Rational Number

What Are Rational Numbers?

The word ‘**rational**’ arises from the term ‘**ratio**’.

‘Rational’ $\Longrightarrow$ ‘Ratio’

We know that a ratio like 4 is to 5 can also be written as $\frac{4}{5}$. Here, 4 and 5 are natural numbers.

Similarly, the ratio of two integers m and n, where n is not equal to zero, that is, m is to n can be written in the form $\frac{m}{n}$. This is the form in which rational numbers are expressed.

A rational number is defined as a number that can be expressed in the form $\frac{m}{n}$, where m and n are integers and n ≠ 0.

For example: $\frac{5}{3}, \frac{22}{57}, \frac{13}{3}$, are the rational numbers.

Numerator and Denominator

In a rational number, the integer above the line is called the **numerator**. The integer below the line is called the **denominator**.

For example: $\frac{5}{3}, \frac{22}{57}, \frac{13}{3}$ in these examples 5, 22, and 13 are the numerators and 3, 57 and 3 are denominator.

Equivalent Rational Numbers

Two rational numbers are said to be equivalent if their standard forms are same.

Let $\frac{a}{b} = \frac{c}{d}$ are two rational numbers then they are equivalent if

$$\frac{a}{b} = \frac{c}{d}$$

Or **ad = bc**

Positive and Negative Rational Numbers

A rational number is said to be **positive** if its numerator and denominator are either either positive or negative.

For example: $\frac{-3}{-4}, \frac{63}{7}, \frac{-25}{-34}$ etc.

So, $\frac{6}{7}$ is a rational number, Note here both the numerator and denominator are positive so it is a positive rational number.

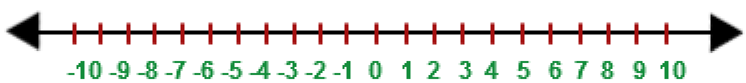
A rational number is said to be **Negative** if its numerator and denominator are such that one of them is a positive integer and other is negative integer.

For example: $\frac{6}{-3}, \frac{-53}{2}, \frac{-5}{44}$ etc.

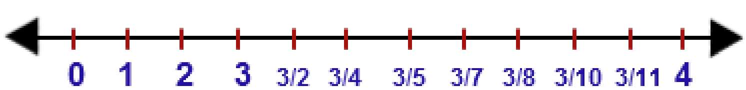
Also, $\left(\frac{-1}{4}\right)$ is a negative rational number as the numerator –1 is a negative integer and the denominator 4 is a positive integer.

Rational Numbers on a Number Line

Rational numbers are terminating or recurring decimal numbers written in the form of fraction ab in which 'a' and 'b' are integers and the denominator 'b' ≠ 0. The numbers from -∞ to +∞ are graphically plotted in a single line called **number line**.



Plotting of Rational Numbers on this number line is the rational numbers on a number line. Rational numbers are both positive and negative. With 0 in the center of the number line the Positive Rational Numbers are plotted in the right of 0 and the Negative Rational Numbers in the left of 0. Between two rational numbers an infinite set of other rational numbers are present.



Rational Numbers in Standard Form

A rational number $\frac{p}{q}$ is said to be in its simplest form if p and q have no common factor except 1 i.e. they are co – prime.

For example: $\frac{5}{4}, \frac{-5}{2}, \frac{-3}{25}$ etc.

Or

A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and p and q do not have any common factor other than 1.

For example: $\frac{2}{3}, \frac{3}{7}, \frac{4}{5}$ etc.

Comparison of Rational Numbers

A. **On Number line:** on the number line, a rational number on the right is always greater than the rational number on its left.

B. **(i) Comparison by equivalent fractions:** Given two rational numbers $\frac{a}{b} = \frac{c}{b}$ with common denominator then

$\frac{a}{b} = \frac{c}{b}$ If a = c

$\frac{a}{b} > \frac{c}{b}$ **If a > c**

$\frac{a}{b} < \frac{c}{b}$ **If a < c**

Where a, b, c are all positive integers respectively.

(ii) Given two rational number $\frac{a}{b} = \frac{c}{d}$ then

$\frac{a}{b} = \frac{c}{d}$ If ad = bc

$\frac{a}{b} > \frac{c}{d}$ **If ad > bc**

$\frac{a}{b} < \frac{c}{d}$ **If ad < bc**

Where ‘b’ and‘d’ are positive integers

Note: To compare two rational numbers we always write the rational numbers with positive denominators.

