Linear Equation in One Variable

Introduction

An algebraic expression consists of variables or constants connected by mathematical operations. Let us observe the following expressions.

$2x + 3y + 19$, $7a - 8b + 11$ and $12xy + 7yz - 8zr$ are examples of algebraic expressions.

$2x - 7 = 11$, $4x + 3y = 24$, $3a - 5b = 15$ and $11m = 55$ are examples of algebraic equations.

Two algebraic expressions connected by an equality (=) sign make an equation.

An equation contains one or more variables. An equation is a statement of equality that involves one or more variables or constants.

In the equation $4x + 3y = 24$, is called the coefficient of x, 3 is called the coefficient of y and 24 is called constant term.

Linear Equation in One Variable:

One way of describing an equation is in terms of its degree. The degree of an equation is equal to the highest power of the variable in it. If an equation has only one variable and the highest power of the variable is one, then it is called a **linear equation in one variable**.

The following algebraic expressions are not linear expressions since the highest power of there is 2 i.e., more than 1.

$2x^2 + 5y^2$, $3x^2 + 5$, $a^2 + 3a - 1$

The following equations are linear equations in one variable, their variables being x, a and n respectively.

$3x + 5 = 11$, $12a - 4 = 20$, $5n = 35$.

Features of an Equation

While working with equations, keep the following important points in mind.

1. RHS and LHS: In an equation, the expression on the left of the equality sing is called **left hand side** (LHS) and the expression on the right of the equality sig is called the **right hand side** (RHS).
For example:

$3x + 5 = 11$

$3x + 5$ = LHS and 11 = RHS

2. Solution of an Equation: In an equation, we can substitute different values for the variable, but the values of LHS and RHS expressions will be equal only for certain values of the variable.

These values of the variable for which the LHS is equal to the RHS are called **solutions of the equation**. All linear equations will have only one solution. The solution can be rational number too.

Let us check whether $x = 5$ is the solution for $3x + 5 = 11$.

Substituting $x = 5$ in $3x + 5$, we get

$$\text{LHS} = (3 \times 5) + 5 = 15 + 5 = 20$$

This is not equal to the RHS which is 11.

3. Mathematical Operations on an equation: An equation is like a balance. It means if you perform the same mathematical operation on both sides of an equation, then the equality remains unchanged.

Solving Linear Equations

When Variable is only on One Side of the Equation

Let us consider on some example, solutions of linear equations having a variable only one side of equality. In this example we will observe that during transposing of a number or expression from one side of the equation to the other, their sign is changed, i.e., negative sign becomes positive and positive sign becomes negative.

Example:

Find the solution of $5x + 4 = 34$

Solution:

Step 1: Subtract 4 from both sides.

$$5x + 4 - 4 = 34 - 4$$

$\Rightarrow 5x = 30$

Step 2: Divide both sides by 5.

$5x/5 = 30/5$

Implies x = 6, which is required solution.

Example:

Find the solution for 4x – 3 = 20

Solution:

Step 1: Transpose – 3 from LHS to RHS.

$4x = 20 + 3$

$4x = 23$

Step 2: Divide both sides by 4.

$4x/4 = 23/4$

Check: We can verify whether our answer is correct by substituting the newly found values of the variable in the equation.

Solution:

x =23/4 in the LHS of 4x – 3 = 20

LHS = (4 x 23/4) – 3

= 23 – 3 = 20

RHS = 20

LHS = RHS

So, the solution is correct.

Example:

Solve 15/4 – 7x = 9

Solution:

We have 15/4 – 7x = 9

– 7x = 9 – 15/4 (transposing 15/4 to R H S)

Or – 7x = 21/4

Or x = 21/ [4 × (–7)] (dividing both sides by – 7)

Or x = - (3 x 7)/ (4 x 7)

Or x = - 3/4 (solution)

Check: LHS = 15/4 - 7 (-3/4)

= 15/4 + 21/4

= 36/4

= 9 = RHS (as required)

Some Applications

Example:

What should be added to twice the rational number -7/3 to get 3/7?

