

## Illustration-Solution

### Illustration-1

The two sides of a wall (2 mm thick, with a cross-sectional area of 0.2 m<sup>2</sup>) are maintained at 30°C and 90°C. The thermal conductivity of the wall material is 1.28 W/(m·°C). Find out the rate of heat transfer through *the wall*?

### Solution-1

**Assumptions:-**

1. Steady-state one-dimensional conduction
2. Thermal conductivity is constant for the temperature range of interest
3. The heat loss through the edge side surface is insignificant
4. The layers are in perfect thermal contact

Given,

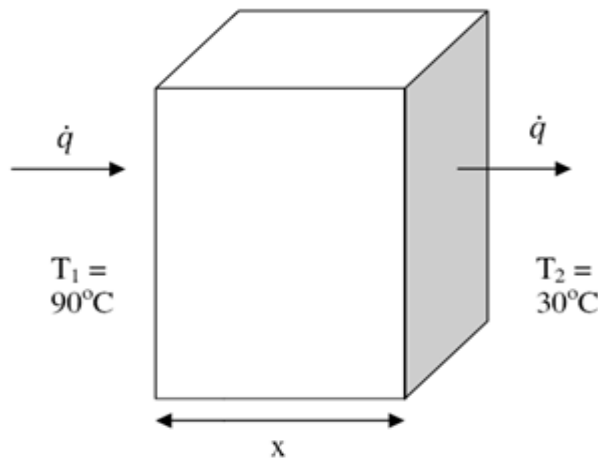


Fig. Illustration

$$k = 1.28 \text{ W/(m}\cdot^\circ\text{C)}$$

$$A = 0.2 \text{ m}^2$$

$$x = 2 \text{ mm} = 0.002 \text{ m}$$

The rate of heat transfer can be written as,

$$\dot{q} = \frac{\Delta T}{x/kA}$$

$$\dot{q} = \frac{90 - 30}{0.002 / 1.28 \times 0.2} = 7680 \text{ W}$$

### **Illustration-2**

*One side of a 1 cm thick stainless steel wall ( $k_1 = 19 \text{ W/m}\cdot^\circ\text{C}$ ) is maintained at  $180^\circ\text{C}$  and the other side is insulated with a layer of 4 cm fibreglass ( $k_2 = 0.04 \text{ W/m}\cdot^\circ\text{C}$ ). The outside of the fibreglass is maintained at  $60^\circ\text{C}$  and the heat loss through the wall is 300 W. Determine the area of the wall?*

### **Solution-2**

#### **Assumptions:-**

1. Steady-state one-dimensional conduction.
2. Thermal conductivity is constant for the temperature range of interest.
3. The heat loss through the edge side surface is insignificant.
4. The layers are in perfect thermal contact.

Given,  $k_1 = 19 \text{ W/m}\cdot^\circ\text{C}$        $k_2 = 0.04 \text{ W/m}\cdot^\circ\text{C}$        $\dot{q} = 300 \text{ W}$

$x_1 = 1 \text{ cm} = 0.01 \text{ m}$        $x_2 = 4 \text{ cm} = 0.04 \text{ m}$

The resistance of the above composite,

$$R = \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}$$

### Illustration-3

Consider a composite wall containing 5-different materials as shown in the fig. 2.7. Calculate the rate of heat flow through the composite from the following data?

$$x_1 = 0.1 \text{ m}$$

$$x_2 = 0.2 \text{ m}$$

$$x_3 = 0.15 \text{ m}$$

$$k_1 = 15 \text{ W/m}^\circ\text{C}$$

$$k_2 = 25 \text{ W/m}^\circ\text{C}$$

$$k_3 = 30 \text{ W/m}^\circ\text{C}$$

$$k_4 = 20 \text{ W/m}^\circ\text{C}$$

$$k_5 = 35 \text{ W/m}^\circ\text{C}$$

$$h_2 = 1 \text{ m}$$

$$h_3 = 3 \text{ m}$$

$$h_4 = 2.5 \text{ m}$$

$$h_5 = 1.5 \text{ m}$$

$$T_A = 120^\circ\text{C}$$

$$T_B = 50^\circ\text{C}$$

### Solution-3

#### **Assumptions:**

1. Steady-state one-dimensional conduction.
2. Thermal conductivity is constant for the temperature range of interest.
3. The heat loss through the edge side surface is insignificant.
4. The layers are in perfect thermal contact.
5. Area in the direction of heat flow is  $1 \text{ m}^2$ .

The height of the first layer is 4 m ( $h_1 = h_2 + h_3$ ).

