AREA UNDER THE CURVE

EXERCISE # 1

Questions based on — Area bounded by curve and x-axis

Q.1 Area bounded by $y = \sec^2 x$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$ and x- axis is-

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\sqrt{\frac{2}{3}}$

Sol. **[A]**

$$A = \int_{\pi/6}^{\pi/3} \sec^2 x \, dx = [\tan x]_{\pi/6}^{\pi/3}$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Q.2 Area bounded by the curve $y = xe^{x^2}$, x- axis and the ordinates $x = 0$, $x = \alpha$ is-

(A) $\frac{e^{\alpha^2}+1}{2}$ sq. units (B) $\frac{e^{\alpha^2}-1}{2}$ sq. units

(C) $e^{\alpha^2} + 1$ sq. units (D) $e^{\alpha^2} - 1$ sq. units

Sol. **[B]**

$$A = \int_0^{\alpha} x e^{x^2} dx$$

Put $x^2 = t$

$$A = \frac{1}{2}\int_0^{\alpha^2} e^t \, dt = \frac{e^{\alpha^2}-1}{2}$$

Q.3 The area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1) \sin (3b + 4)$, then $f(x)$ equals-

(A) $(x - 1) \cos (3x + 4)$

(B) $\sin (3x + 4)$

(C) $\sin (3x + 4) + 3(x - 1) \cos (3x + 4)$

(D) None of these

Sol. **[C]**

Given that $\int_1^b f(x)\, dx = (b - 1) \sin (3b + 4)$

diff. with respect. to b, we have

$f(b) = \sin (3b + 4) + 3(b - 1) \cos (3b + 4)$

$\Rightarrow f(x) = \sin (3x+ 4) + 3(x - 1) \cos (3x + 4)$

Q.4 The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is-

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

Sol. **[C]**

$y = 4x - x^2 - 3$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x - 3)(x - 1) = 0 \Rightarrow x = 1, 3$

$$A = \int_1^3 (4x - x^2 - 3)\, dx$$

$$= \left[2x^2 - \frac{x^3}{3} - 3x\right]_1^3 = 18 - 9 - 9 - 2 + \frac{1}{3} + 3$$

$$= 1 + \frac{1}{3} = \frac{4}{3}$$

Q.5 The area of the region bounded by the curve $y = \sin x$ and the x- axis in $[-\pi, \pi]$ is-

(A) 4 (B) 8 (C) 12 (D) 2

Sol. **[A]**

$$\text{Area} = 2\int_0^{\pi} \sin x \, dx$$

$$= 2[-\cos x]_0^{\pi} = 2[1 + 1] = 4$$

Questions based on — Area bounded by curve and y-axis

Q.6 Area in 1st quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$ is-

(A) $\frac{3}{7}$ (B) $\frac{5}{7}$ (C) $\frac{7}{3}$ (D) $\frac{7}{5}$

Sol. **[C]**

$$A = \frac{1}{2}\int_1^4 \sqrt{y}\, dy$$

$$= \frac{1}{3}\left[y^{3/2}\right]_1^4 = \frac{7}{3}$$

Q.7 The area between the curves $x = 2 - y - y^2$ and y-axis, is-

(A) 9 (B) $\frac{9}{2}$ (C) $\frac{9}{4}$ (D) 3

Sol. **[B]**

$y^2 + y - 2 = 0$

$(y + 2)(y - 1) = 0 \Rightarrow y = 1, -2$

required area = $\int_{-2}^{1} x\,dy$

$= \int_{-2}^{1} (2 - y - y^2)\,dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= 2 - \frac{1}{2} - \frac{1}{3} + 4 + \frac{4}{2} - \frac{8}{3} = \frac{9}{2}$

Questions based on — Area bounded by two curves

Q.8 Area of figure bounded by straight lines x = 0, x = 2 and the curves $y = 2^x$, $y = 2x - x^2$ is-

(A) $3\log_2 e - \frac{3}{4}$ (B) $\frac{3}{\ln 2} - \frac{4}{3}$

(C) $\frac{3}{\ln 2} + \frac{3}{4}$ (D) $\frac{3}{\ln 2} + \frac{4}{3}$

Sol. **[B]**

$A = \int_{0}^{2} (2^x - 2x + x^2)\,dx$

$= \left[\frac{2^x}{\ln 2}\right]_0^2 - \left(x^2\right)_0^2 + \frac{1}{3}\left[x^3\right]_0^2$

$= \frac{3}{\ln 2} - 4 + \frac{8}{3}$

$= \frac{3}{\ln 2} - \frac{4}{3}$

Q.9 Area bounded by $y = x^2 + 1$ and the tangents to it drawn from the origin, is-

(A) $\frac{8}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{10}{3}$

Sol. **[C]**

Parabola is $x^2 = y - 1$

Tangent to it from origin is

$y = 2x$ and $y = -2x$

Intersection point of

$x^2 = y - 1$ and $y = 2x$ is (1, 2)

Area = $2\int_{0}^{1} (x^2 + 1 - 2x)\,dx$

$= 2\int_{0}^{1} (x-1)^2\,dx = 2\left[\frac{(x-1)^3}{3}\right]_0^1 = \frac{2}{3}$

Q.10 The value of a for which the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$ is 1 sq. unit, is-

(A) $\sqrt{3}$ (B) 4 (C) $4\sqrt{3}$ (D) $\frac{\sqrt{3}}{4}$

Sol. **[D]**

Curve is $y^2 = 4ax$ and $x^2 = 4ay$

intersection point is (0, 0) and (4a, 4a)

So Area = $\int_{0}^{4a} \left(\sqrt{4ax} - \frac{x^2}{4a}\right) dx$

Given that Area = 1

$\Rightarrow \int_{0}^{4a} \left(\sqrt{4ax} - \frac{x^2}{4a}\right) dx = 1$

$\Rightarrow \left[\sqrt{4a}.\frac{2}{3}x^{3/2} - \frac{x^3}{12a}\right]_0^{4a} = 1$

$\Rightarrow \frac{2}{3}(4a)^2 - \frac{(4a)^3}{12a} = 1$

$\Rightarrow a = \frac{\sqrt{3}}{4}$

Q.11 The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$, is-

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$

Sol. **[A]**

From curve $y^2 = 4x$ and line $2x - 3y + 4 = 0$

we get x coordinate of intersection point which is x = 1, x = 4

Area = $\int_{1}^{4} \left[\sqrt{4x} - \left(\frac{2x+4}{3}\right)\right] dx$

$= \left[2.\frac{2}{3}x^{3/2} - \frac{1}{3}(x^2 + 4x)\right]_1^4$

$= \frac{32}{3} - \frac{32}{3} - \frac{4}{3} + \frac{5}{3} = \frac{1}{3}$

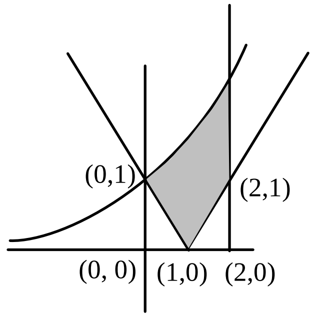
Q.12 The area bounded by the curve $y = e^x$ & the lines $y = |x - 1|$, x = 2 is given by-

(A) $e^2 + 1$ (B) $e^2 - 1$

(C) $e^2 - 2$ (D) None of these

Sol. **[C]**

$y = e^x$, $y = |x - 1|$, x = 2



Required area $= \int_0^2 e^x\,dx - 1$

$= e^2 - 2$

Q.13 The average value of $f(x) = \sec^2 x$ from $x = 0$ to $x = \frac{\pi}{4}$, is-

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{2}{\pi}$ (D) $\frac{4}{\pi}$

Sol. **[D]**

$$\frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sec^2 x\,dx$$

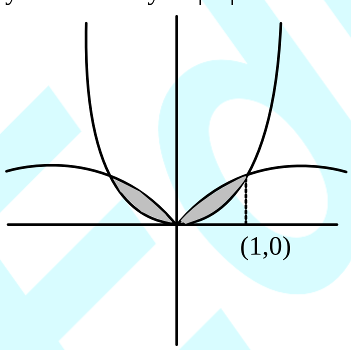
$$\Rightarrow \frac{4}{\pi}\left[\tan x\right]_0^{\pi/4} \Rightarrow \frac{4}{\pi}$$

Q.14 The area of the region(s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is-

(A) 1/3 (B) 2/3 (C) 1/6 (D) 1

Sol. **[B]**

$y = x^2$ and $y^2 = |x|$



$$\text{Area} = 2\int_0^1 (\sqrt{x} - x^2)\,dx = 2\left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$

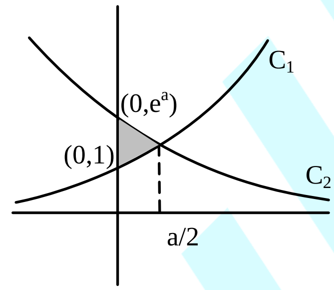
$$= 2\left[\frac{2}{3} - \frac{1}{3}\right] = \frac{2}{3}$$

Q.15 Let 'a' be a positive constant number. Consider two curves $C_1 : y = e^x$, $C_2 : y = e^{a-x}$. Let S be the area of the part surrounding by C_1, C_2 and the y-axis, then $\lim_{a\to 0} \frac{S}{a^2}$ equals-

(A) 4 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{1}{4}$

Sol. **[D]**

$C_1 : y = e^x$ and $C_2 : y = e^{a-x}$



$$\text{Area} = \int_0^{a/2} (e^{a-x} - e^x)\,dx = (-e^{a-x} - e^x)_0^{a/2}$$

$S = e^a - 2\,e^{a/2} + 1$

$$\lim_{a\to 0} \frac{S}{a^2} = \lim_{a\to 0} \frac{e^a - 2e^{a/2} + 1}{a^2} = \frac{1}{4}$$

Questions based on

Parametric Curves

Q.16 The area bounded by the curve $x = a\cos^3 t$, $y = a\sin^3 t$ is-

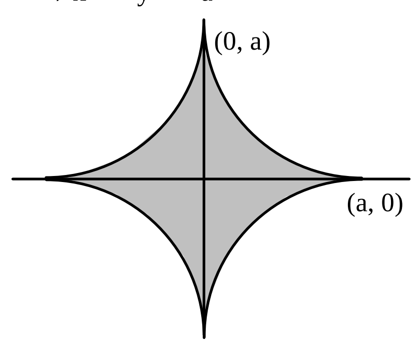
(A) $\frac{3\pi a^2}{32}$ (B) $\frac{\pi a^2}{32}$

(C) $\frac{3\pi a^2}{16}$ (D) None of these

Sol. **[A]**

$x = a\cos^3 t$, $y = a\sin^3 t$

$\Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$



$$\text{Area} = 4\int_0^a y\,dx$$

$\because y = a\sin^3 t$ and $dx = -3a\cos^2 t \sin t\,dt$

$$\text{Area} = \int_{\pi/2}^{0} (a\sin^3 t)(-3a\cos^2 t \sin t)\,dt$$

$$= 3a^2 \int_{0}^{\pi/2} \sin^4 t \cos^2 t \,dt$$

$$3a^2 \cdot \frac{(3.1).1}{6.4.2} \cdot \frac{\pi}{2} = \frac{3\pi a^2}{32}$$

Q.17 The area bounded by the curves x = a (θ – sinθ), y = a(1 – cos θ), 0 ≤ θ ≤ 2π, is-

(A) πa^2 sq. units (B) $2\pi a^2$ sq. units

(C) $3\pi a^2$ sq. units (D) $4\pi a^2$ sq. units

Sol. **[C]**

$$A = a\int_{0}^{2\pi} (1-\cos\theta)\ .\ a(1 - \cos\theta)\,d\theta$$

$$= a^2 \int_{0}^{2\pi} (1+\cos^2\theta + 2\cos\theta)\ d\theta$$

$$= a^2\left[[x]_0^{2\pi} + \frac{1}{2}\left[x + \frac{1}{2}\sin 2\theta\right]_0^{2\pi} + 0\right]$$

$= a^2(2\pi + \pi)$

$= 3\pi a^2$

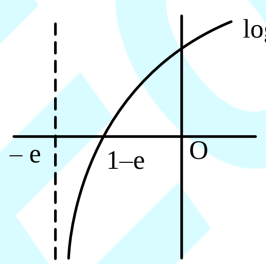
Questions based on

Miscellaneous

Q.18 The area enclosed between the curve $y = \log_e (x + e)$ and the coordinate axes is-

(A) 4 (B) 3 (C) 2 (D) 1

Sol. **[D]**



$$\text{Area} = \int_{1-e}^{0} \log (x + e)\,dx$$

$$= [x\log(x+e)]_{1-e}^{0} - \int_{1-e}^{0} \frac{x}{x+e}dx$$

$$= -\int_{1-e}^{0} \frac{x+e-e}{x+e}dx$$

$$= -\ [x - e\ \log(x+e)]_{1-e}^{0}$$

$= e + 1 – e = 1$

Q.19 The area of the figure bounded by the curves $y = \lambda n x$ & $y = (\lambda n x)^2$ is-

(A) e + 1 (B) e –1

(C) 3 – e (D) 1

Sol. **[C]**

$y = \lambda n\ x$ and $y = (\lambda n\ x)^2$

Intersection at x = 1, and x = e

$$\text{Area} = \int_{1}^{e}\left[(\lambda n\, x) - (\lambda n\ x)^2\right] dx$$

$$= [x\,\lambda n\, x - x]_1^e - \left[x(\lambda n\, x)^2\right]_1^e + 2\int_{1}^{e} \lambda n\ x\ dx$$

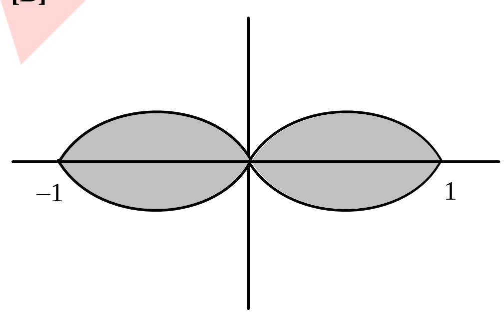
$$= 1 - e + 2\ [x\,\lambda n\, x - x]_1^e$$

$= 1 – e + 2 (e – e + 1) = 3 – e$

Q.20 The area enclosed by the curve $y^2 + x^4 = x^2$, is-

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) $\frac{10}{3}$

Sol. **[B]**



$$\text{Total Area} = 4\int_{0}^{1} y\ \ dx = 4\int_{0}^{1} x\sqrt{1-x^2}\ dx$$

Solving we get

$$\text{Area} = \frac{4}{3}$$

Q.21 Let z be a complex number such that $\text{Re}(z) = \sqrt{x^2+4}$, and $\text{Im}(z) = \sqrt{y-4}$ satisfying $|z| = \sqrt{10}$. Area enclosed by the set of points (x, y) on the complex plane, is-

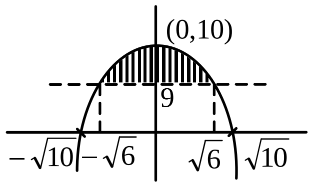
(A) $8\sqrt{6}$ (B) $4\sqrt{6}$

(C) $\frac{20\sqrt{10}}{3}$ (D) $\frac{40\sqrt{10}}{3}$

Sol. **[A]**

$x^2 + 4 + y - 4 = 10$

$x^2 = -(y - 10)$



$y \geq 4$

$$A = 2\left[\int_0^{\sqrt{6}} y\,dx - 4\times\sqrt{6}\right] = 2\int_0^{\sqrt{6}}(10 - x^2)dx = 8\sqrt{6}$$

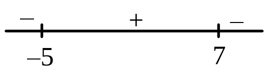
Q.22 Area of the curve $y^2 = (7 - x)(5 + x)$ above x-axis and between the ordinates $x = -5$ and $x = 1$, is-

(A) 9π (B) 18π

(C) 15π (D) None of these

Sol. **[A]**

$y^2 = -(x - 7)(x + 5)$



$$\text{Area} = \int_{-5}^{1}\sqrt{35 + 2x - x^2}\,dx$$

$$= \int_{-5}^{1}\sqrt{36 - (x-1)^2}\,dx$$

$$= \left[\frac{36}{2}\sin^{-1}\frac{x-1}{6} + \frac{x-1}{2}\sqrt{36 - (x-1)^2}\right]_{-5}^{1}$$

$= 9\pi$

Q.23 The area bounded in the first quadrant between the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $3x + 4y = 12$, is-

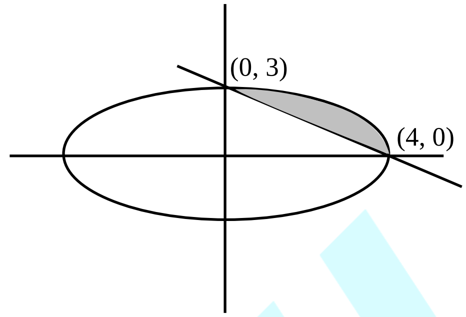
(A) $6(\pi - 1)$ (B) $3(\pi - 2)$

(C) $3(\pi - 1)$ (D) None of these

Sol. **[B]**

Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

line $\frac{x}{4} + \frac{y}{3} = 1$



Area in first quadrant

$= \frac{\pi ab}{4} - \frac{1}{2}ab$

$= \frac{12}{4}\pi - \frac{1}{2} \cdot 12 = 3(\pi - 2)$

Q.24 The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$, is-

(A) $\frac{4 + 3\lambda n\,3}{2}$ (B) $\frac{4 - 3\lambda n\,3}{2}$

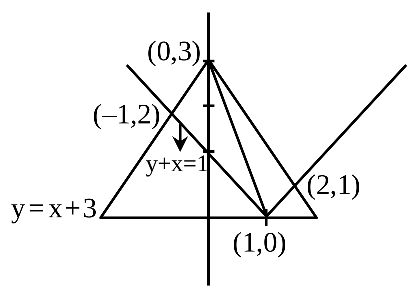
(C) $\frac{3}{2} + \lambda n3$ (D) $\frac{1}{2} + \lambda n3$

Sol. **[B]**

True / False type questions

Q.25 The area of the figure bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is 4 sq. unit.

