Summative Assesment-II

Sample Paper-1

MATHEMATICS

CLASS X

MM : 80 Time: 3-hrs.

GENERAL INSTRUCTIONS :

1. All questions are compulsory.
2. The question paper is divided into four sections
   Section A: 10 qusetions (1 mark each)
   Section B: 8 questions (2marks each)
   Section C: 10 questions (3 marks each)
   Section D: 6 questions (4 marks each)
3. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks and 1 question of four marks each.
4. Use of calculators is not allowed.

SECTION - A

Q1. The distance between two parallel tangents to a circle of radius 5 cm is

(a) 10cm (b) 5cm (c) 8cm (d) 9cm

Q2. The probability of occurrence of event A is denoted by P(A) so the range of P(A) is

(a) $0<P(A)<1$ (b) $0\leq P(A)<1$ (c) $0<P(A)\leq 1$ (d) $0\leq P(A)\leq 1$

Q3. A tree is broken by the wind. The top struck the ground at an angle of 30^o and at a distance of 30 metres from the foot of the tree. The height of the tree in metres is

(a) $25\sqrt{3}$ (b) $30\sqrt{3}$ (c) $35\sqrt{3}$ (d) $40\sqrt{3}$

Q4. The area of a square ABCD, whose vertices are A(5,6),B(1,5),C(2,1) and D(6,2) is given by

(a) 17 sq. units (b) 34 sq. units (c) 10 sq. units (d) 7 sq. units

Q5. If the perimeter and area of a circle are numerically equal, then the radius of the circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

Q6. The ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube is

(a) $\pi:8$ (b) $\pi:6$ (c) $8:\pi$ (d) $6:\pi$

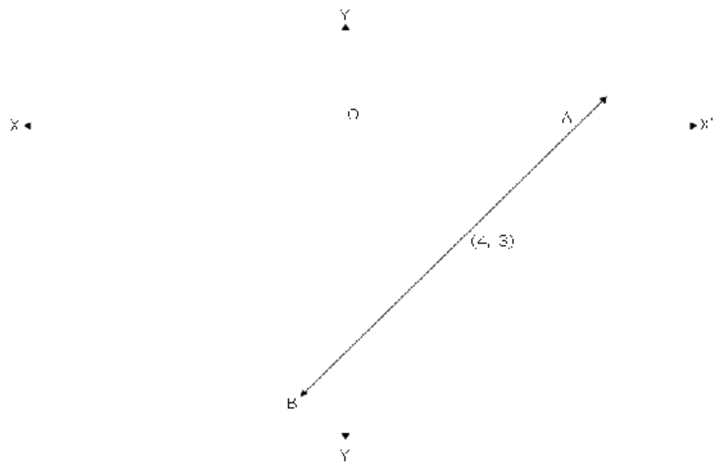
Q7. Which constant should be added and subtracted to solve the quadratic equation $4x^2 - \sqrt{3}x - 5 = 0$ by the method of completing the square?

(a) $\frac{3}{64}$ (b) $\frac{3}{16}$ (c) $\frac{3}{4}$ (d) $\frac{\sqrt{3}}{8}$

Q8. The first and last terms of an AP are 1 and 11. If the sum of all its terms is 36, then the number of terms will be

(a) 5 (b) 6 (c) 7 (d) 8

Q9. The mid-point of the line segment AB in given figure is (4,-3). The respective coordinates of A and B are



(a) (8,0) and (0,6)

(b) (-8,0) and (0,6)

(c) (6,0) and (0,8)

(d) (8,0) and (0,-6)

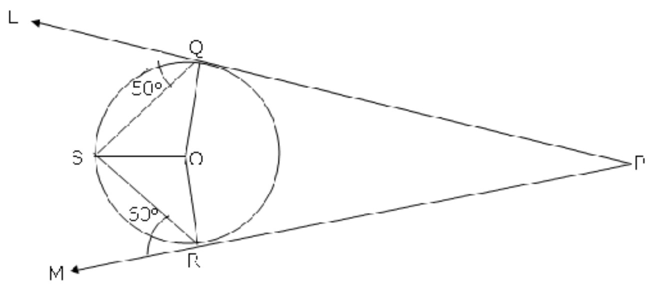
Q10. A funnel is the combination of

(a) Cone and cylinder
(b) Frustum of a cone and cylinder
(c) Hemisphere and cylinder
(d) Hemisphere and cone

SECTION B

Q11. Find the roots of the quadratic equation $2x^2 - \sqrt{5}\,x - 2 = 0$ using the quadratic formula.