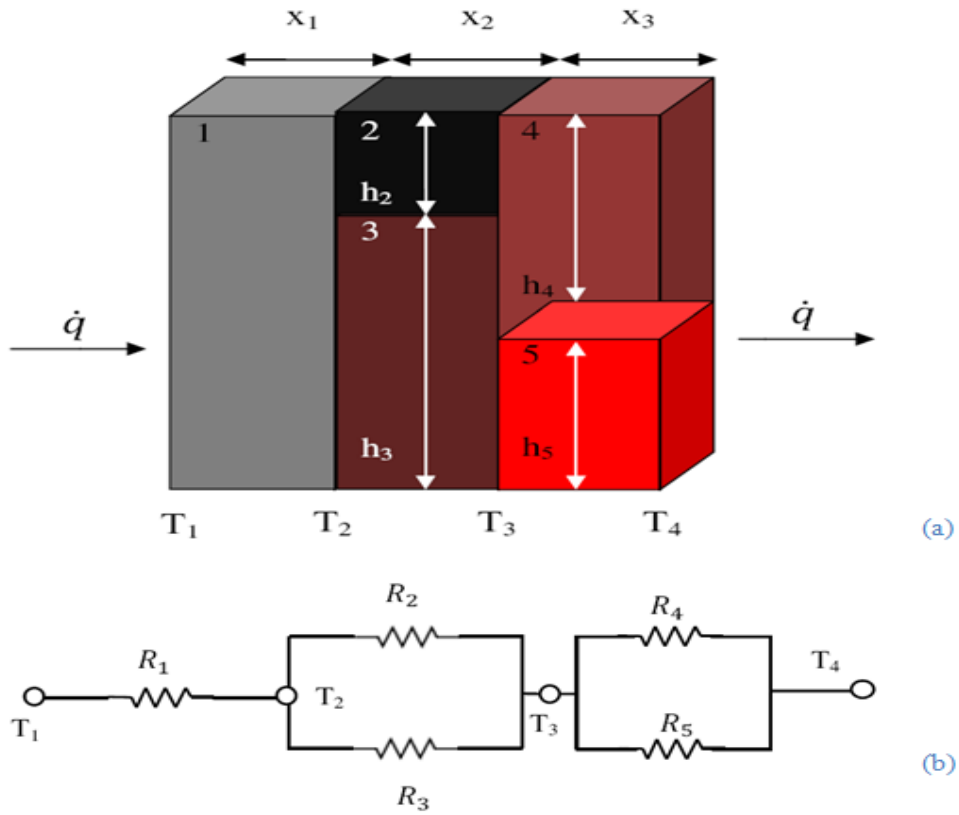
The equivalent circuit diagram of the above composite is,

$$R = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} + \frac{1}{\frac{1}{R_4} + \frac{1}{R_5}}$$

On calculating equivalent resistance with the given data (Note: thickness of layer 2 = thickness of layer 3 and thickness of layer 4 = thickness of layer 5, in the heat flow direction),

$$R = \frac{0.1}{(15)(1)} + \frac{1}{\left(\frac{1}{\left(\frac{0.2}{25\left(\frac{1}{4}\right)}\right)} + \frac{1}{\left(\frac{0.2}{30\left(\frac{3}{4}\right)}\right)}\right)} + \frac{1}{\left(\frac{1}{\left(\frac{0.15}{20\left(\frac{2.5}{4}\right)}\right)} + \frac{1}{\left(\frac{0.15}{35\left(\frac{1.5}{4}\right)}\right)}\right)}$$

$$R = 0.0195 \frac{^\circ\text{C}}{\text{W}}$$



**Fig. 3: Composite of illustration (a) composite, (b) corresponding electrical circuit**

Thus the heat flow rate through the composite,

$$\dot{q} = \frac{T_A - T_B}{R}$$

$$\dot{q} = \frac{120 - 50}{0.0195} = 3.59 \text{ kW}$$

#### Illustration-4

The steady state temperature distribution in a wall is , where  $x$  (in meter) is the position in the wall and  $T$  is the temperature (in °C). The thickness of the wall is 0.2 m and the thermal conductivity of the wall is 1.2 (W/m·°C). The wall dissipates the heat to the ambient at 30 °C. Calculate the heat transfer coefficient at the surface of the wall at 0.2 m.