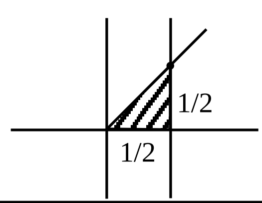
Sol.



$A = 2\sqrt{2}\,.\sqrt{2} = 4$

Q.26 Area of the region bounded by $y = \{x\}$ and $2x - 1 = 0$, $y = 0$ is $\frac{1}{4}$, ({ } stands for fraction part)

Sol.



$A = \frac{1}{8}$

Q.27 The positive value of 'a' for which area covered by figure y = sin ax, y = 0, $x = \frac{\pi}{3a}$ and $x = \frac{\pi}{a}$ is equal to 3, is $\frac{1}{2}$.

Sol. $A = -\frac{1}{a}\left[\cos a x\right]_{\pi/3a}^{\pi/a}$

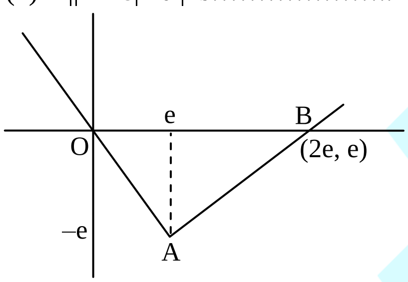
$A = -\frac{1}{a}\left[-1-\frac{1}{2}\right] = \frac{3}{2a}$

$a = \frac{3}{2A} = \frac{1}{2}$ when A = 3

Fill in the blanks type questions

Q.28 The area bounded by x-axis, x = 0, x = 2e and f(x) = ||x – e| –e | is………………..

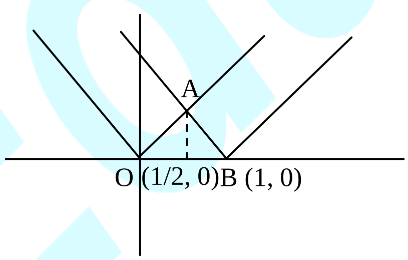
Sol.



area = area of ΔOAB

$= \frac{1}{2}.2e.e = e^2$

Q.29 The area bounded by f(x) = min. (|x|, |x –1|) and x-axis is…………………



Sol.

Area= Area of ΔOAB

$= \frac{1}{2}\left(1\times\frac{1}{2}\right) = \frac{1}{4}$

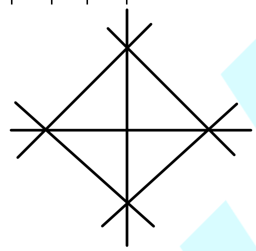
Q.30 The area bounded by x-axis, $x = \frac{\pi}{2}$ and f(x) = {sin x} is…………………

Sol. $A = \int_0^{\pi/2} \sin x\, dx = \left[-\cos x\right]_0^{\pi/2} = 1$

Q.31 Area enclosed by the curve |x – 2| + |y + 1| = 1 is equal to …………………

Sol. | x – 2 | + | y + 1| = 1

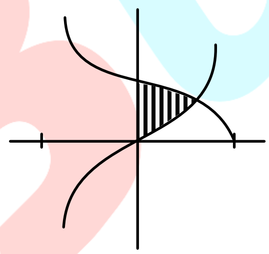
| X | + | Y | = 1



Area = 2

Q.32 Area bounded by the curve $y = \sin^{-1} x$, $y = \cos^{-1} x$ and x = 0 is equal to …………

Sol.



$A = 2\int_0^{\pi/4} \sin y\, dy = -2\left[\cos y\right]_0^{\pi/4}$

$= -2\left[\frac{1}{\sqrt{2}}-1\right] = 2\left[1-\frac{1}{\sqrt{2}}\right]$

$= 2 - \sqrt{2}$

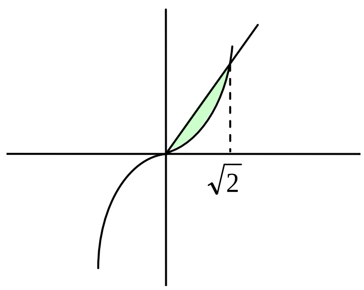
EXERCISE # 2

Part-A Only single correct answer type questions

Q.1 The area between the curve $y = x^3$ and $y = x + |x|$ is-

(A) 0 (B) 2 (C) 1 (D) 3

Sol. **[C]**



required area = $\int_0^{\sqrt{2}} (2x - x^3)\,dx$

$= \left[x^2 - \dfrac{x^4}{4}\right]_0^{\sqrt{2}} = 2 - 1 = 1$

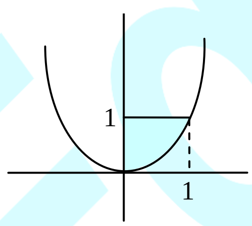
Q.2 The area bounded by $y = x^2$ and $y = [x + 1]$, $x \le 1$ and the y-axis is-

(where [ ] stands for greatest integer function)

(A) 1 (B) 2/3

(C) 1/3 (D) None of these

Sol. **[B]**



required area

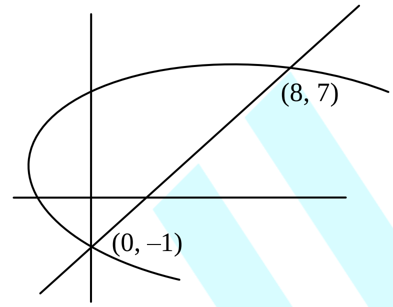
$= \int_0^1 x\,dy = \int_0^1 y^{1/2}\,dy = \dfrac{2}{3}\left[y^{3/2}\right]_0^1 = \dfrac{2}{3}$

Q.3 Area bounded by the curves $y = x - 1$ and $(y - 1)^2 = 4(x + 1)$ is -

(A) 8/3 (B) 16/3

(C) 32/3 (D) 64/3

Sol. **[D]**



required area = $\int_{-1}^{7} (y_1 - y_2)\,dy$

$= \int_{-1}^{7} \left\{\left(\dfrac{1}{4}(y-1)^2 - 1\right) - (y+1)\right\} dy$

$= \int_{-1}^{7} \left\{\dfrac{1}{4}(y-1)^2 - y - 2\right\} dy$

$= \dfrac{1}{12}\left[(y-1)^3\right]_{-1}^{7} - \left[\dfrac{y^2}{2}\right]_{-1}^{7} - 2\,[y]_{-1}^{7}$

$= \dfrac{1}{12}(216 + 8) - \left[\dfrac{49}{2} - \dfrac{1}{2}\right] - 2\,[7 + 1]$

$= \dfrac{224}{12} - \dfrac{48}{2} - 16 = \dfrac{56}{3} - 40 = -\dfrac{64}{3}$

Area = $\dfrac{64}{3}$

Q.4 If $y = mx$ divides the area bounded by lines $x = 0$, $y = 0$, $x = 3/2$ & the curve $y = 1 + 4x - x^2$ in two equal parts, then m is equal to-

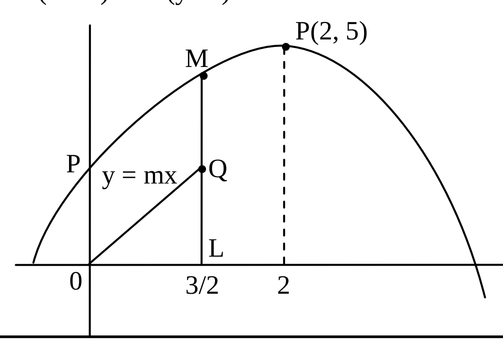
(A) 13/8 (B) 13/4

(C) 13/6 (D) None of these

Sol. **[C]**

$y = 1 + 4x - x^2$

$\Rightarrow (x - 2)^2 = -(y - 5)$



Given that

area of OLMP = 2 area of OQL

$\Rightarrow$ Area of OLMP = $\int_0^{3/2}(1+4x-x^2)\,dx$

$= \left[x + x^2 - \frac{x^3}{3}\right]_0^{3/2} = \frac{39}{8}$

and area of OQL = $\frac{1}{2}\cdot\frac{3}{2}\cdot QL$

$= \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}m = \frac{9}{8}m$

$\Rightarrow \frac{39}{8} = 2\left(\frac{9}{8}m\right)$

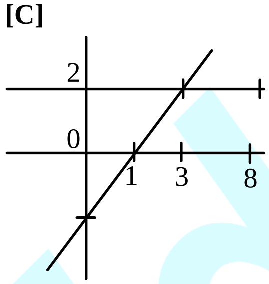
$\Rightarrow m = \frac{13}{6}$

Q.5 The area bounded by the curve $y = \left[\frac{x^2}{64} + 2\right]$, $y = x - 1$ and $x = 0$ above the x axis will be-
(Where [ ] represents greatest integer function.)

(A) 2 (B) 3

(C) 4 (D) None of these

Sol. **[C]**



$A = \int_0^2 (y+1)\,dy = \frac{1}{2}\left[(y+1)^2\right]_0^2$

$= \frac{1}{2} \times 8 = 4$

Q.6 If A_1 is the area enclosed by the curve xy = 1, x-axis and the ordinates x = 1, x = 2; and A_2 is the enclosed by the curve xy = 1, x-axis and the coordinates x = 2, x = 4; then-

(A) $A_2 = 2A_1$ (B) $A_1 = 2A_2$

(C) $A_2 = 3A_1$ (D) $A_1 = A_2$

Sol. **[D]**

Area for $A_1 = \int_1^2 \frac{1}{x}dx$

$= [\lambda n\ x]_1^2 = \lambda n\ 2$

Area for $A_2 = \int_2^4 \frac{1}{x}dx$

$= [\lambda n\ x]_2^4 = \lambda n\ 4 - \lambda n\ 2 = \lambda n\ 2$

Clearly $A_1 = A_2$

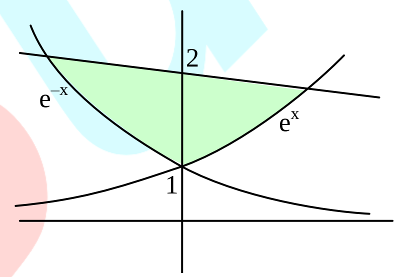
Q.7 Area of the region bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line y = 2 is-

(A) log (4/e) (B) 2 log (4/e)

(C) 4 log (4/e) (D) None of these

Sol. **[B]**

$y = e^x$, $y = e^{-x}$, y = 2



required area = $2\int_1^2 x\,dy$

$= 2\int_1^2 \lambda ny\,dy$

$= 2\,[y\,\lambda n\,y - y]_1^2$

$= 2\,[2\,\lambda n\,2 - 2 + 1]$

$= 2\,\lambda n\,4/e$

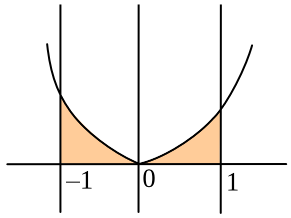
Q.8 Area bounded by curve $y = xe^{|x|}$ and lines | x | = 1, y = 0 will be-

(A) 4 (B) 6 (C) 1 (D) 2

Sol. **[D]**

$y = xe^{|x|}$ and lines |x| = 1, y = 0

$y = \begin{cases} xe^x & x > 0 \\ xe^{-x} & x < 0 \end{cases}$



required area = $2\int_0^1 xe^x dx$

$= 2\,[xe^x - e^x]_0^1 = 2$

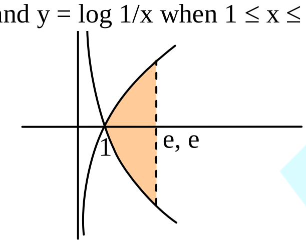
Q.9 The area between the curves y = log x and y = log 1/x when $1 \le x \le e$ is-

(A) log (4/e) (B) 1

(C) 3 (D) $2\int_1^e \log x\, dx$

Sol. **[D]**

y = log x and y = log 1/x when $1 \le x \le e$



required area = $2\int_1^e \log x\, dx$

Q.10 If A is the area between the curve y = sin x and x-axis in the interval [0, π/2], then the area between y = sin 2x and x-axis in this interval will be -

(A) A (B) 2A

(C) A/2 (D) None of these

Sol. **[A]**

$A = \int_0^{\pi/2} \sin x\, dx = -[\cos x]_0^{\pi/2} = 1$

Again area = $2\int_0^{\pi/4} \sin 2x\, dx$

$= -\,[\cos 2x]_0^{\pi/4} = 1 = A$

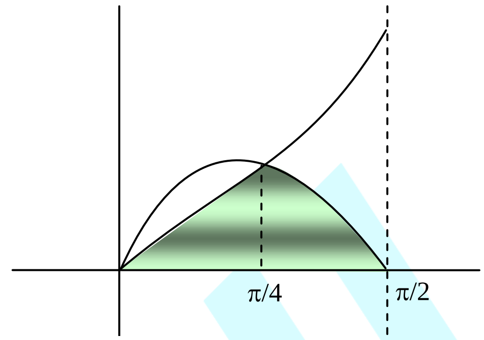
Q.11 The common area bounded by the curves y = sin 2x, y = tan x and y = 0 in [0, π/2], is-

(A) 1/2 log (2/e) (B) 1/2 log 2e

(C) log 2e (D) $\log\sqrt{2} - 1$

Sol. **[B]**

y = sin 2x, y = tan x, y = 0 in [0, π/2]



required area

$= \int_0^{\pi/4} \tan x\, dx + \int_{\pi/4}^{\pi/2} \sin 2x\, dx$

$= -\,[\log\cos x]_0^{\pi/4} - \frac{1}{2}\,[\cos 2x]_{\pi/4}^{\pi/2}$

$= -\log\frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{1}{2}\log 2 + \frac{1}{2} = \frac{1}{2}\log 2e$

Q.12 The ratio of the areas of two curves y = cos x and y = cos 2x between x-axis from x = 0 to x = π/3 is-

(A) 1 : 2 (B) $2\sqrt{3} : 4 - \sqrt{3}$

(C) 1 : 1 (D) None of these

Sol. **[B]**

Let $I_1 = \int_0^{\pi/3} \cos x\, dx$

and $I_2 = \int_0^{\pi/3} \cos 2x\, dx$

$I_1 = [\sin x]_0^{\pi/3} = \frac{\sqrt{3}}{2}$

$I_2 = \int_0^{\pi/4} \cos 2x\, dx - \int_{\pi/4}^{\pi/3} \cos 2x\, dx$

$= \frac{1}{2}\,[(\sin 2x)_0^{\pi/4} - (\sin 2x)_{\pi/4}^{\pi/3}]$

$= \frac{1}{2}\left[1 - \frac{\sqrt{3}}{2} + 1\right] = \frac{4-\sqrt{3}}{4}$

$\frac{I_1}{I_2} = \frac{2\sqrt{3}}{4-\sqrt{3}}$

Part-B **One or more than one correct answer type Questions**

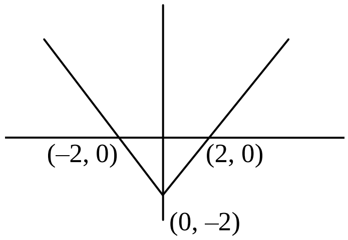
Q.13 Let $f(x) = |x| - 2$ and $g(x) = |f(x)|$.
Now area bounded by x-axis and f(x) is A_1 and area bounded by x-axis and g(x) is A_2 then-

(A) $A_1 = 3$ (B) $A_1 = A_2$
(C) $A_2 = 4$ (D) $A_1 + A_2 = 8$

Sol. **[B, C, D]**

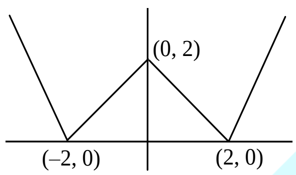
$f(x) = |x| - 2$, $g(x) = |f(x)|$

$f(x) = |x| - 2$,



and

g(x) = |f(x)| =



$$A_1 = f(x) = \left|2\int_0^2 (x-2)\,dx\right| = \left|2\left[\frac{x^2}{2} - 2x\right]_0^2\right|$$

$$= |2[2-4]| = 4$$

A_2 = clearly same as $A_1 = 4$

option B, C and D are correct.

Q.14 Let $L : x - y - 1 = 0$ be a line & $C : y^2 = 2x + 1$ be a parabola then -

(A) area bounded by L and C lying in the upper half plane is $\frac{32}{3}$

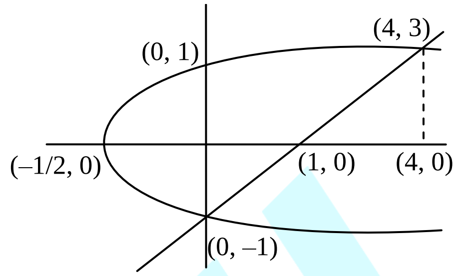
(B) area bounded by L and C lying in the plane is $\frac{16}{3}$

(C) area bounded by L and C in the upper half plane is $\frac{9}{2}$

(D) area bounded by L and C in the lower half plane is $\frac{5}{6}$

Sol. **[B, C, D]**

$L : x - y = 1$ $C : y^2 = 2(x + 1/2)$



Area bounded by L and C in upper half plan is

$$= \int_{-\frac{1}{2}}^{4} \sqrt{2x+1}\,dx - \frac{1}{2} x \times 3 \times 3$$

$$= \int_0^3 t^2\,dt - \frac{9}{2} \qquad \text{where } 2x + 1 = t^2$$

$$= 9 - \frac{9}{2} = \frac{9}{2}$$ option C is correct.

