SOLVED EXAMPLES

Ex. 1 Find the value of x and y for which $(2 + 3i)\, x^2 - (3 - 2i)\, y = 2x - 3y + 5i$ where $x, y \in R$.

Sol. $(2 + 3i)x^2 - (3 - 2i)y = 2x - 3y + 5i$

$\Rightarrow \quad 2x^2 - 3y = 2x - 3y$

$\Rightarrow \quad x^2 - x = 0$

$\Rightarrow \quad x = 0, 1 \quad$ and $\quad 3x^2 + 2y = 5$

$\Rightarrow \quad$ if $x = 0,\ y = \dfrac{5}{2}$ $\quad$ and $\quad$ if $x = 1, y = 1$

$\therefore \quad x = 0, y = \dfrac{5}{2}$ $\quad$ and $\quad x = 1, y = 1$

are two solutions of the given equation which can also be represented as $\left(0, \dfrac{5}{2}\right)$ & $(1, 1)$ $\left(0, \dfrac{5}{2}\right)$, $(1, 1)$

Ex. 2 Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$.

Sol. Let $p(x) = x^3 - 3x^2 + 4x - 12$ be the given polynomial. By factor theorem, $(x - a)$ is a factor of a polynomial $p(x)$ iff $p(A) = 0$. Therefore, in order to prove that $x - 3$ is a factor of $p(x)$, it is sufficient to show that $p(3) = 0$. Now,

$p(x) = x^3 - 3x^2 + 4x - 12$

$\Rightarrow \quad p(3) = 3^3 - 3 \times 3^2 + 4 \times 3 - 12$

$= 27 - 27 + 12 - 12 = 0$

Hence, $(x - 3)$ is a factor of $p(x) = x^3 - 3x^2 + 4x - 12$.

Ex. 3 If a two-digit number is divided by the number having same digits written in reverse order, we get 4 as quotient and 3 as remainder and if the number is divided by the sum of the digits then 8 as a quotient and 7 as a remainder is obtained. Find the number.

Sol. Let $10x+y$ be the required number.

$\therefore \quad 10x+y = 4(10y+x) + 3 \quad$ ...... **(i)**

and $\quad 10x+y = 8(x+y) + 7, \quad$ ...... **(ii)**

on solving **(i)** and **(ii)**

we get $\quad x=7, \quad y=1$

$\therefore \quad$ the number is equal to 71

Ex. 4 If $\left(a + \dfrac{1}{a}\right)^2 = 3$, then find value of $a^3 + \dfrac{1}{a^3}$.

Sol. $a + \dfrac{1}{a} = \pm\sqrt{3}$

$$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right) = \pm 3\sqrt{3}\ \mu\ 3\sqrt{3} = 0.$$

Ex. 5 Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$.

Sol. $a^{9(1/6)(1/3)4} \cdot a^{9(1/3)(1/6)4} = a^2 \cdot a^2 = a^4$.

Ex. 6 Find rational numbers a and b, such that $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

Sol. $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}} = a + b\sqrt{5} \quad \Rightarrow \quad \frac{61+24\sqrt{5}}{-29} = a + b\sqrt{5}$

$a = -\frac{61}{29}, \; b = -\frac{24}{29}$

Ex. 7 What term must be added to each term of the ratio 5 : 37 to make it equal to 1 : 3?

Sol. Let x be added to each term of the ratio 5 : 37.

Then $\frac{x+5}{x+37} = \frac{1}{3} \quad \Rightarrow \quad 3x + 15 = x + 37$ i.e. $x = 11$

Ex. 8 Solve the equation $\frac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \frac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3}$

Sol. $\frac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \frac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3}$

By the process of componendo and dividendo, we have

$$\frac{3x^4}{x^2 - 2x - 3} = \frac{5x^4}{2x^2 - 7x + 3}$$

or $3x^4(2x^2 - 7x + 3) - 5x^4(x^2 - 2x - 3) = 0$

or $x^4[6x^2 - 21x + 9 - 5x^2 + 10x + 15] = 0$

or $x^4(x^2 - 11x + 24) = 0$

$\therefore$ $x = 0$ or $x^2 - 11x + 24 = 0$

$x = 0$ or $(x-8)(x-3) = 0$

$\therefore$ $x = 0, 8, 3$

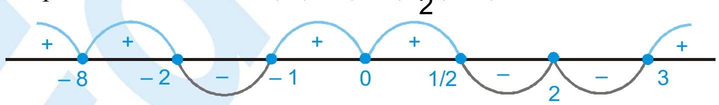
Ex.9 Solve the inequality if

$$f(x) = \frac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5} \text{ is } > 0 \text{ or } < 0.$$

Sol. Let

$$f(x) = \frac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$$

the poles and zeros are $0, 3, -2, -1, \frac{1}{2}, -8, 2$



If $f(x) > 0$, then $x \in (-\infty, -8) \cup (-8, -2) \cup (-1, 0) \cup \left(0, \frac{1}{2}\right) \cup (3, \infty)$

and if $f(x) < 0$, then $x \in (-2, -1) \cup \left(\frac{1}{2}, 2\right) \cup (2, 3)$

Ex. 10 Solve the following linear equations

(i) $x|x| = 4$ **(ii)** $|x - 3| + 2|x + 1| = 4$

Sol. (i) $x|x| = 4$

If $x > 0$

$\therefore$ $x^2 = 4$ $\Rightarrow$ $x = \pm 2$ $\therefore$ $x = 2$ $(\because x \geq 0)$

If $x < 0$ $\Rightarrow$ $-x^2 = 4 \Rightarrow$ $x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

case I : If $x \leq -1$

$\therefore$ $-(x - 3) - 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 - 2x - 2 = 4$ $\Rightarrow$ $-3x + 1 = 4$

$\Rightarrow$ $-3x = 3$ $\Rightarrow$ $x = -1$

case II : If $-1 < x \leq 3$

$\therefore$ $-(x - 3) + 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 + 2x + 2 = 4$

$\Rightarrow$ $x = -1$ which is not possible

case III : If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4$ $\Rightarrow$ $x = 5/3$ which is not possible

$\therefore$ $x = -1$

Ex. 11 If $\sqrt{5} = 2.236$ and $\sqrt{2} = 1.414$, then evaluate : $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{4}{\sqrt{5}-\sqrt{2}}$

Sol. $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{4}{\sqrt{5}-\sqrt{2}}$

$$= \frac{3(\sqrt{5}-\sqrt{2}) + 4(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$$

$$= \frac{3\sqrt{5} - 3\sqrt{2} + 4\sqrt{5} + 4\sqrt{2}}{5-2}$$

$$= \frac{7\sqrt{5}+\sqrt{2}}{5-2} \Rightarrow = \frac{7\sqrt{5}+\sqrt{2}}{3}$$

$$= \frac{7 \times 2.236 + 1.414}{3} \Rightarrow = \frac{15.652 + 1.414}{3}$$

$$= \frac{17.066}{3} = 5.688 \text{ (Approx).}$$

Ex. 12 If $25^{x-1} = 5^{2x-1} - 100$, find the value of x.

Sol. We have,

$\Rightarrow$ $25^{x-1} = 5^{2x-1} - 100$

$\Rightarrow$ $(5^2)^{x-1} = 5^{2x-1} - 100.$ $\Rightarrow$ $5^{2x-2} - 5^{2x-1} = -100$

$\Rightarrow$ $5^{2x-2} - 5^{2x-2}.5^1 = -100$ $\Rightarrow$ $5^{2x-2}(1-5) = -100$

$\Rightarrow$ $5^{2x-2}(-4) = -100$ $\Rightarrow$ $5^{2x-2} = 25$

$\Rightarrow$ $5^{2x-2} = 5^2$ $\Rightarrow$ $2x - 2 = 2$

$\Rightarrow$ $2x = 4$

$x = 2.$

Ex. 13 Find the value of the followings :

(i) $\log_a\left(1-\frac{1}{2}\right)+\log_a\left(1-\frac{1}{3}\right)+.......+\log_a\left(1-\frac{1}{n}\right)$

(ii) $\log_2\frac{75}{16}-2\log_2\frac{5}{9}+\log_2\frac{32}{243}$

(iii) $81^{\frac{1}{\log_5 3}}$

Sol. (i) $\log_a\left(1-\frac{1}{2}\right)+\log_a\left(1-\frac{1}{3}\right)+.......+\log_a\left(1-\frac{1}{n}\right)$

$$=\log_a\left(\frac{1}{2}\right)+\log_a\left(\frac{2}{3}\right)+......+\log_a\left(\frac{n-1}{n}\right)=\log_a\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}.......\frac{n-1}{n}\right)=\log_a\left(\frac{1}{n}\right)=-\log_a n$$

(ii) $\log_2\left(\frac{75}{16}\right)-2\log_2\left(\frac{5}{9}\right)+\log_2\left(\frac{32}{243}\right)$

$$=\log_2\left(\frac{75}{16}\right)-\log_2\left(\frac{25}{81}\right)+\log_2\left(\frac{35}{243}\right)$$

$$=\log_2\left(\frac{75}{16}\times\frac{32}{243}\times\frac{81}{25}\right)=\log_2 2=1$$

(iii) $81^{\frac{1}{\log_5 3}}=81^{\log_3 5}=3^{\log_3 5^4}=5^4=625$

Ex. 14 $\log_2\log_4\log_5 x=0$

Sol. $\log_4\log_5 x=2^0=1$

$\log_5 x=4$

$\Rightarrow$ $x=5^4=625$

Ex. 15 The value of N, satisfying $\log_a[1+\log_b\{1+\log_c(1+\log_p N)\}]=0$.

