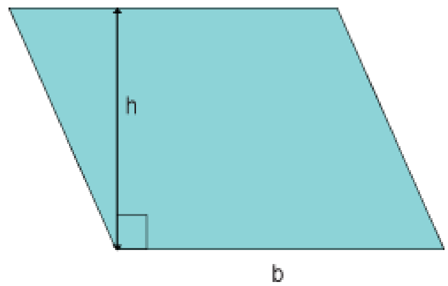
Areas of Parallelograms and Triangles

Parallelogram

A parallelogram is a four sides and two pairs of parallel lines. Opposite sides are equal in length and opposite angles are equal in measure.



Area of Parallelogram = Base x height

= a x h

Where a is the base, h is the height.

Points to Remember

- The diagonals of a parallelogram bisect each other.
- A diagonal of the parallelogram divides it into two congruent triangles.
- Opposite sides and angles of a parallelogram are equal.
- Parallelograms on the same base and between the same parallels are equal in area.
- Parallelograms on the same base that have equal areas are between the same parallels.
- Two triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- Area of Triangle = 1/2× Base × corresponding height.
- Area of a Rhombus= 1/2× product of diagonals.
- Area of a Trapezium= 1/2× (sum of the parallel sides) × (distance between them).
- A median of a triangle divides it into two triangles of equal area.
- The diagonal of a parallelogram divides it into four triangles of equal area.

Example 1: Find the area of a parallelogram with a base of 15 centimeters and a height of 6 centimeters.

Solution:

Area of parallelogram = base x height

= (15 cm) x (6 cm)

= 90 cm^2

Example 2: Find the area of a parallelogram with a base of 7 m and a height of 10 m.

Solution:

Area of parallelogram = base x height

= (7 m) x (10 m)

= 70 m^2

Example 3: The area of a parallelogram is 30 square centimeters and the base is 4 centimeters. Find the height.

Solution:

Area of parallelogram = base x height

30 cm^2 = (4 cm) x h

h = 30/4

= 7.5 cm^2

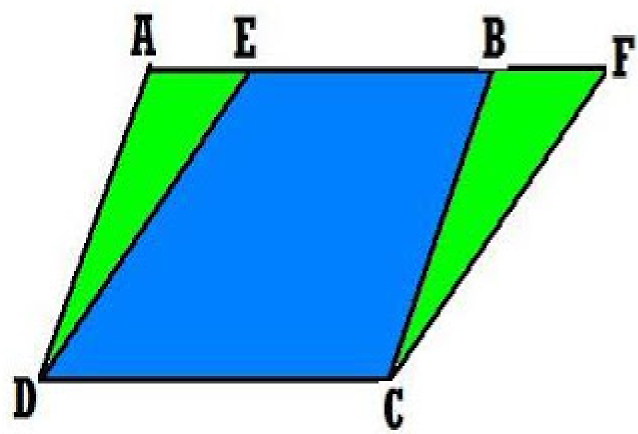
Theorem

Parallelograms on the same base and between the same parallels are equal in area.

Proof

Two parallelograms ABCD and EFCD, on the same base DC and between the same parallels.

We need to prove that ar (ABCD) = ar (EFCD).



In Δ ADE and Δ BCF,

∠ DAE = ∠ CBF (Corresponding angles from AD || BC and transversal AF) (1)

∠ AED = ∠ BFC (Corresponding angles from ED || FC and transversal AF) (2)

Therefore, ∠ ADE = ∠ BCF (Angle sum property of a triangle) (3)

Also, AD = BC (Opposite sides of the parallelogram ABCD) (4)

So, Δ ADE ≅ Δ BCF [By ASA rule, using (1), (3), and (4)]

Therefore, ar (ADE) = ar (BCF) (Congruent figures have equal areas) (5)

Now, adding ar (EDCB) both the sides

ar (ADE) + ar (EDCB) = ar (BCF)+ ar (EDCB)

ar (ABCD) = ar (EFCD)

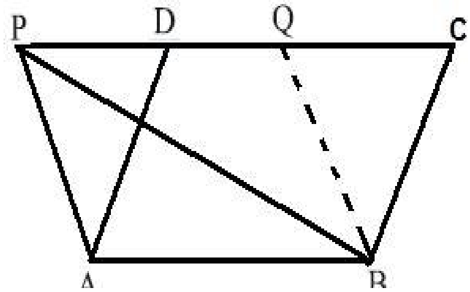
So, parallelograms ABCD and EFCD are equal in area.

Example:

If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

Solution:

Let Δ ABP and parallelogram ABCD be on the same base AB and between the same parallels AB and PC (see Fig).



To prove : ar (PAB) =1/2 ar (ABCD)

Construction:

Draw BQ || AP to obtain another parallelogram ABQP. So that parallelograms ABQP and ABCD will be on the same base AB and between the same parallels AB and PC.

Therefore, ar (ABQP) = ar (ABCD) ----------------------------------- (1)

(By Theorem that parallelograms on the same base and between the same parallels are equal in area)

But Δ PAB ≅ Δ BQP (As diagonal PB divides parallelogram ABQP into two congruent triangles.)

