Rational Number

Whole number

Whole numbers are closed under addition and multiplication, but are not closed under subtraction and division. Let us see what does this mean.

Example:

4 + 7 = 11

5 + 3 = 8

The addition of two or more whole numbers always result in a whole number that is why it is termed as closed.

Similarly, multiplication of two or more whole numbers always results in a whole number.

Example:

2 X 3 = 6,

5 X 7 = 35

On the other hand subtraction and division of two whole numbers may not always result in whole number; hence it is not a closed case.

Example:

5 – 7 = -2 is not a whole number

5 ÷ 7 = 7/5 is not a whole number.

Rational Number

Rational number is a number that is expressed in the form p/q, where p and q are integers and q not equal to zero. There are infinitely many Rational numbers between any two Rational Numbers.

Properties of Rational Numbers

Closure Properties of Rational Numbers

Rational numbers are closed under the operations of addition, subtraction and multiplication. Rational numbers can either be positive or negative.

If a and b are any two rational numbers and

$$a + b = c$$

Then c will always be a rational number.

If a and b are any two rational numbers and

$$a - b = c$$

Then c will always be a rational number

If a and b are any two rational numbers, then

$$a \times b = c$$

Then c will always be a rational number

Rational numbers are not closed under division since division of a rational number by zero is not defined.

Thus, Rational numbers are closed under addition, subtraction and multiplication.

Rational numbers are not closed under division.

Following examples illustrate how rational numbers are closed that is result in a rational number after operations of addition, subtraction and multiplication.

Addition: 1/2 + 1/2 = 1/1 is a rational number.

Subtraction: 3/4 – 1/4 = 2/4 = 1/4 is a rational number.

Multiplication: 3/4 x 1/4 = 3/16 is a rational number.

Division: For any rational number a, a ¸ 0 is not defined, so this is not a closed case.

Numbers	**Closed under**			
	Addition	**Subtraction**	**Multiplication**	**Division**
Rational Numbers	Yes	Yes	Yes	No
Integers	Yes	Yes	Yes	No
Whole Number	Yes	No	Yes	No
Natural Number	Yes	No	Yes	No

Commutative Properties of Rational Numbers

If a and b are any two rational numbers, then

$$a + b = b + a \quad \text{(commutative property)}$$

Example:

3/4 + 1/4 = 3/4 + 1/4

5/2 + 2/5 = 2/5 + 5/2

If a, b and c are any three rational numbers, then

$$a \times b = b \times a \quad \text{(commutative property)}$$

Example:

2/3 x 3/4 = 3/4 x 2/3

5 x 7 = 7 x 5

Thus, Rational numbers are commutative under addition and multiplication.

Rational numbers are not commutative under subtraction and division.

Numbers	**Commutative for**			
	Addition	**Subtraction**	**Multiplication**	**Division**
Rational Numbers	Yes	No	Yes	No
Integers	Yes	No	Yes	No
Whole Number	Yes	No	Yes	No
Natural Number	Yes	No	Yes	No

Associative Properties of Rational Numbers

If a, b, and c are any three rational numbers, then

$$a + (b + c) = (a + b) + c \quad \text{(associative property)}$$

Example:

-2/3 + [3/5 + (-5/6)] = [(-2/3) + 3/5] + (-5/6)

-5/3 + [2/5 + 7/6] = [(-5/3) + 2/5] + 7/6

If a, b and c are any three rational numbers, then

$$a \times (b \times c) = (a \times b) \times c \quad \text{(associative property)}$$

Example:

-2/3 + [3/5 + (-5/6)] = [(-2/3) + 3/5] + (-5/6)

-5/3 + [2/5 + 7/6] = [(-5/3) + 2/5] + 7/6

Thus, Rational numbers are associative under addition and multiplication.

Rational numbers are not associative under subtraction and division.

