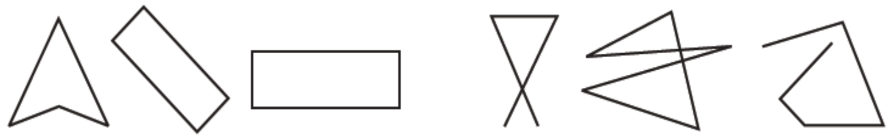
Understanding Quadrilaterals

Polygons

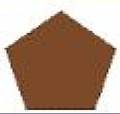
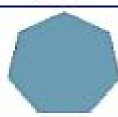
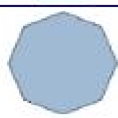
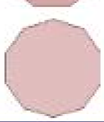
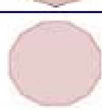
A simple closed curve made up of only line segments is called a **polygon**.



Curves that are polygons   Curves that are not polygons

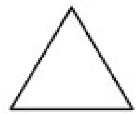
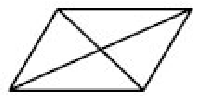
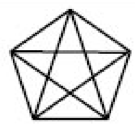
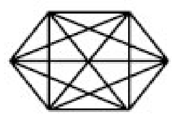
Classification of polygons

Polygons can be classified by the number of sides they have:

Polygon	Graphic	Sides	Angles	Vertices	Diagonals	Number Triangles
Triangle		3	3	3	0	1
Quadrilateral		4	4	4	2	2
Pentagon		5	5	5	5	3
Hexagon		6	6	6	9	4
Heptagon or Septagon		7	7	7	14	5
Octagon		8	8	8	20	6
Nonagon or Novagon		9	9	9	27	7
Decagon		10	10	10	35	8
Dodecagon		12	12	12	54	10
n-gon	---	n	n	n	$\frac{1}{2}n(n-3)$	$(n-2)$

Diagonals

A **diagonal** is a line segment connecting two non-consecutive vertices of a polygon.

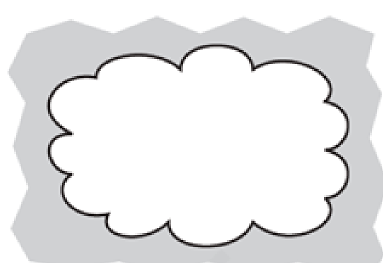
Number of Sides	Diagram	Number of Diagonals
3		0
4		2
5		5
6		9

Interior and exterior of a closed curve

Interior region: The portion of the plane which is inside the curve.

Interior

Exterior region: The portion of the plane which is outside the curve**.**

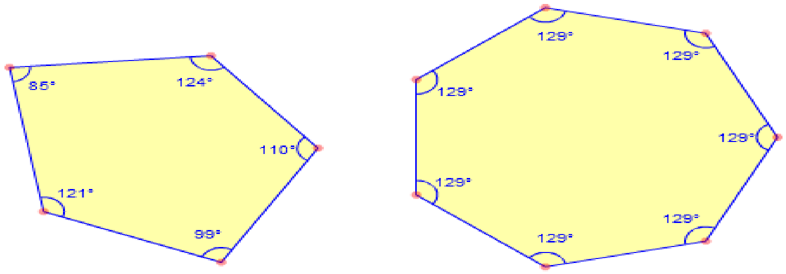


Exterior

Convex and concave polygons

A polygon which has all the interior angles less than 180° is called **convex polygon**.

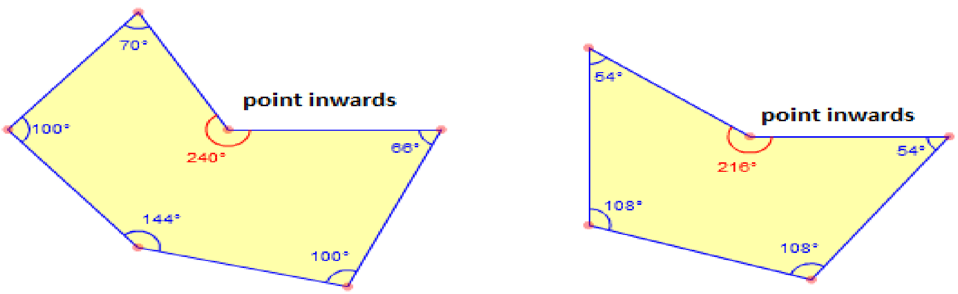
That means all the vertices of the convex polygon point outwards.