Sol. $1+\log_b\{1+\log_c(1+\log_p N)\}=a^0=1$

$\Rightarrow$ $\log_b\{1+\log_c(1+\log_p N)\}=0$ $\Rightarrow$ $1+\log_c(1+\log_p N)=1$

$\Rightarrow$ $\log_c(1+\log_p N)=0$ $\Rightarrow$ $1+\log_p N=1$

$\Rightarrow$ $\log_p N=0$ $\Rightarrow$ $N=1$

Ex. 16 If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then show that $\frac{x^3+a^3}{x^2+a^2}+\frac{y^3+b^3}{y^2+b^2}+\frac{z^3+c^3}{z^2+c^2}=\frac{(x+y+z)^3+(a+b+c)^3}{(x+y+z)^2+(a+b+c)^2}$.

Sol. $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$ (constant)

$\therefore$ $x=ak;\ y=bk;\ z=ck$

Substituting these values of x, y, z in the given expression

$$\frac{x^3+a^3}{x^2+a^2}+\frac{y^3+b^3}{y^2+b^2}+\frac{z^3+c^3}{z^2+c^2}=\frac{(x+y+z)^3+(a+b+c)^3}{(x+y+z)^2+(a+b+c)^2}$$

we obtain

L.H.S. $$=\frac{a^3k^3+a^3}{a^2k^2+a^2}+\frac{b^3k^3+b^3}{b^2k^2+b^2}+\frac{c^3k^3+c^3}{c^2k^2+c^2}=\frac{a^3(k^3+1)}{a^2(k^2+1)}+\frac{b^3(k^3+1)}{b^2(k^3+1)}+\frac{c^3(k^3+1)}{c^2(k^2+1)}$$

$$=\frac{a(k^3+1)}{k^2+1}+\frac{b(k^3+1)}{k^2+1}+\frac{c(k^3+1)}{k^2+1}=\frac{(k^3+1)}{(k^2+1)}\,.\,(a+b+c)$$

Now R.H.S $$=\frac{(ak+bk+ck)^3+(a+b+c)^3}{(ak+bk+ck)^2+(a+b+c)^2}=\frac{k^3(a+b+c)^3+(a+b+c)^3}{k^2(a+b+c)^2+(a+b+c)^2}$$

$$=\frac{(k^3+1)(a+b+c)^3}{(k^2+1)(a+b+c)^2}=\frac{(k^3+1)}{(k^2+1)}\,.\,(a+b+c)$$

We see that L.H.S. = R.H.S.

Ex. 17 If $a^2 + b^2 = 23ab$, then prove that $\log\frac{(a+b)}{5}=\frac{1}{2}(\log a+\log b)$.

Sol. $a^2 + b^2 = (a + b)^2 - 2ab = 23ab$

$\Rightarrow$ $(a + b)^2 = 25ab \Rightarrow a+b = 5\sqrt{ab}$ ....(i)

Using (i)

L.H.S. $=\log\frac{(a+b)}{5}=\log\frac{5\sqrt{ab}}{5}=\frac{1}{2}\log ab=\frac{1}{2}(\log a+\log b)=$ R.H.S.

Ex. 18 If $\frac{z-1}{z+1}$ is purely imaginary, then prove that $|z| = 1$

Sol. $\text{Re}\left(\frac{z-1}{z+1}\right)=0$

$\Rightarrow$ $\frac{z-1}{z+1}+\left(\overline{\frac{z-1}{z+1}}\right)=0$

$\Rightarrow$ $\frac{z-1}{z+1}+\frac{\bar{z}-1}{\bar{z}+1}=0$

$\Rightarrow$ $z\bar{z}-\bar{z}+z-1+z\bar{z}-z+\bar{z}-1=0$

$\Rightarrow$ $z\bar{z}=1$ $\Rightarrow$ $|z|^2=1$

$\Rightarrow$ $|z|=1$ **Hence proved**

Ex. 19 Evaluate : $81^{1/\log_5 3}+27^{\log_9 36}+3^{4/\log_7 9}$

Sol. $81^{\log_3 5}+3^{3\log_9 36}+3^{4\log_9 7}$

$=3^{4\log_3 5}+3^{\log_3 (36)^{3/2}}+3^{\log_3 7^2}$

$=625+216+49=890.$

Ex. 20 If a, b, c are distinct positive real numbers different from 1 such that

$(\log_b a \, . \, \log_c a - \log_a a) + (\log_a b \, . \, \log_c b - \log_b b) + (\log_a c \, . \, \log_b c - \log_c c) = 0$, then find value of abc.

Sol. $(\log_b a \log_c a - 1) + (\log_a b \, . \, \log_c b - 1) + (\log_a c \log_b c - 1) = 0$

$$\Rightarrow \frac{\log a}{\log b}.\frac{\log a}{\log c} + \frac{\log b}{\log a}.\frac{\log b}{\log c} + \frac{\log c}{\log a}.\frac{\log c}{\log b} = 3 \quad \Rightarrow \quad (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a \log b \log c$$

$\Rightarrow$ $(\log a + \log b + \log c) = 0$ [✈If $a^3 + b^3 + c^3 - 3abc = 0$, then $a + b + c = 0$ if $a \neq b \neq c$]

$\Rightarrow$ $\log abc = \log 1$ $\Rightarrow$ $abc = 1$

Ex. 21 If $||x-1|-2| = 5$, then find x.

Sol. $|x-1| - 2 = \pm 5$

$|x-1| = 7, -3$

Case-I : When $|x-1| = 7$ $\Rightarrow$ $x - 1 = \pm 7$ $\Rightarrow$ $x = 8, -6$

Case-II : When $|x-1| = -3$ (reject)

Ex. 22 Solve the inequality $\log_{1/3}(5x-1) > 0$.

Sol. by using the basic property of logarithm.

$$\begin{cases} 5x - 1 < 1 \\ 5x - 1 > 0 \end{cases} \Rightarrow \begin{cases} 5x < 2 \\ 5x > 1 \end{cases} \Rightarrow \begin{matrix} x < \dfrac{2}{5} \\ x > \dfrac{1}{5} \end{matrix}$$

$\Rightarrow$ The solution of the inequality is given by $\left(\frac{1}{5}, \frac{2}{5}\right)$ Ans.

Ex. 23 Solve the inequality $\log_{(2x+3)} x^2 < \log_{(2x+3)}(2x+3)$.

Sol. The given inequality is equivalent to the collection of the systems

$$\left[\begin{array}{l} \begin{cases} 0 < 2x + 3 < 1 \\ x^2 > 2x + 3 \end{cases} \quad \text{.........(i)} \\ \begin{cases} 2x + 3 > 1 \\ 0 < x^2 < 2x + 3 \end{cases} \quad \text{.........(ii)} \end{array}\right.$$

Solving system **(i)** we obtain

$$\begin{cases} \dfrac{-3}{2} < x < -1 \\ (x-3)(x+1) > 0 \end{cases} \quad \text{.........(iii)}$$

System **(iii)** is equivalent to the collection of two systems

$$\left[\begin{array}{ll} \frac{-3}{2} < x < -1, \quad x > 3 & \text{.........(iv)} \\ \frac{-3}{2} < x < -1, \quad x < -1 & \text{.........(v)} \end{array}\right.$$

system **(iv)** has no solution. The solution of system **(v)** is $x \in \left(\frac{-3}{2}, -1\right)$,

solving system **(ii)** we obtain.

$$\begin{cases} x > -1 \\ (x-3)(x+1) < 0 \end{cases} \quad \text{or} \quad \begin{cases} x > -1 \\ -1 < x < 3 \end{cases} \quad \Rightarrow \quad x \in (-1, 3)$$

$$x \in \left(\frac{-3}{2}, -1\right) \cup (-1, 3)$$

Ex. 24 Solve the inequation $\log_{\left(\frac{x^2-12x+30}{10}\right)}\left(\log_2 \frac{2x}{5}\right) > 0.$

Sol. This inequation is equivalent to the collection of following systems.

$$\begin{cases} \frac{x^2-12x+30}{10} > 1, \\ \log_2\left(\frac{2x}{5}\right) > 1, \end{cases} \quad \text{and} \quad \begin{cases} 0 < \frac{x^2-12x+30}{10} < 1, \\ 0 < \log_2\left(\frac{2x}{5}\right) < 1 \end{cases}$$

Solving the first system we have.

$$\begin{cases} x^2 - 12x + 20 > 0 \\ \frac{2x}{5} > 2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} (x-10)(x-2) > 0 \\ x > 5 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x < 2 \text{ or } x > 10 \\ x > 5 \end{cases}$$

Therefore the system has solution $x > 10$

Solving the second system we have.

$$\Rightarrow \quad \begin{cases} 0 < x^2 - 12x + 30 < 10 \\ 1 < \frac{2x}{5} < 2 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x^2 - 12x + 30 > 0 \quad \text{and} \quad x^2 - 12x + 20 < 0 \\ \frac{5}{2} < x < 5 \end{cases}$$

$$\Rightarrow \quad \begin{cases} x < 6 - \sqrt{6} \quad \text{or} \quad x > 6 + \sqrt{6} \quad \text{and} \quad 2 < x < 10 \\ \frac{5}{2} < x < 5 \end{cases}$$

$\therefore$ The system has solutions $\frac{5}{2} < x < 6 - \sqrt{6}$ combining both systems, then solution of the original in equation is.

$$x \in \left(\frac{5}{2}, 6 - \sqrt{6}\right) \cup (10, \infty)$$

Ex. 25 If $|x-1|+|x+1|=2$, then find x.