Q12. In given figure, PQL and PRM are tangents to the circle with centre O at the points Q and R respectively. S is a point on the circle such that $\angle SQL = 50^o$ and $\angle SRM = 60^o$. Find the value of $\angle QSR$.



Q13. Two dice are thrown simultaneously. Find the probability of getting a doublet of even number.

OR

Three unbiased coins are tossed together. Find the probability of getting at least two heads.

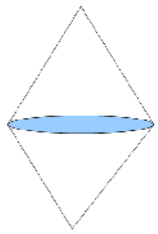
Q14. Find the value of k for which 2k + 7, 6k – 2 and 8k + 4 form 3 consecutive terms of an AP.

Q15. If d_1, d_2 ($d_2 > d_1$) are the diameters of two concentric circles and c is the length of a chord of a circle which is tangent to the other circle, then prove that $d_2^2 = c^2 + d_1^2$.

Q16. Find the length of the median drawn through A on BC of a ΔABC whose vertices are A(7,-3), B(5,3)and C(3,-1).

Q17. In a circle of radius 10cm, an arc subtends an angle of 90^o at the centre. Find the area of the major sector.

Q18. Calculate the area of the shaded region in the given figure, which is common between the two quadrants of circles of radius 8 cm each.



SECTION C

Q19. Find the values of k for which the given equation has real and equal roots:

$2x^2-10x+k=0$.

OR

Solve for x : $\left(\frac{4x-3}{2x+1}\right) - 10\left(\frac{2x+1}{4x-3}\right)=3 \qquad \left(x \neq \frac{-1}{2}; x \neq \frac{3}{4}\right)$

Q20. What is the probability of having 53 Thursdays in a non-leap year?

Q21. If the points A(7,-2), B(5,1) and C(3,k) are collinear, the find the value of k.

Q22. If A (-2,-1) ,B (a,0), C (4,b) and D(1,2) are the vertices of a parallelogram, find the values of a and b.

OR

If the mid point of the line segment joining the points A (3,4) and B (k, 6) is P (x,y) and x+y-10=0, then find the value of k.

Q23. Find three terms in AP such that their sum is 3 and product is -8.

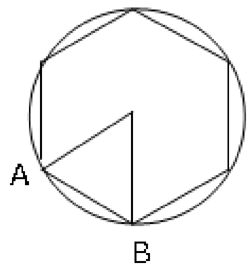
Q24. A circle touches the side BC of a triangle ABC at P and the extended sides AB and AC at Q and R respectively. Prove that AQ = $\frac{1}{2}$ (BC+CA+AB)

Q25. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 metres towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of the tower.

OR

The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. The height of the second tower is 60 m find the height of the first tower.

Q26. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the design at the rate of Rs. 0.35 per cm^2.



Q27. A cone of maximum size is carved out from a cube of edge 14cm. Find the surface area of the cone and of the remaining solid left out after the cone is carved out.

Q28. A canal is 300 cm wide and 120 cm deep. The water in the canal is flowing with a speed of 20km/h. How much area will it irrigate in 20 minutes if 8 cm of standing water is desired?

SECTION D

Q29. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

OR

A takes 6 days less than B to finish a piece of work. If both A and B together can finish the work in 4 days, find the time taken by B to finish the work.

Q30. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Q31. Given a rhombus ABCD in which AB=4cm and $\angle ABC = 60$, divide it into two triangles say, ABC and ADC. Construct the triangle AB'C' similar to ΔABC with scale factor $\frac{2}{3}$. Draw a line segment C'D' parallel to CD where D' lies on AD. Is AB'C'D' a rhombus? Give reasons.

Q32. How many terms of the sequence 13,11,9 ….. make the sum 45? Explain the answer.

Q33. A container is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the container. Also, find the cost of the milk which can completely fill the container, at the rate of Rs 25 per litre (use π = 3.14).

Q34. A round balloon of radius r subtends an angle θ at the eye of the observer while the angle of elevation of its centre is ϕ. Prove that the height of the centre of the balloon is $r \sin\phi . \text{cosec} \frac{\theta}{2}$.