Area in lower half plane is

$$= \int_{-\frac{1}{2}}^{0} \sqrt{2x+1}\,dx + \frac{1}{2} \times 1 \times 1$$

$$= \int_0^1 t^2\,dt + \frac{1}{2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

option D is correct.

Area bounded by L and C lying in the plane is

$$= \frac{9}{2} + \frac{5}{6} = \frac{32}{6} = \frac{16}{3}$$

Option B is correct. So option B, C, and D are correct.

Q.15 If $C_1 \equiv y = \frac{1}{1+x^2}$ and $C_2 \equiv y = \frac{x^2}{2}$ be two curve lying in XY plane. Then -

(A) area bounded by curve $y = \frac{1}{1+x^2}$ & $y = 0$ is π

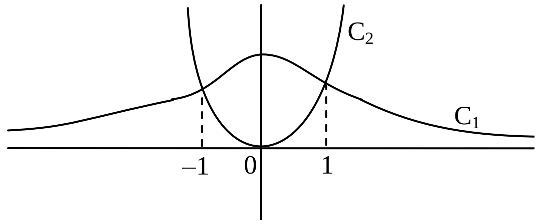
(B) area bounded by C_1 and C_2 is $\frac{\pi}{2} - \frac{1}{3}$

(C) area bounded by C_1 and C_2 is $1 - \frac{\pi}{2}$

(D) area bounded by curve $y = \frac{1}{1+x^2}$ & x-axis is $\frac{\pi}{2}$

Sol. **[A, B]**

$C_1 \equiv y = \frac{1}{1+x^2}$ and $C_2 \equiv y = \frac{x^2}{2}$



(A) area bounded by the curve $y = \frac{1}{1+x^2}$ and $y = 0$

$= 2\int_0^{\infty} \frac{1}{1+x^2}dx = 2\left[\tan^{-1}x\right]_0^{\infty} = \frac{\pi}{2}$

(B) area bounded by C_1 and C_2 is

$= \int_{-1}^{1}\left(\frac{1}{1+x^2} - \frac{x^2}{2}\right)dx = 2\int_0^{1}\left(\frac{1}{1+x^2} - \frac{x^2}{2}\right)dx$

$= 2\left[\tan^{-1}x - \frac{x^3}{6}\right]_0^1 = 2\left[\frac{\pi}{4} - \frac{1}{6}\right]$

$= \frac{\pi}{2} - \frac{1}{3}$

$\Rightarrow$ options A, B are correct.

Part-C Assertion-Reason type Questions

The following questions consist of two statements each, printed as Statement-1 and Statement-2. While answering these questions you are to choose any one of the following four responses.

(A) If both Statement-1 and Statement-2 are true and the Statement-2 is correct explanation of the Statement-1.

(B) If both Statement-1 and Statement-2 are true but Statement-2 is not correct explanation of the Statement-1.

(C) If Statement-1 is true but the Statement-2 is false.

(D) If Statement-1 is false but Statement-2 is true.

Q.16 **Statement-1 :** The area of the curve $y = \sin^2 x$ from 0 to π will be more than that of curve $y = \sin x$ from 0 to π.

Statement-2 : $t^2 > t$ if $t > 1$.

Sol. **[D]**

Q.17 **Statement-1 :** Area formed by curve $y = \cos x$ with $y = 0$, $x = 0$ and $x = \frac{3\pi}{4}$ is $2 - \frac{1}{\sqrt{2}}$.

Statement-2 : Area of curve $y = f(x)$ with x-axis between ordinates $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$.

Sol. **[A]**

$A = \left(\sin x\right)_0^{\pi/2} + \left|\left(\sin x\right)_{\pi/2}^{3\pi/4}\right|$

$= 1 + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \frac{1}{\sqrt{2}}$

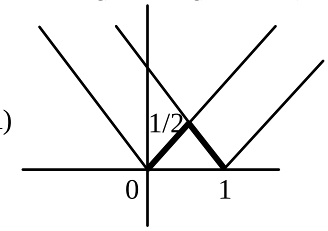
Part-D Column Matching type Questions

Q.18 Let $f(x) = |x|$, $g(x) = |x - 1|$ and $h(x) = |x + 1|$.

Column I	**Column II**
(A) Area bounded by min (f(x), g(x)) and x-axis is	(P) $\frac{1}{8}$ sq. unit
(B) Area bounded by min (f(x), h(x)) and x-axis is	(Q) $\frac{1}{4}$ sq. unit
(C) Area bounded by min (f(x), g(x), h(x)) and x-axis is	(R) $\frac{1}{2}$ sq. unit
(D) Area bounded by min (f(x), g(x), h(x)), and $y = \frac{1}{2}$ is	(S) 3/4 sq. unit

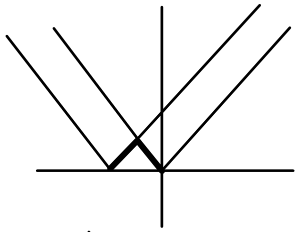
Sol. **A $\rightarrow$ Q, B $\rightarrow$ Q, C $\rightarrow$ R, D $\rightarrow$ S**

(A)

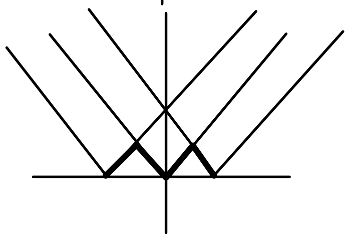


$A = \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$

(B) $A = \frac{1}{4}$

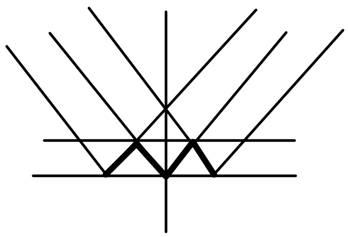


(C)



$A = \frac{1}{2}$

(D)



$A = \frac{3}{4}$

Q.19 **Column I** **Column II**

(A) Let area of the figure bounded by $y = -3x^2 - |x| + 2$, $x = 0$ and $y = 0$ is $\frac{22}{a^3}$ sq. units then a is (P) 1/3

(B) Let $I_{n,\,m} = \int \frac{\sin^n x}{\cos^m x}\,dx$, $m \neq 1$, if (Q) 3

$I_{n,m} = \frac{\sin^{n-1} x}{(m-1)\cos^{m-1} x} + \frac{a(n-1)}{2(m-1)}\, I_{n-2,\,m-2}$

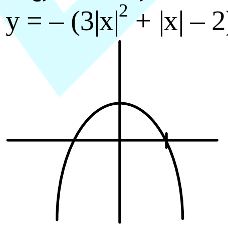
then a is

(C) The common area of the curves $y = \sqrt{x}$ and $x = \sqrt{y}$ is (R) –2

(D) Let $f_n(x) = \int \cot^n x\,dx$, then $3(f_2(3\pi/4) + f_4(3\pi/4))$ is (S) 1

Sol. **A → Q; B → R; C → P, R; D → S**

(A) $y = -(3|x|^2 + |x| - 2)$



$$A = 2\int_0^{2/3} (-3x^2 - x + 2)\,dx$$

$$= 2\left[-\left[x^3\right]_0^{2/3} - \frac{1}{2}\left[x^2\right]_0^{2/3} + 2\left[x\right]_0^{2/3}\right]$$

$$= 2\left[-\frac{8}{27} - \frac{2}{9} + \frac{4}{3}\right]$$

$$= 2 \times \frac{22}{27}$$

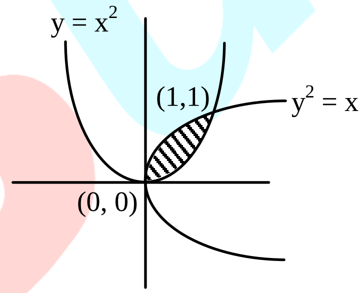
a = 3

(B) $I_{m,n} = \int \frac{\sin^{n-1} x \sin x\,dx}{\cos^m x}$

$$I_{m,n} = \frac{\sin^{n-1} x}{(m-1)\cos^{m-1} x} - \frac{(n-1)}{(m-1)}\int \frac{\sin^{n-2} x}{\cos^{m-1} x}$$

$\frac{a}{2} = -1 \Rightarrow a = -2$

(C)



$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

(D) $I_n = \int \cot^{n-2} x\,(\mathrm{cosec}^2 x - 1)\,dx$

$$I_n = \frac{-\cot^{n-1} x}{(n-1)} - I_{n-2}$$

$$I_n + I_{n-2} = \frac{\cot^{n-1} x}{1-n}$$

n = 4

$x = \frac{3\pi}{4}$

$I_4 + I_2 = \frac{1}{3}$

$3(I_4 + I_2) = 1$

Q.20 Match the column

Column I **Column II**

(A) Area bounded by $y = x^3$ with x-axis between x = 1 and x = 2, is (P) 1/3

(B) Area bounded by $y \leq 4x - x^2$ in I quadrant is (Q) 15/4

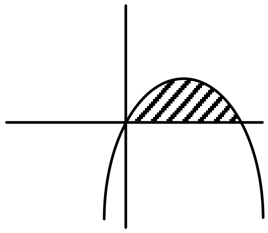
(C) Area of region $\{(x, y): x^2 \le y \le |x|\}$ is — (R) 3/2

(D) Area of portion cut off by $y = x^3$, $y = x^2$ and ordinates $x = 0$, $x = 2$, is — (S) 32/3

Sol. $\mathbf{A \to Q, B \to S, C \to P, D \to R}$

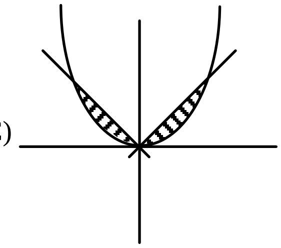
(A) $A = \frac{1}{4}\left(x^4\right)_1^2 = \frac{15}{4}$

(B)



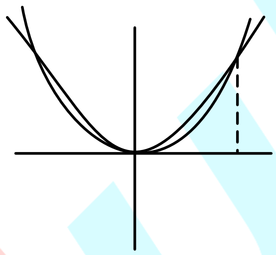
$$A = \int_0^4 (4x - x^2)\, dx = 2 \times 16 - \frac{1}{3}(64) = \frac{32}{3}$$

(C)



$$A = 2\int_0^1 (x - x^2)\, dx = 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{3}$$

(D)



$$A = \int_0^1 (x^2 - x^3)\, dx + \int_1^2 (x^3 - x^2)\, dx$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{15}{4} - \frac{7}{3}$$

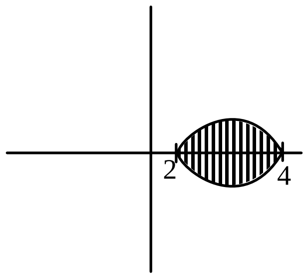
$$= -2 + \frac{7}{2} = \frac{3}{2}$$

EXERCISE # 3

Part-A Subjective Type Questions

Q.1 Find area of the loop of the curve $y^2 = (x-2)(x-4)^2$.

Sol.



$A = 2\int_2^4 (x-4)\sqrt{x-2}\,dx$

$x - 2 = t^2$

$= 2\int_0^{\sqrt{2}} (t^2-2).t.2t\,dt$

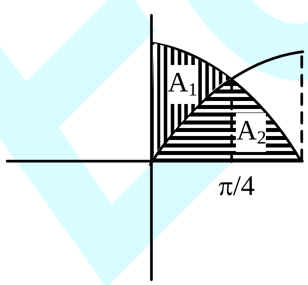
$= 4\int_0^{\sqrt{2}} (t^4-2t^2)\,dt$

$= \left[\frac{4}{5}.4\sqrt{2} - \frac{8}{3}2\sqrt{2}\right]$

$= \frac{2}{15}\times 16\sqrt{2}$

Q.2 Find the ratio in which the area enclosed by the curve y = cos x ($0 \le x \le \pi/2$) in the first quadrant is divided by the curve y = sin x.

Sol.



$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = \sqrt{2} - 1$

$A_2 = \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$

$= -\left(\frac{1}{\sqrt{2}} - 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \sqrt{2}$

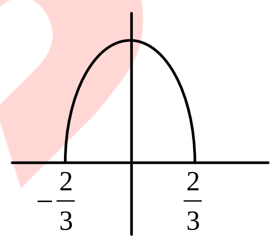
$\frac{A_1}{A_2} = \frac{\sqrt{2}-1}{2-\sqrt{2}} = \frac{(\sqrt{2}-1)(2+\sqrt{2})}{2}$

$= \frac{2\sqrt{2}+2-2-\sqrt{2}}{2}$

$= \frac{1}{\sqrt{2}}$

Q.3 Find the area of the figure bounded by $y = -3x^2 - |x| + 2$ and $y = 0$.

Sol.



$A = 2\int_0^{2/3} (-3x^2 - x + 2)\,dx$

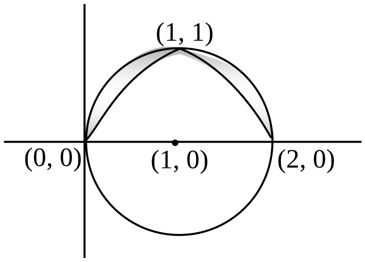
$= -2\left[x^3\right]_0^{2/3} - \left[x^2\right]_0^{2/3} + 4\left[x\right]_0^{2/3}$

$= \frac{-16}{27} - \frac{4}{9} + \frac{8}{3} = \frac{44}{27}$

Q.4 Find the area bounded by $x^2 + y^2 - 2x = 0$ and $y = \sin(\pi x/2)$ in the upper half of the circle.

Sol. $x^2 + y^2 - 2x = 0$ and $y = \sin\left(\frac{\pi x}{2}\right)$

$\Rightarrow (x-1)^2 + y^2 = 1$ and $y = \sin\left(\frac{\pi x}{2}\right)$



$$\text{required area} = \frac{\pi}{2} - \int_0^2 \sin\frac{\pi x}{2}\,dx$$

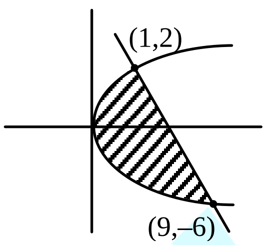
$$= \frac{\pi}{2} + \frac{2}{\pi}\left[\cos\frac{\pi x}{2}\right]_0^2$$

$$= \frac{\pi}{2} + \frac{2}{\pi}[-1-1]$$

$$= \left(\frac{\pi}{2} - \frac{4}{\pi}\right) \text{sq. unit}$$

Q.5 Find the area of the region bounded by the parabola $y^2 = 4x$ and the normal to it at one of the ends of its latus rectum

Sol.



Line $y - 2 = -1(x - 1)$

$x = 3 - y$

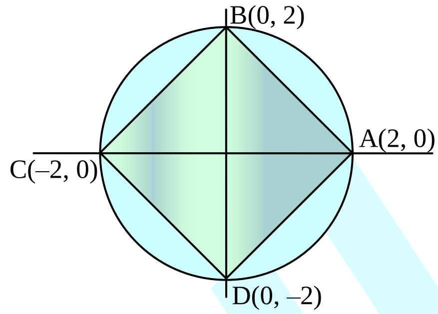
$$A = \int_{-6}^{2}(3-y)\,dy - \int_{-6}^{2}\frac{y^2}{4}\,dy$$

$$= 24 + 16 - \frac{56}{3} = \frac{64}{3}$$

Q.6 Calculate the area enclosed by the curve $4 \le x^2 + y^2 \le 2(|x| + |y|)$.

Sol. Given $x^2 + y^2 \ge 4$ and $2(|x| + |y|) \ge 4$

$\Rightarrow x^2 + y^2 \ge 4$ and $|x| + |y| \ge 2$



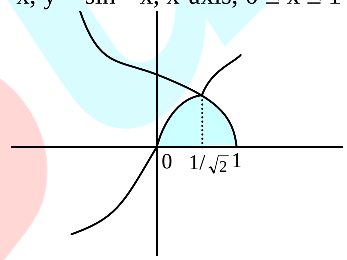
Common area = Area of square ABCD

$= \frac{1}{2}$ (product of diagonal)

$= \frac{1}{2}(16) = 8$ sq. unit.

Q.7 Find the area of the region bounded by the axis of x, $0 \le x \le 1$, y = arc (cos x) and y = arc (sin x).

Sol. $y = \cos^{-1} x$, $y = \sin^{-1} x$, x-axis, $0 \le x \le 1$



$$\text{required area} = \int_0^{1/\sqrt{2}} \sin^{-1} x\,dx + \int_{1/\sqrt{2}}^{1} \cos^{-1} x\,dx$$

$$= [x\sin^{-1}x]_0^{1/\sqrt{2}} - \int_0^{1/\sqrt{2}} \frac{x}{\sqrt{1-x^2}}\,dx$$

$$+ [x\cos^{-1}x]_{1/\sqrt{2}}^{1} + \int_{1/\sqrt{2}}^{1} \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \left(\frac{\pi}{4\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right) - \int_0^{1/\sqrt{2}} \frac{x}{\sqrt{1-x^2}}\,dx$$

$$+ \int_{1/\sqrt{2}}^{1} \frac{x}{\sqrt{1-x^2}}\,dx$$

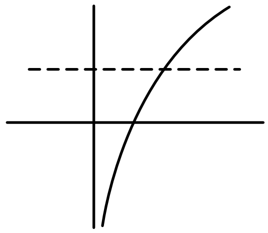
$$= -\int_1^{1/\sqrt{2}} \frac{x}{\sqrt{1-x^2}}\,dx + \int_{1/\sqrt{2}}^{1} \frac{x}{\sqrt{1-x^2}}\,dx$$

Put $1 - x^2 = t^2$ and solving, we get

area = $(\sqrt{2} - 1)$ sq. unit.

Q.8 Find the area enclosed by the curve $y = \frac{d}{dx}(x\, l\, n\, x)$ and the coordinate axes.

