Arithmetic Progression

Arithmetic Progression in short AP is a sequence of numbers or terms in which each term except the first term is obtained by adding a fixed number or constant to the preceding term.

Common difference is given by $d = t_2 - t_1 = t_3 - t_2 = \ldots\ldots$

The n^{th} term is also denoted with l or b.

General term of AP is $t_n = a + (n - 1)d$

Where a is the first term, d is the common difference and n is the number of term.

The common difference is the difference between two successive terms that is $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$

The sum of first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}[a + l]$

Or

$$\frac{n}{2}[first\ term + last\ term]$$

Where **l = a + (n - 1)d** is called the **last term**.

Let us consider the following set of members:
1, 5, 9, 13, 17…….
9, 12, 15, 18, 21 …….
-5, 0, 5, 10, 15……
1.3, 1.6, 1.9, 2.2 ……..…

All these sets follow certain rules. In first set 5 - 1 = 9 - 5 = 13 - 9 = 17 - 13 = 4

In second set 12 - 9 = 15 - 12 = 18 - 15 = 21 - 18 = 3

and so on. In first set the number after 17 in 17 + 4 = 21 and in second set number after 21 is 21 + 3 = 24. In this way we find that in first set second number is 1+ 4 = 5, third number is 5 + 4 = 9 = 1 + 2 x 4 and so on.

On the basis of above discussion we can consider the following series

a, a + d, a + 2d, a + 3d, ……

Here a = 1, d = 4

a + d = 1 + 4 = 5

a + 2d = 1 + 2 x 4 = 9

and so on

Thus we can say that

a = First term

a + d = Second term

a + 2d = Third term

a + 3d = Fourth term and son on

$\therefore$ n^{th} term = a + (n - 1)d

Here First term = t_1 = a

Second term = t_2 = a + d

and hence, t_n = a + (n - 1)d

d is called common difference and the series is called arithmetic progression

Let s = 1 + 2 + 3 + 4 + 5 + 6 + 7 + + 8 + 9 + 10 and writting in reversed order

S = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

Adding these two we get 2s = 11 + 11 + 11 + 11 + 11 + 11 + 11 = 11 + 11 + 11

= 10 X 11

$\therefore$ s = (10 X 11)/2 = 55

In similar way, if

$$Sn = a + (a+d) + (a+2d) + \ldots\ldots\ldots\ldots + \{a + (n-1)d\}$$
$$Sn = \{a + (dn-1)\} + \{a + (n-2)d + \}\ldots\ldots\ldots\ldots + (a+2d) + (a+d) + a$$

Adding we get

$$2\,Sn = \{2a + (n-1)d\} + \{2a + (n-1)d\} + \ldots\ldots\ldots \text{to terms}$$

$$= n\{2a + (n-1)d\}$$

$$\therefore\ s = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{a + (a + n - 1)d\}$$

$$\therefore = \frac{n}{2}(t_1 + t_n)$$

Example 1.

Find the common difference of the A.P. 2, 5, 8, 11, 14…….....

Solution

Here a = 2, a + d = 5, a + 2d = 8

$$\therefore\ d = 14 - 11 = 11 - 8 = 8 - 5 = 5 - 2 = 3$$

Example 2

Show that p - q, p and p + q from consecutive terms of an A.P.

Solution:- p + q - p = p - (p - q) = q = c.d.

Hence p - q, p and p + q from consective terms of an A.P.

Example 3.

Find A.P. whose 5th term is 24 and difference between 7th term and 10th term is 15.

Solution

$t_5 = a + 4d = 24$

$t_7 = a + 6d$

$t_{10} = a + 9d$

$t_{10} - t_7 = 15$

$$(a+9d)-(a+6d)=15$$
$$3d=15$$
$$\therefore d=5$$
$$a+4d=24$$
$$a+4\times5=24$$
$$a+20=24$$
$$\therefore a=24-20=4$$
$$\therefore a=4, d=5$$

Hence the A. P. is 4, 9, 14, 19, 24, 29, 34, ...

Example 4.

Find the sum of first n terms of odd natural number.

Solution:- Given series is 1, 3, 5, 7, 9,----------

a = 1, d = 2

$$Sn=\frac{n}{2}\{2a+(n-1)d\}$$
$$=\frac{n}{2}\{2\times1+(n-1)\times2\}$$
$$=\frac{n}{2}\{2+2n-2\}$$
$$=n^2$$

Example 5.

Common difference of an A.P. is -2 and first term is 80. Find the sum if last term is 10.

Solution

Here a = 80, d = -2, t_n = 10

Using, t_n = a + (n - 1)d

10 = 80 + (n - 1) (-2)

10 - 80 = -2n + 2

-70 - 2 = -2n

n = 36

$$\therefore S = \frac{n}{2}(a + t_n)$$
$$= \frac{36}{2}(80 + 10)$$
$$= 18 \times 90$$
$$= 1620$$

Example 6

Find the sum of first 30 terms of an A. P. whose n^{th} term is 2 + 1/2n

Solution

$t_n = 2 + 1/2n$

$t_{n-1} = 2 + 1/2(n - 1)$

$t_n - t_n - 1 = 2 + 1/2n - 2 - 1/2(n - 1)$

$d = 1/2$

$a = t_1 = 2 + 1/2 \text{ X } 1 = 5/2$

n = 30

$$s = \frac{n}{2}\{2a + (n-1)d\}$$
$$s = \frac{30}{2}\left\{2 \times \frac{5}{2} + (30-1) \times \frac{1}{2}\right\}$$
$$= 15\left(5 + \frac{29}{2}\right)$$
$$= 15 \times \frac{39}{2}$$
$$= \frac{585}{2}$$
$$= 292\frac{1}{2}$$