SOLUTIONS

SECTION A

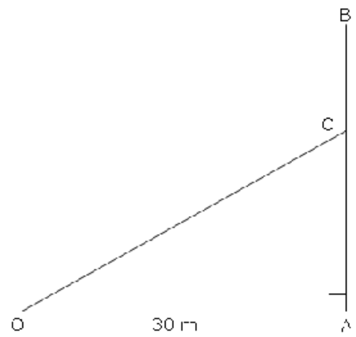
Ans1. Option (a)

Two tangents of a circle are parallel if they are drawn at the end points of a diameter. Therefore, distance between them is the diameter of the circle = 2x5 cm =10cm 1 mark

Ans2. Option (d)

The range of P(A) is $0 \le P(A) \le 1$. 1 mark

Ans3. Option (b)



Let AB be the tree broken at a point C such that the broken part CB takes the position CO and touches the ground at O. OA=30m , $\angle AOC = 30^o$. Let AC = x and BC=CO=y.

In ΔAOC,

$$\tan 30^o = \frac{AC}{OA}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$x = 10\sqrt{3}$$

Again, in ΔAOC,

$$\text{Cos}30^o = \frac{OA}{OC}$$

$$\frac{\sqrt{3}}{2} = \frac{30}{y}$$

$y = 20\sqrt{3}$

Height of the tree = (x+y)

$= 10\sqrt{3} + 20\sqrt{3}$

$= 30\sqrt{3}$ metres 1 mark

Ans4. Option (a)

In a square all the sides are equal, i.e., AB=BC=CD=DA

Distance AB= $\sqrt{(1-5)^2 + (5-6)^2} = \sqrt{17}$ units

Area of square ABCD = $AB^2 = \left(\sqrt{17}\right)^2$ = 17 sq. units 1 mark

Ans5. Option (a)

Given, $2\pi r = \pi r^2$

r=2

Thus, the radius of the circle is 2 units. 1 mark

Ans6. Option (d)

Let x be the edge of the cube. Then, x is also the diameter of the sphere.

Ratio of the volume of the cube to that of the sphere = $x^3 : \frac{4}{3} \times \pi \times \frac{x^3}{8}$

$= 1 : \frac{4\pi}{24} = 6 : \pi$ 1 mark

Ans7. Option (b)

$4x^2 - \sqrt{3}x - 5 = 0$

$x^2 - \frac{\sqrt{3}}{4}x - \frac{5}{4} = 0$

$x^2 - 2 \times \frac{\sqrt{3}}{8}x - \frac{5}{4} = 0$

$$x^2 - 2 \times \frac{\sqrt{3}}{8} x + \left(\frac{\sqrt{3}}{8}\right)^2 - \left(\frac{\sqrt{3}}{8}\right)^2 - \frac{5}{4} = 0$$

Thus, the constant to be added is $\left(\frac{\sqrt{3}}{8}\right)^2 = \frac{3}{64}$ 1 mark

Ans8. Option (b)

Given that the first and last terms of an AP are 1 and 11 i.e. a=1 and l =11.

Let the sum of its n terms is 36, then,

$$S_n = \frac{n}{2} \text{ x } (a+l)$$

$$36 = \frac{n}{2} \text{ x } (1+11)$$

$$n = \frac{36}{6} = 6$$

Thus, the number of terms in the AP is 6. 1 mark

Ans9. Option (d)

The points A and B respectively lie on x and y axis. Let the coordinates of A and B be (x,0) and (0,y) respectively.

It is given that (4,-3) is the mid-point of AB. By mid-point formula,

$$4 = \frac{(x+0)}{2} \text{ and } -3 = \frac{(0+y)}{2}$$

x=8 and y=-6

Thus, the respective coordinates of points A and B are (8,0) and (0,-6). 1 mark

Ans10. Option (b)

A funnel is a combination of frustum of a cone and a cylinder. 1 mark

SECTION B

Ans11. The given quadratic equation is $2x^2 - \sqrt{5}\,x - 2 = 0$.

$b^2 - 4ac = 5 - 4 \text{ x } 2 \text{ x } (-2) = 21$ 1 mark

The roots of the given equation are given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{5} \pm \sqrt{21}}{4}$$

Thus, the roots of the given equation are $\frac{\sqrt{5}+\sqrt{21}}{4}$ and $\frac{\sqrt{5}-\sqrt{21}}{4}$ 1 mark

Ans12. In given figure, O is the centre of the circle.