Sol. **Case-I**

If $x \le -1$

$-(x-1)-(x+1)=2$

$\Rightarrow \quad -x+1-x-1=2$

$\Rightarrow \quad -2x=2 \quad \Rightarrow \quad x=-1$ ....(i)

Case-II

If $-1<x<1$

$-(x-1)+(x+1)=2$

$\Rightarrow \quad -x+1+x+1=2$

$\Rightarrow \quad 2=2 \quad \Rightarrow \quad -1<x<1$ ....(ii)

Case-III

If $x \ge 1$

$x-1+x+1=2$

$\Rightarrow \quad x=1$ ....(iii)

Thus from (i), (ii) and (iii) $-1 \le x \le 1$

Ex. 26 Solve : $x|x+3|+2|x+2|=0$

Sol. **Case-I**

$x<-3$

$-x(x+3)-2(x+2)=0$

$x^2+5x+4=0 \quad \Rightarrow \quad x=-1,-4$

$\Rightarrow \quad x=-4. \quad \because \quad x=-1$ (reject)

Case- II

$-3<x<-2$

$(x)(x+3)-2x-4=0$

$x^2+x-4=0$

$$\Rightarrow \quad x=\frac{-1+\sqrt{17}}{2}, \frac{-1-\sqrt{17}}{2}$$

$$\Rightarrow \quad x=\frac{-1-\sqrt{17}}{2} \quad \because \quad x=\frac{-1+\sqrt{17}}{2} \text{ (reject)}$$

Case-III

$x>-2$

$x(x+3)+2x+4=0$

$x^2+5x+4=0$

$\Rightarrow \quad x=-1,-4.$

$\Rightarrow \quad x=-1 \quad \because \quad x=-4$ (reject)

Hence $x=-4, \dfrac{-1-\sqrt{17}}{2}, -1.$

Ex. 27 Solve the equation $[x] + \{-x\} = 2x$, (where [.] and { . } represents greatest integer function and fractional part function respectively).

Sol. **Case-I**

$x \in I$

$x + 0 = 2x \quad \Rightarrow \quad x = 0$

Case-II

$x \notin I$

$[x] + 1 - \{x\} = 2x$

$[I + f] + 1 - \{I + f\} = 2(I + f)$

$I + 1 - f = 2I + 2f$

$\frac{1-I}{3} = f$ as $\quad 0 < f < 1$

$0 < \frac{1-I}{3} < 1$

$0 < 1 - I < 3$

$-1 < -I < 2$

$-2 < I < 1 \quad \Rightarrow \quad I = -1, 0$

$f = \frac{2}{3}, \frac{1}{3}$

Here $x = -\frac{1}{3}, \frac{1}{3}$

$\therefore$ Solutions are $x = 0, -\frac{1}{3}, \frac{1}{3}$

Ex. 28 Find the value of expression $x^4 - 4x^3 + 3x^2 - 2x + 1$ when $x = 1 + i$ is a factor of expression.

Sol. $x = 1 + i$

$\Rightarrow \quad x - 1 = i \quad \Rightarrow \quad (x-1)^2 = -1$

$\Rightarrow \quad x^2 - 2x + 2 = 0$

Now $\quad x^4 - 4x^3 + 3x^2 - 2x + 1 = (x^2 - 2x + 2)(x^2 - 3x - 3) - 4x + 7$

$\therefore$ when $x = 1 + i$

i.e. $\quad x^2 - 2x + 2 = 0$

$x^4 - 4x^3 + 3x^2 - 2x + 1 = 0 - 4(1 + i) + 7 = -4 + 7 - 4i = 3 - 4i$

Ex. 29 Solve for z if $z^2 + |z| = 0$

Sol. Let $\quad z = x + iy$

$\Rightarrow \quad (x + iy)^2 + \sqrt{x^2 + y^2} = 0$

$\Rightarrow \quad x^2 - y^2 + \sqrt{x^2 + y^2} = 0 \quad$ and $\quad 2xy = 0$

$\Rightarrow \quad x = 0$ or $y = 0$

when $x = 0 \quad -y^2 + |y| = 0$

$\Rightarrow \quad y = 0, 1, -1 \quad \Rightarrow \quad z = 0, i, -i$

when $\quad y = 0 \quad x^2 + |x| = 0$

$\Rightarrow \quad x = 0 \quad \Rightarrow \quad z = 0$

$z = 0, z = i, z = -i$ **Ans.**

Ex. 30 Find square root of $9 + 40i$

Sol. Let $x + iy = \sqrt{9+40i}$

$(x + iy)^2 = 9 + 40i$

$\therefore \quad x^2 - y^2 = 9$ .....**(i)**

and $xy = 20$ .....**(ii)**

squaring **(i)** and adding with 4 times the square of **(ii)**

we get $x^4 + y^4 - 2x^2 y^2 + 4x^2 y^2 = 81 + 1600$

$\Rightarrow \quad (x^2 + y^2)^2 = 1681$

$\Rightarrow \quad x^2 + y^2 = 41$ .....**(iii)**

from **(i)** + **(iii)** we get

$x^2 = 25 \quad \Rightarrow \quad x = \pm 5$

and $y^2 = 16 \quad \Rightarrow \quad y = \pm 4$

from equation **(ii)** we can see that x & y are of same sign

$\therefore \quad x + iy = (5 + 4i)$ or $-(5 + 4i)$

$\therefore$ sq. roots of $a + 40i = \pm (5 + 4i)$

$\pm (5 + 4i)$ **Ans.**

Ex. 31 Show that $\log_4 18$ is an irrational number.

Sol. $\log_4 18 = \log_4(3^2 \times 2) = 2\log_4 3 + \log_4 2 = 2\frac{\log_2 3}{\log_2 4} + \frac{1}{\log_2 4} = \log_2 3 + \frac{1}{2}$

assume the contrary, that this number $\log_2 3$ is rational number.

$\Rightarrow \quad \log_2 3 = \frac{p}{q}$. Since $\log_2 3 > 0$ both numbers p and q may be regarded as natural number

$\Rightarrow \quad 3 = 2^{p/q} \Rightarrow 2^p = 3^q$

But this is not possible for any natural number p and q. The resulting contradiction completes the proof.

Ex. 32 If in a right angled triangle, a and b are the lengths of sides and c is the length of hypotenuse and $c - b \neq 1$, $c + b \neq 1$, then show that $\log_{c+b} a + \log_{c-b} a = 2\log_{c+b} a \cdot \log_{c-b} a$.

