Summative Assesment-II
MATHEMATICS
CLASS X

MM 80 TIME : 3- hrs.

GENERAL INSTRUCTIONS:

1. All questions are compulsory.
2. The question paper is divided into four sections
   Section A: 10 questions (1 mark each)
   Section B: 8 questions (2 marks each)
   Section C: 10 questions (3 marks each)
   Section D: 6 questions (4 marks each)
3. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks and 2 questions of four marks each.
4. Use of calculators is not allowed.

SECTION A

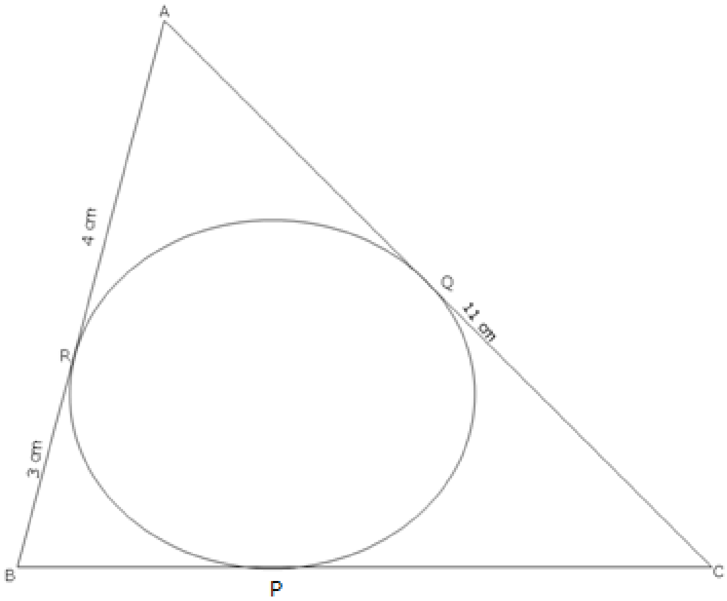
Q1. If the area of a circle is 301.84 cm^2, then its circumference is
(a) 4.6cm (b) 9.8cm (c) 61.6cm (d) 59.4cm

Q2. A letter is chosen at random from the word "PROBABILITY". The probability that it is a vowel is
(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{3}{11}$ (d) $\frac{4}{11}$

Q3. If the length of a shadow cast by a pole is $\sqrt{3}$ times the length of the pole, then the angle of elevation of the sun is
(a) 45^o (b) 30^o (c) 60^o (d) 90^o

Q4. The mid-point of the line segment joining P(-2,8) and Q(-6,-4) is
(a) (-4,2) (b) (4,2) (c) (4,-2) (d) (-4,-2)

Q5. In given figure, the length of BC is



(a) 4 units (b) 6 units (c) 8 units (d) 10 units

Q6. A spherical steel ball is melted to make 8 new identical balls. Then the radius of each new ball is how much times the radius of the original ball?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Q7. If $\frac{1}{2}$ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the other root of the quadratic equation is

(a) $-\frac{5}{2}$ (b) -2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Q8. If the sum of n terms of an AP is $3n^2+n$ and its common difference is 6, then its first term is

(a) 2 (b) 3 (c) 1 (d) 4

Q9. The condition that the point (x,y) may lie on the line joining (3,4) and (-5,-6) is

(a) $5x-4y+1=0$
(b) $5x+4y+1=0$
(c) $-5x+4y+1=0$
(d) $-5x-4y+1=0$

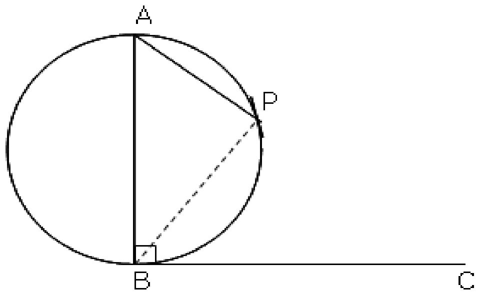
Q10. A largest sphere is carved out of a cube of side 7cm. The volume of the sphere is

(a) 179.67 cu.cm (b) 180.5 cu.cm (c) 182 cu.cm (d) 176.42 cu.cm

SECTION B

Q11. Find the roots of the equation $6x^2 - \sqrt{2}x - 2 = 0$ by the factorization of the corresponding quadratic polynomial.

Q12. AB is the chord of circle with centre O, BC is the tangent at B as shown in the given figure. Show that $\angle PBC = \angle BAP$.



Q13. What is the probability that a number selected from the numbers 1,2,3,….,25 is a prime number, when each of the given numbers is equally likely to be selected?