Numbers	**Associative for**			
	Addition	**Subtraction**	**Multiplication**	**Division**
Rational Numbers	Yes	No	Yes	No
Integers	Yes	No	Yes	No
Whole Number	Yes	No	Yes	No
Natural Number	Yes	Yes	Yes	No

The role of zero (0)

The rational number 0 is the **additive identity** for rational numbers.
For given rational number a,

$$a + 0 = a$$

In general,

$a + 0 = 0 + a = a$ (where a is a whole number)
$b + 0 = 0 + b = b$ (where b is an integer)
$c + 0 = 0 + c = c$ (where c is a rational number)

Zero is called the identity for the addition of rational numbers, integers and whole numbers.

The role of 1

The rational number 1 is the **multiplicative identity** for rational numbers.

For example:

5 × 1 = 5 = 1 × 5 (Multiplication of 1 with a whole number)
2/7× 1 = 2/7
3/8 × 1 = 3/8

Negative of a number

The **additive inverse** of the rational number (a / b) is (- a / b) and vice versa is also true.

For example:

(2/3) + (-2/3) = 0

(-1/2) + (1/2) = 0

Reciprocal

The **reciprocal** or **multiplicative inverse** of the rational number is the numbers which result in 1 after multiplication with the number. I.e. a/b x b/a =1

Example:

2/3 x 3/2 = 1

Distributive Property for Rational Numbers

Multiplication over addition

If x, y and z are any three rational numbers, then

a x (b + c) = (a x b) + (a x c)

Multiplication over Subtraction

If x, y, and z are any three rational numbers, then

a x (b – c) = (a x b) – (a x c)

For example:

-3/4 x {2/3 + (-5/6)}

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\left(\frac{-5}{6}\right)\right\}=\frac{-3}{4}\times\left\{\frac{(4)+(-5)}{6}\right\}$$

$$=\frac{-3}{4}\times\left(\frac{-1}{6}\right)=\frac{3}{24}=\frac{1}{8}$$

$$\frac{-3}{4}\times\frac{2}{3}=\frac{-3\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$$

$$\frac{-3}{4}\times\frac{-5}{6}=\frac{5}{8}$$

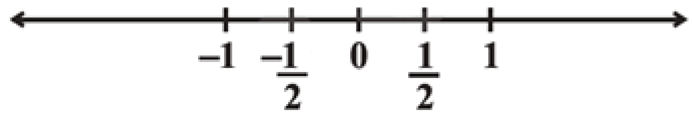
$$\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)=\frac{-1}{2}+\frac{5}{8}=\frac{1}{8}$$

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)$$

Representation of Rational Numbers on the Number Line

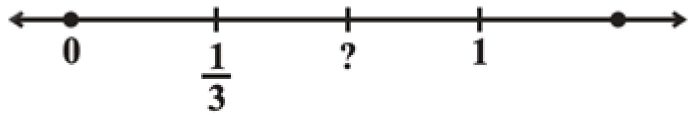
Rational numbers

(i)



The point on the number line (i) which is half way between 0 and 1 has been labelled 1/2.

(ii)

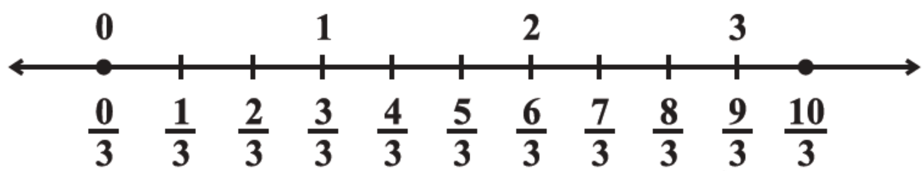


Also, the first of the equally spaced points that divides the distance between 0 and 1 into three equal parts can be labelled 1/ 3, as on number line (ii).

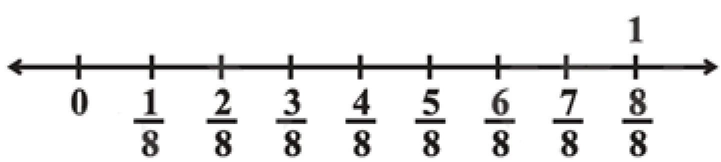
How would you label the second of these division points on number line (ii)?