Sol. We know that in a right angled triangle

$c^2 = a^2 + b^2$

$c^2 - b^2 = a^2$ .....**(i)**

$$\text{LHS} = \frac{1}{\log_a(c+b)} + \frac{1}{\log_a(c-b)} = \frac{\log_a(c-b) + \log_a(c+b)}{\log_a(c+b).\log_a(c-b)}$$

$$= \frac{\log_a(c^2 - b^2)}{\log_a(c+b).\log_a(c-b)} = \frac{\log_a a^2}{\log_a(c+b).\log_a(c-b)} \quad \textbf{(using (i))}$$

$$= \frac{2}{\log_a(c+b).\log_a(c-b)} = 2\log_{(c+b)} a \cdot \log_{(c-b)} a = \text{RHS}$$

Ex. 33 Solve the following equation for x : $\frac{6}{5}a^{\log_a x.\log_{10} a.\log_a 5} - 3^{\log_{10}(x/10)} = 9^{\log_{100} x + \log_4 2}$

Sol. Let $A = \log_a x . \log_{10} a . \log_a 5 = \log_{10} x . \log_a 5 = \log_a (5)^{\log_{10} x}$

$\therefore$ $a^A = 5^{\log_{10} x} = 5^{\lambda}$ (say $\log_{10} x = \lambda$)

Let $B = \log_{10}(x/10) = \log_{10} x - 1 = \lambda - 1$

$\therefore$ $3^B = 3^{\lambda - 1} = \frac{3^{\lambda}}{3}$

Let $C = \log_{100} x + \log_4 2 = \log_{10^2} x^1 + \log_{2^2} 2^1 = \frac{1}{2}\log_{10} x + \frac{1}{2} = \frac{\lambda + 1}{2}$

$\therefore$ $9^C = 9^{\frac{\lambda+1}{2}} = 3^{\lambda+1} = 3.3^{\lambda}$

According to question $\frac{6}{5}.5^{\lambda} - \frac{3^{\lambda}}{3} = 3.3^{\lambda}$

$\Rightarrow$ $6.5^{\lambda-1} = 3^{\lambda}\left(\frac{1}{3} + 3\right)$ $\Rightarrow$ $6.5^{\lambda-1} = 3^{\lambda-1}(10)$ $\Rightarrow$ $5^{\lambda-2} = 3^{\lambda-2}$

which is possible only when $\lambda = 2$ $\Rightarrow$ $\log_{10} x = 2$ $\Rightarrow$ $x = 10^2 = 100$

Ex. 34 Solve : $\log_{(2x-1)}\left(\frac{x^4+2}{2x+1}\right) = 1$

Sol. $\frac{x^4+2}{2x+1} = 2x - 1$ $\Rightarrow$ $x^4 + 2 = 4x^2 - 1$

$\Rightarrow$ $x^4 - 4x^2 + 3 = 0$ $\Rightarrow$ $x^2 = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} = 3, 1$

$\Rightarrow$ $x = \pm\sqrt{3}, \pm 1$ .... **(i)**

Substituting $x = -\sqrt{3}$ and -1 in $\log_{2x-1}\left(\frac{x^4+2}{2x+1}\right)$ we get $2x - 1$ negative. And if $x = 1$ in $2x - 1$

we get base = 1 $\Rightarrow$ reject $x = \pm 1, -\sqrt{3}$

Hence $x = \sqrt{3}$

Exercise # 1 [Single Correct Choice Type Questions]

1. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to -

(A) 1/2 **(B)** 1 **(C)** 2 **(D)** 4

2. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ is equal to-

(A) abc **(B)** $\frac{1}{abc}$ **(C)** 0 **(D)** 1

3. Value of x satisfying $\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = \log_{10}\sqrt{1-x^2} + 2$ is

(A) $0 < x < 1$ **(B)** $-1 < x < 1$ **(C)** $-1 < x < 0$ **(D)** none of these

4. The number of real solution of the equation $\log_{10}(7x-9)^2 + \log_{10}(3x-4)^2 = 2$ is -

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

5. Given system of simultaneous equations $2^x . 5^y = 1$ and $5^{x+1} . 2^y = 2$. Then -

(A) $x = \log_{10}5$ and $y = \log_{10}2$ **(B)** $x = \log_{10}2$ and $y = \log_{10}5$

(C) $x = \log_{10}\left(\frac{1}{5}\right)$ and $y = \log_{10}2$ **(D)** $x = \log_{10}5$ and $y = \log_{10}\left(\frac{1}{2}\right)$

6. The value of $3^{\log_4 5} + 4^{\log_5 3} - 5^{\log_4 3} - 3^{\log_5 4}$ is -

(A) 0 **(B)** 1 **(C)** 2 **(D)** none of these

7. The natural number n for which the expression $y = 5(\log_3 n)^2 - \log_3 n^{12} + 9$, has the minimum value is

(A) 2 **(B)** 3 **(C)** $3^{6/5}$ **(D)** 4

8. The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to -

(A) $a^2 - a - 1$ **(B)** $a^2 + a - 1$ **(C)** $a^2 - a + 1$ **(D)** $a^2 + a + 1$

9. The value of the expression, $\log_4\left(\frac{x^2}{4}\right) - 2\log_4(4x^4)$ when $x = -2$ is -

(A) -6 **(B)** -5 **(C)** -4 **(D)** meaningless

10. Which one of the following denotes the greatest positive proper fraction ?

(A) $\left(\frac{1}{4}\right)^{\log_2 6}$ **(B)** $\left(\frac{1}{3}\right)^{\log_3 5}$ **(C)** $3^{-\log_3 2}$ **(D)** $8^{\left(\frac{1}{-\log_3 2}\right)}$

11. The equation, $\log_2(2x^2) + (\log_2 x).\, x^{\log_x(\log_2 x+1)} + \frac{1}{2}\log_4^2(x^4) + 2^{-3\log_{1/2}(\log_2 x)} = 1$ has -

(A) exactly one real solution **(B)** two real solutions

(C) 3 real solutions **(D)** no solution

12. If $p = \frac{s}{(1+k)^n}$, then n equals -

(A) $\log\frac{s}{p(1+k)}$ **(B)** $\frac{\log(s/p)}{\log(1+k)}$ **(C)** $\frac{\log s}{\log p(1+k)}$ **(D)** $\frac{\log p(1+k)}{\log(s/p)}$

13. Let $A = \{x \mid x^2 + (m-1)x - 2(m+1) = 0, x \in R\}$
$B = \{x \mid (m-1)x^2 + mx + 1 = 0, x \in R\}$
Number of values of *m* such that $A \cup B$ has exactly 3 distinct elements, is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

14. Let ABC be a triangle right angled at C. The value of $\frac{\log_{b+c} a + \log_{c-b} a}{\log_{b+c} a \cdot \log_{c-b} a}$ $(b + c \neq 1, c - b \neq 1)$ equals

(A) 1 **(B)** 2 **(C)** 3 **(D)** 1/2

15. Given that $\log_p x = \alpha$ and $\log_q x = \beta$, then value of $\log_{p/q} x$ equals -

(A) $\frac{\alpha\beta}{\beta - \alpha}$ **(B)** $\frac{\beta - \alpha}{\alpha\beta}$ **(C)** $\frac{\alpha - \beta}{\alpha\beta}$ **(D)** $\frac{\alpha\beta}{\alpha - \beta}$

16. If α and β are the roots of the equation $(\log_2 x)^2 + 4(\log_2 x) - 1 = 0$ then the value of $\log_\beta \alpha + \log_\alpha \beta$ equals

(A) 18 **(B)** -16 **(C)** 14 **(D)** -18

17. $\log_A B$, where $B = \frac{12}{3 + \sqrt{5} + \sqrt{8}}$ and $A = \sqrt{1} + \sqrt{2} + \sqrt{5} - \sqrt{10}$ is -

(A) a negative integer **(B)** a prime integer **(C)** a positive integer **(D)** an even–natural

18. If $\log_c 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_c 10$ where $c > 0$; $c \neq 1$; $b > 1$; $b \neq 1$ determine b -

(A) 25 **(B)** 5 **(C)** 625 **(D)** 16

19. The value of p for which both the roots of the quadratic equation, $4x^2 - 20px + (25p^2 + 15p - 66)$ are less than 2 lies in :

(A) $(4/5, 2)$ **(B)** $(2, \infty)$ **(C)** $(-1, 4/5)$ **(D)** $(-\infty, -1)$

20. Number of cyphers after decimal before a significant figure comes in $\left(\frac{5}{3}\right)^{-100}$ is -

(A) 21 **(B)** 22 **(C)** 23 **(D)** none

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Which of the following are correct ?

(A) $\log_3 19 . \log_{1/7} 3.\log_4 1/7 > 2$ (B) $\log_5 (1/23)$ lies between -2 & -1

(C) if $m = 4^{\log_4 7}$ and $n = \left(\frac{1}{9}\right)^{-2\log_3 7}$ then $n = m^4$

(D) $\log_{\sqrt{5}} \sin\left(\frac{\pi}{5}\right) . \log_{\sqrt{\sin\frac{\pi}{5}}} 5$ simplifies to an irrational number

2. If x & y are real numbers and $\frac{y}{x} = x$, then 'y' cannot take the value(s) :

(A) – 1 (B) 0 (C) 1 (D) 2

3. The solution set of the system of equations, $\log_{12} x \left(\frac{1}{\log_x 2} + \log_2 y\right) = \log_2 x$ and $\log_2 x.(\log_3 (x + y)) = 3 \log_3 x$ is -

(A) x = 6 ; y = 2 (B) x = 4 ; y = 3 (C) x = 2 ; y = 6 (D) x = 3 ; y = 4

4. If x_1 and x_2 are the solution of the equation $x^{3\log_{10}^3 x - \frac{2}{3}\log_{10} x} = 100\sqrt[3]{10}$ then -

(A) $x_1 x_2 = 1$ (B) $x_1 . x_2 = x_1 + x_2$ (C) $\log_{x_2} x_1 = -1$ (D) $\log (x_1 . x_2) = 0$