Main points to compare two rational numbers

i. Express each one of the two given rational numbers with positive denominator.

ii. Take the L.C.M of these positive denominators.

iii. Express each rational number (obtained in (i)) with this L.C.M as common denominator.

iv. The number having greater numerator is greater.

Rational Numbers between Two Rational Numbers

Between any two rational numbers, we can always find rational numbers.

Suppose we are given two rational numbers $\left(\frac{5}{7}\right)$ and $\left(\frac{3}{5}\right)$. First we convert them to rational numbers with same denominators.

L.C.M of 7 and 5 is 35,

So, $\left(\frac{5}{7}\right) = \left(\frac{25}{35}\right)$

And $\left(\frac{3}{5}\right) = \left(\frac{21}{35}\right)$

We can definitely find rational numbers between $\left(\frac{21}{35}\right)$ and $\left(\frac{25}{35}\right)$.

So, $\left(\frac{21}{35}\right)$, $\left(\frac{22}{35}\right)$, $\left(\frac{23}{35}\right)$ are rational numbers between $\left(\frac{5}{7}\right)$ and $\left(\frac{3}{5}\right)$.

To find more rational numbers between them, we increase the denominator. See how.

$$\frac{3\times 70}{5\times 70}=\frac{210}{350} \text{ and } \frac{5\times 50}{7\times 50}=\frac{250}{350}$$

So,

$$\frac{210}{350}<\frac{211}{350}<\frac{212}{350}<\ldots<\frac{248}{350}<\frac{249}{350}<\frac{250}{350}$$

We can find **infinite number of rational numbers between any two rational numbers.**

Operations on Rational Numbers

Addition of rational numbers

Following are the main points which are to be kept in mind.

- Convert each rational number with a positive denominator.

- It the denominators of the rational numbers are equal, we add their numerators and kept the same denominators.

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

- If the denominators of the rational numbers are different, we change them to equivalent rational numbers with same denominator and then add numerator.

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

Additive Inverse

The additive inverse (negative) of a rational number is another rational number which when added to the given number, the sum becomes zero.

It is obtained by multiplying the original rational number by negative

I.e. The additive inverse of $\left(\frac{a}{b}\right) = \left(-\frac{a}{b}\right)$

For example: The additive inverse of $\left(\frac{7}{3}\right) = \left(-\frac{7}{3}\right)$

Subtraction of Rational Number

Subtraction is the inverse of addition that is we add the negative (additive inverse) of the number.

For two rational numbers $\frac{a}{b}, \frac{c}{d}$ the subtraction is

$$\frac{a}{b}-\frac{c}{d}=\frac{a}{b}+\left(-\frac{c}{d}\right)$$

Now, proceed as explained earlier.

Multiplication of Rational Number

It is similar to multiplication of fractions. Thus,

Product of two rational numbers = $\dfrac{\textit{Product of their Numerators}}{\textit{Product of their denominators}}$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Note: if the given numbers are in mixed form then convert them into standard form and then multiply as shown below.

Example: multiply $3\frac{7}{8}$ by $2\frac{3}{5}$

Solution: we have

$$3\frac{7}{8} \times 2\frac{3}{5} = \frac{31}{8} \times \frac{13}{5}$$

$$\Rightarrow \quad = \frac{31 \times 13}{8 \times 5}$$

$$\Rightarrow \quad = \frac{403}{40}$$

$$\Rightarrow \quad = 10\frac{3}{40}$$

Product of Reciprocals

If $\frac{p}{q}$ and $\frac{m}{n}$ are two rational numbers such that $\frac{p}{q} \times \frac{m}{n} = 1$, then each is called the **multiplicative Inverse or Reciprocal** of the other with b, d ≠ 0.

For example: the multiplicative inverse of $\frac{21}{5}$ is $\frac{5}{21}$

Division of Rational number

Division is the inverse of multiplication, so, to divide a rational number by another rational number we need to multiply the first number by the multiplicative inverse (Reciprocal) of the second number.

Thus, if $\frac{p}{q}$ and $\frac{m}{n}$ are two rational numbers and $\frac{m}{n} \neq 0$ then

$$\frac{p}{q} \div \frac{m}{n} = \frac{p}{q} \times \frac{n}{m}$$