OR

Two dice are thrown simultaneously. Find the probability of getting an even number as the sum.

Q14. Find the sum of the first 20 terms of the AP: -6,0,6,12,……..

Q15. If a,b,c are the lengths of the sides of a right triangle, where c is the hypotenuse, then prove that the radius, r, of the circle which touches the sides of the triangle is given by $r = \frac{a+b-c}{2}$.

Q16. Find the area of triangle whose vertices are (2,3),(-1,0) and (2,-4).

Q17. A wheel has diameter 84cm. Find how many complete revolutions it must make to cover a distance of 792 metres?

Q18. An umbrella has 10 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 40 cm, find the area between the two consecutive ribs of the umbrella.

SECTION C

Q19. Check whether the equation $6x^2 - 7x + 2 = 0$ has real roots, and if it has, find them by the method of completing the squares.

OR

For what value(s) of k, will the equation $4x^2 - 2(k+1)x + (k+4) = 0$ have repeated roots?

Q20. Two unbiased coins are tossed. Calculate the probability of getting

(i) Exactly two heads

(ii) At least two tails

(iii) No tail

Q21. Find the ratio in which the line $2x+y-4=0$ divides the line segment joining the points A (2,-2) and B (3,7). Also, find their point of intersection.

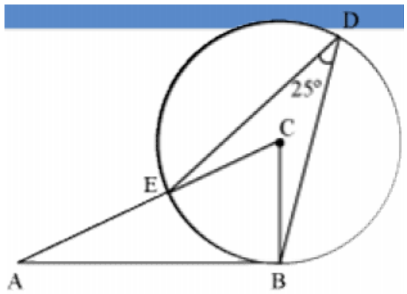
Q22. Prove that the diagonals of a rectangle bisect each other and are equal.

OR

Find the point on the y-axis, which is equidistant from the points (12,3) and (-5,10).

Q23. The mth term of an AP is n and the nth term is m. Find the rth term of the AP.

Q24. In the given figure, AB is a tangent to the circle with centre C at the point B. Find the measure of $\angle BAC$.

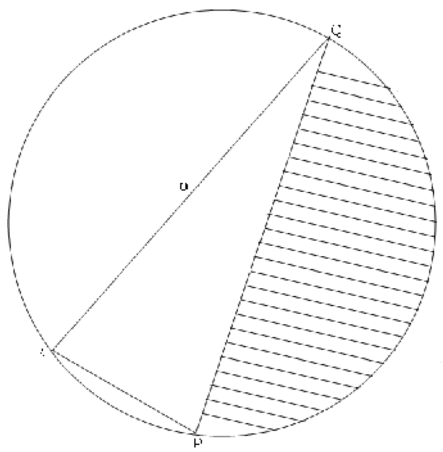


Q25. The length of the shadow of a tower standing on level plane is found to be 2y metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of the tower is $y(\sqrt{3}+1)$ metres.

OR

As observed from the top of a lighthouse, 100 metres high above sea level, the angle of depression of a ship moving directly towards it, changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation.

Q26. Find the area of the shaded region if PQ=24cm,PR=7cm and O is the centre of the circle.



Q27. A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5cm and height 3cm. Find the number of cones so formed.

Q28. Water flows at the rate of 10m per minute through a cylindrical pipe having diameter 5mm. How much time will it take to fill a conical vessel whose base is of diameter 40cm and depth 24cm?

SECTION D

Q29. Two water taps together can fill a tank in $\frac{75}{8}$ hrs. The bigger tap takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

OR

One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Q30. Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so the AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.

Q31. Is it possible to locate a point X on the line segment PQ such that PQ: QX = $\frac{\sqrt{7}}{\sqrt{4}} : \frac{\sqrt{4}}{\sqrt{7}}$? If yes, then construct it. Also, justify the construction.

Q32. Three numbers are in the ratio 3:7:9. If 5 is subtracted from the second, the resulting numbers are in AP. Find the original numbers.

OR

The interior angles of a polygon are in AP. The smallest angle is 52° and the common difference is 8°. Find the number of sides of the polygon.

Q33. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4cm and the diameter of its base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.

Q34. The angle of elevation of a cloud from a point 60 metres above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.

SOLUTIONS

SECTION A

Ans1. Option (c)

Area of the circle = 301.84 cm^2

$\pi r^2 = 301.84$

$\Rightarrow r^2 = \frac{7}{22} \times 301.84$

$\Rightarrow r^2 = 96.04$

$\Rightarrow r = 9.8cm$

$\Rightarrow$ Circumference = $2\pi r = 2 \times \frac{22}{7} \times 9.8 = 61.6$ cm

1 mark

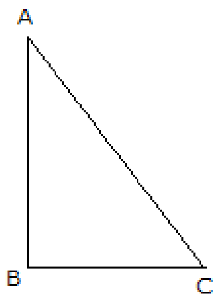
Ans2. Option (d)

In the word "probability", there are 11 letters out of which 4 are vowels (o,a,i,i,).