Solution:

Twice the rational number - 7/3 is 2 x (-7/3) = -14/3. Suppose *x* added to this number gives 3/7 i.e.

X + (-14/3) = 3/7

X – 14/3 = 3/7

X = 3/7 + 14/3 (transposing 14/ 3 to RHS)

= (3 x 3) + (14 x 7)/21

= (9 + 98)/ 21 = 107/21

Thus, 107/21 should be added to 2 x (-7/3) to give 3/7.

Example:

The perimeter of a rectangle is 13 cm and its width is2 (3/4) cm. Find its length.

Solution:

Assume the length of the rectangle to be X cm.

The perimeter of the rectangle = 2 × (length + width)

= 2 × [X + 2 (3/4)]

= 2 x (X + 11/4)

The perimeter is given to be 13 cm. Therefore,

2 (X + 11/4) = 13

X + 11/4 = 13/2 (dividing both sides by 2)

X = 13 /11 - 2/ 4

= 26/4 -11/4

= 15/4

=3 (3/4)

The length of the rectangle is 3 (3/4) cm.

Solving Equations having the Variable on both Sides

Observe the following example that deal with the solution of equations having a variable on both sides of the equality.

Example:

Simplify 5(x – 7) = 4(x + 3)

Solution:

Step 1: Simplifying both side of the equality.

5x – 35 = 4x + 12

Step2: Bringing all the terms with variables to one side or to the LHS and taking the constants to the other side.

$5x - 4x = 12 + 35$

$x = 47$

Example:

Solve $5x + 7/2 = 3x/2 - 14$

Solution:

Multiply both sides of the equation by 2. We get

$2 \times (5x + 7/2) = 2 \times (3x/2 - 14)$

$(2 \times 5x) + (2 \times 7/2) = (2 \times 3x/2) - (2 \times 14)$

Or $10x + 7 = 3x - 28$

Or $10x - 3x + 7 = -28$ (transposing 3x to LHS)

Or $7x + 7 = -28$

Or $7x = -28 - 7$

Or $7x = -35$

Or $x = -35/7$

Or $x = -5$ (this is a solution)

Some More Applications

Example:

Five years ago, a man was seven times as old as his son. Five years hence, the father will be three times as old as his son. Find their present ages.

Solution:

Suppose that son's age five years ago be x years

Then father's age five years ago = (7x) years

Son's present age = (x + 5) years

Father's present age = (7x + 5) years

Son's age after five years = 7x + 5 + 5 = (7x + 10) years

According to the given condition,

Father's age after five years = Three times son's age after five years

Or, $7x + 10 = 3(x + 10)$

Or, $7x + 10 = 3x + 30$

Or, $7x - 3x = 30 - 10$

Or, $4x = 20$

Or, $x = 20/4$

$X = 5$

Son's age five years ago = 5 years

Son's preset age = x + 5 = 5 + 5 = 10 years

Father's present age = $7x \times 5$

$= 7.5 \times 5$

$= 35 + 5$

= 40 years.

Check: Son's age after five years = 10 + 5

= 15 years

Father's age after five years = 40 + 5

= 45 years

Now, $45 = 3(15)$

Hence, father's age = three times son's age

Therefore, the solution is correct.

Example:

The present ages of Isha and Raghu are in the ratio 5 : 7. Four year later, their ages will be in the ratio 3 : 4. Find their present ages.

Solution:

Suppose that present ages of Isha and Raghu is 5x years and 7x respectively.

Isha's age after four years = (5x + 4) years

Raghu's age after four years = (7x + 4) years.

According to the given condition,

$$5x + 4/7x + 4 = 3/4$$

Cross-multiplying, we have

$$4(5x + 4) = 3(7x + 4)$$

Or, $20x + 16 = 21x + 12$

Or $-21x + 20x = 12 - 16$

Or $-x = -4$

Therefore, Isha's present age = 5x = 5 ´ 4 = 20 years

Raghu's present age = 7x = 7 ´ 4 = 28 years

Check: Isha's ager after four years = 20 + 4

= 24 years

Raghu's age after four years = 28 + 4

= 32 years

The ratio between Isha's and Raghu's age is 24 : 32 = 3 : 4

Hence, the solution is correct.