Sol. $y = 1 + \lambda n\ x$



$A = \left[x\,\lambda n x\right]_0^{1/e}$

$= -\frac{1}{e} - \lim\limits_{x\to 0} x\,\lambda n\ x$

$|A| = \frac{1}{e}$

Q.9 Find the area of curve enclosed by [x] + [y] = 4 in the Ist quadrant.

Sol. [x] + [y] = 4

[x] = 4 – [y]

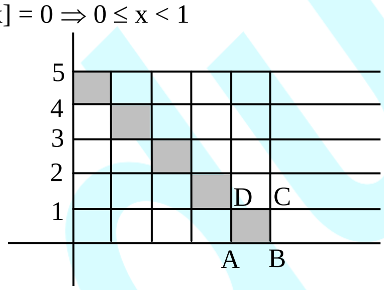
when y = 0, [x] = 4 ⇒ 4 ≤ x < 5

y = 1, [x] = 3 ⇒ 3 ≤ x < 4

y = 2, [x] = 2 ⇒ 2 ≤ x < 3

y = 3, [x] = 1 ⇒ 1 ≤ x < 2

y = 4, [x] = 0 ⇒ 0 ≤ x < 1

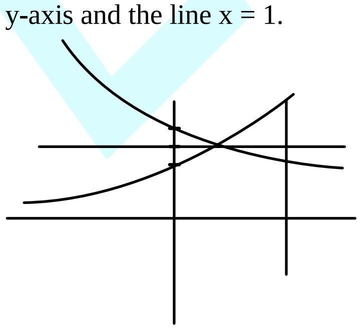


required area = 5 (area of square ABCD)

= 5 × 1 = 5

Q.10 Let f(x) = minimum $\{e^x, 3/2, 1+ e^{-x} \mid 0 \le x \le 1\}$ Find the area bounded by y = f(x), x-axis, y-axis and the line x = 1.

Sol.



$e^x = 1 + e^{-x}$

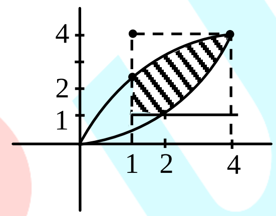
$t - \frac{1}{t} - 1 = 0$

$t^2 - t - 1 = 0$

$t = \frac{1 \pm \sqrt{5}}{2}$

Q.11 Let $y^2 = 4[\sqrt{y}]\,x$ and $x^2 = 4[\sqrt{x}]\,y$ be two curves, where [ ] represents greatest integer function. Find area bounded by these two curves within the square formed by the lines x = 1,y = 1, x = 4, y = 4.

Sol.



$A = 2\int_1^4 \sqrt{x}\,dx - 1 - \int_2^4 \frac{x^2}{4}dx$

$= \frac{4}{3}\left(x^{3/2}\right)_1^4 - 1 - \frac{1}{12}\left(x^3\right)_2^4$

$= \frac{28}{3} - 1 - \frac{56}{12}$

$= \frac{11}{3}$

Q.12 Find out the area enclosed by circle |z| = 2, parabola $y = x^2 + x + 1$, the curve $y = \left[\sin^2\frac{x}{4} + \cos\frac{x}{4}\right]$ and x-axis, where [.] denotes the greatest integer function.

Sol. $\Theta\ y = \left[\sin^2\frac{x}{4} + \cos\frac{x}{4}\right]$

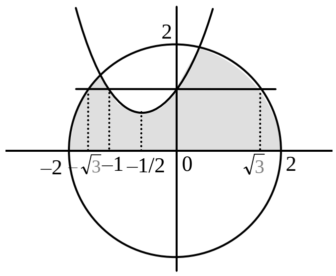
$1 < \sin^2\frac{x}{4} + \cos\frac{x}{4} < 2 \quad \Rightarrow y = 1$

so area enclose by the curves

$\Theta\ |z| = 2 \Rightarrow x^2 + y^2 = 4$

and $y = x^2 + x + 1 \Rightarrow \left(y - \frac{3}{4}\right) = \left(x + \frac{1}{2}\right)^2$

and y = 1



required area

$= \sqrt{3} \times 1 + (\sqrt{3} - 1) \times 1 + \int_{-1}^{0} (x^2 + x + 1)\,dx$

$+ \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\,dx$

$= (2\sqrt{3} - 1) + \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]_{-1}^{0}$

$+ 2\left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$

solving we get

area $= \left(\frac{2\pi}{3} + \sqrt{3} - \frac{1}{6}\right)$ sq. unit

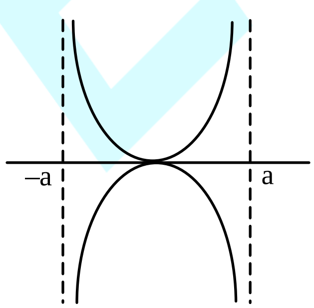
Q.13 Compute the area of the loop of the curve $y^2 = x^2[(1 + x)/(1 - x)]$.

Sol. $(2 - \pi/2)$ units^2

Q.14 Find the area included between the curve $x^2y^2 = a^2 (y^2 - x^2)$ and its asymptotes.

Sol. $y^2 = \frac{a^2x^2}{a^2 - x^2}$

$y = \pm\frac{ax}{\sqrt{a^2 - x^2}}$



$A = 4\int_{0}^{a} \frac{ax}{\sqrt{a^2 - x^2}}dx = -2a\int_{0}^{a} \frac{-2x}{\sqrt{a^2 - x^2}}dx$

$= -2a \times 2\left(\sqrt{a^2 - x^2}\right)_0^a$

$= -4a\,(-a) = 4a^2$

Part-B Passage based Question

Passage I (Q. 15 to 17)

Let there are three functions described here :

$f(x) = \{\sin x\}$, $g(x) = \{\cos x\}$, $h(x) = [x/\pi]$. Where $[x]$ is greatest integral part of x & $\{x\}$ is the fractional part of x.

On the basis of above information, answer the following questions-

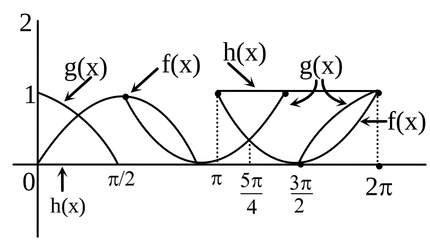
Q.15 If the area bounded by x-axis, $x = 0$, $x = \pi$ and f(x) is t_1 and the area bounded by x-axis, $x = \pi$, $x = 2\pi$ and f(x) is t_2 then-

(A) $\frac{t_2}{t_1} > \frac{1}{2}$ (B) $0 < \frac{t_2}{t_1} < \frac{1}{2}$

(C) $\frac{t_2}{t_1} > 1$ (D) None of these

Sol. **[A]**

$f(x) = \{\sin x\}$, $g(x) = \{\cos x\}$, $h(x) = [x/\pi]$



$t_1 = \int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi} = 1 + 1 = 2$

and $t_2 = 2\int_{\pi}^{3\pi/2} (1 + \sin x)\,dx = 2\,[x - \cos x]_{\pi}^{3\pi/2}$

$= 2\left[\frac{3\pi}{2} - 0 - \pi - 1\right] = \pi - 2$

$\frac{t_2}{t_1} = \frac{\pi - 2}{2} = \frac{\pi}{2} - 1 = \frac{3.14}{2} - 1 = 1.57 - 1$

$\frac{t_2}{t_1} = 0.57$ (approx.)

Clearly $\frac{t_2}{t_1} > \frac{1}{2}$

Q.16 The area bounded by f(x), h(x), x = π and x = 2π is

(A) 2 (B) π – 2 (C) $\frac{\pi - 2}{2}$ (D) None

Sol. **[A]**

Area bounded by f(x), h(x), x = π and x = 2π from Q.1, we have

Area bounded by x-axis, x = π, x = 2π and and area of rectangle made by f(x) is = π – 2 x = π, x = 2π and x-axis is π × 1 = π

required area = π – (π –2) = 2

Q.17 The area bounded by f(x), g(x), x-axis, x = π and $x = \frac{3\pi}{2}$ is-

(A) $\frac{\pi}{2} - \frac{1}{\sqrt{2}}$ (B) $\frac{\pi}{2} - \sqrt{2}$

(C) $\frac{\pi}{2} - \frac{1}{2}$ (D) None of these

Sol. **[B]**

Required area

$= \int_{\pi}^{5\pi/4} (1 + \cos x)\, dx + \int_{5\pi/4}^{3\pi/2} (1 + \sin x)\, dx$

$= [x + \sin x]_{\pi}^{5\pi/4} + [1 - \cos x]_{5\pi/4}^{3\pi/2}$

$= \frac{5\pi}{4} - \frac{1}{\sqrt{2}} - \pi + \frac{3\pi}{2} - \frac{5\pi}{4} - \frac{1}{\sqrt{2}}$

$= \frac{\pi}{2} - \sqrt{2}$

Passage II (Q. 18 to 20)

Five curves defined as follows :

C_1 : $|x + y| \le 1$

C_2 : $|x - y| \le 1$

C_3 : $|x| \le \frac{1}{2}$

C_4 : $|y| \le \frac{1}{2}$

C_5 : $3x^2 + 3y^2 = 1$

Q.18 The ratio of region bounded by C_1, C_2 and C_3, C_4 is -

(A) 1.5 (B) 2 (C) $\frac{1}{2}$ (D) None

Sol. **[B]**

A_1 = area bounded by C_1, C_2

= area of square ABCD = 2

A_2 = Area bounded by C_3, C_4

= Area of square EFGH = 1

$\Rightarrow \frac{A_1}{A_2} = \frac{2}{1} = 2$

Q.19 The area bounded by C_1 and C_2 which does not contain the area of C_5 is

(A) $2 - \frac{\pi}{4}$ (B) $2 - \frac{\pi}{6}$

(C) $2 - \frac{\pi}{3}$ (D) None of these

Sol. **[C]**

Θ area bounded by C_1 and C_2 = 2

∴ area bounded by $C_5 = \frac{\pi}{3}$

⇒ area bounded by C_1 and C_2 which does not contain the area of C_5

$= 2 - \frac{\pi}{3}$

Q.20 That part of area which is bounded by C_1 and C_2 but not bounded by C_3 and C_4 is-

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) None

Sol. **[A]**

$\because$ Area bounded by C_1 and $C_2 = 2$

$\therefore$ area bounded by C_3 and $C_4 = 1$

$\Rightarrow$ area bounded by C_1 and C_2 but not bounded by C_3 and $C_4 = 2 - 1 = 1$

Passage III (Q. 21 to 23)

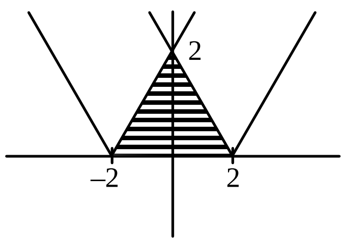
Let there are two function defined here: $f(x) = \min(|x-2|, |x+2|)$ and $g(x) = \min(e^x, e^{-x})$. Now the root of the equation $e^{-x} + x - 2 = 0$ is α, where $\alpha \in R$.

Q.21 The area bounded by f(x) and x-axis is

(A) 1 sq. unit (B) 4 sq. unit

(C) 6 sq. unit (D) None of these

Sol. **[B]**



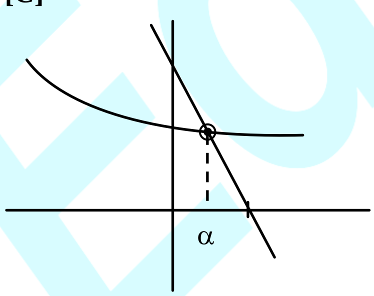
$$\frac{1}{2} \times 4 \times 2 = 4$$

Q.22 Which statement is correct -

(A) $\alpha \in (2, 3)$ (B) $\alpha \in (-1, 0)$

(C) $\alpha \in (0, 2)$ (D) None of these

Sol. **[C]**

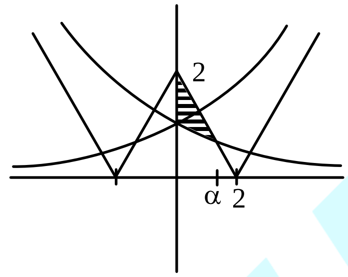


Q.23 The area bounded by f(x), g(x) and x = 0 in first quadrant is

(A) $e^{-\alpha} - 1$ (B) $2 - e^{-\alpha}$

(C) $1 + e^{-\alpha}$ (D) None of these

Sol. **[D]**



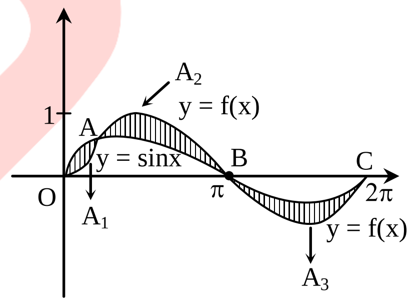
$$A = \int_0^{\alpha} (2 - x - e^{-x})\,dx$$

$$= 2\alpha - \frac{1}{2}\alpha^2 + e^{-\alpha} - 1$$

Passage IV (Q. 24 to 26)

In the adjacent figure, the graphs of two functions y = f(x) and y = sin x are given. They intersect at origin, A(a , f(a)), B(π, 0) and C(2π, 0). A_i (i =1, 2, 3) is the area bounded by the curves as shown in the figure respectively for $x \in (0, a)$, $x \in (a, \pi)$, $x \in (\pi, 2\pi)$.

If $A_1 = 1 + (a-1)\cos a - \sin a$.



Q.24 The function f(x) is-

(A) $x^2 \sin x$ (B) $x \sin x$

(C) $2x \sin x$ (D) $x^3 \sin x$

Sol. **[B]**

From fig. we have

$$\int_0^a (\sin x - f(x))\,dx = A_1$$

$$\Rightarrow \int_0^a (\sin x - f(x))\,dx = 1 + (a-1)\cos a - \sin a$$

diff. with respect to a, we get

$\sin a - f(a) = \cos a - (a-1)\sin a - \cos a$

$\Rightarrow f(a) = a \sin a$

$\Rightarrow f(x) = x \sin x$

Q.25 Value of A_2 is-

(A) $(\pi - 1)$ units^2 (B) $(\pi/2 - 1)$ units^2

(C) $(\pi - \sin 1 - 1)$ units^2 (D) $\pi/2$units^2

Sol. **[C]**

Θ $f(x) = x \sin x$ and $y = \sin x$

$\Rightarrow (a, f(a)) \equiv (1, a \sin a)$

$$A_2 = \int_1^{\pi} (x \sin x - \sin x)\, dx$$

$$= [-x\cos x]_1^{\pi} + [\sin x]_1^{\pi} + [\cos x]_1^{\pi}$$

$$= \pi + \cos 1 - \sin 1 - 1 - \cos 1$$

$$= (\pi - \sin 1 - 1)\text{ unit}^2$$

Q.26 Value of A_3 is-

(A) $(2\pi - 1)$ units^2 (B) $(3\pi - \sin 2)$ units^2

(C) $(3\pi - 2)$ units^2 (D) $(\pi - 2)$ units^2

Sol. **[C]**

$$A_3 = \left| \int_{\pi}^{2\pi} (\sin x - x \sin x)\, dx \right|$$

$$= \left| [-\cos x]_{\pi}^{2\pi} + [\cos x]_{\pi}^{2\pi} - \int_{\pi}^{2\pi} \cos x\, dx \right|$$

$$= \left| -1 - 1 + 2\pi + \pi - [\sin x]_{\pi}^{2\pi} \right|$$

$$= (3\pi - 2)\text{ unit}^2$$

Passage V (Q. 27 to 29)

Area enclosed by curve $y = f(x)$ and $y = x^2 + 2$ between the abscissa $x = 2$ and $x = \alpha$ is given as $(\alpha^3 - 4\alpha^2 + 8)$ sq. unit. It is known that curve $y = f(x)$ lies below the parabola $y = x^2 + 2$.

