Sample Paper – III

Mathematical Methods in Engineering & Science

Attempt All Questions | **Max Marks-100**

Each Question Carries Equal Marks | **Time: 3 Hrs.**

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1. Given any constants a_0, b_0, find every solution of the differential equation

$$y' = a_0 y + b_0 y^3.$$

2. Find the series solution of the following IVPs

(i) $\ddot{x} + t\dot{x} - 2x = 0, \quad x(0) = 1, \quad \dot{x}(0) = 0$

(ii) $t(2-t)\ddot{x} - 6(t-1)\dot{x} - 4x = 0, \quad x(1) = 1, \quad \dot{x}(1) = 0$

(iii) $\ddot{x} + e^t\dot{x} + (1+t^2)x = 0, \quad x(0) = 1, \quad \dot{x}(0) = 0$

(iv) $\ddot{x} - (\sin t)x = 0, \quad x(\pi) = 1, \quad \dot{x}(\pi) = 0$

3. Find every solution of the equation $t\,y' = 3y + t^5\,y^{1/3}$.

4. Let $\mathbf{v} = (1,0,0)$ and $\mathbf{w} = (a,0,0)$ be vectors in $\mathbb{R}^3$. Show that $\|\mathbf{w}\| = |a|\,\|\mathbf{v}\|$.

5. Let $\mathbf{v} = (a,b,c)$ and $\mathbf{w} = (3a,3b,3c)$ be vectors in $\mathbb{R}^3$. Show that $\|\mathbf{w}\| = 3\,\|\mathbf{v}\|$.

6. Find the distance between the parallel planes $10x+2y-2z = 5$ and $5x+y-z = 1$.

7. The block B has eigenvalues $1, 2$ and C has eigenvalues $3, 4$ and D has eigenvalues $5, 7$. Find the eigenvalues of the 4 by 4 matrix A:

$$A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}.$$

8. Find the rank and the four eigenvalues of A and C:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

9. (a) Find the eigenvalues and eigenvectors of A. They depend on c:

$$A = \begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix}.$$

(b) Show that A has just one line of eigenvectors when $c = 1.6$.

(c) This is a Markov matrix when $c = .8$. Then A^n will approach what matrix A^∞?

10. Find the complex Fourier series of the function

$$f(t) = \begin{cases} 0 & -\pi < t < 0, \\ 1 & 0 < t < \pi. \end{cases}$$