P(getting a vowel) = $\frac{4}{11}$

1 mark

Ans3. Option (b)



Let AB be the pole of height h.

Length of the shadow = BC = $\sqrt{3}$ h

If θ denotes the angle of elevation of the sun, then $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^o$

1 mark

Ans4. Option (a)

Let R be the mid point of PQ, then, the coordinates of mid point of PQ, i.e., R are

$\left[\frac{(-2-6)}{2}, \frac{(8-4)}{2}\right] = (-4,2)$

1 mark

Ans5. Option (d)

Given, AR=4cm, BR=3cm and AC =11cm

We know that the lengths of tangents drawn to the circle from an external point are equal.

Therefore, AR=AQ=4cm, BR=BP=3cm and PC=QC=AC-AQ = 11cm-4cm=7cm

BC= BP+PC = 3cm+7cm =10 cm

1 mark

Ans6. Option (a)

Let r be the radius of the original steel ball and r' be the radius of each of the new balls formed after melting.

$\frac{4}{3}\pi r^3 = 8x\ \frac{4}{3}\pi r'^3 \Rightarrow r^3=8r'^3 \Rightarrow r' = \frac{1}{2}r$

Hence, the radius of each new ball is $\frac{1}{2}$ times the radius of the original ball.

1 mark

Ans7. Option (a)

It is given that $\frac{1}{2}$ is a root of the equation $x^2+kx-\frac{5}{4}=0$.

$\therefore \left(\frac{1}{2}\right)^2 + k \times \frac{1}{2} - \frac{5}{4} = 0$

1+2k-5=0 or k=2

Putting the value of k in given equation we get,

$4x^2+8x-5=0$

$\Rightarrow 4x^2+10x-2x-5=0$

$\Rightarrow (2x+5)(2x-1)=0$

$\Rightarrow x = \frac{1}{2}$ or $-\frac{5}{2}$

Hence, the other root of the given equation is $\frac{-5}{2}$.

1 mark

Ans8. Option (d)

Given that the sum of n terms of the AP is $3n^2+n$.

$S_n = 3n^2+n$

$a_1 = S_1 = 3(1)^2 + 1 = 4$

Thus, the first term of the AP is 4.

1 mark

Ans9. Option (a)

If three points lie on a line then the area of the triangle formed by them is 0.

$\frac{1}{2}[x(4+6) + 3(-6 - y) - 5(y-4)] = 0$

$10x - 18 - 3y - 5y + 20 = 0$

$10x - 8y + 2 = 0$

$5x - 4y + 1 = 0$

1 mark

Ans10. Option (a)

Diameter of the sphere = Side of the cube = 7 cm

Volume of the sphere = $\frac{4}{3} \times \pi \times \left(\frac{7}{2}\right)^3$ = 179.67 cu. cm

1 mark

SECTION B

Ans11. The corresponding quadratic polynomial can be factorized as below.

$6x^2 - \sqrt{2}x - 2 \Rightarrow 6x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 2$

$\Rightarrow 3x(2x-\sqrt{2})+\sqrt{2}(2x-\sqrt{2})$

$\Rightarrow (3x+\sqrt{2})(2x-\sqrt{2})$

1 mark

Now, $6x^2-\sqrt{2}x-2=0$ gives $(3x+\sqrt{2})(2x-\sqrt{2})=0$, i.e., $(3x+\sqrt{2})=0$ or $(2x-\sqrt{2})=0$

Therefore, the roots of the given quadratic equation are $-\frac{\sqrt{2}}{3}$ and $\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$

1 mark

Ans12. $\angle ABC=90^o$, since AB being diameter is perpendicular to tangent BC at the point of contact.

$\frac{1}{2}$ mark

So, $\angle ABP + \angle PBC = 90^o$ (i)

Also, $\angle APB = 90^o$ (angle in the semi circle)

$\frac{1}{2}$ mark

So $\angle BAP + \angle ABP = 90^o$ (ii)

From (i) and (ii),

$\angle PBC = \angle BAP$ 1 mark

Ans13. Total number of outcomes = 25

Let A be the event of getting a prime number.

Prime numbers from 1 to 25 are 2, 3, 5, 7, 11, 13, 17, 19, 23.