Q.27 Area enclosed by curve $y = f(x)$ with x-axis, $x = 0$, $x = 1$ is

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) $\frac{16}{7}$ (D) $\frac{4}{3}$

Sol. **[B]**

$$\alpha^3 - 4\alpha^2 + 8 = \int_2^{\alpha} (x^2 + 2 - f(x))\, dx$$

Diff. w.r.to α

$3\alpha^2 - 8\alpha = \alpha^2 + 2 - f(\alpha)$

$f(\alpha) = -2\alpha^2 + 8\alpha + 2$

$f(x) = -2x^2 + 8x + 2$

Now

$$A = \int_0^1 (-2x^2 + 8x + 2)\, dx$$

$$= -\frac{2}{3} + 4 + 2 = \frac{16}{3}$$

Q.28 If $f(x)$ lies above x-axis in $x \in (p, q)$, then $(q + p)$ is equal to

(A) 2 (B) 3 (C) 4 (D) 8

Sol. **[C]**

$2x^2 - 8x + 2 = 0 \begin{cases} p \\ q \end{cases}$

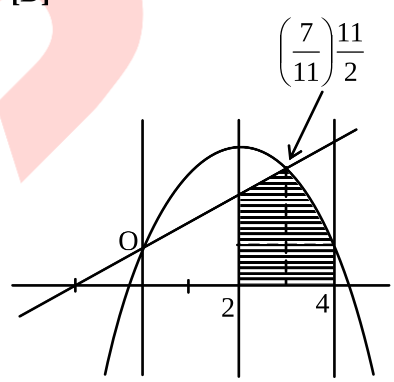
$$p + q = -\frac{(-8)}{2} = 4$$

Q.29 Value of area bounded by line $y = x + 2$ and $y = f(x)$, $x = 2$ and $x = 4$ is

(A) $\frac{36}{5}$ (B) $\frac{7}{5}$

(C) $\frac{123}{13}$ (D) None of these

Sol. **[D]**



$-2x^2 + 8x + 2 = x + 2$

$2x^2 - 7x = 0$

$x(2x - 7) = 0$

$$A = \frac{1}{2} \times \frac{3}{2} \times \frac{11}{2} + \left| \int_{\frac{7}{2}}^{4} (-2x^2 + 8x + 2)\, dx \right|$$

$$= \frac{33}{8} + \left| \left(-\frac{2}{3}\left(64 - \frac{343}{8}\right) \right) + 4\left(16 - \frac{49}{4}\right) + 2\left(4 - \frac{7}{2}\right) \right|$$

$$= \frac{33}{8} + \left[-\frac{338}{24} + 15 + 1 \right] = \frac{33}{8} + \frac{46}{24}$$

$$= \frac{145}{24}$$

EXERCISE # 4

Old IIT-JEE Questions

Q.1 The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinates axes, lies in the first quadrant. If its area is 2, then the value of b is - **[IIT Scr. 2001]**

(A) – 1 (B) 3 (C) – 3 (D) 1

Sol. **[C]**

Let $y = f(x) = x^2 + bx - b$

$\frac{dy}{dx} = 2x + b$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2 + b$

equation of tangent at (1, 1) is

$\Rightarrow (y - 1) = (2 + b)(x - 1)$

$\Rightarrow (2 + b)x - y + 1 - 2 - b = 0$

$\Rightarrow (2 + b)x - y = 1 + b$

$\Rightarrow \frac{x}{\frac{1+b}{2+b}} - \frac{y}{1+b} = 1$

Intercept made by this line with coordinate axis is $\frac{1+b}{2+b}, -(1+b)$

Area $= -\frac{1}{2} \cdot \left(\frac{1+b}{2+b}\right)(1+b) = 2$ given

$\Rightarrow -8 - 4b = 1 + b^2 + 2b$

$\Rightarrow (b + 3)^2 = 0 \Rightarrow b = -3$

Q.2 Let $b \neq 0$ and for j = 0, 1, 2 ....n, let S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots\ldots. S_n$ are in geometric progression. Also, find their sum for a = –1 and b = π. **[IIT-2001]**

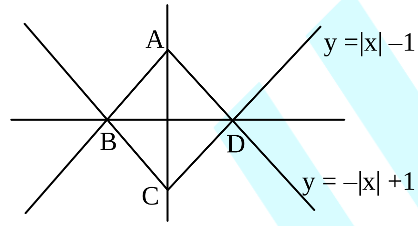
Sol. $\frac{(e+1)\pi(e^{n+1}-1)}{(\pi^2+1)(e-1)}$ sq. units.

Q.3 The area bounded by the curves y = |x| – 1 and y = – |x| + 1 is **[IIT Scr.2002]**

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

Sol. **[B]**

y = |x| – 1 and y = –|x| + 1



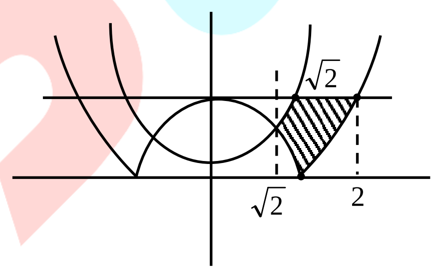
ABCD is a square

where AC = 2

Area $= \frac{1}{2}(AC)^2 = 2$

Q.4 Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and y = 2, which lies to the right of the line x = 1. **[IIT-2002]**

Sol.



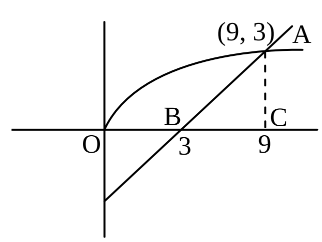
$$A = \int_1^{\sqrt{2}} x^2\,dx + 2(2-\sqrt{2}) - \int_1^{2} |x^2 - 2|\,dx$$

Q.5 Area of the region bounded by $y = \sqrt{x}$, x = 2y + 3 & x-axis lying in 1st quadrant is- **[IIT Scr.2003]**

(A) $2\sqrt{3}$ (B) 18 (C) 9 (D) 34/3

Sol. **[C]**

$y = \sqrt{x}$, x – 2y = 3



required area = $\int_0^9 \sqrt{x}\,dx - \frac{1}{2} \times 6 \times 3$

$= \frac{2}{3}[x^{3/2}]_0^9 - 9$

$= 18 - 9 = 9$

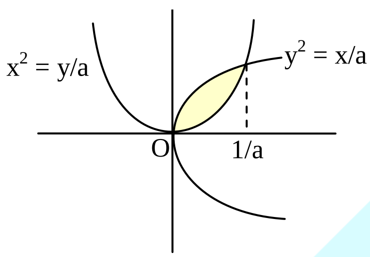
Q.6 If area bounded by the curve $x = ay^2$ & $y = ax^2$ is 1, then a is equal to - **[IIT Scr.2004]**

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3

Sol. **[A]**

$x = ay^2$ and $y = ax^2$

$\Rightarrow y^2 = \frac{x}{a}$ and $x^2 = \frac{y}{a}$



required area = $\int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1$ given

$\Rightarrow \left[\frac{1}{\sqrt{a}}\frac{2}{3}x^{3/2} - \frac{ax^3}{3}\right]_0^{1/a} = 1$

$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1$

$\Rightarrow a^2 = \frac{1}{3}$ $\quad \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

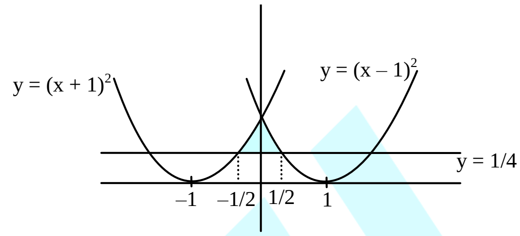
from option $a = \frac{1}{\sqrt{3}}$

Q.7 The area between the curves $y = (x - 1)^2$ $y = (x + 1)^2$ and $y = 1/4$ is **[IIT Scr.2005]**

(A) 1/3 (B) 2/3 (C) 4/3 (D) 1/6

Sol. **[A]**

$y = (x - 1)^2$; $y = (x + 1)^2$ and $y = 1/4$



required area = $2\int_0^{1/2}\left((x-1)^2 - 1/4\right)dx$

$= 2\left[\frac{(x-1)^3}{3} - \frac{1}{4}x\right]_0^{1/2}$

$= 2\left(-\frac{1}{24} - \frac{1}{8} + \frac{1}{3}\right) = \frac{1}{3}$

Q.8 f(x) be a quadratic polynomial & a, b, c are three distinct real numbers, such that:

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

V is the point where f(x) attains maximum. A & B are the points on f(x) such that f(x) cuts x-axis at A in the first quadrant and chord AB subtends right angle at V. Find the area bounded by curve y = f(x) and chord AB.

[IIT-2005]

Sol. $f(x) = \alpha x^2 + \beta x + \gamma$

$4a^2 f(-1) + 4a f(1) + f(2) = 3a^2 + 3a$

$a^2[4 f(-1) - 3] + a[4f(1) - 3] + f(2) = 0$ ..(1)

Similarly

$b^2 (4f(-1) - 3) + b(4f(1) - 3) + f(2) = 0$ ….(2)

$c^2 [4f(-1) - 3] + c [4f(-1) - 3] + f(2) = 0$ ….(3)

here, a, b, c satisfy the equation

$[4f(-1) - 3] x^2 + [4f(1) - 3]x + f(2) = 0$

hence it is an identity

$\left.\begin{array}{l} f(-1) = \frac{3}{4} \\ f(1) = \frac{3}{4} \end{array}\right\} f(2) = 0$

$\Rightarrow \beta = 0$

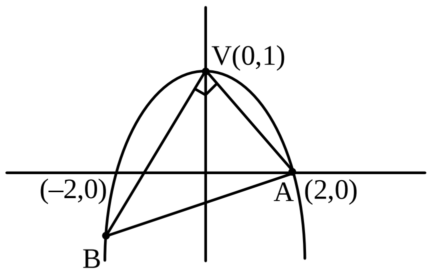
So, $f(1) = \alpha + \gamma = \frac{3}{4}$

f(2) = 4α + γ = 0

$\alpha = -\dfrac{1}{4}$

γ = 1

$f(x) = -\dfrac{1}{4}x^2 + 1$



$\left(x, -\frac{1}{4}x^2 + 1\right)$

$$\frac{-\frac{1}{4}x^2}{x} \times \left(-\frac{1}{2}\right) = -1$$

x = – 8

so B(– 8, – 15)

Line AB

$$y - 0 = \frac{-15-0}{-8-2}(x-2)$$

$$y = \frac{3}{2}(x-2)$$

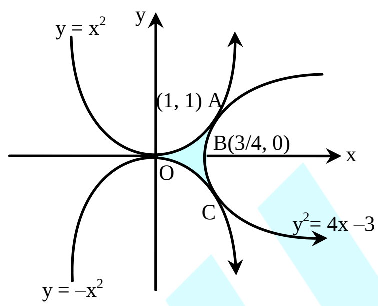
$$\text{Area} = \int_{-8}^{2}\left[\left(-\frac{1}{4}x^2 + 1\right) - \frac{3}{2}(x-2)\right]dx$$

$$= \int_{-8}^{2}\left(-\frac{1}{4}x^2 - \frac{3}{2}x + 4\right)dx$$

Q.9 Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ & $y^2 = 4x - 3$. **[IIT-2005]**

Sol. The region bounded by the curves $y = x^2$, $y = -x^2$ and $y^2 = 4x - 3$ is symmetrical about x- axis.

Where y = 4x – 3 meets at (1, 1)



Hence, area (OABCO)

$$= 2\left\{\int_0^1 x^2\,dx - \int_{3/4}^{1}(\sqrt{4x-3})\,dx\right\}$$

$$= 2\left\{\left(\frac{x^3}{3}\right)_0^1 - \left(\frac{(4x-3)^{3/2}}{32/4}\right)_{3/4}^1\right\}$$

$$= 2\left\{\frac{1}{3} - \frac{1}{6}\right\} = 1.\ \frac{1}{6} = \frac{1}{3} \text{ square units.}$$

Q.10 The area of the region between the curves $y = \sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y = \sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and $x = \dfrac{\pi}{4}$ is- **[IIT 2008]**

(A) $\displaystyle\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

(B) $\displaystyle\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\displaystyle\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(D) $\displaystyle\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

Sol. **[B]**

$$A = \int_0^{\pi/4}\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\,dx$$

$$= \int_0^{\pi/4}\frac{\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| - \left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|}{\sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/4} \frac{2\sin\frac{x}{2}}{\sqrt{\cos x}}\,dx$$

$$= \int_0^{\pi/4} \frac{2\tan\frac{x}{2}\cos\frac{x}{2}}{\sqrt{2\cos^2\frac{x}{2}-1}}\,dx$$

$$= \int_0^{\pi/4} \frac{2\tan\frac{x}{2}}{\sqrt{2-\sec^2\frac{x}{2}}}\,dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

$$I = \int_0^{\sqrt{2}-1} \frac{2.t\,(2\,dt)}{(1+t^2)\sqrt{2-(1+t^2)}}$$

$$= \int_0^{\sqrt{2}-1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$$

Passage I (Q.11 to 13)

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$. **[IIT-2008]**

Q.11 If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

Sol. **[B]**

$3y^2y' - 3y' + 1 = 0$

$$y' = \frac{1}{3(1-y^2)} = -\frac{1}{21}$$

$6y\,.\,(y')^2 + 3y^2\,y'' - 3y'' = 0$

$$\Rightarrow \frac{12\sqrt{2}}{(21)^2} + 21y'' = 0$$

$$y'' = \frac{12\sqrt{2}}{-3^2.7^3}$$

Q.12 The area of the region bounded by the curve $y = f(x)$, the x- axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3((f(x))^2-1)}\,dx + bf(b) - af(a)$

(B) $-\int_a^b \frac{x}{3((f(x))^2-1)}\,dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2-1)}\,dx - bf(b) + af(a)$

(D) $-\int_a^b \frac{x}{3((f(x))^2-1)}\,dx - bf(b) + af(a)$

Sol. **[A]**

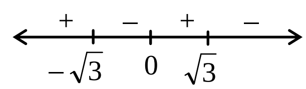
$y^3 - 3y + x = 0$

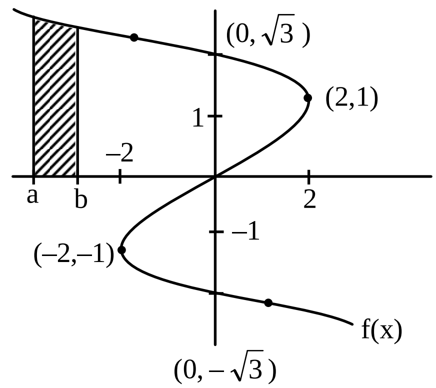
$x = 3y - y^3 = y(3 - y^2)$

$(0, 0), (0, \sqrt{3}), (0, -\sqrt{3})$

$$y' = \frac{-1}{3(y^2-1)}$$

odd symmetric





$y' = \infty$ at $y = \pm 1$

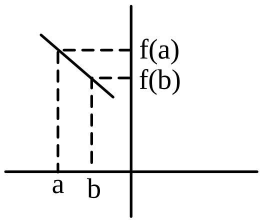
$$A = \int_a^b 1.y\,dx$$

$= [x f(x)]_a^b - \int_a^b x f'(x)\,dx$

$= b\,f(b) - a\,f(a) - \int_a^b \frac{x(-1)}{3(y^2-1)}\,dx$

$= b\,f(b) - a\,f(a) + \int_a^b \frac{x}{3(f^2(x)-1)}\,dx$

Alternate :



$- a\,f(a) + b\,f(b) - \int_{f(b)}^{f(a)} x\,dy$

as $\frac{dy}{dx} = \frac{-1}{3(y^2-1)}$

$-a\,f(a) + b\,f(b) - \int_a^b \frac{x(-dx)}{3(y^2-1)}$

Q.13 $\int_{-1}^{1} g'(x)\,dx =$

(A) 2g (–1) (B) 0

(C) –2g (1) (D) 2g(1)

Sol. **[D]**

$\int_{-1}^{1} g'(x)\,dx = (g(x))_{-1}^{1} = g(1) - g(-1)$

g(x) is odd function

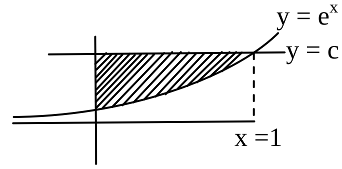
= g(1) + g(1) = 2g(1)

Q.14 Area of the region bounded by the curve $y = e^x$ and lines x = 0 and y = e is **[IIT 2009]**

(A) e – 1 (B) $\int_1^e \ln(e+1-y)\,dy$

(C) $e - \int_0^1 e^x dx$ (D) $\int_1^e \ln y\,dy$

Sol. **[B, C, D]**



$y = e^x$

Area $= \int_1^e (\ln y)\,dy$

$\int_1^e \ln(1+e-y)\,dy = |y(\ln y - 1)|_1^e = 0 - (-1) = 1$

(C) $e - \int_0^1 e^x\,dx = e - |e^x|_0^1 = e - [e-1] = 1$

Passage II (Q.15 to 17)

Consider the polynomial

$f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of f(x) and let t = |s|.

[IIT-2010]

Q.15 The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$

(C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

Sol. **[C]**

$f(x) = 4x^3 + 3x^2 + 2x + 1$

$f'(x) = 12x^2 + 6x + 2$ is always positive

f(0) = 1, f(–1/2) = 1/4, f(–3/4) = $-\frac{1}{2}$

so root $\in \left(\frac{-3}{4}, \frac{-1}{2}\right)$ $\because$ the equation have only one real root so s $\in \left(\frac{-3}{4}, \frac{-1}{2}\right)$ and t $\in \left(\frac{1}{2}, \frac{3}{4}\right)$

Q.16 The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$

(C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

Sol. **[A]**

$A(t) = \int_0^t f(x)\,d(x) = t^4 + t^3 + t^2 + t$

$= t\left(\frac{1-t^4}{1-t}\right)$

$A(1/2) = 15/16$ & $A(3/4) = 3\left(\frac{175}{256}\right)$

So $A(t) \in \left(\frac{3}{4}, 3\right)$

Q.17 The function $f'(x)$ is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in $(-t, t)$

(D) decreasing in $(-t, t)$

Sol. **[B]**

$f'(x) = 12x^2 + 6x + 2$

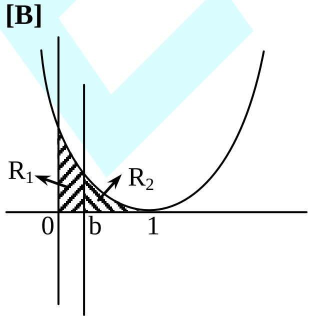
$f'(x) \uparrow \left(-\frac{1}{4}, \infty\right)$

$\downarrow \left(-\infty, -\frac{1}{4}\right)$

Q.18 Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$, and $x = 0$ into two parts $R_1(0 \le x \le b)$ and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals **[IIT 2011]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Sol. **[B]**



$R_1 - R_2 = \frac{1}{4}$

$\int_0^b (x-1)^2\,dx - \int_b^1 (x-1)^2\,dx = \frac{1}{4}$

$\left[\frac{(x-1)^3}{3}\right]_0^b - \left[\frac{(x-1)^3}{3}\right]_b^1 = \frac{1}{4}$

$\frac{(b-1)^3}{3} + \frac{1}{3} - 0 + \frac{(b-1)^3}{3} = \frac{1}{4}$

$\frac{2(b-1)^3}{3} = \frac{1}{4} - \frac{1}{3} = \frac{-1}{12}$

$(b-1)^3 = -\frac{1}{8}$

$b - 1 = -\frac{1}{2} \Rightarrow b = \frac{1}{2}$

Q.19 Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x\, f(x)dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then **[IIT 2011]**

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$

(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Sol. **[C]**

$R_1 = \int_{-1}^{2} x\, f(x)\, dx$ … (i)

$R_1 = \int_{-1}^{2} (1-x)\, f(1-x)\, dx$

$= \int_{-1}^{2} (1-x)\, f(x)\, dx$ … (ii)

(i) + (ii)

$2R_1 = \int_{-1}^{2} f(x)\,dx = R_2$

$\therefore 2R_1 = R_2$

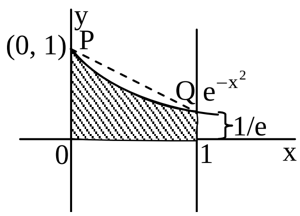
Q.20 Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then **[IIT 2012]**

(A) $S \ge \frac{1}{e}$ (B) $S \ge 1 - \frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$

(D) $S \le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Sol. **[A, B, D]**



$$\int_0^1 e^{-x^2}dx > \int_0^1 e^{-x}dx$$

$= 1-\frac{1}{e}$ (A), (B)

Area above x-axis by PQ line $y = 1 + x\left(\frac{1}{e}-1\right)$

$S \le \int_0^1 y\,dx = \frac{1+e}{2e}$ < (D) also (B) > (C)

Hence (C) not possible.

