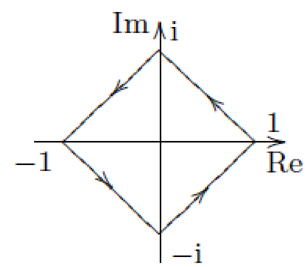
Sample Paper – I
Mathematical Methods in Engineering & Science

1. The figure on the right shows a closed curve α, Give an explicit parametrization for α and calculate

$$\frac{1}{2\pi\mathrm{i}} \int_\alpha \frac{1}{z}\, dz \ .$$



2. Let $\alpha : [0, \pi] \to \mathbb{C}$ be defined by

$$\alpha(t) := \exp(\mathrm{i}t)$$

and $\beta : [0, 2] \to \mathbb{C}$ by

$$\beta(t) = \begin{cases} 1 + t(-\mathrm{i} - 1) & \text{for } t \in [0, 1] \ , \\ 1 - t + \mathrm{i}(t - 2) & \text{for } t \in [1, 2] \ . \end{cases}$$

Sketch α and β, and calculate

$$\int_\alpha \frac{1}{z}\, dz \qquad \text{and} \qquad \int_\beta \frac{1}{z}\, dz \ .$$

3. Let $a \in \mathbb{C}$, $\varepsilon > 0$. The punctured disk

$$\dot{U}_\varepsilon(a) := \{ z \in \mathbb{C} \ ; \quad 0 < |z - a| < \varepsilon \} \ ,$$

is a domain.

Deduce: If $D \subset \mathbb{C}$ is a domain and $z_1, \ldots, z_m$ are finitely many points, then the set $D' := D \setminus \{z_1, \ldots, z_m\}$ is also a domain.

4. Let $\emptyset \neq D \subset \mathbb{C}$ be open. The continuous function

$$f : D \longrightarrow \mathbb{C} \ , \quad z \longmapsto \overline{z} \ ,$$

5. Compute

$$\int_\alpha z \exp(z^2)\, dz \ ,$$

where

(a) α is the line between the point 0 and the point $1 + \mathrm{i}$,
(b) α is the piece of the parabola with equation $y = x^2$, which lies between the points 0 and $1 + \mathrm{i}$.

6. Compute

$$\int_\alpha \sin z \, dz \ ,$$

where α is the piece of the parabola with equation $y = x^2$, which lies between the points 0 and $-1 + \mathrm{i}$.

For Exercises 7-8, find the distance d from the point P to the line L.

7. $P = (1, -1, -1)$, $L: x = -2 - 2t,\ y = 4t,\ z = 7 + t$

8. $P = (0, 0, 0)$, $L: x = 3 + 2t,\ y = 4 + 3t,\ z = 5 + 4t$

Exercise 9

Find the rank of each of the following matrices

(a)

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{pmatrix}$$

(b)

$$B = \begin{pmatrix} 3 & 1 & 0 & 1 & -9 \\ 0 & -2 & 12 & -8 & -6 \\ 2 & -3 & 22 & -14 & -17 \end{pmatrix}$$

Exercise 10

Write down the inverses of the following elementary matrices:

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{pmatrix}, E_3 = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$