Example:

A motorboat covers a certain distance downstream in a river in five hours. It covers the same distance upstream in six hour. The speed of water is 2 km/h. find the speed of the boat in still water.

Solution: Suppose that the speed of the boat in still water be 'x' km/h

Speed of water = 2 km/h

Speed of the boat downstream = (x – 2) km/h

Distance covered in 5 hours = Speed × Time

= 5(x – 2) km

(The relative speed when the direction of the boat and the flow of water is the same = the sum of the speeds of the boat and water).

Distance covered in 6 hours = Speed × Distance = 6(x – 2) km

Speed of the boat upstream = (x – 2) km/h

(The relative speed when the boat travels opposite to the flow of water = the difference in speeds of the boat and water.

But the boat covers the same distance upstream and downstream.

Therefore,

$$6(x - 2) = 5(x - 2)$$

$$6x - 12 = 5x - 10$$

Or $$6x - 5x = 12 + 10$$

Or $$x = 22$$

Hence, the speed of the boat in still water = 22 km/h

Check: Distance covered in 5 hours (downstream) = Speed × Time

$$= 5(22 + 2) = 120 \text{ km}$$

Distance covered in 6 hours (upstream) = Speed × Time

$$= 6(22 - 2)$$

$$= 120 \text{ km}$$

In both cases, the distance is same. Hence the solution is correct.

Reducing Equations to Simpler Form

Example:

Solve 3(t - 3) = 5 (2t + 1)

Solution:

We have 3(t - 3) = 5 (2t + 1)

Now, open the brackets we get

$\Rightarrow$ 3t - 9 = 10t + 5

$\Rightarrow$ 3t - 10t = 5 + 9 = 14

$\Rightarrow$ - 7t = 14

$\Rightarrow$ t = - 2

Example:

Solve 15(y - 4) - 2(y - 9) + 5(y + 6) = 0

Solution:

We have 15(y - 4) - 2(y - 9) + 5(y + 6) = 0

Now, open the brackets we get

$\Rightarrow$ 15y - 60 - 2y + 18 + 5y + 30 = 0

$\Rightarrow$ 18y - 12 = 0

$\Rightarrow$ 18y = 12

$\Rightarrow$ y = 12/18 = 2/ 3

Example:

Solve 3(5z - 7) - 2(9z - 11) = 4(8z - 13) – 17

Solution:

We have 3(5z - 7) - 2(9z - 11) = 4(8z - 13) – 17

Now, open the brackets we get

$\Rightarrow$ 15z - 21- 18z + 22 = 32z - 52 – 17

$\Rightarrow$ - 3z + 1 = 32z – 69

$\Rightarrow$ 32z - (- 3z) = 1+ 69 = 70

$\Rightarrow$ 35z = 70

$\Rightarrow$ z = 2

Equations Reducible to the Linear Form

Example: Hari and Harry's age are in the ratio of 5:7. Four years later the ratio of their ages will be 3:4. Find their current age.

Solution:

Let Hari's Age = 5x and Harry's Age = 7x

Four years from now Hari's Age= 5x+4

Four years from now Harry's Age = 7x+4

Then we get following equation:

(5x + 4)/ (7x + 4) = 3 by 4

$\Rightarrow$ 20x + 16 = 21x + 12

$\Rightarrow$ 21x - 20x = 16 – 12

$\Rightarrow$ x = 4

So, Hari's Age 5x =20 years and Harry's Age 7x = 28 years

Example:

If in a rational number denominator is greater than numerator by 8. If you increase the numerator by 17 and decrease the denominator by 1, you get 3/2 as result. Find the number.

Solution:

Let the numerator be x, then denominator = x + 8

So, the equation we get is as follows:

(x + 17)/ (x + 7) = 3 by 2

$\Rightarrow$2x + 34 = 3x + 21

$\Rightarrow$3x - 2x = x = 34 - 21 = 13

Numerator = 13 and Denominator = 13 + 8=21

The rational number = 21/13