Number of favourable outcomes = 9

1 mark

$P(A) = \frac{9}{25}$

1 mark

OR

S=[(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)]

Total number of outcomes when two dice are thrown = 6x6= 36 $\frac{1}{2}$ mark

Let A be the event of getting an even number as the sum.

Favourable outcomes are
(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3)(5,5),(6,2),(6,4),(6,6)

Number of favourable outcomes =18
1 mark

$P(A) = \frac{18}{36} = \frac{1}{2}$ $\frac{1}{2}$ mark

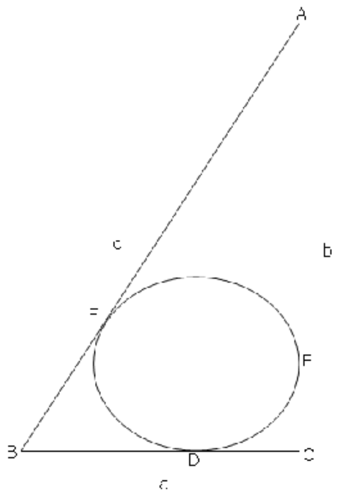
Ans14. We know that sum of first n terms of an AP a, a+d,a+2d.......... is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

Here, n=20, a=-6 and d=6 $\frac{1}{2}$ mark

Therefore, $S_{20} = 10[2(-6) +(20-1)6] = 10[-12+114] = 1020$ $1\frac{1}{2}$ marks

Ans15.



Let the circle touches the sides BC,CA,AB of the right triangle ABC at D,E and F respectively, where BC=a, CA=b and AB=c.

Then, AE=AF and BD=BF. Also CE=CD=r.

∴b-r =AF, a-r=BF
1 mark

or AB =c=AF+BF = b-r+a-r

$\Rightarrow r = \frac{a+b-c}{2}$

1 mark

Ans16. Area of the triangle $= \frac{1}{2}$ [2(0+4)+(-1)(-4-3)+2(3-0)]

1 mark

$= \frac{1}{2}$ [8+7+6]

$= \frac{21}{2}$ = 10.5 sq. units

1 mark

Ans17. Diameter = 2r =84cm

Distance covered in one revolution = circumference = $2\pi r$

$= \frac{22}{7} \times 84$ = 264cm

1 mark

Thus, to cover a distance of 264cm, number of revolution =1

$\therefore$ To cover a distance of 792 metres = 79200cm, number of revolutions $= \frac{79200}{264}$ = 300

1 mark

Ans18. There are 10 ribs in an umbrella. The area between two consecutive ribs is subtending an angle of $\frac{360^0}{10} = 36^0$ at the centre of the assumed flat circle.

1 mark

Area between two consecutive ribs of circle $= \frac{36^0}{360^0} x \pi r^2$

$= \frac{36^0}{360^0} x \frac{22}{7} x (40)^2$

$= \frac{1}{10} x \frac{22}{7} x 40 x 40 = 502.86 cm^2$ 1 mark

SECTION C

Ans19. Discriminant = $b^2 - 4ac$ = 49 – 4 x 6 x 2 = 1 > 0

1 mark

So, the given equation has two distinct real roots.

Now, $6x^2 - 7x + 2 = 0$

$\Rightarrow 36x^2 - 42x + 12 = 0$

$\Rightarrow \left(6x - \frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$

$\Rightarrow \left(6x - \frac{7}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0$

$\Rightarrow \left(6x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

1 mark

The roots are given by $6x - \frac{7}{2} = \pm \frac{1}{2}$

$\Rightarrow 6x = 4,3$

$\Rightarrow x = \frac{2}{3}, \frac{1}{2}$

1 mark

OR

A quadratic equation $ax^2+bx+c=0$ will have repeated roots only when D=0.

i.e., $b^2\text{-}4ac=0 \Rightarrow [-2(k+1)]^2 - 4\times4(k+4)=0$

1 mark

$\Rightarrow 4(k^2+2k+1) - 16(k+4)=0$

$\Rightarrow k^2+2k+1-4k-16 = 0$

$\Rightarrow k^2-2k-15=0$

1 mark

$\Rightarrow (k-5)(k+3)=0 \Rightarrow k= 5,-3$

1 mark

Ans20. When two unbiased coins are tossed the possible outcomes are HH,HT,TH,TT.

Total number of outcomes = 4

(i) Number of favourable outcomes of getting exactly two heads =1 (HH)

P(exactly two heads) = $\frac{1}{4}$

1 mark

(ii) Number of favourable outcomes of getting at least two tails = 1 (TT)

P(at least two tails) = $\frac{1}{4}$

1 mark

(iii) Number of favourable outcomes of getting no tail = 1(HH)

P(no tail) = $\frac{1}{4}$

1 mark

Ans21. Let $2x+y-4=0$ divides the line joining A (2,-2) and B (3,7) at P in ratio k:1.