The point to be labelled is twice as far from and to the right of 0 as the point Labeled 1/3. So it is two times 1/3, i.e., 2/3. You can continue to label equally-spaced points on the number line in the same way. The next marking is 1. You can see that 1 is the same as 3/3. Then comes 4/3, 5/3, 6/2 (or 2), 7/3 and so on as shown on the number line (iii)

(iii)

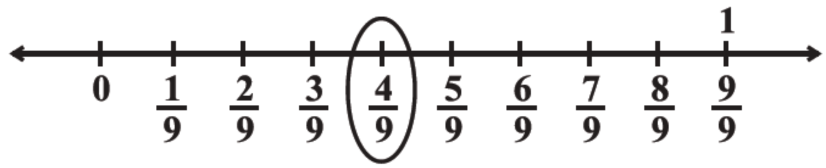


Similarly, to represent 1/8, the number line may be divided into eight equal parts as shown:
We use the number 1/8 to name the first point of this division. The second point of division will be labelled 2/ 8, the third point 3/ 8, and so on as shown on number line (iv)

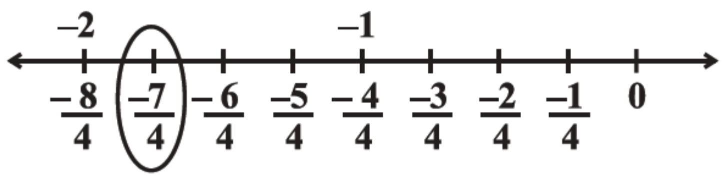


Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells ‘how many’ of these parts are considered. So, a rational number such as 4/9 means four of nine equal parts on the right of 0 (number line v) and for -7/4, we make 7 markings of distance 1/ 4 each on the left of zero and starting from 0. The seventh marking is -7/4 [number line (vi)].

(v)



(vi)



Rational Numbers between Two Rational Numbers

Between any two integers, there are countless numbers, and sometimes, there is no number at all.

For example:

Between 12 and 20 there are seven integers (13, 14, 15, 16, 17, 18, and 19),

And between 10 and 11 there are no integers.

Between 100 and 1000 there are 899 numbers.

However, we can not observe this limitation in rational numbers. In fact, between any two rational numbers, we can find countless rational numbers.

Check this fact through the following example.

FIRST METHOD:

Example:

How many rational numbers are there between 1/5 and 4/5?

Solution:

You may say there are only two – 2/5, 3/5

But if you rewrite 1/5 as 5/25 and 4/5 as 20/25, then you would have 6/25, 7/25, 8/25, 9/25, 10/25………..10/25.

In all, there are 14 rational numbers and all of them lie between 1/5 and 4/5.

Now if we write 1/5 as 100/500 and 4/5 as 400 500, we would have a new set of rational numbers occurring between 1/5 and 4/5.

These are 101/500, 102/500, 103/500, 104/500, 105/500,…………..397/500, 398/500, 399/500.

And, you can go on endlessly finding new sets of rational numbers between 1/5 and 4/5.

This method of finding rational numbers can be applied to any two randomly selected rational numbers.

Therefore, it follows that there are countless rational numbers between any two rational numbers.

Second method

This method is also called the arithmetic mean method. In this method, first we find the mean of the two given rational numbers, and then we go on finding more mean values of either the new and original rational number combination or to the tow mean values.

Suppose two given rational numbers are a and b. We will find rational numbers q1, q2, q3, q4…..between a and b as follows.

$$q1 = 1/2\ (a + b)$$

$$q2 = 1/2\ (a + q1)$$

$$q3 = 1/2\ (q1 + b)$$

$$q4 = 1/2\ (q1 + q2)$$

Counting this procedure, we can get the required number of rational numbers between the rational numbers and b.

Observe the following example that is based on this method.

Example:

Find four ration numbers between 1 and 2, by method of arithmetic mean.

Solution:

q1 = 1/2 (1 +2) = 1/2 × 3 = 3/2

q2 = 1/2 (1 + 3/2) = 1/2 × (2 + 3/2) = 5/4

q3 = 1/2 (3/2 + 2) = 1/2 × (3 + 4/2) = 7/4

q4 = 1/2 (3/2 + 5/4) = 1/2 (6 +5/4) = 11/8

If we represent these numbers on the number line, you will notice that all these numbers lies between 1 and 2.