Therefore, $\angle OQL = \angle ORM = 90^o$ (radius is perpendicular to tangent at the point of contact)

$\angle OSQ = \angle OQS = 90° - 50° = 40°$ 1 mark

$\angle RSO = \angle SRO = 90° - 60° = 30°$ $\frac{1}{2}$ mark

Thus , $\angle QSR = \angle OSQ + \angle OSR = 40° + 30° = 70°$ $\frac{1}{2}$ mark

Ans13.

S = [(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4), (3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)]

Total number of outcomes when two dice are thrown = 6x6= 36 $\frac{1}{2}$ mark

Let A be the event of getting a doublet of even number. Doublets of even number are (2,2),(4, 4),(6,6).

Number of favourable outcomes =3 $\frac{1}{2}$ mark

$P(A) = \frac{3}{36} = \frac{1}{12}$ 1mark

OR

S=[HHH,HHT,HTH,THH,HTT,THT,TTH,TTT]

Total number of outcomes = 8 $\frac{1}{2}$ mark

Outcomes of getting at least two heads = HHH,HHT,HTH,THH

Number of favourable outcomes =4 $\frac{1}{2}$ mark

P(getting at least two heads) = $\frac{4}{8} = \frac{1}{2}$ 1mark

Ans14. We know that three terms p,q,r form consecutive terms of AP if and only if 2q = p+r

Thus, 2k + 7, 6k – 2 and 8k + 4 will form consecutuve terms of an AP is 2(6k-2) = (2k+7) + (8k+4) 1mark

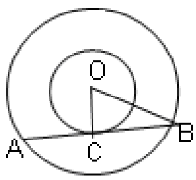
Now, 2(6k-2) = (2k+7) + (8k+4)

$\Rightarrow 12k - 4 = 10k + 11$

$\Rightarrow 2k = 15$

$\Rightarrow k = \frac{15}{2}$ 1 mark

Ans15.



Let AB = c be a chord of the larger circle, of diameter d_2, which touches the other circle at C. Then ΔOCB is a right triangle. $\frac{1}{2}$ mark

By Pythagoras theorem,

$OC^2 + BC^2 = OB^2$

i.e. , $\left(\frac{1}{2}d_1\right)^2 + \left(\frac{1}{2}c\right)^2 = \left(\frac{1}{2}d_2\right)^2$ (as C bisects AB) 1mark

Therefore, $d_2^2 = c^2 + d_1^2$ $\frac{1}{2}$ mark

Ans16. Let D be the mid point of the side BC. Then, the coordinates of D are

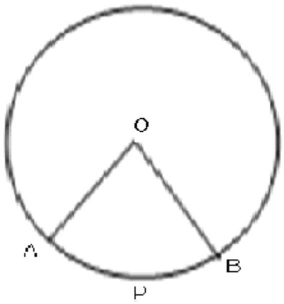
$\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = (4,1)$ 1mark

Therefore, length of median AD is given by:

AD = $\sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = 5$units

1 mark

Ans17.



Area of sector OAPB = $\frac{\pi r^2 \theta}{360} = \frac{22}{7} \times \frac{10x10x90}{360} = \frac{550}{7}$ 1mark

Area of major sector = area of circle – area of sector OAPB

$= \pi r^2 - \frac{550}{7}$

$= \frac{22}{7} x10x10 - \frac{550}{7}$

$= (\frac{2200}{7} - \frac{550}{7}) = \frac{1650}{7} cm^2$ 1mark

Ans18. Required shaded region = area of two quadrants – area of square

$= 2(\frac{1}{4}\pi 8^2) - 8x8$ 1 mark

$= \frac{1}{2}(\frac{22}{7} x\ 64) - 64$

$= \frac{4}{7} x\ 64 = \frac{256}{7} cm^2$ 1mark

SECTION C

Ans19. The given quadratic equation is $2x^2-10x+k=0$.

Here, a=2, b=-10 and c=k

Therefore, $D=b^2-4ac = (-10)^2 - 4x2xk = 100-8k$ $1\frac{1}{2}$ marks

The equation will have real and equal roots, if

$D=0 \Rightarrow 100-8k = 0 \Rightarrow K=\frac{100}{8} = \frac{25}{2}$ $1\frac{1}{2}$ marks

OR

The given quadratic equation is $\left(\frac{4x-3}{2x+1}\right) - 10\left(\frac{2x+1}{4x-3}\right)=3.$