So, the point $P\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$ lies on $2x+y-4=0$.

1mark

$$\therefore 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$\Rightarrow 6k+4+7k-2-4k-4 = 0$

$\Rightarrow 9k = 2$ or $k = \frac{2}{9}$

Therefore, the required ratio is 2:9.

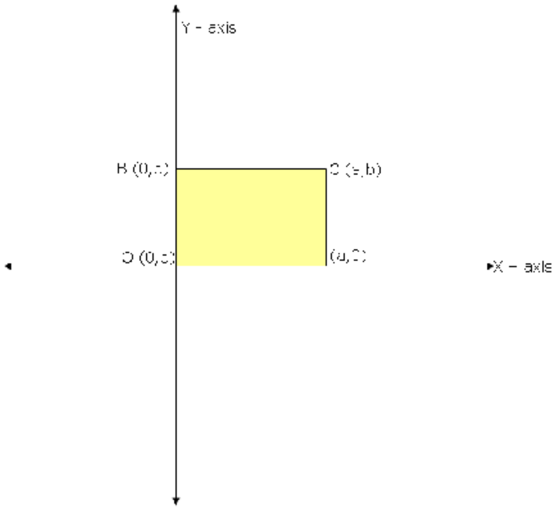
1 mark

The coordinates of the point of intersection, i.e., the coordinates of P are

$$\left(\frac{3\times\frac{2}{9}+2}{\frac{2}{9}+1}, \frac{7\times\frac{2}{9}-2}{\frac{2}{9}+1}\right) = \left(\frac{3\times2+2\times9}{2+9}, \frac{7\times2-2\times9}{2+9}\right) = \left(\frac{24}{11}, \frac{-4}{11}\right)$$

1mark

Ans22.



Let OACB be a rectangle such that OA is along x-axis and OB is along y-axis.
Let OA = a and OB = b.

1 mark

Then, the coordinates of A and B are (a,0) and (0,b) respectively.
Since, OACB is a rectangle. Therefore,

AC = OB

AC = b

Thus, we have

OA = a and AC = b

So, the coordinates of C are (a,b).

The coordinates of the mid – point of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid – point of AB are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

1 mark

Clearly, coordinates of the mid – point of OC and AB are same.
Hence, OC and AB bisect each other.

Also, OC = $\sqrt{(a^2+b^2)}$ and AB = $\sqrt{(a-0)^2+(0-b)^2} = \sqrt{(a^2+b^2)}$

Therefore, OC = AB
1 mark

OR

Let the required point on y-axis be (0,y). $\frac{1}{2}$ mark

Distance between (0,y) and (12,3) = Distance between (0,y) and (-5,10)

$\sqrt{(12-0)^2+(3-y)^2} = \sqrt{(-5+0)^2+(10-y)^2}$

1 mark

$144+9+y^2-6y=25+100+y^2-20y$

$14y=-28$ or $y=-2$

1 mark

Hence the required point on y-axis is (0,-2). $\frac{1}{2}$ mark

Ans23. Let a and d respectively be the first term and the common difference of the AP.

According to the given conditions,

$a+(m-1)d = n$ ....(i)

$a+(n-1)d = m$ ....(ii)

1 mark

On solving (i) and (ii), we get,

$d = -1$; $a = m+n-1$

1 mark

Therefore, r^{th} term = $a+(r-1)d = (m+n-1) - (r-1) = m+n-r$

1 mark

Ans24. It is known that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle ECB = 2\angle EDB$

$\Rightarrow \angle ECB = 2 \times 25° = 50°$ (1 mark)

It is also known that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle ABC = 90°$ (1 mark)

Applying angle sum property of triangles in ΔABC:

$\angle ABC + \angle ACB + \angle BAC = 180°$

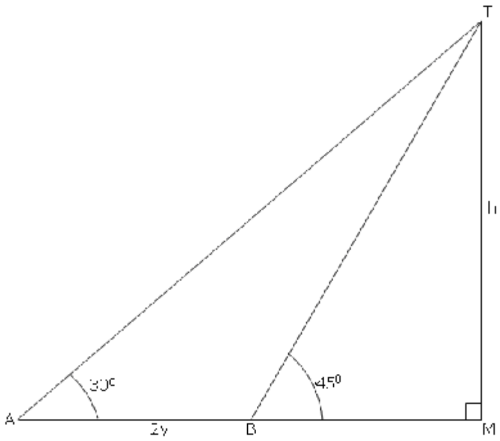
$\Rightarrow 90° + 50° + \angle BAC = 180°$

$\Rightarrow 140° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 140° = 40°$

Thus, the measure of $\angle BAC$ is 40°. (1 mark)

Ans25.