Convex polygons

A polygon which has one or more interior angles greater than 180° is called **concave polygon**.

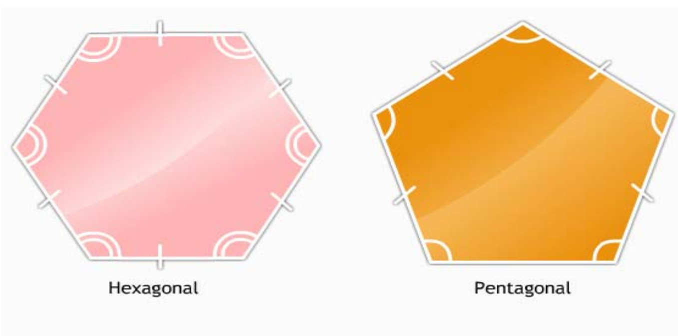
That means at least one vertex of the concave polygon point inwards.



Concave polygons

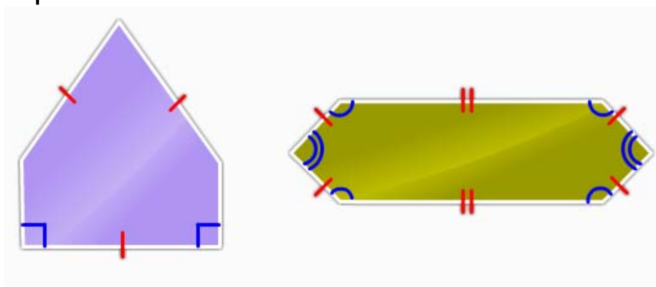
Regular and irregular polygons

Regular polygons: A polygon is said to be regular if all the angles are equal and all the sides are equal.



Regular polygon

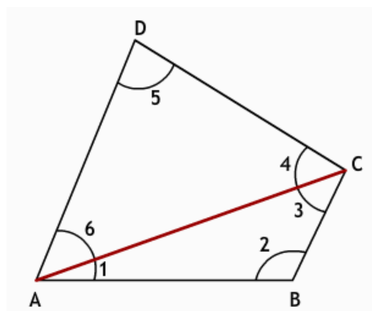
Irregular polygons: A polygon is said to be irregular if all the angles are not equal and all the sides are not equal.



Polygons that are not regular

Angle sum property

Theorem: The sum of four angles of a quadrilateral is 360°.



Proof: In $\triangle ABC$,

$\angle 1 + \angle 2 + \angle 3 = 180°$ …… (1) (From the angle sum property of the triangle)

In $\triangle ACD$,
$\angle 4 + \angle 5 + \angle 6 = 180°$ ……… (2) (From the angle sum property of the triangle)

From (1) and (2)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

It can be written as:

$(\angle 1 + \angle 6) + \angle 2 + (\angle 3 + \angle 4) + (\angle 5) = 360°$

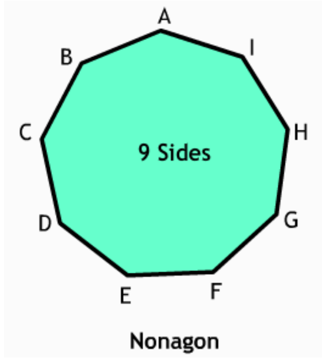
$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$ Hence Proved.

For a regular n-sided polygon,

Each interior angle = $\{[(2n - 4) \times 90]/n\}°$

Example:

Find the sum of the interior angles of a nonagon?



Solution:

For a nonagon, the number of sides, n = 9.