Hence A, B, D

EXERCISE # 5

Q.1 The slope of the tangent to the curve y = f(x) at a point (x, y) is 2x + 1 and the curve passes through (1, 2). The area of the region bounded by the curve, the x-axis and the line x = 1 is- **[IIT-1995]**

(A) 5/3 units (B) 5/6 units

(C) 6/5 units (D) 6 units

Sol. **[B]**

Here $\frac{dy}{dx} = 2x + 1$

Integrating both side

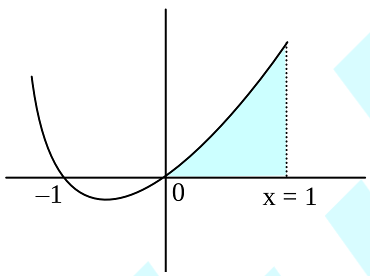
$\int dy = \int (2x+1)\, dx$

$\Rightarrow y = x^2 + x + c$

which passes through (1, 2) so

Θ $2 = 1 + 1 + c \Rightarrow c = 0$

$\therefore y = x^2 + x$



required area bounded by curve, x-axis and x = 1 is

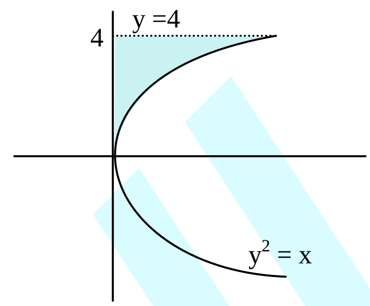
$\int_0^1 (x^2 + x)\, dx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$

$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ sq. units.

Q.2 The area bounded by the parabola $y^2 = x$, the line y = 4 and the y-axis is - **[REE- 1995]**

(A) 64/3 (B) 32/3 (C) 16/3 (D) 128/3

Sol. **[A]**



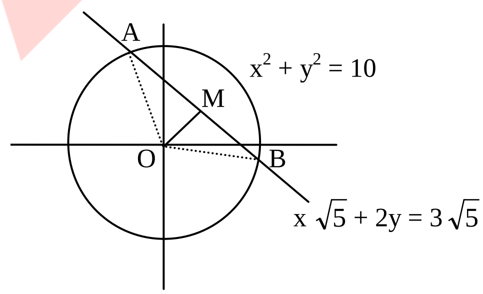
required area = $\int_0^4 x\, dy$

$= \int_0^4 y^2\, dy = \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3}$

Q.3 The area of the triangle formed by joining the origin to the points of intersection of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and circle $x^2 + y^2 = 10$, is - **[REE- 1998]**

(A) 5 (B) 4 (C) 3 (D) 6

Sol. **[A]**



We want to find the area of ΔOAB for this, we draw a perpendicular from (0, 0) the given line

$\Rightarrow OM = \left|\frac{-3\sqrt{5}}{\sqrt{5+4}}\right| = \sqrt{5}$

$OM = \sqrt{5}$

given that $OB = \sqrt{10}$

$\Rightarrow (MB)^2 = (OB)^2 - (OM)^2$

$= 10 - 5 = 5$

$\Rightarrow MB = \sqrt{5}$

$\Rightarrow AB = 2MB = 2\sqrt{5}$

Area of ΔOAB

$= \frac{1}{2} . AB. OM = \frac{1}{2} . 2\sqrt{5} . \sqrt{5} = 5$

Q.4 For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2.

[IIT- 1999]

(A) –4 (B) –2 (C) 2,–4 (D) 4, – 2

Sol. **[C]**

$y = x - x^2$ and $y = mx$

$\Rightarrow mx = x - x^2 \Rightarrow x^2 + x(m-1) = 0$

$\Rightarrow x = 0, x = 1 - m$

required area = $\int_0^{1-m} (x - x^2 - mx)\,dx$

$= \int_0^{1-m} (x(1-m) - x^2)\,dx$

$= \left[\frac{x^2}{2}(1-m) - \frac{x^3}{3}\right]_0^{1-m}$

$= \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \frac{(1-m)^3}{6}$

But given that area = $\frac{9}{2}$

$\Rightarrow \frac{(1-m)^3}{6} = \pm\frac{9}{2}$

Solving, we get

$(1 - m)^3 = \pm 27$

$1 - m = \pm 3$

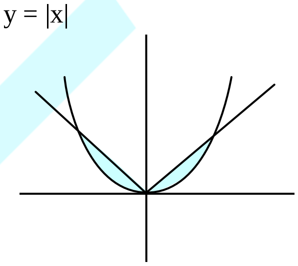
$m = 4, -2$

Q.5 The area of the region bounded by the curves $y = x^2$ and $y = |x|$ is - **[REE- 1999]**

(A) 5/3 (B) 1/3 (C) 5/6 (D) 1/6

Sol. **[B]**

$y = x^2$ and $y = |x|$



$\because y = x^2$ and $y = x$

$\Rightarrow x(x - 1) = 0 ; \quad x = 0, 1$

required area = $2\int_0^1 (x - x^2)\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

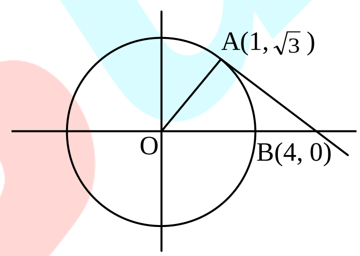
$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2. \frac{1}{6} = \frac{1}{3}$

Q.6 The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is............

[IIT 1989]

Sol.



Equation of tangent at $(1, \sqrt{3})$ is

$x + \sqrt{3}\,y = 4$

ΔOAB is a right triangle

$\Rightarrow$ Area of $\Delta OAB = \frac{1}{2}$ (OA) (AB)

$\because AB = \sqrt{9+3} = \sqrt{12}$

OA = 2

Area $= \frac{1}{2} . 2. \sqrt{12} = 2\sqrt{3}$

Q.7 Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

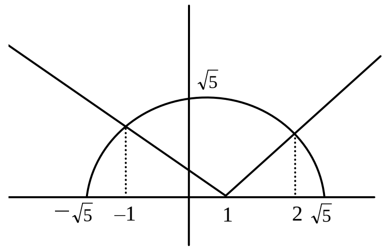
[IIT-1985]

Sol. $y = \sqrt{5 - x^2}$ and $y = |x - 1|$

point of intersection are

$5 - x^2 = (x - 1)^2 \Rightarrow x = 2, -1$

sketch is as follows



required area

$= \int_{-1}^{2}\sqrt{5-x^2}\,dx - \int_{-1}^{1}(-x+1)\,dx - \int_{1}^{2}(x-1)\,dx$

$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[-\frac{x^2}{2}+x\right]_{-1}^{1}$

$- \left[\frac{x^2}{2} - x\right]_{1}^{2}$

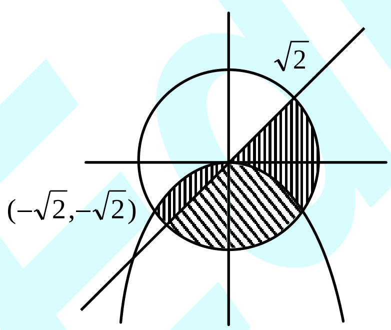
$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$

Solving we get

$\frac{5}{2}\sin^{-1}(1) - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2}$

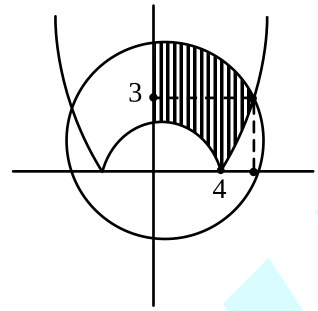
Q.8 Find the area bounded by the curves: $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}\,.y$ and $x = y$. **[IIT 1986]**

Sol. $\frac{1}{\sqrt{2}}\int_{0}^{\sqrt{2}}\sqrt{4-x^2}\,dx - \frac{1}{2}\,.\,\sqrt{2}\,.\,\sqrt{2} - \left|\int_{0}^{\sqrt{2}}\frac{-x^2}{\sqrt{2}}\,dx\right|$



Q.9 Find the area bounded by the curves $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ above the x-axis. **[IIT 1987]**

Sol.



$y = \frac{x^2-4}{4} \Rightarrow x^2 = 4y + 4$

$y^2 + 4y - 21 = 0$

$y^2 + 7y - 3y - 21 = 0$

$(y - 3)(y + 7) = 0$

$A = \int_{0}^{4}\sqrt{25-x^2}\,dx - \int_{0}^{4}\frac{|4-x^2|}{4}\,dx$

$A = \frac{1}{2}\left[x\sqrt{25-x^2} + 25\sin^{-1}\frac{x}{5}\right]_{0}^{4}$

$- \int_{0}^{2}\frac{4-x^2}{4}\,dx - \int_{2}^{4}\frac{x^2-4}{4}\,dx$

$= \frac{1}{2}\left[12 + 25\sin^{-1}\frac{4}{5}\right] - 2 + \frac{1}{12}\,.\,8 - \frac{1}{12}\,.\,56 + 2$

$= 6 + \frac{25}{2}\sin^{-1}\frac{4}{5} - 4$

Q.10 Find the area of the region bounded by the curve C : y = tan x, tangent drawn to C at $x = \pi/4$ and the x-axis. **[IIT-1988]**

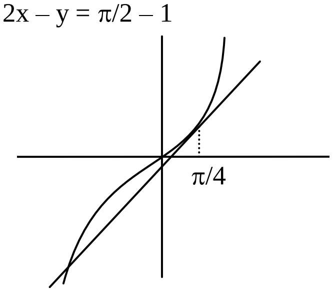
Sol. $y = \tan x$

at $x = \pi/4 \Rightarrow y = 1$

$\frac{dy}{dx} = \sec^2 x \Rightarrow \left(\frac{dy}{dx}\right)_{(\pi/4-1)} = 2$

Hence tangent at $\left(\frac{\pi}{4}, 1\right)$ is $(y - 1) = 2\left(x - \frac{\pi}{4}\right)$

$\Rightarrow 2x - y = \pi/2 - 1$



required area

$$= \int_0^{\pi/4} \tan x\, dx - \int_0^{\pi/4} (2x + 1 - \pi/2)\, dx$$

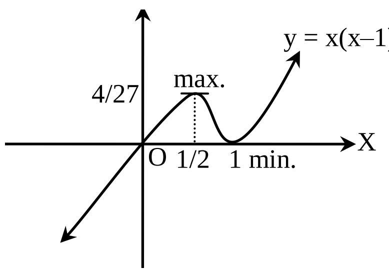
$$= \left(\log|\sec x|\right)_0^{\pi/4} - \left(x^2 + x - \pi/2\right)_0^{\pi/4}$$

$$= \log \sqrt{2} - \left(\frac{\pi}{4} - \frac{\pi^2}{16}\right)$$

$$= \frac{\pi^2}{16} - \frac{\pi}{4} - \frac{1}{2}\log 2$$

Q.11 Find all maxima and minima of the function $y = x(x-1)^2$, $0 \le x \le 2$. Also determine the area bounded by the curve $y = x(x-1)^2$, the x-axis and the line $x = 2$. **[IIT-1989]**

Sol. $y = x(x-1)^2$



$\Rightarrow \frac{dy}{dx} = x.\ 2(x-1) + (x-1)^2$

$= (x-1).\ \{2x + x - 1\}$

$= (x-1)(3x-1)$

+ – +

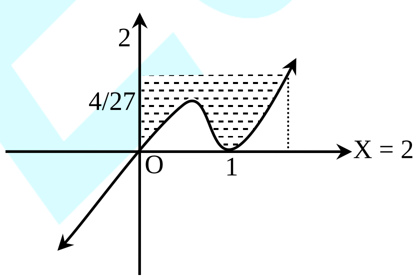
1/3 1

$\therefore$ maximum at $x = 1/3 \Rightarrow y_{max} = \frac{1}{3}\left(-\frac{2}{3}\right)^2 = \frac{4}{27}$

minimum at $x = 1 \Rightarrow y_{min} = 0$

Now, to find the area bounded by the curve $y = x(x-1)^2$,

The y-axis and line $x = 2$.



$\Rightarrow$ Area $= \int_0^2 x(x-1)^2.dx$

$$= \int_0^2 (x^3 - 2x^2 + x).dx = \left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right)_0^2$$

$$= \left(4 - \frac{16}{3} + 2\right) = 6 - \frac{16}{3} = \frac{2}{3} \text{ sq. units.}$$

Q.12 Compute the area of the region bounded by the curves $y = ex\,\lambda n\,x$ and $y = \frac{\lambda n x}{ex}$, where $\lambda n\, e = 1$.

[IIT 1990]

Sol. $ex\,\lambda n\,x = \frac{\lambda n\,x}{ex}$

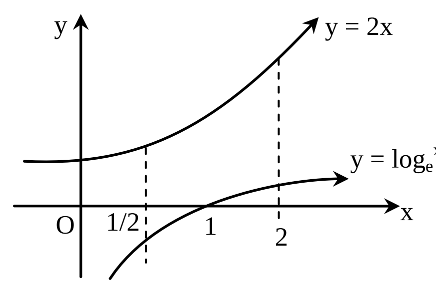
$\frac{\lambda n\,x}{ex}(e^2x^2 - 1) = 0$

$x = 1,\ x = \frac{1}{e}$

$$A = \int_{1/e}^{1} \left(ex\,\lambda nx - \frac{\lambda nx}{ex}\right) dx$$

Q.13 Sketch the curves and identify the region bounded by $x = 1/2$, $x = 2$, $y = \lambda nx$ and $y = 2^x$. Find the area of this region. **[IIT- 1991]**

Sol. The required area is the shaded portion in following figure.



In the region $\frac{1}{2} \le x \le 2$ the curve $y = 2^x$ lies above as compared to $y = \log_e x$

Hence, the required area

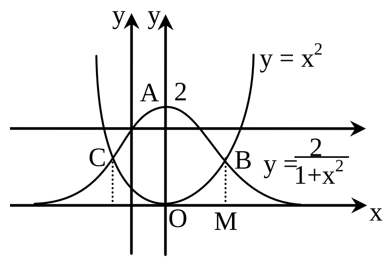
$$= \int_{1/2}^{2} (2^x - \log x)\, dx$$

$$= \left(\frac{2^x}{\log 2} - (x \log x - x)\right)_{1/2}^{2}$$

$$= \frac{4-\sqrt{2}}{\log 2} - \frac{5}{2}\log 2 + \frac{3}{2}$$

Q.14 Sketch the region bounded by the curves $y = x^2$ and $y = 2/(1 + x^2)$, Find its area. **[IIT-1992]**

Sol. The curve $y = x^2$ is a parabola. It is symmetric about x-axis and has its vertex at (0, 0) and the curve $y = \frac{2}{1+x^2}$ is a bell shaped curve, x-axis is its asymptote and it is symmetric about y-axis and its vertex is (0, 2).



To obtain the area, we need point of intersection of

$y = x^2$ ... (1)

and $y = \frac{2}{1+x^2}$ .... (2)

$\Rightarrow$ $y = \frac{2}{1+y}$

$\Rightarrow$ $y + y^2 = 2$

$\Rightarrow$ $y^2 + y - 2 = 0$

$\Rightarrow$ $y^2 + 2y - y - 2 = 0$

$\Rightarrow$ $y(y + 2) - 1(y + 2) = 0$

$\Rightarrow$ $(y - 1)(y + 2) = 0$

$y = -2, 1$ but $y \geq 0$ so $y = 1 \Rightarrow x = \pm 1$

Therefore coordinates of C are (–1, 1) and coordinates of B are (1, 1)

The area is OBACO = 2 area curve OB/AO

$$= 2\left[\int_0^1 \frac{2}{1+x^2}dx - \int_0^1 x^2 dx\right]$$

$$= 2[2\tan^{-1}x]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{2\pi}{4} - \frac{1}{3}\right] = \pi - \frac{2}{3}$$

Q.15 In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? **[IIT- 1994]**

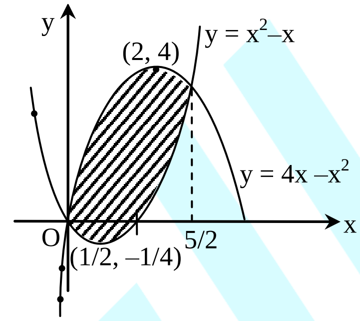
Sol. $y = 4x - x^2$ (given)

$= -(x^2 - 4x + 4 - 4)$

$= -(x^2 - 4x + 4) + 4$

$y = -(x - 2)^2 + 4$

$y - 4 = -(x - 2)^2$



Therefore, it is a vertically downward parabola with vertex at (2, 4) and its axis is x = 2.

and $y = x^2 - x$ (given)

$\Rightarrow$ $y = x^2 - x + \frac{1}{4} - \frac{1}{4}$

$\Rightarrow$ $y = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$

$\Rightarrow$ $y + \frac{1}{4} = (x - 1/2)^2$

This is a parabola having its vertex at $\left(\frac{1}{2}, -\frac{1}{4}\right)$ its axis at $x = \frac{1}{2}$ and opening upwards.