1 mark

Let TM be the tower of height h. It is given that AB=2y

In ΔBMT,

$\tan 45° = \frac{h}{BM} \Rightarrow h = BM$ .....(i)

1 mark

In ΔAMT,

$\tan 30° = \frac{h}{2y + BM}$

$\Rightarrow 2y + BM = h\sqrt{3}$

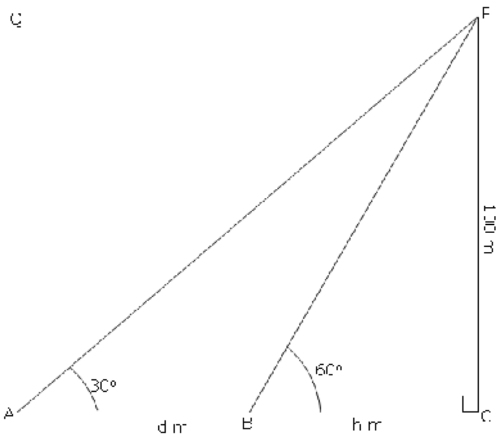
$\Rightarrow h\left(\sqrt{3} - 1\right) = 2y$

$\Rightarrow h = y\left(\sqrt{3} + 1\right)$ m

Thus, the height of the tower is y($\sqrt{3}$ +1) metres.

1 mark

OR



$\frac{1}{2}$ mark

Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation,i.e., AB=d metres.

Suppose that the observer is at the point P. It is given that PC = 100m.

Let h be the distance (in metres) from B to C.

In triangle PCA,

$\frac{d+h}{100}$ = cot30° = $\sqrt{3}$

d+h = 100$\sqrt{3}$ ..(i) 1 mark

In triangle PCB,

$\frac{h}{100}$ = cot60° = $\frac{1}{\sqrt{3}}$

$\Rightarrow$ h = $\frac{100}{\sqrt{3}}$ m $\frac{1}{2}$ mark

Putting the value of h in (i) we get,

d= 100 $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) = \frac{200}{\sqrt{3}}$ = 115.47 (approx.)

Thus, the distance travelled by the ship from A to B is approximately 115.47 metres.

1 mark

Ans26. Angle in a semi-circle is 90°, therefore, ∠RPQ = 90°.

In ΔQPR, by Pythagoras theorem, we have,

$QR^2=PR^2+PQ^2 = 7^2+24^2=49+576=625$

QR=25cm

Diameter of the circle = 25cm

Radius of the circle = $\frac{25}{2}$cm

1 mark

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22\times 25\times 25}{2\times 7\times 2\times 2} = \frac{11\times 625}{28}cm^2$

Area of ΔPQR = $\frac{1}{2}$ PR x PQ = $\frac{1}{2}\times 7\times 24 = 84\ cm^2$

1 mark

Area of shaded region = $\frac{11\times 625}{28}$ - 84 = $\frac{6875-2352}{28} = \frac{4523}{28} = 161.54\ cm^2$

1 mark

Ans27. Volume of the solid metallic sphere = $\frac{4}{3}\pi(10.5)^3\ cm^3$

1 mark

Volume of each cone = $\frac{1}{3}\pi(3.5)^2\times 3\ cm^3$

1 mark

Number of cones so formed = $\dfrac{\frac{4}{3}\pi(10.5)^3}{\frac{1}{3}\pi(3.5)^2\times 3}$ =126

1 mark

Ans28. Amount of water required to fill the conical vessel = volume of the conical vessel

$= \frac{1}{3}\pi(20)^2 \times 24 = 3200\pi$ cu. cm....(i)

1 mark

Amount of water that flows out of the cylindrical pipe in 1 minute

$= \pi \text{ x } \left(\frac{5}{20}\right)^2 \times 10 \times 100 = 62.5\pi$ cu.cm ....(ii)

1 mark

From (i) and (ii) we get,

Time taken to fill the vessel = $\frac{3200\pi}{62.5\pi}$ = 51.2 minutes

1 mark

SECTION D

Ans29. Let bigger tap fill the tank in x hours.

Therefore, smaller tap fills the tank in (x+10) hours.

So, bigger tap fills $\frac{1}{x}$ of the tank in one hour and smaller tap fills $\frac{1}{x+10}$ of tank in one hour.