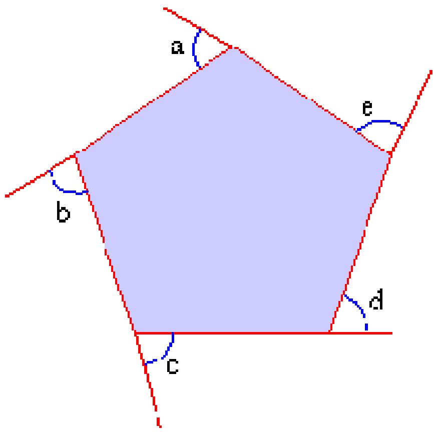
Sum of interior angles = $(2n - 4) \times 90°$

$= (2 \times 9 - 4) \times 90°$

$= 14 \times 90°$

$= 1260°$

Sum of the Measures of the Exterior Angles of a Polygon



Figure

Figure shows a pentagon. Its external angles are named from 'a' to e. The aim is to find the sum of these five angles.

It is known that the sum of internal angles of a Pentagon

$= (5 - 2) \times 180°$

$= 3 \times 180°$

$= 540°$

∴Each interior angle of the pentagon measures

540° / 5 = 108°

The interior and exterior angles form linear pairs and hence are supplementary.

∴Each exterior angle measures 180° - 108° = 72°

∴ Sum of five exterior angles = 5 × 72° = 360°

It can be proved that the sum of the exterior angles for any polygon is $360^{0.}$

Sum of interior angles of an n sided polygon = (n - 2) 180°.

∴ Measure of each internal angle = $\frac{(n-2)180^0}{n}$

∴ Each exterior angle = $180 - \frac{(n-2)180^0}{n}$

∴ Sum of n exterior angles = $n\left[180 - \frac{(n-2)180}{n}\right]^0$

$= 180n - 180n + 360^0$

$= 360^0$

Conclusion: The sum of interior angles of a polygon is dependent on the number of sides but the sum of the exterior angles is always 360°.

Example: One exterior angle of a regular polygon is 60°. How many sides does it have?

Solution:

Let the number of sides be n.

The polygon is regular so one exterior = 360°/n

360°/n = 60°

It gives n = 6.

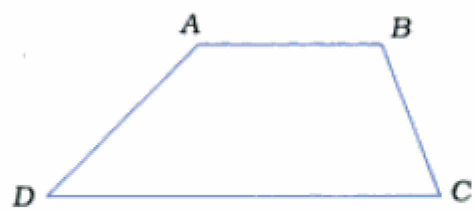
So the polygon has 6 sides and it is hexagon.

Kinds of Quadrilaterals

Based on the nature of the sides or angles of a quadrilateral, it gets special names.

Trapezium

A trapezium is a quadrilateral in which only one pair of opposite sides is parallel and the other two sides are non-parallel.

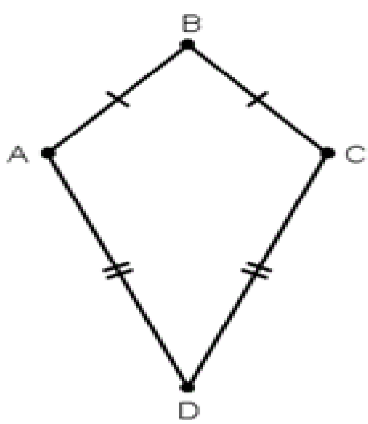


Trapezium

In the given figure, ABCD is a trapezium in which AB II CD, but AD is not parallel to BC.

Kite

A kite is a quadrilateral which has two pairs of equal adjacent sides, but unequal opposite sides.

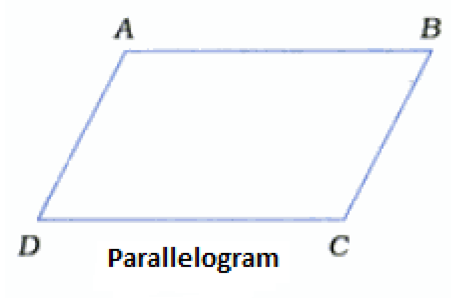


Kite

In the given figure, ABCD is a kite in which AB = BC, AD = CD, AD ≠ BC and CD ≠ AB.

Parallelogram

A parallelogram is a quadrilateral in which each pair of opposite sides is parallel.

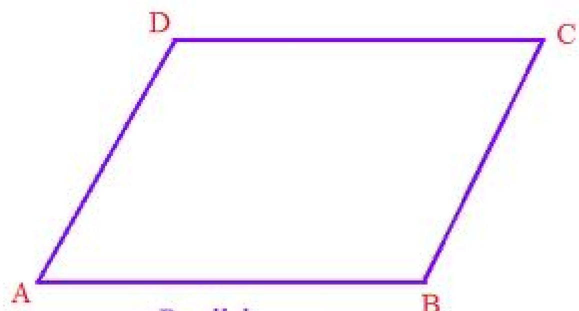


Parallelogram

In the given figure, ABCD is a parallelogram in which AB II DC and AD II BC.