5. If $\log_k x. \log_5 k = \log_x 5$, $k \neq 1$, $k > 0$, then x is is equal to -

(A) k (B) $\frac{1}{5}$ (C) 5 (D) none of these

6. If $x^{3/4(\log_3 x)^2 + \log_3 x - 5/4} = \sqrt{3}$, then x has -

(A) one positive integral value (B) one irrational value

(C) two positive rational values (D) none of these

7. If $\sin^2\beta = \sin\alpha \cos\alpha$ then $\cos 2\beta$ has the value equal to :

(A) $1 + \sin 2\alpha$ (B) $2\sin^2\left(\frac{\pi}{4} - \alpha\right)$ (C) $1 - \sin 2\alpha$ (D) $2\cos^2\left(\frac{\pi}{4} + \alpha\right)$

8. Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :

(A) a natural number (B) a prime number (C) a rational number (D) an integer

9. If $a^x = b$, $b^y = c$, $c^z = a$ and $x = \log_b a^2$; $y = \log_c b^3$ & $z = \log_a c^k$, where a, b, c > 0 & a, b, c ≠ 1, then k is equal to -

(A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (C) $\log_{64} 2$ (D) $\log_{32} 2$

10. Which of the following statements are true

(A) $\log_2 3 < \log_{12} 10$ (B) $\log_6 5 < \log_7 8$

(C) $\log_3 26 < \log_2 9$ (D) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

11. If $a \neq 0$, then the inequation $|x - a| + |x + a| < b$

(A) has no solutions if $b \leq 2|a|$

(B) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b > 2|a|$

(C) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b < 2|a|$

(D) All above

12. Which of the following when simplified reduces to an integer ?

(A) $\frac{2\log 6}{\log 12 + \log 3}$ **(B)** $\frac{\log 32}{\log 4}$ **(C)** $\frac{\log_5 16 - \log_5 4}{\log_5 128}$ **(D)** $\log_{1/4}\left(\frac{1}{16}\right)^2$

13. Difference of squares of two distinct odd natural numbers is always a multiple of.

(A) 4 **(B)** 3 **(C)** 6 **(D)** 8

14. The equation $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has -

(A) no integral solution **(B)** one natural solution **(C)** two real solution **(D)** one irrational solution

15. Values of x satisfying the equation $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(A) 1 **(B)** 5 **(C)** $\frac{1}{25}$ **(D)** 3

16. If $y = \log_{7-a}(2x^2 + 2x + a + 3)$ is defined $\forall\ x \in R$, then possible integral value(s) of a is/are

(A) –3 **(B)** –2 **(C)** 4 **(D)** 5

17. If $|z_1 + z_2|^2 = |z_1|^2 + |z_z|^2$ then

(A) $\frac{z_1}{z_2}$ is purely real

(B) $\frac{z_1}{z_2}$ is purely imaginary

(C) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

(D) amp $\frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$

18. Which of the following when simplified, vanishes ?

(A) $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8}$

(B) $\log_2\left(\frac{2}{3}\right) + \log_4\left(\frac{9}{4}\right)$

(C) $-\log_8 \log_4 \log_2 16$

(D) $\log_{10} \cot 1° + \log_{10} \cot 2° + \log_{10} \cot 3° + \ldots\ldots\ldots + \log_{10} \cot 89°$

19. Which of the following when simplified, reduces to unity ?

(A) $\log_{10}5 \,.\, \log_{10}20 + \log_{10}^2 2$

(B) $\frac{2\log 2 + \log 3}{\log 48 - \log 4}$

(C) $-\log_5 \log_3 \sqrt{\sqrt[5]{9}}$

(D) $\frac{1}{6} \log_{\frac{\sqrt{3}}{2}}\left(\frac{64}{27}\right)$

20. The inequality $[2 - x] + 2[x - 1] \geq 0$ is satisfied by (where [.] denotes greatest integer function) :

(A) $x \in \{0\}$ **(B)** $x \in W$ **(C)** $x \in N$ **(D)** $x \in [1, \infty)$

21. The number $N = \dfrac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$ when simplified reduces to -

(A) a prime number
(B) an irrational number
(C) a real which is less than $\log_3 \pi$
(D) a real which is greater than $\log_7 6$

22. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has :

(A) one irrational solution
(B) no prime solution
(C) two real solutions
(D) one integral solution

23. If $\dfrac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\dfrac{1}{\sqrt{10}}$
(B) x lies between $\dfrac{1}{100}$ and $\dfrac{1}{\sqrt{10}}$
(C) minimum value of x is $\dfrac{1}{10}$
(D) minimum value of x is $\dfrac{1}{100}$

24. If p, q $\in$ N satisfy the equation $x^{\sqrt{x}} = \left(\sqrt{x}\right)^x$ then p & q are -

(A) relatively prime
(B) twin prime
(C) coprime
(D) if $\log_q p$ is defined then $\log_p q$ is not & vice versa

25. Values of x satisfying $\left(\dfrac{1}{3}\right)^{\{x\}} > \dfrac{1}{\sqrt{3}}$ are (where {.} denotes the fractional part function)

(A) π
(B) $-1 + \dfrac{1}{\sqrt{2}}$
(C) $2 + \dfrac{1}{\sqrt[3]{9}}$
(D) $\dfrac{e}{2}$

26. The expression, $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\sqrt[p]{P}}}}}_{n \text{ radical sign}}$ where $p \ge 2$, $p \in N$, when simplified is -

(A) independent of p, but dependent on n
(B) independent of n, but dependent on p
(C) dependent on both p & n
(D) negative

27. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x+y) = \dfrac{2}{3}$ is :

(A) {6, 3}
(B) {3, 6}
(C) {6, 12}
(D) {12, 6}

28. Which of the following numbers are positive ?

(A) $\log_{\log_3 2}\left(\dfrac{1}{2}\right)$
(B) $\log_2\left(\dfrac{2}{3}\right)^{-2/3}$
(C) $\log_{10}\log_{10} 9$
(D) $\log_{10}\sin 25°$

29. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution
(D) complex roots.

30. Let $y = \dfrac{\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x + \cos 6x + \cos 7x}{\sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x + \sin 6x + \sin 7x}$ then which of the following hold good?

(A) The value of y when $x = \pi/8$ is not defined.
(B) The value of y when $x = \pi/16$ is 1.
(C) The value of y when $x = \pi/32$ is $\sqrt{2} - 1$.
(D) The value of y when $x = \pi/48$ is $2 + \sqrt{3}$.

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).
(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** Minimum value of $|x - 2| + |x - 5| + |x + 3|$ is 8.
Statement-II : If $a < b < c$, then the minimum value of $|x - a| + |x - b| + |x - c|$ is $|b - a| + |b - c|$.

2. **Statement-I :** If $a = y^2$, $b = z^2$, $c = x^2$, then $8\log_a x^3 . \log_b y^3 . \log_c z^3 = 27$

Statement-II : $\log_b a . \log_c b = \log_c a$, also $\log_b a = \dfrac{1}{\log_a b}$

3. **Statement-I :** $[x] + [-x] = x^2 - 5x + 6$ has only two real solution.

Statement-II : $[x] + [-x] = \begin{cases} -1 \ , & x \notin I \\ 0 \ , & x \in I \end{cases}$

4. **Statement-I :** If $\log_{(\log_5 x)} 5 = 2$, then $x = 5^{\sqrt{5}}$
Statement-II : $\log_x a = b$, if $a > 0$, then $x = a^{1/b}$

5. **Statement-I :** The equation $\log_{\frac{1}{2+|x|}}(5+x^2) = \log_{(3+x^2)}(15+\sqrt{x})$ has real solutions.

Statement-II : $\log_{1/b} a = -\log_b a$ (where a,b > 0 and b ≠ 1) and if number and base both are greater than unity then the number is positive.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1.

Column – I	Column – II
(A) Set of all values of x satisfying the inequation $\frac{5x+1}{(x+1)^2} < 1$ is	(p) $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$
(B) Set of all values of x satisfying the inequation $\lvert x\rvert + \lvert x-3\rvert > 3$ is	(q) $(-\infty, -5) \cup (-3, 3) \cup (5, \infty)$
(C) Set of all values of x satisfying the inequation $\frac{1}{\lvert x\rvert - 3} < \frac{1}{2}$ is	(r) $(-\infty, -1) \cup (-1, 0) \cup (3, \infty)$
(D) Set of all values of x satisfying the inequation $\frac{x^4}{(x-2)^2} > 0$ is	(s) $(0, 3) \cup (4, \infty)$
	(t) $(-\infty, 0) \cup (3, \infty)$

2. Match the column for values of x which satisfy the equation in Column-I

Column-I	Column-II
(A) $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$	(p) 5
(B) $x^{\log x + 4} = 32$, where base of logarithm is 2	(q) 100
(C) $5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$ where the base of logarithm is 10	(r) 2
(D) $9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3	(s) $\frac{1}{32}$

3.