Both the taps together fill $\left(\frac{1}{x}+\frac{1}{x+10}\right)$ of tank in one hour. 1 mark

it is given that both the taps take $\frac{75}{8}$ hours to fill the tank, so, they fill $\frac{8}{75}$ of tank in one hour.

Hence, $\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75} \Rightarrow \frac{x+10+x}{x(x+10)} = \frac{8}{75}$ $\frac{1}{2}$ mark

$8x^2+80x = 150x+750$

$8x^2-70x-750=0$ 1 mark

D= 4900-4(8)(-750) = 28900

$\sqrt{D}$ = 170

$x = \frac{70 \pm 170}{16}$

$x = 15, -\frac{25}{4}$ 1 mark

x being time cannot be negative, therefore, x=15.

Thus, the bigger tap fills the tank in 15 hours and smaller tap fills the tank in 25 hours.

$\frac{1}{2}$ mark

OR

Let the total number of camels be x.

Number of camels seen in the forest = $\frac{x}{4}$

Number of camels gone to mountains = $2\sqrt{x}$

remaining number of camels on the bank of river =15

Total number of camels = $15 + 2\sqrt{x} + \frac{x}{4}$

1 mark

$15 + 2\sqrt{x} + \frac{x}{4} = x$

$3x-8\sqrt{x}-60=0$ $\frac{1}{2}$ mark

Let $x= y^2$, we get,

$3y^2-8y-60=0$ $\frac{1}{2}$ mark

$3y^2-18y+10y-60=0$

$3y(y-6)+10(y-6)=0$

$(3y+10)(y-6)=0$

$y=6$ or $y=-\frac{10}{3}$

1 mark

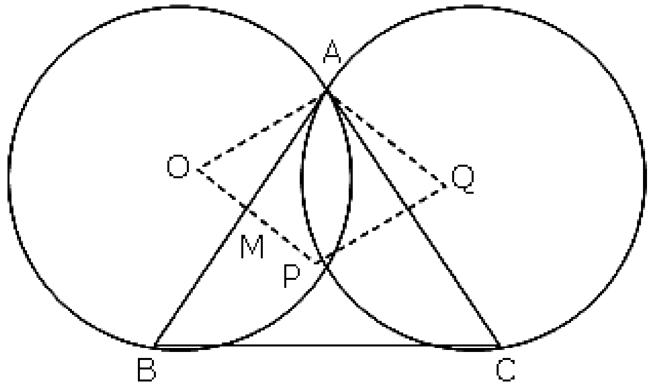
Now, $y = -\frac{10}{3} \Rightarrow x=\frac{100}{9}$, which is not possible

Therefore, $y=6 \Rightarrow x=36$

Hence, the number of camels is 36.

1 mark

Ans30.



1 mark

It is known that the radius through the point of contact is $\perp$ to the tangent.
$\therefore$ AQ $\perp$ AB

1 mark

Also, AQ || OP (Opposite sides of a || gm are parallel)
$\therefore$ OP $\perp$ AB ($\because$ AQ || OP and AQ $\perp$ AB)

Let OP intersects AB at M.
$\therefore$ OM $\perp$ AB

It is also known that the perpendicular from the centre of a circle to a chord bisects the chord.
$\therefore$ AM = MB

1 mark

Thus, OM and hence OP is the perpendicular bisector of AB. Similarly, PQ is the perpendicular bisector of AC.

Now in ΔABC,
OP is the perpendicular bisector of side AB.

$\therefore$ PA = PB
($\because$ Any point of the perpendicular bisector is equidistant form the fixed points)
Similarly, PA = PC.
Hence, PA = PB =PC

Thus, P is equidistant from the three vertices of ΔABC.

Now, the circle with P as centre and its distance from any vertex as radius passes through the three vertices of ΔABC and thus the point P is the circumcentre of ΔABC.

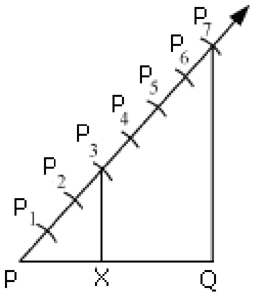
1 mark

Ans31. $PQ : QX = \frac{\sqrt{7}}{\sqrt{4}} : \frac{\sqrt{4}}{\sqrt{7}} = \frac{7}{\sqrt{7} \times \sqrt{4}} : \frac{4}{\sqrt{7} \times \sqrt{4}} = 7 : 4$

Here, 7 and 4, both are positive integers. So, a point X can be located on the line segment PQ such that $PQ : QX = \frac{\sqrt{7}}{\sqrt{4}} : \frac{\sqrt{4}}{\sqrt{7}} = 7 : 4$.