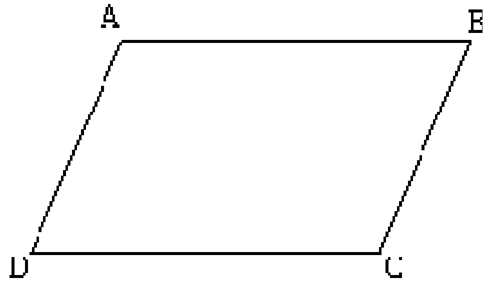
Elements of a parallelogram

- There are four sides in a parallelogram that are AB, BC, CD and DA.
- Four angles are ∠A, ∠B, ∠C and ∠D.
- There are two pairs of opposite sides that are AB and DC, DA and CB.
- AB and BC, BC and CD, CD and DA, DA and AB are the **adjacent sides**.
- The **opposite sides** of a parallelogram have equal length.
- (∠A and ∠B) and (∠B and ∠C) are **adjacent angles.** They are at the ends of the same side.



Parallelogram

Property: In a parallelogram, the opposite sides are of equal length.

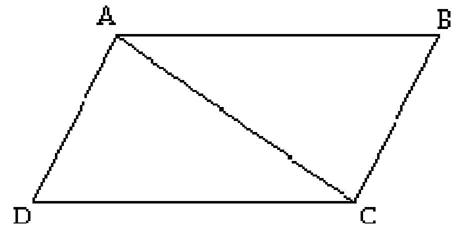


Given: AB || CD and AD || BC

To prove: AB = CD, AD = BC

Construction: Draw in the diagonal AC.

Proof: consider ΔABC and ΔCDA,



AB II CD (given)

AD II BC (given)

∠BAC = ∠DCA (alternate interior angles)

∠DAC = ∠BCA (alternate interior angles)

And AC = AC (common side)

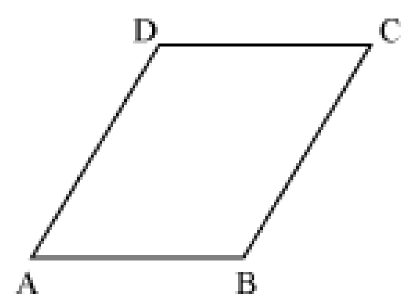
So, ΔABC ≅ ΔCDA (by ASA congruent rule)

Hence, AB = CD (CPCT)

And AD = BC (CPCT)

Angles of a parallelogram

Property: The adjacent angles in a parallelogram are supplementary.



Given: ABCD is a parallelogram.

To Prove: $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180^\circ$

Proof: ABCD is a parallelogram

∴ AD||BC (Opposite sides of parallelogram are parallel)

⇒ $\angle DAB + \angle ABC = 180^\circ$ (Sum of consecutive interior angles is supplementary)

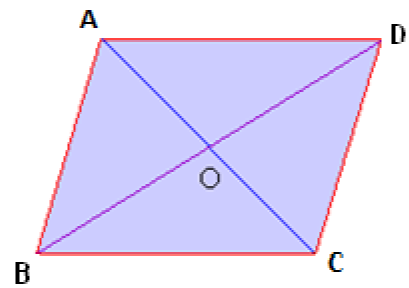
∴ $\angle A + \angle B = 180^\circ$

Similarly, $\angle B + \angle C = 180^\circ$, $\angle C + \angle D = 180^\circ$ and $\angle D + \angle A = 180^\circ$.

Thus, the adjacent angles of a parallelogram are supplementary.

Diagonals of a parallelogram

Property: The diagonals of a parallelogram bisect each other.



Parallelogram

Given: ABCD is a parallelogram.

