now $[\vec{d}\,\vec{j}\,\vec{k}][\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix}\vec{d}\cdot\vec{a} & \vec{d}\cdot\vec{b} & \vec{d}\cdot\vec{c}\\ \vec{j}\cdot\vec{a} & \vec{j}\cdot\vec{b} & \vec{j}\cdot\vec{c}\\ \vec{k}\cdot\vec{a} & \vec{k}\cdot\vec{b} & \vec{k}\cdot\vec{c}\end{vmatrix}=0,$

$\rightarrow$ the first row is zero.

But since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors

$\therefore [\vec{a}\,\vec{b}\,\vec{c}]\neq 0 \Rightarrow d_1=0.$

Hence $[\vec{d}\,\vec{j}\,\vec{k}]=0.$

It can similarly be shown that $d_2=0,\ d_3=0.$

Hence $\vec{d}$ is a zero vector.

**S2 : Since** $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, therefore $[\vec{a}\,\vec{b}\,\vec{c}] \neq 0$.

Now $\{\vec{a}+\vec{b}+\vec{c}\}\cdot\{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\}$

$= \vec{a}\cdot(\vec{a}\times\vec{b})+\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{c}\times\vec{a})$

$+\vec{b}\cdot(\vec{a}\times\vec{b})+\vec{b}\cdot(\vec{b}\times\vec{c})+\vec{b}\cdot(\vec{c}\times\vec{a})$

$+\vec{c}\cdot(\vec{a}\times\vec{b})+\vec{c}\cdot(\vec{b}\times\vec{c})+\vec{c}\cdot(\vec{c}\times\vec{a})$

$=0+[\vec{a}\,\vec{b}\,\vec{c}]+0+0+0+[\vec{b}\,\vec{c}\,\vec{a}]+[\vec{c}\,\vec{a}\,\vec{b}]+0$

$+0=3[\vec{a}\,\vec{b}\,\vec{c}]\neq 0.$

Hence $\vec{a}+\vec{b}+\vec{c}$ is not perpendicular to $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

**S3 : Since** $\vec{a}\perp\vec{c}$ $\therefore$ $\vec{a}\cdot\vec{c}=0$.

Now, $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\,\vec{b}-(\vec{a}\cdot\vec{b})\,\vec{c}$

$=0-(\vec{a}\cdot\vec{b})\,\vec{c}=-(\vec{a}\cdot\vec{b})\,\vec{c}$;

and $(\vec{a}\times\vec{b})\times\vec{c}=-\{(\vec{c}\cdot\vec{b})\,\vec{a}-(\vec{c}\cdot\vec{a})\,\vec{b}\}$

$=-(\vec{c}\cdot\vec{b})\,\vec{a}-0=-(\vec{b}\cdot\vec{c})\,\vec{a}$.

Hence $\{\vec{a}\times(\vec{b}\times\vec{c})\}\cdot\{(\vec{a}\times\vec{b})\times\vec{c}\}=$ $(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})\,\vec{c}\cdot\vec{a}=0$

$\therefore$ $(\vec{a}\times\vec{b})\times\vec{c}$ is perpendicular to $\vec{a}\times(\vec{b}\times\vec{c})$

**S4 :** Writing $\vec{a}\times\vec{b}=\vec{p}$, we have,

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{p}\times(\vec{c}\times\vec{d})$

$=(\vec{p}\cdot\vec{d})\,\vec{c}-(\vec{p}\cdot\vec{c})\,\vec{d}$

$=(\vec{a}\times\vec{b}\cdot\vec{d})\vec{c}-(\vec{a}\times\vec{b}\cdot\vec{c})\,\vec{d}=0\vec{c}-0\vec{d}=\vec{0}$;

**Since** $\vec{a}\cdot\vec{b}\times\vec{d}=0$, because $\vec{a},\vec{b},\vec{d}$ are coplanar and $\vec{a}\cdot\vec{b}\times\vec{c}=0$, because $\vec{a},\vec{b},\vec{c}$ are coplanar.