Column-I	Column-II
(A) Interval containing all the solutions of the inequality $3 - x > 3\sqrt{1-x^2}$ is	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing all the solutions of the inequality $\left(\frac{1}{3}\right)^{\sqrt{x+2}} < 3^{-x}$ is	(q) (π, π^2)
(C) Interval containing all the solutions of the inequality $\log_5(x-3) + \frac{1}{2}\log_5 3 < \frac{1}{2}\log_5(2x^2 - 6x + 7)$ is	(r) $(-\pi, \pi)$
(D) Interval containing all the solutions of the equation $7^{x+2} - \frac{1}{7} \cdot 7^{x+1} - 14.7^{x-1} + 2.7^{x} = 48$ is	(s) $(-e, e)$
	(t) $([\pi], -[-\pi^2])$, where [.] is G.I.F.

Part # II [Comprehension Type Questions]

Comprehension # 1

The general procedure for solving equation containing modulus function is to split the domain into subintervals and solve the various cases. But there are certain structures of equations which can be solved by a different approach. For example, for solving the equation $|f(x)| + |g(x)| = f(x) - g(x)$ one can follow this method. First find the permissible set of values of x for the equation.

Since LHS $\geq 0 \Rightarrow f(x) - g(x) \geq 0$. Now squaring both sides, we get

$f^2 + g^2 + 2|f.g| = f^2 + g^2 - 2fg$

$\Rightarrow$ $|fg| = -fg$. The equation can hold if $f.g \leq 0$ and $f \geq g$. This can be simplified to $f \geq 0, g \leq 0$.

Answer the following questions on the basis of this method

1. The complete solution of the equation $|x^3 - x| + |2 - x| = x^3 - 2$ is

(A) $[2, \infty)$ **(B)** $[-1, 0] \cup [2, \infty)$ **(C)** $\left[2^{\frac{1}{3}}, \infty\right)$ **(D)** none of these

2. The complete solution set of the equation $|x^2 - x| + |x + 3| = |x^2 - 2x - 3|$ is

(A) $[1, \infty)$ **(B)** $[-3, 0] \cup [1, \infty)$ **(C)** $(-\infty, -3]$ **(D)** $(-\infty, -3] \cup [0, 1]$

3. All the condition(s) for which $|f(x) - g(x)| = |f(x)| + |g(x)|$ is true, is

(A) $f(x) \geq 0, g(x) \leq 0$ **(B)** $f(x) \leq 0, g(x) \geq 0$ **(C)** $f(x) . g(x) \leq 0$ **(D)** $f(x) . g(x) = 0$

Comprehension # 2

In comparison of two numbers, logarithm of smaller number is smaller, if base of the logarithm is greater than one. Logarithm of smaller number is larger, if base of logarithm is in between zero and one. For example $\log_2 4$ is smaller than $\log_2 8$ and $\log_{\frac{1}{2}} 4$ is larger than $\log_{\frac{1}{2}} 8$.

On the Basis of Above Information, Answer the Following Questions

1. Identify the correct order :-

(A) $\log_2 6 < \log_3 8 < \log_3 6 < \log_4 6$ **(B)** $\log_2 6 > \log_3 8 > \log_3 6 > \log_4 6$

(C) $\log_3 8 > \log_2 6 > \log_3 6 > \log_4 6$ **(D)** $\log_3 8 > \log_4 6 > \log_3 6 > \log_2 6$

2. $\log_{\frac{1}{20}} 40$ is-

(A) greater than one **(B)** smaller than one

(C) greater than zero and smaller than one **(D)** none of these

3. $\log_{\frac{2}{3}} \frac{5}{6}$ is-

(A) less than zero **(B)** greater than zero and less than one

(C) greater than one **(D)** none of these

Comprehension # 3

Let $a_1 < a_2 < a_3 < ....< a_n$, n is an odd natural number and m, k $\in$ N

Consider the equation

$$|x - a_1| + |x - a_2| + |x - a_3| + ... + |x - a_n| = kx + d.$$

Case-1 When $2m - n = k$ for some $m < n$, then the equation has

(i) no solution if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$

(ii) infinite solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$

(iii) two solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$

Case-2 Let when $2m - n \neq k$ for any $m < n$

Two cases arise

(A) If $|k| > n$, then there is one solution.

(B) If $|k| < n$, then there is m such that $2m - (n - 1) = k$.

(i) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$ no solution

(ii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$ one solution

(iii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$ two solutions

1. Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 9| = 3x + 5$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

2. Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 10| = 2x + 1$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

3. Number of solutions of $|x - 1| + |x - 2| + |x - 4| + |x - 6| + |x - 7| = 2x + 10$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

Exercise # 4 [Subjective Type Questions]

1. Prove that $\frac{\log_a N}{\log_{ab} N} = 1 + \log_a b$ & indicate the permissible values of the letters.

2. The positive integers p, q and r are all primes if $p^2 - q^2 = r$, then find all possible values of r.

3. Solve the system of equations :
$$\log_a x \log_a (xyz) = 48$$
$$\log_a y \log_a (xyz) = 12, a > 0, a \neq 1$$
$$\log_a z \log_a (xyz) = 84$$

4. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then find the value of $\frac{2a^4 b^2 + 3a^2 c^2 - 5e^4 f}{2b^6 + 3b^2 d^2 - 5f^5}$ in terms of a and b.

5. Given $a^2 + b^2 = c^2$ & $a > 0$; $b > 0$; $c > 0$, $c - b \neq 1$, $c + b \neq 1$,
Prove that : $\log_{c+b} a + \log_{c-b} a = 2 . \log_{c+b} a . \log_{c-b} a$

6. Solve the simultaneous equations $|x + 2| + y = 5$, $x - |y| = 1$

7. Find a rational number which is 50 times its own logarithm to the base 10.

8. Solve the equations $\log_{100} |x + y| = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ for x and y.

9. Solve for x : $\log 4 + \left(1 + \frac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3} + 27\right)$.

10. Find the values of x satisfying the equation
$|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2\log_x 9$.

11. Solve the following equation for x & y : $\log_{100} |x + y| = \frac{1}{2}$, $\log_{10}y - \log_{10}|x| = \log_{100}4$.

12. Find the real values of x and y for which the following equation is satisfied :
$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

13. If the numbers 296, 436 and 542 divided by a positive number 'p' leaving the remainder 7, 11 and 15 respectively, then find the largest value of p.

14. Solve the following inequalities :

(i) $\log_{1/5} \frac{4x+6}{x} \geq 0$

(ii) $\log_2(4^x - 2.2^x + 17) > 5$.

(iii) $\log^2 x \geq \log x + 2$

(iv) $\log_{0.5} (x + 5)^2 > \log_{1/2} (3x - 1)^2$.

(v) $\log_{(3x^2+1)} 2 < \frac{1}{2}$

(vi) $\log_{x^2}(2+x) < 1$

15. Solve the following inequalities :

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$ **(ii)** $x - \sqrt{1-|x|} < 0$ **(iii)** $\sqrt{x^2-x-6} < 2x-3$

(iv) $\sqrt{x^2-6x+8} \le \sqrt{x+1}$ **(v)** $\sqrt{x^2-7x+10} + 9\log_4\left(\frac{x}{8}\right) \ge 2x + \sqrt{14x-20-2x^2} - 13$

(vi) $\sqrt{\log_{1/2}^2 x + 4\log_2\sqrt{x}} < \sqrt{2}\,(4-\log_{16}x^4)$.

16. Solve the following equations

(i) $|x| + 2 = 3$ **(ii)** $|x| - 2x + 5 = 0$

(iii) $x|x| = 4$ **(iv)** $||x-1|-2| = 1$

(v) $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$ **(vi)** $|x-3| + 2|x+1| = 4$

(vii) $||x-1|-2| = |x-3|$

17. Find the values of x which satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$

18. Solve for x : $\log_{3/4}\log_8(x^2+7) + \log_{1/2}\log_{1/4}(x^2+7)^{-1} = -2$.

19. Prove that product of four consecutive positive integers increased by 1 is a perfect square.

20. Find value(s) of 'x' satisfying equation $|2x-1| = 3[x] + 2\{x\}$. (where [.] and { . } denote greatest integer and fractional part function respectively).