To Prove: OB = OD and OA = OC

Proof: consider two triangles AOB and COB

Then, ∠ (AOB) = ∠ (COD) (vertically opposite angles)

Side AB = side CD (since, opposite sides of parallelogram are equal)

And, ∠ (ABO) = ∠ (CDO) (since, AB parallel to CD and transversal BD intersects them

So, ∠ (ABO) = ∠ (CDO) [alternate angles]

Therefore, triangles AOB and COB are congruent (By AAS rule)

Therefore OB = OD (By CPCT)

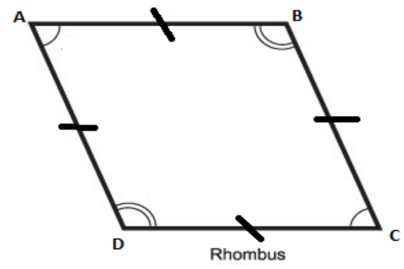
And OA = OC (By CPCT)

Therefore, diagonals of a parallelogram bisect each other.

Some Special Parallelograms

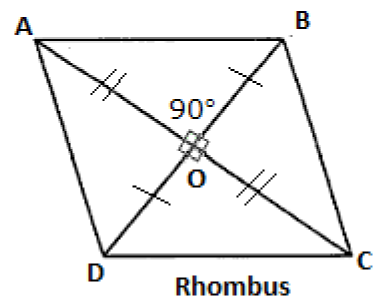
Rhombus

- A rhombus is a quadrilateral with all sides having equal length.
- The opposite sides of a rhombus are of equal length i.e., AB = DC, AD = BC.
- The opposite angles are also equal i.e., ∠A = ∠C, ∠B = ∠D so it has all the properties of a parallelogram and of a kite.



Property: Diagonals of a Rhombus bisect each other at right angles

In the given rhombus, ABCD, AC and BD are the diagonals that meet at point O.

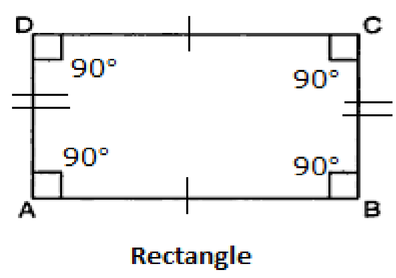


It is observed that AO = CO, BO = DO and ∠DOC = ∠COB = ∠BOA = ∠AOD = 90°.

Hence, Diagonals of a Rhombus bisect each other at right angles.

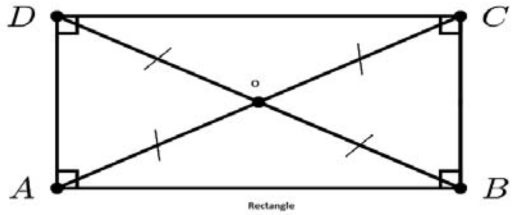
A rectangle

A quadrilateral with opposite sides equal that are AB = DC, AD = BC and all the four angles equal to right angle that are ∠A = ∠B = ∠C = ∠D = 90° is called **rectangle**.



Property: Diagonals of a rectangle are equal and bisect each other.

In the given rectangle ABCD,

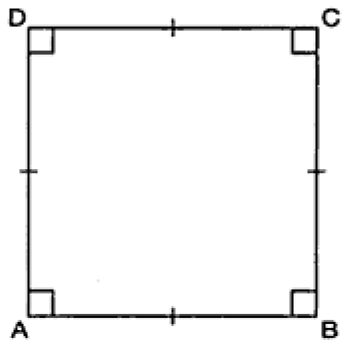


It can be observed that AO = CO, BO = DO, And AC = BD.

So that diagonals of a rectangle are equal and bisect each other.

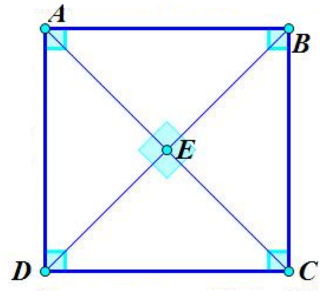
A square

A quadrilateral with all its four sides equal that are AB = BC = CD = DA and each of its four angles a right angle that are ∠A = ∠B = ∠C = ∠D = 90° is called a **square**.



Square

Property: The diagonals of a square are perpendicular bisectors of each other.



Diagonals of a square are equal i.e. AC = BD, and perpendicular to each other i.e. AC ⊥ BD and bisects each other."