So, ar (PAB) = ar (BQP) - ------------(2)

Therefore, ar (PAB) =1/2ar (ABQP) [From (2)] --------------------- (3)

This gives ar (PAB) =1/2 ar (ABCD) [From (1) and (3)]

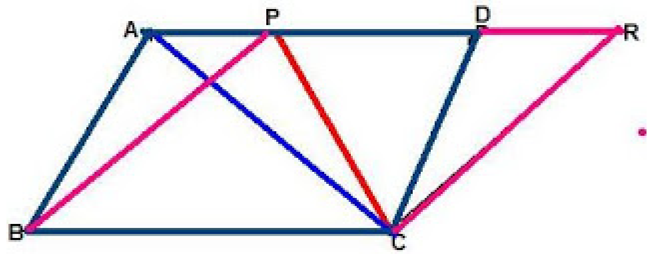
Theorem

Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Let two triangles ABC and PBC on the same base BC and between the same parallels BC and AP

Construction :

Draw CD || BA and CR || BP such that D and R lie on line AP(see Fig) to get two parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR.



Now, Parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR. Therefore, ar (ABCD) = ar (PBCR) ----------(1)

Now Δ ABC ≅ Δ CDA and Δ PBC ≅ Δ CRP (The diagonals of a parallelogram divides it into two congruent triangles)

So, ar(Δ ABC) = ar(Δ CDA) and ---------(ii)

ar (ABCD) = ar(Δ ABC) + ar(Δ CDA) --------------(iii)

ar (PBCR) = ar(Δ PBC) = ar(ΔCRP) ----------------(iv)

From (1),(ii),(iii) and (iv) , we have

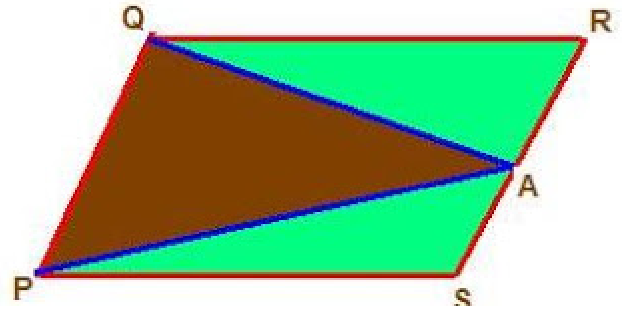
So, ar (ABC) =1/2 ar (ABCD)

and ar (PBC) =1/2 ar (PBCR)

Therefore, ar (ABC) = ar (PBC)

Example:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?



A divides the field into three parts. These parts are triangular in shape

ΔPSA, ΔPAQ and ΔQRA

So, Area of parallogram (PQRS)= Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA (1)

$\therefore$ Area (ΔPAQ) = 1/2 area (PQRS) . .. (2)

[a parallelogram and triangle are on the same base and between the same parallels, the area of triangle is half the area of the parallelogram]

From equations (1) and (2), we get

Area (ΔPSA) + area (ΔQRA) + 1/2 area (PQRS) = area (PQRS)

Area (ΔPSA) + area (ΔQRA)= area (PQRS) -1/2 area (PQRS)

Area (ΔPSA) + area (ΔQRA)= 1/2 area (PQRS)

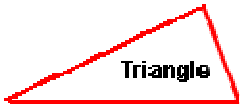
Area (ΔPSA) + area (ΔQRA)= Area (ΔPAQ)

Clearly, farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular part PSA and QRA and pulses in triangular parts PAQ.

Area of Triangle

The area of a polygon is the number of square units inside that polygon. Area is 2-dimensional like a carpet or an area rug.

A triangleis a three-sided polygon. We will look at several types of triangles in this lesson



To find the area of a triangle, multiply the base by the height, and then divide by 2. The division by 2 comes from the fact that a parallelogram can be divided into 2 triangles. For example, in the diagram to the left, the area of each triangle is equal to one-half the area of the parallelogram. Since the area of a parallelogram is , the area of a triangle must be one-half the area of a parallelogram. Thus, the formula for the area of a triangle is:

$$A = \frac{1}{2} \cdot b \cdot h$$

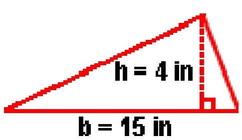
or

$$A = \frac{b \cdot h}{2}$$

Where is the base, is the height and • means multiply.

The base and height of a triangle must be perpendicular to each other. In each of the examples below, the base is a side of the triangle. However, depending on the triangle, the height may or may not be a side of the triangle. For example, in the right triangle in Example 2, the height is a side of the triangle since it is perpendicular to the base. In the triangles in Examples 1 and 3, the lateral sides are not perpendicular to the base, so a dotted line is drawn to represent the height.

Example 1: Find the area of an acute triangle with a base of 15 inches and a height of 4 inches.



Solution:

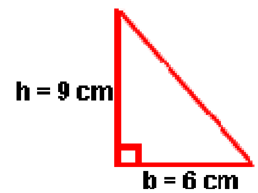
$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (15 \text{ in}) \cdot (4 \text{ in})$$

$$A = \frac{1}{2} \cdot (60\ in^2)$$

$$A = 30\ in^2$$

Example 2: Find the area of a right triangle with a base of 6 centimeters and a height of 9 centimeters.



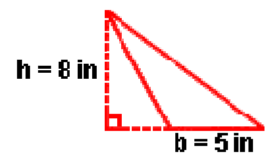
Solution: $A = \frac{1}{2} \cdot b \cdot h$

$$A = \frac{1}{2} \cdot (6\ cm) \cdot (9\ cm)$$

$$A = \frac{1}{2} \cdot (54\ cm^2)$$

$$A = 27\ cm^2$$

Example 3: Find the area of an obtuse triangle with a base of 5 inches and a height of 8 inches.



Solution: $A = \frac{1}{2} \cdot b \cdot h$

$$A = \frac{1}{2} \cdot (5\ in) \cdot (8\ in)$$

$$A = \frac{1}{2} \cdot (40\ in^2)$$

$$A = 20\ in^2$$