1 mark

The construction can be done by locating a point X on PQ such that it divides PQ in the ratio 3: 4. It can be done as follows:



2 marks

The construction can be justified as follows:

Since P_3X is parallel to P_7Q, $\frac{PP_3}{P_3P_7} = \frac{PX}{XQ}$ (Basic Proportionality Theorem)

By construction:

$$\frac{PP_3}{P_3P_7} = \frac{3}{4}$$

$$\therefore \frac{PX}{XQ} = \frac{3}{4}$$

This shows that point X divides PQ in the ratio 3: 4, i.e., PQ: QX = 7: 4.

1 mark

Ans32. Let the three numbers which are in the ratio 3:7:9 be 3x,7x and 9x.

Now, on subtracting 5 from the second number, we get the three numbers as

3x, 7x-5 and 9x

1 mark

Since, these numbers are in AP, therefore, we have

2(7x-5) = 3x+9x

1 mark

$\Rightarrow$ 14x-10 = 12x

$\Rightarrow$ 2x=10
$\Rightarrow$ x=5
1 mark
$\Rightarrow$ Hence, the three original numbers are 3x =15, 7x=35 and 9x =45
1 mark

OR

Here, a=52° and d=8°
$\frac{1}{2}$ mark

Let the polygon have n sides. Then, the sum of interior angles of the polygon is (n-2)180°. $\frac{1}{2}$ mark

$\Rightarrow S_n = (n-2)180$

$\Rightarrow \frac{n}{2}[2a+(n-1)d] = (n-2)180$

$\Rightarrow \frac{n}{2}[104+(n-1)8] = (n-2)180$

1 mark

$\Rightarrow \frac{n}{2}[8n+96] = (n-2)180$

$\Rightarrow 8n^2+96n=360n-720$

$\Rightarrow 8n^2-264n+720=0$

$\Rightarrow n^2-33n+90=0$

$\Rightarrow (n-30)(n-3)=0$

$\Rightarrow n=3,30$

Hence, there are either 3 or 30 sides of the polygon. 2 marks

Ans33. Let r = 4 cm be the radius of the hemisphere and the cone and h = 4 cm be the height of the cone.

Volume of the toy = volume of the hemisphere + volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{2\times 22}{3\times 7} \text{ x } 4^3 + \frac{1\times 22}{3\times 7}\times 4^2 \times 4 = \frac{1408}{7} \text{ cm}^3$$

1 mark

A cube circumscribes the given solid. Therefore, edge of the cube should be 8 cm.

Volume of the cube = 8^3 cm^3 = 512 cm^3
1 mark

Difference in the volumes of the cube and the toy = $512 - \frac{1408}{7}$ = 310.86 cm^3

Total surface area of the toy = curved surface area of cone + curved surface area of hemisphere

$= \pi rl + 2\pi r^2$, where $l = \sqrt{h^2 + r^2} = 4\sqrt{2}$

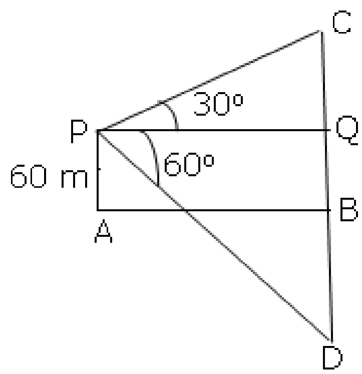
1mark

$= \pi r(l+2r)$

$= \frac{22}{7} \times 4 \times (4\sqrt{2} + 2 \times 4)$

$= \frac{88}{7}(4\sqrt{2} + 8) = 171.48cm^2$ 1 mark

Ans34.



1 mark

Let C be the cloud and D be its reflection. Let the height of the cloud be H metres. BC=BD=H

BQ=AP= 60m. Therefore CQ= H-60 and DQ = H+60

In ΔCQP,

$\frac{PQ}{CQ} = \cot 30° \Rightarrow \frac{PQ}{H-60} = \sqrt{3} \Rightarrow PQ = (H-60)\sqrt{3}$ m ....(i) 1 mark

In ΔDQP,

$\frac{PQ}{DQ} = \cot 60° \Rightarrow \frac{PQ}{H+60} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{(H+60)}{\sqrt{3}}$ ....(ii) 1 mark

From (i) and (ii),

$(H-60)\sqrt{3} = \frac{(H+60)}{\sqrt{3}} \Rightarrow 3H - 180 = H+60 \Rightarrow H=120$

Thus, the height of the cloud is 120m.