Let $\frac{2x+1}{4x-3} = y$

the given equation becomes $\frac{1}{y}-10y = 3$ $\frac{1}{2}$ mark

$\Rightarrow 10y^2+3y-1=0$
$\Rightarrow 10y^2+5y-2y-1=0$
$\Rightarrow 5y(2y+1) - 1(2y+1) =0$
$\Rightarrow (2y+1)(5y-1)=0$
$\Rightarrow y = -\frac{1}{2}, \frac{1}{5}$ $1\frac{1}{2}$ marks

Hence, $\frac{2x+1}{4x-3} = -\frac{1}{2}$ or $\frac{2x+1}{4x-3} = \frac{1}{5}$

$x=\frac{1}{8}$ or $x=\frac{-4}{3}$ 1 mark

Ans20. In a non- leap year, there are 365 days, i.e. 52 weeks.

52 weeks = 364 days

1year = 52weeks and 1 day

This extra one day can be mon,tue,wed,thu,fri,sat,or sun. 1 mark

Total number of outcomes = 7

Number of favourable outcomes =1 1 mark

P(having 53 Thursdays) $=\frac{1}{7}$ 1 mark

Ans21. If three points A, B and C are collinear, then the area of triangle ABC = 0. 1 mark

$\therefore \frac{1}{2}[7(1-k)+5(k+2)+3(-2-1) = 0$ 1 mark

$7-7k+5k+10-9=0$

$-2k+8=0$

k=4

Thus, the given points are collinear for k=4. 1 mark

Ans22. We know that the diagonals of a parallelogram bisect each other, i.e., the mid point of AC will be the same as that of BD. $\frac{1}{2}$ mark

$\therefore \left[\left(\frac{-2+4}{2}\right),\left(\frac{-1+b}{2}\right)\right] = \left[\left(\frac{a+1}{2}\right),\left(\frac{0+2}{2}\right)\right]$ 1 mark

$\left[1,\left(\frac{b-1}{2}\right)\right] = \left[\left(\frac{a+1}{2}\right),1\right]$

$\left(\frac{a+1}{2}\right) = 1$ and $\left(\frac{b-1}{2}\right) = 1$

a = 1 and b = 3 $1\frac{1}{2}$ marks

OR

Coordinates of the mid point of the line segment joining A (3,4) and B (k,6)

$= \left(\frac{3+k}{2},\frac{4+6}{2}\right) = \left(\frac{3+k}{2},5\right)$ 1mark

$\therefore \left(\frac{3+k}{2},5\right) = (x,y)$

$\left(\frac{3+k}{2}\right) = x$ and $5 = y$

Since, x+y-10=0

So, $\frac{3+k}{2} + 5 - 10 = 0$ 1mark

3+k=10

k = 7 1 mark

Ans23. Let a – d, a and a + d be three terms in AP.

According to the question,

a – d + a + a + d = 3

3a = 3 or a = 1 1 mark

(a – d) (a) (a + d) = -8

$a(a^2 - d^2) = -8$ 1 mark

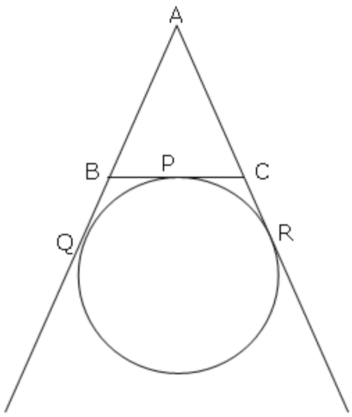
Putting the value of a = 1, we get,

$1 - d^2 = -8$

$d^2 = 9$ or $d = \pm 3$

Thus, the required three terms are -2, 1, 4 or 4,1,-2. 1 mark

Ans24.



BQ = BP (lengths of tangents drawn from an external point to a circle are equal)

Similarly, CP=CR , and AQ=AR 1 mark

2AQ= AQ+AR

= (AB+BQ) + (AC+CR)

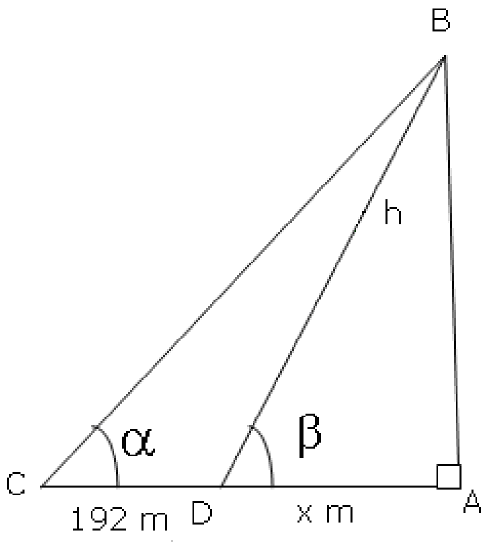
= AB+BP+AC+CP 1 mark

= (BP+CP) +AC+AB

2AQ = BC+CA+AB

$\therefore AQ = \frac{1}{2}(BC+CA+AB)$ 1 mark

Ans25.