To obtain the x-coordinate of the points of intersection we solve $y = 4x - x^2$ and $y = x^2 - x$

$\Rightarrow$ $4x - x^2 = x^2 - x$

$\Rightarrow 2x^2 = 5x \Rightarrow 2x^2 - 5x = 0 \Rightarrow x(2 - 5x) = 0$

$\Rightarrow x = 0, \frac{5}{2}$

Also $y = x^2 - x$, meets x- axis at (0, 0) and (1, 0)

Now area,

$$A_1 = \int_0^{5/2} [(4x - x^2)] - [(x^2 - x)]\,dx$$

$$= \int_0^{5/2} (5x - 2x^2)\,dx$$

$$= \left[\left(\frac{5}{2}x^2 - \frac{2}{3}x^3\right)\right]_0^{5/2}$$

$$= \frac{5}{2}\left(\frac{5}{2}\right)^2 - \frac{2}{3}\cdot\left(\frac{5}{2}\right)^3$$

$= \frac{5}{2} \cdot \frac{25}{4} - \frac{2}{3} \cdot \frac{125}{8}$

$= \frac{125}{8}\left(1 - \frac{2}{3}\right) = \frac{125}{24}$

This area is considering above and below x-axis both. Now for area below x-axis separately. We consider

$A_2 = -\int_0^1 (x^2 - x)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Therefore net area above the x- axis

$= A_1 - A_2 = \frac{125-4}{24} = \frac{121}{24}$

Hence, ratio of area above the x- axis and area below x- axis

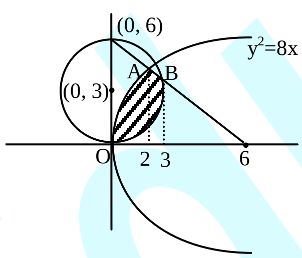
$= \frac{121}{24} : \frac{1}{6} = 121 : 4$

Q.16 Find the area given by $x + y \le 6$, $x^2 + y^2 \le 6y$ and $y^2 \le 8x$. **[REE-1995]**

Sol. $x + y \le 6$, $x^2 + (y-3)^2 \le 9$, $y^2 \le 8x$

We take

$x + y = 6$, $x^2 + (y-3)^2 = 9$, $y^2 = 8x$



Solving $y^2 = 8x$ and $x + y = 6$, we get A(2, 4) and solving $x^2 + (y-3)^2 = 9$ and $x + y = 6$, we get B (3, 3)

required area

$= \int_0^2 \left(2\sqrt{2x} - (3 - \sqrt{9 - x^2}\right)dx +$

$\int_2^3 \left[(6-x) - (3 - \sqrt{9 - x^2})\right]dx$

$= 2\sqrt{2}\int_0^2 \sqrt{x}\,dx + \int_2^3 (6-x)\,dx - \int_0^3 (3 - \sqrt{9-x^2})\,dx$

$= 2\sqrt{2} \cdot \frac{2}{3}\left[x^{3/2}\right]_0^2 + \left[6x - \frac{x^2}{2}\right]_2^3$

$- \left[3x - \frac{x}{2}\sqrt{9-x^2} - \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$

Solving, we get

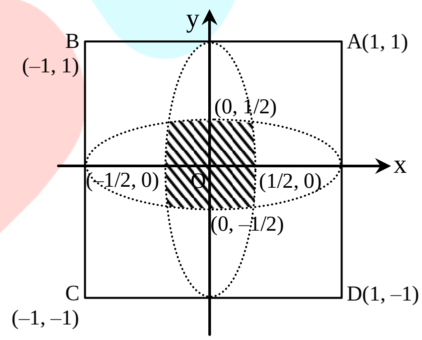
Area $= \frac{1}{12}(27\pi - 2)$

Q.17 Consider a square with vertices at (1, 1), (–1, 1), (–1, –1) and (1, –1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area. **[IIT- 1995]**

Sol. The equations of the sides of the square are as follows:

AB : y = 1, BC : x = –1, CD : y = –1, DA : x = 1

Let the region be S and (x, y) is any point inside it. Then according to given conditions,



$\sqrt{x^2 + y^2} < |1-x|, |1+x|, |1-y|, |1+y|$

$\Rightarrow x^2 + y^2 < (1-x)^2, (1+x)^2, (1-y)^2, (1+y)^2$

$\Rightarrow x^2 + y^2 < x^2 - 2x + 1, x^2 + 2x + 1, y^2 - 2y + 1, y^2 + 2y + 1$

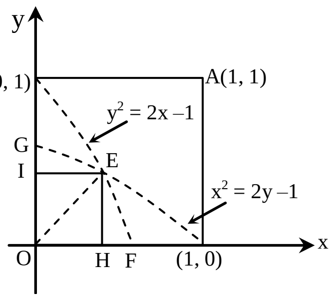
$\Rightarrow y^2 < 1 - 2x, y^2 < 1 + 2x, x^2 < 1 - 2y$ and $x^2 < 2y + 1$

Now in $y^2 = 1 - 2x$ and $y^2 = 1 + 2x$, the first equation represents a parabola with vertex at (1/2, 0) and second equation represents a parabola with vertex (–1/2, 0).

And in $x^2 = 1 - 2y$ and $x^2 = 1 + 2y$, the first equation represents a parabola with vertex at (0, –1/2).

Therefore, the region S is the region lying inside the four parabolas

$y^2 = 1 - 2x, y^2 = 1 + 2x, x^2 = 1 + 2y, x^2 = 1 - 2y$



where S is the shaded region.

Now, S is symmetrical in all four quadrants, therefore, S = 4 × area lying in the first quadrant.

Now, $y^2 = 1 - 2x$ and $x^2 = 1 - 2y$ intersect on the line y = x. The point of intersection is $E(\sqrt{2} - 1, \sqrt{2} - 1)$

Area of the region OEFO

= area of ΔOEH + area of HEFH

$= \frac{1}{2}(\sqrt{2}-1)^2 + \int_{\sqrt{2}-1}^{1/2} \sqrt{1-2x}\,dx$

$= \frac{1}{2}(\sqrt{2}-1)^2 + \left[(1-2x)^{3/2}\frac{2}{3}\cdot\frac{1}{2}(-1)\right]_{\sqrt{2}-1}^{1/2}$

$= \frac{1}{2}(2 + 1 - 2\sqrt{2}) + \frac{1}{3}(1 + 2 - 2\sqrt{2})^{3/2}$

$= \frac{1}{2}(3 - 2\sqrt{2}) + \frac{1}{3}(3 - 2\sqrt{2})^{3/2}$

$= \frac{1}{2}(3 - 2\sqrt{2}) + \frac{1}{3}[(\sqrt{2}-1)^2]^{3/2}$

$= \frac{1}{2}(3 - 2\sqrt{2}) + \frac{1}{3}(\sqrt{2}-1)^3$

$= \frac{1}{2}(3 - \sqrt{2}) + \frac{1}{3}[2\sqrt{2} - 1 - 3\sqrt{2}(\sqrt{2}-1)]$

$= \frac{1}{2}(3 - 2\sqrt{2}) + \frac{1}{3}[5\sqrt{2} - 7]$

$= \frac{1}{6}[9 - 6\sqrt{2} + 10\sqrt{2} - 14] = \frac{1}{6}[4\sqrt{2} - 5]$

Similarly, area OEGO $= \frac{1}{6}(4\sqrt{2} - 5)$

Therefore, area of S lying in first quadrant

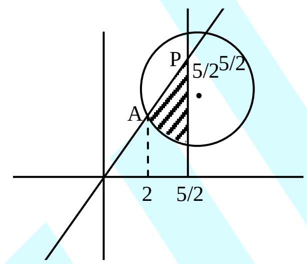
$= \frac{2}{6}(4\sqrt{2} - 5) = \frac{1}{3}(4\sqrt{2} - 5)$

Hence, S $= \frac{4}{3}(4\sqrt{2} - 5) = \frac{1}{3}(16\sqrt{2} - 20)$

Q.18 Find the area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \le 0$, $y \le x$ and $x \le \frac{5}{2}$. **[REE- 1996]**

Sol. $x^2 + y^2 - 6x - 4y + 12 \le 0$, $y \le x$, and $x \le \frac{5}{2}$

$\Rightarrow (x-3)^2 + (y-2)^2 = 1$, $y = x$, $x = \frac{5}{2}$



required area

$= \int_2^{5/2}\left[x - \left(2 - \sqrt{1-(x-3)^2}\right)\right]dx$

$= \left[\frac{x^2}{2} - 2x + \frac{x-3}{2}\sqrt{1-(x-3)^2} + \frac{1}{2}\sin^{-1}(x-3)\right]_2^{5/2}$

$= \frac{25}{8} - 5 - \frac{\sqrt{3}}{8} - \frac{\pi}{12} - 2 + 4 - \frac{\pi}{4}$

$= \frac{1}{8} - \frac{\sqrt{3}}{8} + \frac{\pi}{6}$

Q.19 Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \pi/4$.

Prove that for $n \ge 2$, $A_n + A_{n-2} = \frac{1}{n-1}$ and deduce $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$. **[IIT- 1996]**

Sol. We have, $A_n = \int_0^{\pi/4}(\tan x)^n\,dx$

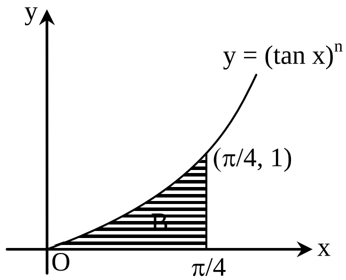
Since, $0 < \tan x < 1$, when $0 < x < \pi/4$

We have

$0 < (\tan x)^{n+1} < (\tan x)^n$ for each $n \in N$

$\Rightarrow \int_0^{\pi/4}(\tan x)^{n+1}dx < \int_0^{\pi/4}(\tan x)^n\,dx$

$\Rightarrow A_{n+1} < A_n$



Now, for $n > 2$

$$A_n + A_{n+2} = \int_0^{\pi/4} [(\tan x)^n + (\tan x)^{n+2}]dx$$

$$= \int_0^{\pi/4} (\tan x)^n + (1 + \tan^2 x)\,dx$$

$$= \int_0^{\pi/4} (\tan x)^n \sec^2 x\,dx$$

$$= \left[\frac{1}{(n+1)}(\tan x)^{n+1}\right]_0^{\pi/4}$$

$$\left[\Theta \int f(x)^n f'(x)\,dx = \frac{f(x)^{n+1}}{n+1}\right]$$

$$= \frac{1}{(n+1)}(1-0) = \frac{1}{n+1}$$

Since $A_{n+2} < A_{n+1} < A_n$, we get

$$A_n + A_{n+2} < 2A_n$$

$$\Rightarrow \frac{1}{n+1} < 2A_n$$

$$\Rightarrow \frac{1}{2n+2} < A_n \qquad \text{... (1)}$$

Also for $n > 2$

$$A_n + A_n < A_n + A_{n-2} = \frac{1}{n-1}$$

$$\Rightarrow 2A_n < \frac{1}{n-1}$$

$$\Rightarrow A_n < \frac{1}{2n-2} \qquad \text{... (2)}$$

From (1) and (2) $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$.

Q.20 Let O(0, 0), A(2, 0) and B$\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy d(P, OA) ≤ min {d(P, OB), d(P, AB)}, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area. **[IIT 1997]**

Sol. Point P lies inside ΔOAB & closest to OA

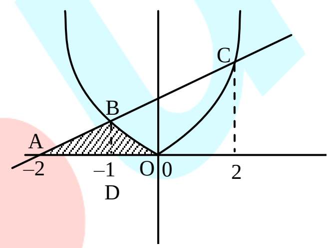
$$A = \frac{1}{2} \times 2 \times \tan 15° = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$A = \frac{4 - 2\sqrt{3}}{2}$$

$$A = 2 - \sqrt{3}$$

Q.21 Indicate the region bounded by the curves $x^2 = y$ and $y = x + 2$ and obtain the area enclosed by them. **[REE- 1997]**

Sol. $x^2 = y$, $y = x + 2$, x- axis



required area = area of ΔABD + Area of region BOD

$$= \frac{1}{2} \times |x| + \left|\int_{-1}^{0} x^2\,dx\right|$$

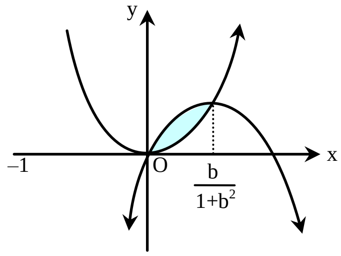
$$= \frac{1}{2} + \left|\left[\frac{x^3}{3}\right]_{-1}^{0}\right| = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Q.22 Find all possible values of $b > 0$, so that the area of the bounded region enclosed between the parabola $y = x - bx^2$ and $y = x^2/b$ is maximum. **[IIT- 1997]**

Sol. Eliminating y from $y = \frac{x^2}{b}$ and $y = x - bx^2$, we get

$$x^2 = bx - b^2x^2$$

$$\Rightarrow \quad x = 0, \frac{b}{1+b^2}$$



Thus, the area enclosed between the parabolas,

$$A = \int_0^{b/1+b^2} \left(x - bx^2 - \frac{x^2}{b} \right) dx,$$

$$= \left\{ \frac{x^2}{2} - \frac{x^3}{3} \cdot \frac{1+b^2}{b} \right\}_0^{b/1+b^2}$$

$$= \frac{1}{6} \cdot \frac{b^2}{(1+b^2)^2}$$

For maximum, value of A, $\frac{dA}{db} = 0$

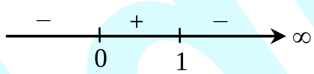
But $\frac{dA}{db} = \frac{1}{6} \cdot \frac{(1+b^2)^2.2b - 2b^2.(1+b^2).2b}{(1+b^2)^4}$

$$= \frac{1}{3} \cdot \frac{b(1-b^2)}{(1+b^2)^3}$$

Hence, $\frac{dA}{db} = 0$ gives b= –1, 0, 1 since b > 0

$\therefore$ we consider only b = 1

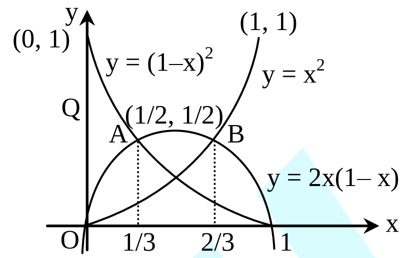
Sign scheme for $\frac{dA}{db}$ around b = 1 is as below



from scheme it is clear A is maximum at b = 1.

Q.23 Let f(x) = maximum $\{x^2, (1 - x)^2, 2x(1 - x)\}$ where $0 \le x \le 1$. Determine the area of the region bounded by the curves y = f(x), x-axis, x = 0 and x = 1. **[IIT- 1997]**

Sol. We can draw the graph of $y = x^2$, $y = (1 - x^2)$ and $y = 2x(1 - x)$ in following fig. Now, to get the point of intersection of $y = x^2$ and $y = 2x(1 - x)$. We solve both the equations.



We get $x^2 = 2x(1 - x)$

$\Rightarrow x^2 = 2x - 2x^2$

$\Rightarrow 3x^2 = 2x$

$\Rightarrow 3x^2 - 2x = 0$

$\Rightarrow x(3x - 2) = 0$

$\Rightarrow x = 0, 2/3$

Similarly, we can find the coordinate of the points of intersection of

$y = (1 - x^2)$ and $y = 2x(1 - x)$ are x = 1/3 and x = 1.