21. If $(2+i)(2+2i)(2+3i) ...... (2+ni) = x+iy$, then find the value of $5.8.13. ....(4+n^2)$

22. Solve the equation $|x+1| - |x+3| + |x-1| - 2|x-2| = x+2$.

23. Find all possible solutions of equation $||x^2-6x+5| - |2x^2-3x+1|| = 3|x^2-3x+2|$

24. Solve the following inequalities :

(i) $|x-3| \ge 2$ **(ii)** $||x-2|-3| \le 0$

(iii) $||3x-9|+2| > 2$ **(iv)** $|2x-3| - |x| \le 3$

25. Find the number of solutions of equation $3^{|x|} - |2-|x|| = 1$.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\,\omega$ is equal to : **[AIEEE 2003]**

(A) 1 **(B)** – 1 **(C)** i **(D)** – i

2. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **[AIEEE 2003]**

(A) x = 4 n, where n is any positive integer
(B) x = 2 n, where n is any positive integer
(C) x = 4 n + 1, where n is any positive integer
(D) x = 2n + 1, where n is any positive integer

3. Let z,w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals : **[AIEEE 2004]**

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{3\pi}{4}$ **(D)** $\frac{5\pi}{4}$

4. The conjugate of a complex number is $\frac{1}{i-1}$. Then, that complex number is- **[AIEEE 2008]**

(A) $-\frac{1}{i-1}$ **(B)** $\frac{1}{i+1}$ **(C)** $-\frac{1}{i+1}$ **(D)** $\frac{1}{i-1}$

5. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that : **[AIEEE- 2011]**

(A) $\beta \in (0, 1)$ **(B)** $\beta \in (-1, 0)$ **(C)** $|\beta| = 1$ **(D)** $\beta \in (1, \infty)$

6. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is :

(A) 3 **(B)** – 4 **(C)** 6 **(D)** 5 **[Main- 2016]**

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Find sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ **[IIT-1997]**

2. Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$. **[IIT-1997]**

3. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has **[IIT-JEE-1997]**

(A) no solution **(B)** one solution
(C) two solutions **(D)** more than two solutions

4. Find all real numbers x which satisfy the equation

$2\log_2\log_2 x + \log_{1/2}\log_2(2\sqrt{2}\ x) = 1.$ **[JEE - 1999]**

5. **(1)** If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $|z_1+z_2+z_3|$ is : **[IIT-JEE-2000]**

(A) equal to 1 **(B)** less than 1 **(C)** greater than 3 **(D)** equal to 3

(2) If arg (z) < 0, then arg (– z) – arg (z) = **[IIT-JEE-2000]**

(A) π **(B)** $-\pi$ **(C)** $-\frac{\pi}{2}$ **(D)** $\frac{\pi}{2}$

6. Solve the equation $\log_{3/4}\log_8(x^2+7)+\log_{1/2}\log_{1/4}(x^2+7)^{-1}=-2$. **[JEE– 2000]**

7. Number of solutions of $\log_4(x-1)=\log_2(x-3)$ is **[JEE 2001 (Screening)]**
(A) 3 **(B)** 1 **(C)** 2 **(D)** 0

8. The set of all real numbers x for which $x^2-|x+2|+x>0$ is **[IIT-JEE-2002]**
(A) $(-\infty,-\sqrt{2})\cup(2,\infty)$ **(B)** $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$
(C) $(-\infty,-1)\cup(1,\infty)$ **(D)** $(\sqrt{2},\infty)$

9. If z_1 and z_2 are two complex numbers such that $|z_1|<1<|z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$. **[IIT-JEE-2003]**

10. Let $f(x)=\frac{x^2-6x+5}{x^2-5x+6}$ **[IIT-JEE 2007]**

	Column – I		Column – II
(A)	If $-1<x<1$, then f(x) satisfies	**(p)**	$0<f(x)<1$
(B)	If $1<x<2$, then f(x) satisfies	**(q)**	$f(x)<0$
(C)	If $3<x<5$, then f(x) satisfies	**(r)**	$f(x)>0$
(D)	If $x>5$, then f(x) satisfies	**(s)**	$f(x)<1$

11. Let (x_0, y_0) be the solution of the following equations **[JEE 2011]**

$$(2x)^{\ln 2}=(3y)^{\ln 3}$$
$$3^{\ln x}=2^{\ln y}$$

Then x_0 is

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 6

12. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}\right)$ is **[JEE 2012]**

13. If $3^x=4^{x-1}$, then x = **[IIT-JEE Ad.-2013]**

(A) $\frac{2\log_3 2}{2\log_3 2-1}$ **(B)** $\frac{2}{2-\log_2 3}$ **(C)** $\frac{1}{1-\log_4 3}$ **(D)** $\frac{2\log_2 3}{2\log_2 3-1}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. Greatest integer less than or equal to the number $\log_2 15.\ \log_{1/6} 2.\ \log_3 1/6$ is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

2. The solution set of the inequation $1 + \log_{\frac{1}{3}}(x^2 + x + 1) > 0$ is

(A) $(-\infty, -2) \cup (1, \infty)$ **(B)** $[-1, 2]$ **(C)** $(-2, 1)$ **(D)** $(-\infty, \infty)$

3. If $|x^2 - 9| + |x^2 - 4| = 5$, then the set of values of x is

(A) $(-\infty, -3) \cup (3, \infty)$ **(B)** $(-\infty, -2) \cup (3, \infty)$ **(C)** $(-\infty, 3)$ **(D)** $[-3, -2] \cup [2, 3]$

4. Solution set of $|x^2 - 5x + 7| + |x^2 - 5x - 14| = 21$ is

(A) $[-2, 7]$ **(B)** $(-\infty, -2] \cup [7, \infty)$ **(C)** $[7, \infty)$ **(D)** $(-\infty, -2]$

5. Consider the following statements :

S_1 : $x = \sqrt{\log_{11} 7}$ and $y = \sqrt{\log_7 11}$, then $e^{y \bullet n7 - x \bullet n11}$ is equal to 1.

S_2 : $\log_x 3 > \log_x 2$ is true for all values of $x \in (0, 1) \cup (1, \infty)$

S_3 : $|x - 2| = [-\pi]$, then x is 6, –2

S_4 : $\log_{25}(2 + \tan^2\theta) = 0.5$, then θ may be $\frac{4\pi}{3}$ or $\frac{2\pi}{3}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) T F F T **(B)** F F T T **(C)** F T F T **(D)** T T F T

6. The product of all the solutions of the equation $(x - 2)^2 - 3|x - 2| + 2 = 0$ is

(A) 2 **(B)** – 4 **(C)** 0 **(D)** none of these

7. The number of solutions of $|[x] - 2x| = 4$ is (where [.] denotes greatest integer function)

(A) 2 **(B)** 4 **(C)** 3 **(D)** Infinite

8. Number of values of x satisfying the equations $5\{x\} = x + [x]$ and $[x] - \{x\} = \frac{1}{2}$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

9. Solution set of the inequality $\log_e^2 [2x] - \log_e [2x] \le 0$ is

(A) $[1, 3)$ **(B)** $(0, 3)$ **(C)** $\{1, 2\}$ **(D)** $\left[\frac{1}{2}, \frac{3}{2}\right)$

10. $N = \dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$, then $\log_2 N$ has the value

(A) 0 **(B)** 1 **(C)** –1 **(D)** None of these

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $\ell n(x+z)+\ell n(x-2y+z)=2\,\ell n(x-z)$, then

(A) $y=\frac{2xz}{x+z}$ **(B)** $y^2=xz$ **(C)** $2y=x+z$ **(D)** $\frac{x}{z}=\frac{x-y}{y-z}$

12. $\log_{10}5\,.\,\log_{10}20+(\log_{10}2)^2$ when simplified reduces to

(A) an odd prime number **(B)** an even prime

(C) a rational number **(D)** an integer

13. If $\frac{|x+2|-x}{x}<2$, then the set of values of x is

(A) $(-\infty,1)\cup(2,\infty)$ **(B)** $(-\infty,0)\cup(1,\infty)$ **(C)** $(-\infty,-1)\cup(0,\infty)$ **(D)** none of these

14. Which of the following when simplified reduces to unity ?

(A) $\log_3\log_{27}\log_4 64$ **(B)** $2\log_{18}(\sqrt{2}+\sqrt{8})$

(C) $\log_2\sqrt{10}+\log_2\left(\frac{2}{\sqrt{5}}\right)$ **(D)** $-\log_{\sqrt{2}-1}(\sqrt{2}+1)$

15. If $x, y, z\in R^+$ and $z\in R$ then the system $x+y+z=2,\ 2xy-z^2=4$

(A) is satisfied for $x=2, y=2, z=-2$ **(B)** has only one real solution

(C) has only two real solution **(D)** has infinite solutions.

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** If 2, 3 & 6 are the sides of a triangle then it is an obtuse angled triangle.

Statement-II : If $b^2>a^2+c^2$, where b is the greatest side, then triangle must be obtuse angled.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** Minimum value of $|x-2|+|x-5|+|x+3|$ is 8.

Statement-II : If $a<b<c$, then the minimum value of $|x-a|+|x-b|+|x-c|$ is $|b-a|+|b-c|$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** $\log_e x>\log_e y \quad\Rightarrow\quad x>y,\ x, y>0$

Statement-II : If $\log_a x>\log_a y$, then $x>y$ $(x, y>0\ \&\ a>0, a\neq1)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** $[x] + [-x] = x^2 - 5x + 6$ has only two real solution.