$\frac{1}{2}$ mark

Let AB be the tower of height h metres. Let AD=x metres, CD=192 metres.

$\tan\alpha = \frac{5}{12}$, $\tan\beta = \frac{3}{4}$

In ΔBAC,

$\tan\alpha = \frac{AB}{AC} \Rightarrow \frac{5}{12} = \frac{h}{(x+192)}$ .... (i) 1mark

In ΔDAB,

$\tan\beta = \frac{AB}{AD} \Rightarrow \frac{3}{4} = \frac{h}{x}$ or $x = \frac{4h}{3}$ .... (ii) $\frac{1}{2}$ mark

Using (ii) in (i)

$$\frac{5}{12} = \left(\frac{h}{192 + \frac{4h}{3}}\right)$$

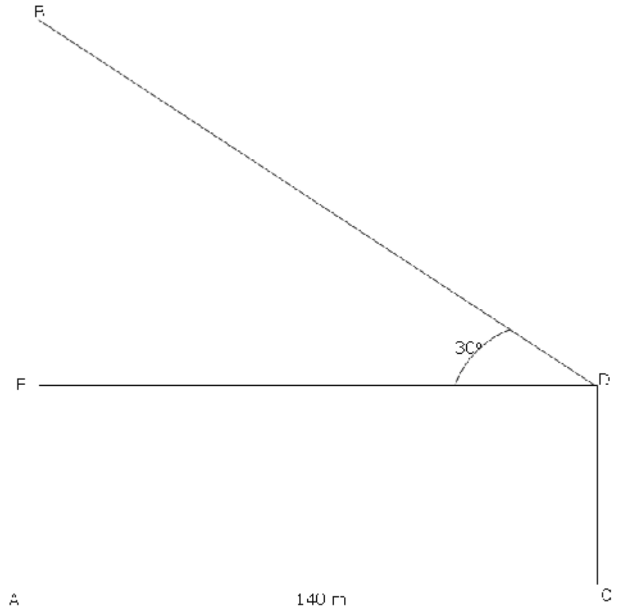
$$5\left(192 + \frac{4h}{3}\right) = 12h$$

$2880 + 20h = 36h$

$16h = 2880$ or $h = 180$

Hence, the height of the tower is 180 metres. 1mark

OR



$\frac{1}{2}$ mark

Let AB and CD be two towers of height h m and 60 m respectively.

AC=140m and $\angle BDE = 30^0$.

In ΔDEB,

$\tan 30° = \frac{BE}{DE}$

$\frac{1}{\angle 3} = \frac{BE}{140}$ (DE=AC=140m)

$BE = \frac{140}{\sqrt{3}} = 80.83m$ $1\frac{1}{2}$ marks

Thus, the height of the first tower is

AB= AE+BE = CD+BE =60+80.83 = 140.83m 1 mark

Ans26. Area of one design $= \frac{\pi r^2 \theta}{360} = \frac{60}{360} \times \pi \times 28^2$ – area of ΔOAB

$= \pi \times \frac{28^2}{6} - \frac{\sqrt{3}}{4} \times 28^2$

$= 28^2 \left(\frac{11}{21} - \frac{17}{40}\right) cm^2$ 2 marks

Total cost of making the design = Rs 6x $28^2 \left(\frac{11}{21} - \frac{17}{40}\right)$ x 0.35

= Rs 28x28x$\frac{83}{400}$ = Rs. 162.68 1 mark

Ans27. The cone of maximum size that is carved out from a cube of edge 14 cm will be of base radius 7 cm and the height 14 cm.