**14.** $(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})]=\vec{b}+\vec{a}$

$\{(\vec{a}-\vec{b})\cdot(2\vec{a}+\vec{b})\}(\vec{b}+\vec{a})-\{(\vec{a}-\vec{b})\cdot(\vec{b}+\vec{a})\}(2\vec{a}+\vec{b})=\vec{b}+\vec{a}$

$\Rightarrow$ $(2-\vec{a}\cdot\vec{b}-1)(\vec{b}+\vec{a})=\vec{b}+\vec{a}$

$\Rightarrow$ either $\vec{b}+\vec{a}=\vec{0}$ or $1-\vec{a}\cdot\vec{b}=1$

$\Rightarrow$ either $\vec{b}=-\vec{a}$ or $\vec{a}\cdot\vec{b}=0$

$\Rightarrow$ either $\theta=\pi$ or $\theta=\dfrac{\pi}{2}$

**15. (A)** $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=\vec{a}\times[(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}]$

$=-(\vec{a}\cdot\vec{a})(\vec{a}\times\vec{b})$

$\therefore$ **(A)** is not correct

**(B)** $\vec{v}\cdot\vec{a}=\vec{0}\Rightarrow\vec{v}=\vec{0}$ or $\vec{v}\perp\vec{a}$

$\vec{v}\cdot\vec{b}=\vec{0}\Rightarrow\vec{v}=\vec{0}$ or $\vec{v}\perp\vec{b}$

$\vec{v}\cdot\vec{c}=\vec{0}\Rightarrow\vec{v}=\vec{0}$ or $\vec{v}\perp\vec{c}$

$\therefore$ $\vec{v}=\vec{0}$ or $\vec{v}\perp\vec{a},\vec{b},\vec{c}$

$\therefore$ $\vec{v}=\vec{0}$

**(C)** $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\vec{0}$

$\therefore$ statement is incorrect

**(D)** $\vec{a}\times\vec{b}'+\vec{b}\cdot\vec{c}'+\vec{c}\cdot\vec{a}'=0$.

(Property of reciprocal system)

**(D)** is incorrect

**16.** Both the statements are correct but statement–2 is not correct explanation of statement–1 because vectors $\vec{b},\vec{c},\vec{d}$ in statement–1 are coplanar.

**17.** Statement–1 is false and Statement–2 is true.

**Since** $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplanar

**18.** Statement-1 is correct and Statement – 2 is correct but Statement – 2 is not correct explanation of Statement – 1

**19.** $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=$

$(2\vec{a}-2\vec{b})+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})$

$=-2\overrightarrow{AB}+5\overrightarrow{AC}-6\overrightarrow{AD}=\vec{0}$

$\therefore$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are linearly dependent,

Hence by statement-2, the statement-1 is true.

**20.** Statement - 1 $\vec{b}_1=\left(\dfrac{(2\hat{i}+\hat{j}-3\hat{k})\cdot(3\hat{i}-\hat{j})}{|3\hat{i}-\hat{j}|}\right)\dfrac{3\hat{i}-\hat{j}}{|3\hat{i}-\hat{j}|}$

$=\dfrac{3\hat{i}}{2}-\dfrac{\hat{j}}{2}$

$\therefore$ $\vec{b}_2=2\hat{i}+\hat{j}-3\hat{k}-\dfrac{3\hat{i}}{2}+\dfrac{\hat{j}}{2}=\dfrac{\hat{i}}{2}+\dfrac{3\hat{j}}{2}-3\hat{k}$

$\therefore$ statement is false

Statement - 2 is true

**21.** $\vec{a}_1 = \left[(2\hat{i}+3\hat{j}-6\hat{k})\cdot\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right]\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

$= \frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k})$

$\vec{a}_2 = \frac{-41}{49}\left((2\hat{i}-3\hat{j}+6\hat{k})\cdot\frac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}\right)$

$\frac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7} = \frac{-41}{(49)^2}(-4-9+36)(-2\hat{i}+3\hat{j}+6\hat{k})$

$= \frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

**22.** $\vec{a}_1 \cdot \vec{b} = \frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k}) \cdot (2\hat{i}-3\hat{j}+6\hat{k}) = -41$

**23.** $\vec{a}, \vec{a}_1, \vec{b}$ are coplanar, because $\vec{a}_1, \vec{b}$ are collinear.

**24.** The diagonals are

$\vec{d}_1 = 3\hat{a}-2\hat{b}+2\hat{c} + (-\hat{a}-2\hat{c}) = 2\hat{a}-2\hat{b}$

$\vec{d}_2 = 3\hat{a}-2\hat{b}+2\hat{c} - (-\hat{a}-2\hat{c}) = 4\hat{a}-2\hat{b}+4\hat{c}$

Angle between them $= \cos^{-1}\frac{\vec{d}_1.\vec{d}_2}{|\vec{d}_1|.|\vec{d}_2|}$

$= \cos^{-1}\left(\frac{8+4}{2\sqrt{2}\,(6)}\right) = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