From the figure it is clear that

$$f(x) = \begin{cases} (1-x)^2 , & 0 \le x \le 1/3 \\ 2x(1-x) , & 1/3 \le x \le 2/3 \\ x^2 , & 2/3 \le x \le 1 \end{cases}$$

The required area A is given by

$$A = \int_0^1 f(x)\,dx$$

$$= \int_0^{1/3} (1-x)^2\,dx + \int_{1/3}^{2/3} 2x(1-x)\,dx + \int_{2/3}^{1} x^2\,dx$$

$$= \left[-\frac{1}{3}(1-x)^3 \right]_0^{1/3} + \left[x^2 - \frac{2x^3}{3} \right]_{1/3}^{2/3} + \left[\frac{1}{3}x^3 \right]_{2/3}^{1}$$

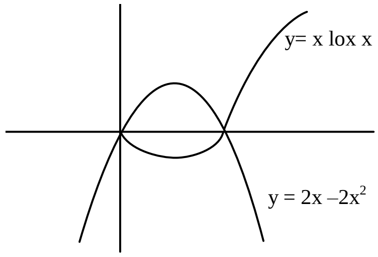
$$= -\frac{1}{3}\left(\frac{2}{3}\right)^3 + \frac{1}{3}\left(\frac{2}{3}\right)^2 - \frac{2}{3}\left(\frac{2}{3}\right)^3$$

$$-\left(\frac{1}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^2 + \frac{1}{3}(1) - \frac{1}{3}\left(\frac{2}{3}\right)^3 = \frac{17}{27} \text{ sq. units}$$

Q.24 Indicate the region bounded by the curves $y = x \log x$ and $y = 2x - 2x^2$ and obtain the area enclosed by them. **[REE- 1998]**

Sol. $y = x \log x$, $y = 2x - 2x^2$

$\Rightarrow y = x \log x$, $\left(x - \frac{1}{2}\right)^2 = -\frac{1}{2}\left(y - \frac{1}{2}\right)$



required area

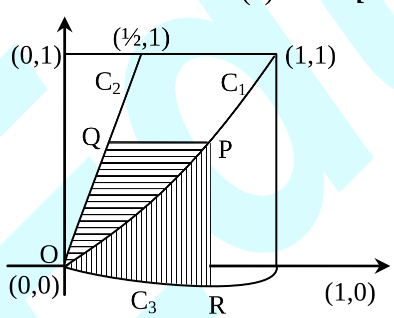
$$= \int_0^1 (2x - 2x^2)\,dx + \left|\int_0^1 x \log x \, dx\right|$$

$$= \left[x^2 - \frac{2}{3}x^3\right]_0^1 + \left|\left[\frac{x^2}{2}\log x\right]_0^1 - \int_0^1 \frac{x}{2}\,dx\right|$$

$$= 1 - \frac{2}{3} + \left|\left[-\frac{x^2}{4}\right]_0^1\right|$$

$$= 1 - \frac{2}{3} + \frac{1}{4} = \frac{7}{12}$$

Q.25 Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes meet C_2 and C_3 at Q and R respectively (see in figure). If for every position of P(on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function f(x). **[IIT- 1998]**



Sol. Refer to the fig. in the question. Let the coordinates of P be (x, x^2), where $0 \le x \le 1$.
For the area (OPRO), upper boundary $y = x^2$
lower boundary : y = f(x)
lower limit of x : 0
upper limit of x : x

$$\therefore \text{ area (OPRO)} = \int_0^x t^2\,dt - \int_0^x f(t)\,dt$$

$$= \left[\frac{t^3}{3}\right]_0^x - \int_0^x f(t)\,dt$$

$$= \frac{x^3}{3} - \int_0^x f(t)\,dt - \int_0^{x^2} \frac{t}{2}\,dt$$

For the area (OPQO) the upper curve: $x = \sqrt{y}$
the lower curve : $x = y/2$
lower limit of y : 0 and upper limit of y : x^2

$$\therefore \text{ area (OPQO)} = \int_0^{\pi^2} \sqrt{t}\,dt - \int_0^{\pi^2} \frac{t}{2}\,dt$$

$$= \frac{2}{3}\left[t^{3/2}\right]_0^{x^2} - \frac{1}{4}\left[t^2\right]_0^{x^2}$$

$$= \frac{2}{3}x^3 - \frac{1}{4}x^4$$

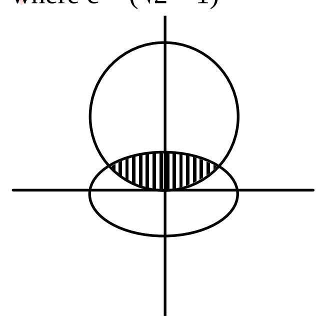
according to the given condition,

$$\frac{1}{3}x^3 - \int_0^x f(t)\,dt = \frac{2}{3}x^3 - \frac{x^4}{4}$$

Differentiating both sides w.r.t. x, get
$x^2 - f(x).\,1 = 2x^2 - x^3$
$\Rightarrow f(x) = x^3 - x^2,\ 0 \le x \le 1$

Q.26 Find the area of the region lying inside $x^2 + (y - 1)^2 = 1$ and outside $c^2x^2 + y^2 = c^2$ where $c = (\sqrt{2} - 1)$ **[REE- 1999]**

Sol.



$$(y-1)^2 - \frac{y^2}{c^2} = 0$$

$$y - 1 = \frac{y}{c} \quad \& \quad y - 1 = -\frac{y}{c}$$

$$y\frac{(c+1)}{c} = 1$$

$$y = \left(\frac{c}{c+1}\right) = \frac{\sqrt{2}-1}{\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$$

$$x = \pm\frac{1}{\sqrt{2}}$$

$$A = c\int_{-\frac{1}{\sqrt2}}^{1/\sqrt2}\sqrt{1-x^2}\,dx - \int_{-\frac{1}{\sqrt2}}^{1/\sqrt2} 1.dx - \int_{-\frac{1}{\sqrt2}}^{1/\sqrt2}\sqrt{1-x^2}\,dx$$

$$A = \frac{(c-1)}{2}\left[x\sqrt{1-x^2}+\sin^{-1}x\right]_{-\frac{1}{\sqrt2}}^{\frac{1}{\sqrt2}} - \sqrt2$$

$$= \frac{(c-1)}{2}\left\{\frac12+\frac{\pi}{4}+\frac12+\frac{\pi}{4}-\sqrt2\right\}$$

$$= \frac{(c-1)}{2}\left(1+\frac{\pi}{2}\right)-\sqrt2$$

$$= \frac{c-1}{2}+(c-1)\frac{\pi}{4}-\sqrt2$$

$$= \frac{\sqrt2-2}{2}+\left(\sqrt2-2\right)\frac{\pi}{4}-\sqrt2$$

$$= \frac{1}{\sqrt2}-1-\sqrt2+\frac{\pi}{2\sqrt2}-\frac{\pi}{2}$$

Q.27 Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x, & |x|\le 1 \\ x^2+ax+b, & |x|>1 \end{cases}$. Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$ **[IIT- 1999]**

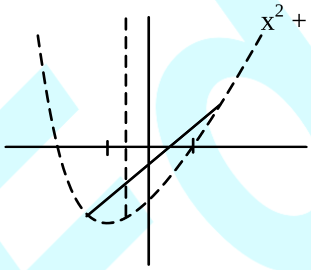
Sol. $2 = 1 + a + b$

$a + b = 1$ …..(1)

$-2 = 1 - a + b$

$a - b = 3$ …..(2)

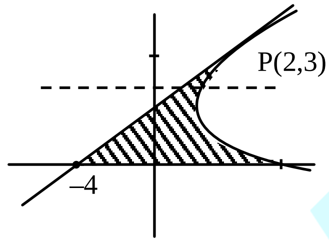
$a = 2$ & $b = -1$



Q.28 Find the area enclosed by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at (2, 3) and x-axis. **[REE- 2000]**

Sol. Tangent at P(2, 3)

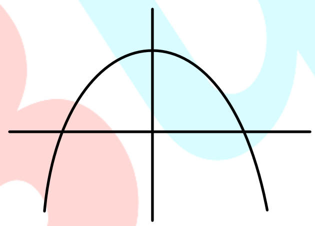
$2y = x + 4$



$$A = \int_0^3 \left\{(y-2)^2+1\right\}-(2y-4)\,dy$$

Q.29 Consider the collection of all curve of the form $y = a - bx^2$ that pass through the point (2, 1), where a and b are positive constants. Determine the value of a and b that will minimise the area of the region bounded by $y = a - bx^2$ and x-axis. Also find the minimum area.

Sol.



$1 = a - 4b$

$$A = \int_{-\sqrt{\frac{a}{b}}}^{\sqrt{\frac{a}{b}}}(a-bx^2)\,dx = \int_{-\sqrt{\frac{4b+1}{b}}}^{\sqrt{\frac{4b+1}{b}}}(1+4b-bx^2)\,dx$$

Diff. w.r. to b

$$A = \int_{-\sqrt{\frac{a}{b}}}^{\sqrt{\frac{a}{b}}}(4-x^2)\,dx = 4.2\sqrt{\frac{a}{b}}-\frac13\,2\left(\frac{a}{b}\right)^{3/2}$$

$$\sqrt{\frac{a}{b}}\left[8-\frac23\left(\frac{a}{b}\right)\right]=0$$

$$\frac{a}{b}=12 \Rightarrow \quad a = 12b$$

$$b = \frac18,\ a = \frac32$$

$$A = \int_{-\sqrt{\frac{a}{b}}}^{\sqrt{\frac{a}{b}}}\frac32-\frac18x^2\,dx = \frac32\ .\ 2\sqrt{\frac{a}{b}}-\frac{2}{24}\left(\frac{a}{b}\right)^{3/2}$$

$$= 3\sqrt{12}-\frac{1}{\sqrt{12}}(12)^{3/2}$$

$= 2\sqrt{12} = 4\sqrt{3}$

Q.30 For what value of 'a' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight line $y = 0$, $x = 0$ and $x = 1$ the least ?

Sol. $A = \int_0^1 y\,dx = \frac{a^2}{3} + \frac{a}{2} + 1$

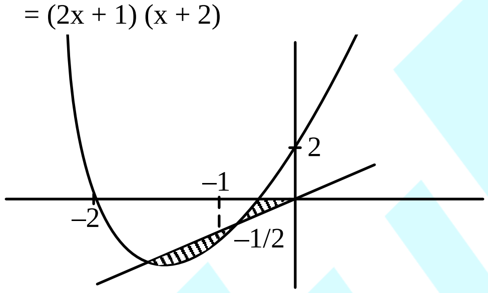
$\frac{dA}{da} = \frac{2a}{3} + \frac{1}{2} = 0$

$a = -\frac{3}{4}$

Q.31 The tangent drawn from the origin to the curve, $y = 2x^2 + 5x + 2$ meets the curve at a point whose y-coordinate is negative. Find the area of the figure bounded by the tangent between the point of contact and origin, the x-axis and the parabola.

Sol. $y = 2x^2 + 5x + 2$

$= (2x + 1)(x + 2)$



$P(x_1, y_1)$

tangent at P

$\frac{y + y_1}{2} = 2x\,x_1 + \frac{5}{2}(x + x_1) + 2$

O lies on it

$y_1 = 5x_1 + 4$

$5x_1 + 4 = 2x_1^2 + 5x_1 + 2$

$x_1 = \pm 1$

$x_1 = -1,\ y_1 = -1$

$(-1, -1)$

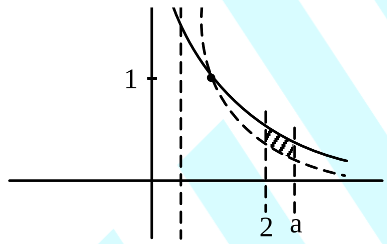
$y = x$

$A = \frac{1}{2} . 1 . 1 + \int_{-1}^{-1/2} (2x^2 + 5x + 2)\,dx$

Q.32 For what value of 'a' is the area of the figure bounded by the lines, $y = \frac{1}{x}$, $y = \frac{1}{2x-1}$, $x = 2$ and $x = a$ equal to $\lambda n \frac{4}{\sqrt{5}}$?

Sol. $\frac{1}{x} = \frac{1}{2x-1}$

$x = 1$



Case 1 $a > 2$

$A = \int_2^a \left(\frac{1}{x} - \frac{1}{2x-1}\right)dx = \lambda n \frac{4}{\sqrt{5}}$

$\left[\lambda n \frac{x}{\sqrt{2x-1}}\right]_2^a = \lambda n \frac{4}{\sqrt{5}}$

$\lambda n \frac{a}{\sqrt{2a-1}} - \lambda n \frac{2}{\sqrt{3}} = \lambda n \frac{4}{\sqrt{5}}$

$\frac{\sqrt{3}a}{2\sqrt{2a-1}} - \frac{4}{\sqrt{5}} \Rightarrow 15a^2 = 64(2a - 1)$

$\Rightarrow 15a^2 - 128a + 64 = 0 \Rightarrow a = 8$

Case 2. $1 < a < 2$

$\int_a^2 \left(\frac{1}{x} - \frac{1}{2x-1}\right)dx = \lambda n \frac{4}{\sqrt{5}}$

$\frac{2\sqrt{2a-1}}{\sqrt{3}a} = \frac{4}{\sqrt{5}} \Rightarrow 10a - 5 = 12a^2$

$\Rightarrow 12a^2 - 10a + 5 = 0$

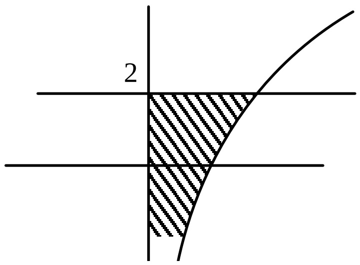
$D < 0$

Case 3 $\frac{1}{2} < a < 1$

$\int_a^1 \left(\frac{1}{2x-1} - \frac{1}{x}\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{2x-1}\right)dx = \lambda n \frac{4}{\sqrt{5}}$

Q.33 Let f be a real valued function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $\lim_{x \to 0} \frac{f(1+x)}{x} = 2$. The area bounded by curve $y = f(x)$, y-axis and the line $y = 2$ is $\frac{\lambda e}{3}$, then $\lambda =$

Sol.



Let $f(x) = \lim x^n$

$$\lim_{x\to 0} \frac{n\,\lambda n\,(x+1)}{x} = 2$$

$n = 2$

$f(x) = \lambda n\, x^2 = 2\lambda n\, x$

$$A = \int_{-\infty}^{2} e^{y/2}\,dy$$

Put $\frac{y}{2} = t$ then $2\int_{-\infty}^{1} e^t\,dt = 2\left[e^t\right]_{-\infty}^{1}$

$$= 2\,.\,e = \frac{\lambda e}{3} \Rightarrow \lambda = 6$$

Q.34 Let $f : R \to R$ be a differentiable function such that $f(0) = 1$ and $f(xy + 1) = f(x)\, f(y) - f(y) - x + 2\ \forall\ x,y \in R$ then area bounded by $f(x)$ and $g(x) = x^2 + 1$ can be expressed as p/q where p and q are relatively prime find (p + q).

Sol. $f'(xy + 1)(y) = f'(x) - 1$

$= f(y)\, f'(x) - 1$

put $y = 0$

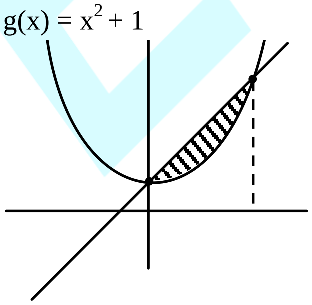
$f'(x) = 1$

$f(x) = x + c$

$c = 1$

$f(x) = x + 1$

$g(x) = x^2 + 1$



$$A = \int_0^1 x + 1 - x^2 - 1\,dx$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} = \frac{p}{q}$$

$p - q = 7$

ANSWER KEY

EXERCISE # 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	C	A	C	B	B	C	D	A	C	D	B	D	A	C	D	C	B
Q.No.	**21**	**22**	**23**	**24**																
Ans.	A	A	B	B																

25. True

26. False

27. True

28. e^2 sq. unit

29. 1/4 sq. unit

30. 1 sq. unit

31. 2 sq. unit

32. $(2 - \sqrt{2})$ sq. unit

EXERCISE # 2

PART-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	C	B	D	C	C	D	B	D	D	A	B	B

PART-B

Qus.	13	14	15
Ans.	B,C,D	B,C,D	A,B

PART-C

16. D

17. D

PART-D

18. $A \to Q$, $B \to Q$, $C \to R$, $D \to S$

19. $A \to Q$, $B \to R$, $C \to P, R$, $D \to S$

20. $A \to Q$, $B \to S$, $C \to P$, $D \to R$

EXERCISE # 3

2. $\sqrt{2}$	**3.** 44/27	**4.** $(\pi/2 - 4/\pi)$sq. units	**6.** 8 sq. units
7. $(\sqrt{2}-1)$ sq. units	**8.** 1/e	**9.** 5	**10.** $2 + \lambda n \left(\frac{4}{3\sqrt{3}}\right) - \frac{1}{e}$.
11. 11/3	**12.** $\left(\sqrt{3}+\frac{2\pi}{3}-\frac{1}{6}\right)$ units^2	**13.** $(2 - \pi/2)$ units^2	**14.** $4a^2$ units^2
15. A	**16.** A	**17.** B	**18.** B
19. C	**20.** A	**21.** B	**22.** C
23. D	**24.** B	**25.** C	**26.** C
27. B	**28.** C	**29.** D	

EXERCISE # 4

1. C	**2.** $\frac{(e+1)\pi(e^{n+1}-1)}{(\pi^2+1)(e-1)}$ sq. units.		**3.** B	**4.** $\frac{20-12\sqrt{2}}{3}$
5. C	**6.** A	**7.** A	**8.** 125/3 sq. unit	**9.** 1/3 sq. unit
10. B	**11.** B	**12.** A	**13.** D	**14.** B, C, D
15. C	**16.** A	**17.** B	**18.** B	**19.** C
20. A, B, D				

EXERCISE # 5

1. B

2. A

3. A

4. D

5. B

6. $2\sqrt{3}$ sq. unit

7. $\frac{5\pi}{4} - \frac{1}{2}$

8. π + (1/3) sq. units

9. 25. $\sin^{-1}$ (4/5) + 4 sq. units

10. $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\log 2$

11. $\frac{2}{3}$ sq. units

12. $(e^2-5)/4e$ units^2

13. $\frac{4-\sqrt{2}}{\log 2} - \frac{5}{2}\log 2 + \frac{3}{2}$

14. $\pi - \frac{2}{3}$

15. 121 : 4

16. 1/12 (27 π – 2)

17. 4/3 $(4\sqrt{2} - 5)$

18. $\frac{3(1-\sqrt{3})+4\pi}{24}$ unit^2

20. $(2-\sqrt{3})$ sq. unit

21. 9/2 sq. unit

22. b = 1

23. 17/27 sq. units

24. 7/12 sq. units

25. $f(x) = -x^2 + x^3$

26. $\left(\pi - \frac{\sqrt{2}}{4}\pi + \frac{1}{\sqrt{2}}\right)$ units^2

27. $\frac{257}{192}$ sq. units

28. 9 sq. units

29. b = 1/8, $A_{minimum} = 4\sqrt{3}$ sq. units

30. $a = -\frac{3}{4}$

31. $\frac{5}{24}$

32. a = 8 or $\frac{2}{5}(6-\sqrt{21})$

33. 6

34. 7