Surface area of the cone = $\pi rl + \pi r^2$

$= \frac{22}{7} \times 7 \times \sqrt{7^2 + 14^2} + \frac{22}{7} (7)^2$

$= \frac{22}{7} \times 7 \times \sqrt{245} + 154 = \left(154\sqrt{5} + 154\right) cm^2 = 154\left(\sqrt{5} + 1\right) cm^2$ $1\frac{1}{2}$ marks

Surface area of the cube = 6 x $(14)^2$ = 6 x 196 = 1176 cm^2

$\frac{1}{2}$ mark

So, surface area of the remaining solid left out after the cone is carved out

$= \left(1176 - 154 - 154\sqrt{5}\right) cm^2 = \left(1022 - 154\sqrt{5}\right) cm^2$ 1 mark

Ans28. Volume of water that flows in the canal in one hour = width of the canal x depth of the canal x speed of the canal water = 3 x 1.2 x 20 x 1000m^3 = 72000m^3 1 mark

In 20 minutes the volume of water in the canal = 72000 x $\frac{20}{60}$ m^3 = 24000m^3 1 mark

Area irrigated in 20 minutes, if 8 cm, i.e., 0.08 m standing water is required

$= \frac{24000}{0.08}$ m^2 = 300000 m^2 = 30 hectares 1 mark

SECTION D

Ans29. Let the original average speed of the train be x km/h. Therefore,

$\frac{63}{x} + \frac{72}{x+6} = 3$ 1 mark

$\frac{7}{x} + \frac{8}{x+6} = \frac{3}{9} = \frac{1}{3}$

$\frac{7(x+6) + 8x}{x(x+6)} = \frac{1}{3}$

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$21(x + 6) + 24x = x(x + 6)$

$21x + 126 + 24x = x^2 + 6x$

$x^2 - 39x - 126 = 0$ 1 mark

$(x + 3)(x - 42) = 0$

$x = -3$ or $x = 42$ $1\frac{1}{2}$ marks

Since x is the average speed of the train, x cannot be negative.

Therefore, $x = 42$

So, the original average speed of the train is 42 km/h. $\frac{1}{2}$ mark

OR

Suppose B alone takes x days to finish the work. Then, A alone can finish it in $(x - 6)$ days.

Now, (A's one day's work) + (B's one day's work) $= \frac{1}{x} + \frac{1}{x-6}$

(A + B)'s one day's work $= \frac{1}{4}$

Therefore, $\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$ 1 mark

$$\frac{x - 6 + x}{x(x-6)} = \frac{1}{4}$$

$8x - 24 = x^2 - 6x$

$x^2 - 14x + 24 = 0$ 1 mark

$x^2 - 12x - 2x + 24 = 0$

$(x - 12)(x - 2) = 0$

$x = 12$ or $x = 2$ $1\frac{1}{2}$ marks

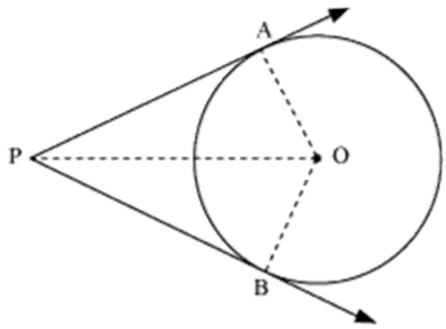
But, x cannot be less than 6. So, $x = 12$.

Hence, B alone can finish the work in 12 days. $\frac{1}{2}$ mark

Ans30. Given: A circle with centre O; PA and PB are two tangents to the circle drawn from an external point P.

To prove: PA = PB

Construction: Join OA, OB, and OP.



2 marks

It is known that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OA $\perp$ PA and OB $\perp$ PB ... (1)

In ΔOPA and ΔOPB:

$\angle$OAP = $\angle$OBP (Using (1))

OA = OB (Radii of the same circle)

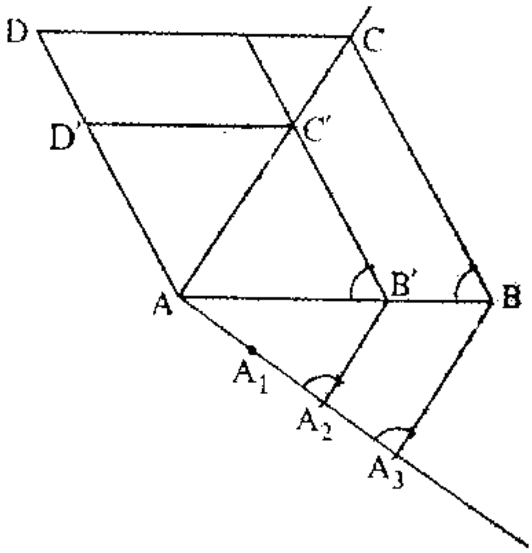
OP = PO (Common side)

Therefore, ΔOPA $\cong$ ΔOPB (RHS congruency criterion)

$\therefore$PA = PB (Corresponding parts of congruent triangles are equal)

Thus, it is proved that the lengths of the two tangents drawn from an external point to a circle are equal. 2 marks

Ans31.