**25.** $\vec{x}+\vec{y} = 2\hat{b}-3\hat{c}$ and $\vec{y}+\vec{z} = -2\hat{a}+3\hat{b}-3\hat{c}$

$\therefore (\vec{x}+\vec{y}) \times (\vec{y}+\vec{z}) = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ 0 & 2 & -3 \\ -2 & 3 & -3 \end{vmatrix} = 3\hat{a}+6\hat{b}+4\hat{c}$

$\therefore$ requied unit vector $= \frac{3\hat{a}+6\hat{b}+4\hat{c}}{\sqrt{61}}$

**26.** $\begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ x & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2(4-1)+3(2+x)+4(-1-2x) = 0 \Rightarrow x = \frac{8}{5}$

**27.** $\vec{r}\times\vec{x} = \vec{y}\times\vec{x} \Rightarrow (\vec{r}-\vec{y})\times\vec{x} = \vec{0} \Rightarrow \vec{r} = \vec{y}+\lambda\vec{x}$

$\vec{r}\times\vec{y} = \vec{x}\times\vec{y} \Rightarrow (\vec{r}-\vec{x})\times\vec{y} = \vec{0}$

$\Rightarrow \vec{r} = \vec{x}+\mu\vec{y}$

$\vec{y}+\lambda\vec{x} = \vec{x}+\mu\vec{y}$

$(2\vec{a}-\vec{b})+\lambda(\vec{a}+\vec{b}) = (\vec{a}+\vec{b})+\mu(2\vec{a}-\vec{b})$

$\Rightarrow 2+\lambda = 1+2\mu,\ -1+\lambda = 1-\mu \Rightarrow \mu = 1, \lambda = 1$

The point of intersection is $3\vec{a}$

**28.** $\hat{a}\times\hat{b} = \hat{c} \Rightarrow \hat{c}.\hat{a}\times\hat{b} = \hat{c}.\hat{c} = 1$

$\Rightarrow \hat{a}.(\hat{b}\times\hat{c}) + \hat{b}.(\hat{c}\times\hat{a}) + \hat{c}.(\hat{a}\times\hat{b}) = 3$

**29. (A)** $\vec{a}+\vec{b} = \hat{j}$ and $2\vec{a}-\vec{b} = 3\hat{i}+\frac{\hat{j}}{2}$

$\therefore \vec{a} = \hat{i}+\frac{\hat{j}}{2},\ \vec{b} = -\hat{i}+\frac{\hat{j}}{2} \quad \therefore \cos\theta = -\frac{3}{5}$

**(B)** $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{6}$

$\Rightarrow \vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a} = 6$

$\therefore |\vec{a}| = 1$

**(C)** $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = -2\hat{i}-14\hat{j}-10\hat{k}$

$\therefore$ Area $= 5\sqrt{3}$

**(D)** $\vec{a}$ is perpendicular

$\vec{b}+\vec{c} \Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c} = 0$ .... (i)

$\vec{b}$ is perpendicular $\vec{a}+\vec{c} \Rightarrow \vec{b}\cdot\vec{c}+\vec{b}\cdot\vec{c} = 0$ .... (ii)

$\vec{c}$ is perpendicular $\vec{a}+\vec{b} \Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b} = 0$ .... (iii)

from (i), (ii) and (iii) we get

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$\therefore |\vec{a}+\vec{b}+\vec{c}| = 7$

**30. (A)** $\overrightarrow{OA} = \hat{i}+2\hat{j}+3\hat{k}$, $\overrightarrow{OB} = -2\hat{i}+\hat{j}-4\hat{k}$,

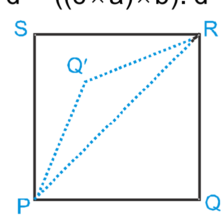
$\overrightarrow{OC} = 3\hat{i}+4\hat{j}-2\hat{k}$

Area $= \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{\sqrt{1218}}{2}$

**(B)** $((\vec{a}\times\vec{b})\times\vec{c}).\vec{d} + ((\vec{b}\times\vec{c})\times\vec{a}).\vec{d} + ((\vec{c}\times\vec{a})\times\vec{b}).\vec{d} = 0$

**(C)** taking P as origin position vector of Q, R and S are

$P\hat{i},\ P\hat{i}+P\hat{j},\ P\hat{j}$




equations of PQ′ and RS are $\vec{r} = t(\hat{i}+\hat{j}+\sqrt{2}\hat{k}),\ \vec{r} = P\hat{i}+P\hat{j}+\lambda\hat{i}$

$\therefore$ shortest distance $= \frac{2P}{\sqrt{6}} \quad \therefore k = 2$

**(D)** $(\vec{a}.\vec{c})(\vec{b}.\vec{d}) - (\vec{b}.\vec{c})(\vec{a}.\vec{d}) = 21$