Statement-II : $[x] + [-x] = \begin{cases} -1 & , \ x \notin I \\ 0 & , \ x \in I \end{cases}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** If $ax^2 + \alpha x + \beta = 0$, where $\alpha, \beta \in R$, has roots a, b and $\log_a b < 0$, then $\alpha + \beta < 0$.
Statement-II : If $ax^2 + \alpha x + \beta = 0$, where $\alpha, \beta \in R$, has roots a, b and $\log_a b < 0$, then $a + \alpha + \beta$ must be negative.
(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the following**

Column – I		Column – II	
(A)	If $\log_{\sin x}(\log_3(\log_{0.2} x)) < 0$, then	**(p)**	$x \in [-1, 1]$
(B)	If $\dfrac{(e^x - 1)(2x-3)(x^2+x+2)}{(\sin x - 2)\, x(x+1)} \le 0$, then	**(q)**	$x \in [-3, 6)$
(C)	If $\lvert 2 - \lvert [x] - 1 \rvert \rvert \le 2$, then (where [.] represents greatest integer function).	**(r)**	$x \in \left(0, \dfrac{1}{125}\right)$
(D)	If $\lvert \sin^{-1}(3x - 4x^3) \rvert \le \dfrac{\pi}{2}$, then	**(s)**	$x \in (1, \infty)$
		(t)	$x \in (-\infty, -1) \cup \left[\dfrac{3}{2}, \infty\right)$

22. **Match the column**

Column- I		Column-II	
(A)	Interval containing all the solutions of the inequality $3 - x > 3\sqrt{1 - x^2}$ is	**(p)**	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
(B)	Interval containing all the solutions of the inequality $\left(\dfrac{1}{3}\right)^{\sqrt{x+2}} < 3^{-x}$ is	**(q)**	(π, π^2)
(C)	Interval containing all the solutions of the inequality $\log_5(x-3) + \dfrac{1}{2}\log_5 3 < \dfrac{1}{2}\log_5(2x^2 - 6x + 7)$ is	**(r)**	$(-\pi, \pi)$
(D)	Interval containing all the solutions of the equation $7^{x+2} - \dfrac{1}{7} . 7^{x+1} - 14.7^{x-1} + 2.7^x = 48$ is	**(s)**	$(-e, e)$
		(t)	$([\pi], -[-\pi^2])$, where [.] is G.I.F.

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

The general procedure for solving equation containing modulus function is to split the domain into subintervals and solve the various cases. But there are certain structures of equations which can be solved by a different approach. For example, for solving the equation $|f(x)| + |g(x)| = f(x) - g(x)$ one can follow this method. First find the permissible set of values of x for the equation.

Since LHS $\geq 0 \Rightarrow f(x) - g(x) \geq 0$. Now squaring both sides, we get

$f^2 + g^2 + 2|f.g| = f^2 + g^2 - 2fg$

$\Rightarrow |fg| = -fg$. The equation can hold if $f.g \leq 0$ and $f \geq g$. This can be simplified to $f \geq 0, g \leq 0$.

Answer the following questions on the basis of this method

1. The complete solution of the equation $|x^3 - x| + |2 - x| = x^3 - 2$ is

(A) $[2, \infty)$ **(B)** $[-1, 0] \cup [2, \infty)$ **(C)** $\left[2^{\frac{1}{3}}, \infty\right)$ **(D)** none of these

2. The complete solution set of the equation $|x^2 - x| + |x + 3| = |x^2 - 2x - 3|$ is

(A) $[1, \infty)$ **(B)** $[-3, 0] \cup [1, \infty)$ **(C)** $(-\infty, -3]$ **(D)** $(-\infty, -3] \cup [0, 1]$

3. All the condition(s) for which $|f(x) - g(x)| = |f(x)| + |g(x)|$ is true, is

(A) $f(x) \geq 0, g(x) \leq 0$ **(B)** $f(x) \leq 0, g(x) \geq 0$ **(C)** $f(x) . g(x) \leq 0$ **(D)** $f(x) . g(x) = 0$

24. Read the following comprehension carefully and answer the questions.

An equation of the form $2m \log_a f(x) = \log_a g(x), a > 0, a \neq 1, m \in N$

$\begin{cases} f(x) > 0 \\ f^{2m}(x) = g(x) \end{cases}$ is equivalent to the system

1. The number of solutions of $2 \log_e 2x = \log_e(7x - 2 - 2x^2)$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** Infinite

2. The number of solutions of $\ln 2x = 2 \ln(4x - 15)$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** Infinite

3. The number of solutions of $\log(3x^2 + x - 2) = 3 \log(3x - 2)$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0

25. Read the following comprehension carefully and answer the questions.

Let $a_1 < a_2 < a_3 < ....< a_n$, n is an odd natural number and $m, k \in N$

Consider the equation

$$|x - a_1| + |x - a_2| + |x - a_3| + ... + |x - a_n| = kx + d.$$

Case-1 When $2m - n = k$ for some $m < n$, then the equation has

(i) no solution if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$

(ii) infinite solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$

(iii) two solutions if $(a_{m+1} + a_{m+2} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$

Case-2 Let when $2m - n \neq k$ for any $m < n$

Two cases arise

(A) If $|k| > n$, then there is one solution.

(B) If $|k| < n$, then there is m such that $2m - (n - 1) = k$.

(i) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) > d$ no solution

(ii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) = d$ one solution

(iii) If $(a_{m+2} + a_{m+3} + ... + a_n) - (a_1 + a_2 + ... + a_m) < d$ two solutions

1. Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 9| = 3x + 5$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

2. Number of solutions of $|x - 1| + |x - 3| + |x - 4| + |x - 7| + |x - 10| = 2x + 1$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

3. Number of solutions of $|x - 1| + |x - 2| + |x - 4| + |x - 6| + |x - 7| = 2x + 10$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

SECTION - VI : INTEGER TYPE

26. The inequality $\log_{x^2} |x - 1| > 0$ is not defined for some integral values of x, find the sum of their magnitudes.

27. If set of all real values of x satisfying $|x^2 - 3x - 1| < |3x^2 + 2x + 1| + |2x^2 + 5x + 2|$, $x^2 - 3x - 1 \neq 0$ is $(-\infty, -a) \cup (-b, \infty)$, then find the value of $a + \log ab$.

28. If value of $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\sqrt[p]{.....\sqrt[p]{p}}}}}}_{2008 \text{ times}}$ is $-\lambda$, then find the value of λ.

29. Let $f(x) = \begin{cases} 1 & , \quad -2 \leq x \leq -1 \\ x + 2 & , \quad -1 < x < 1 \\ 4 - x & , \quad 1 \leq x \leq 2 \end{cases}$

Find number of solutions of $\{f(x)\} = \frac{1}{2}$ (where {.} denotes fractional part function)

30. Find the number of integral solution of the equation $\sqrt{\left[x + \left[\frac{x}{2}\right]\right]} + \left[\sqrt{\{x\}} + \left[\frac{x}{3}\right]\right] = 3.$

(where [ ] denotes greatest integer function)

ANSWER KEY

EXERCISE - 1

1. B **2.** D **3.** D **4.** B **5.** C **6.** A **7.** D **8.** D **9.** A **10.** C **11.** D **12.** B **13.** D
14. B **15.** A **16.** D **17.** C **18.** B **19.** D **20.** B

EXERCISE - 2 : PART # I

1. ABC **2.** AB **3.** AC **4.** ACD **5.** BC **6.** ABC **7.** BCD **8.** ABCD **9.** BC
10. BC **11.** AB **12.** AD **13.** AD **14.** BC **15.** ABC **16.** BCD **17.** BCD **18.** ABCD
19. ABC **20.** ABCD **21.** CD **22.** ABCD **23.** ABD **24.** ACD **25.** ACD **26.** AD **27.** AB
28. AB **29.** ABCD **30.** BD

PART - II

1. A **2.** B **3.** A **4.** A **5.** D

EXERCISE - 3 : PART # I

1. $A \to r$, $B \to t$, $C \to q$, $D \to p$ **2.** $A \to q$, $B \to r\,s$, $C \to q$, $D \to p$
3. $A \to prs$, $B \to rs$, $C \to t$, $D \to prs$

PART - II

Comprehension # 1 : **1.** A **2.** D **3.** C
Comprehension # 2 : **1.** B **2.** B **3.** B
Comprehension # 3 : **1.** C **2.** A **3.** C

EXERCISE - 5 : PART # I

1. D **2.** A **3.** C **4.** C **5.** D **6.** A

PART - II

3. A **5.** **1** A **2** A **7.** B **8.** B **10.** $A \to prs$, $B \to qs$, $C \to qs$, $D \to prs$ **12.** 4
13. ABC

MOCK TEST

1. C **2.** C **3.** D **4.** A **5.** A **6.** C **7.** B **8.** A **9.** D
10. A **11.** AD **12.** CD **13.** B **14.** BD **15.** AB **16.** D **17.** A **18.** C
19. A **20.** A **21.** $A \to r$, $B \to t$, $C \to pqr$, $D \to pq$ **22.** $A \to prs$, $B \to rs$, $C \to t$, $D \to prs$
23. **1** A **2** D **3** C **24.** **1** B **2** B **3** D **25.** **1** C **2** A **3** C
26. 4 **27.** 2 **28.** 2008 **29.** 3 **30.** 1