3 marks

From the figure

$$\frac{AB'}{AB} = \frac{2}{3} = \frac{A'C'}{AC}$$

Also, $\frac{AC'}{AC} = \frac{C'D'}{CD} = \frac{AD'}{AD} = \frac{2}{3}$

Therefore, AB' =B'C' =C'D' =AD' = $\frac{2}{3}$AB

Thus, AB'C'D' is a rhombus. 1 mark

Ans32. Let the sum of first n terms be 45. Then,

$S_n = 45$

$\Rightarrow S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \frac{n}{2}[2 \times 13 + (n-1)(-2)] = 45$ $1\frac{1}{2}$ marks

$\Rightarrow \frac{n}{2}[26-2(n-1)] = 45$

$\Rightarrow 13n - n(n-1) = 45$

$\Rightarrow n^2 - 14n + 45 = 0$ 1 mark

$\Rightarrow (n-9)(n-5) = 0$

$\Rightarrow n = 5$ or 9 1 mark

Hence, the sum of first 5 or the first 9 terms is 45. $\frac{1}{2}$ mark

Ans33. Capacity (or volume) of the container = $\frac{\pi h}{3}[r^2_1 + r^2_2 + r_1r_2]$

Here, h = 30 cm, r_1 = 20 cm and r_2 = 10 cm

So, the capacity of container = $3.14 \times \frac{30}{3}[20^2 + 10^2 + 20 \times 10]\ cm^3$ = 21.980 liters

$1\frac{1}{2}$ marks

Cost of 1 litre of milk = Rs 25

Cost of 21.980 litres of milk = Rs 21.980 x 25 = Rs 549.50 $\frac{1}{2}$ mark

Surface area of the bucket = curved surface area of the bucket + surface area of the bottom

$= \pi l(r_1 + r_2) + \pi r^2_2$

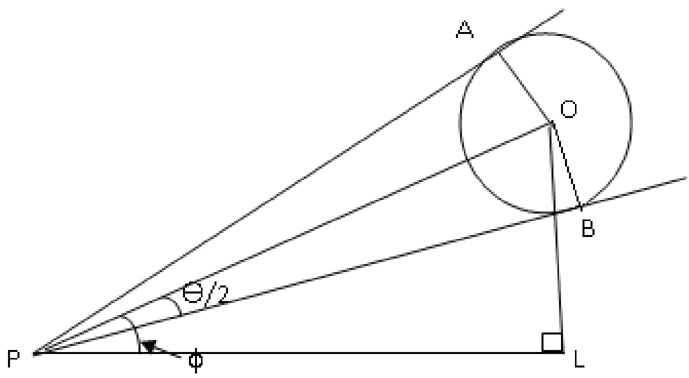
Now, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$l = \sqrt{900 + 100}$ cm = 31.62 cm $\frac{1}{2}$ mark

Therefore, surface area of the bucket = 3.14 x 31.62 (20+10) + 3.14x$(10)^2$

= 3.14 x 1048.6 cm^2 = 3292.6 cm^2(approx.) $1\frac{1}{2}$ marks

Ans34.



1 mark

Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA and PB be tangents from P to the balloon. $\angle APB = \theta$. Therefore, $\angle APO = \angle BPO = \frac{\theta}{2}$

Let OL be perpendicular from O to the horizontal.

$\angle OPL = \phi$. (1 mark)

In ΔOAP,

$$\sin\frac{\theta}{2} = \frac{OA}{OP} = \frac{r}{OP}$$

$\Rightarrow OP = r \operatorname{cosec}\frac{\theta}{2}$ ...(i) (1 mark)

In ΔOPL,

$$\sin\phi = \frac{OL}{OP}$$

$\Rightarrow OL = OP \sin\phi$

$\Rightarrow OL = r \sin\phi \operatorname{cosec}\frac{\theta}{2}$ (from (i)) (1 mark)

Thus, the height of the centre of the balloon is $r \sin\phi \operatorname{cosec}\frac{\theta}{2}$.