#### Solution-4

The rate of heat transfer through the wall by conduction will be equal to the rate of heat transfer from the surface to the ambient by convection at steady state,

Rate of heat transfer by conduction at  $x=0.2$  is given by,

$$-kA \frac{dT}{dx} = h A (T_x - T_a)$$

where  $T_a$  is the ambient temperature.

$$T = 300 - 3050x^2$$

$$T_{x=0.2} = 300 - 3050(0.2^2) = 178 \text{ °C}$$

$$\frac{dT}{dx} = -6100x$$

$$h = \frac{-kA \frac{dT}{dx}}{A(T_x - T_a)} \\ = \frac{(-k)(-6100x)}{T_x - T_a}$$

On putting the values and solving,

$$= \frac{(1.2)(6100)(0.2)}{178-30}$$

$$h = 9.89 \text{ W/(m}^2\text{·°C)}$$

### **Illustration-5**

Warm methanol flowing in the inner pipe of a double pipe heat exchanger is being cooled by the flowing water in the outer tube of the heat exchanger. The thermal conductivity of the exchanger, inner and outer diameter of the inner pipe are 45 W/(m·°C), 26 mm, and 33 mm, respectively. The individual heat transfer coefficients are:

	Coefficient (W/(m <sup>2</sup> ·°C))
Methanol, $h_i$	1000
Water, $h_o$	1750

Calculate the overall heat transfer coefficient based on the outside area of the inner tube.

### **Solution-5**

Using following equation,

$$U_o = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o \ln(r_o/r_i)}{k} + \frac{1}{h_o}}$$

It is apparent that all the values are known. Thus, on putting the values the  $U_o$  is 519 W/(m<sup>2</sup>·°C).

### Illustration-6

A steel pipe having inner diameter as 78 mm and outer diameter as 89 mm has 10 external longitudinal rectangular fins of 1.5 mm thickness. Each of the fins extends 30 mm from the pipe. The thermal conductivity of the fin material is 50 W/m °C. The temperature of the pipe wall and the ambient are 160 °C, and 30 °C, respectively, whereas the surface heat transfer coefficient is 75 W/m<sup>2</sup> °C. What is the percentage increase in the rate of heat transfer after the fin arrangement on the plane tube?

### Solution-6

As the fins are rectangular, the perimeter of the fin,  $P = 2(b + t)$ . The thickness ( $t$ ) of the fin is quite small as compared to the width ( $b$ ) of the fin. Thus,  $P = 2b$  as well as we may assume that there is no heat transfer from the tip of the fin. Under such condition we can treat it as case-III, where there was no heat transfer to the atmosphere due to insulated fin tip.

Using eq. 3.30,

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{75 \times 2b}{50 \times t \times b}} = 44.7$$

Thus,

$$\eta_f = \frac{\tanh ml}{ml} = \frac{\tanh(44.7 \times 0.03)}{44.7 \times 0.03} = 0.65$$

As the pipe length is not given, we will work-out considering the length of the pipe as 1 m and henceforth the breadth of the fins should also be considered as 1 m. We have to consider the area of the fins in order to consider the heat dissipation from the fins. However, we may neglect the fin area at the y-z plane and x-y plane (refer fig. 3.8) as compared to the area of x-z plane.

The area of all the fins = (number of fins) (2 faces) (1) (0.03) = 0.6 m<sup>2</sup>

The maximum rate of heat transfer from the fins

$$h A \Delta T = 75 \times 0.6 \times (160 - 30) = 5850 \text{ W}$$

$$\text{Actual rate of heat transfer} = \eta_f \times 5850 = 3802.5 \text{ W}$$

The total rate of heat transfer from the finned tube will be the sum of actual rate of heat transfer from the fins and the rate of heat transfer from the bare pipe, the pipe portion which is not covered by the fins. Therefore, the remaining area will be calculated as follows,

The remaining area = Total pipe area - base area covered by the 10 fins

Pipe are =

$$2\pi \text{ outer radius} \times 1 = 2\pi \left(\frac{89}{2} 10^{-3}\right)(1) = 0.28 \text{ m}^2$$

Attached area of 10 fins = (10) (1) (0.0015) = 0.015 m<sup>2</sup>

The remaining area comes out to be (0.28 - 0.015) = 0.265 m<sup>2</sup>

The corresponding heat transfer =  $(75)(0.265)(160-30) = 2583.75 \text{ W}$

The total heat transfer from the finned tube =  $3802.5 + 2583.75 = 6386.25 \text{ W}$

Rate of heat transfer from the tube if it does not have any fins =  $(75)(0.28)(160-30) = 2730 \text{ W}$

The percentage increase in the heat transfer =  $\frac{6386.25-2730}{2730} = 134\%$



### Illustration-7

Pressurized air is to be heated by flowing into a pipe of 2.54 cm diameter. The air at 200°C and 2 atm pressure enters in the pipe at 10 m/s. The temperature of the entire pipe is maintained at 220°C. Evaluate the heat transfer coefficient for a unit length of a tube considering the constant heat flux conditions are maintained at the pipe wall. What will be the bulk temperature of the air at the end of 3 m length of the tube?

The following data for the entering air (at 200°C) has been given,

Pr number	0.681
Viscosity	$2.57 \times 10^{-5} \text{ kg/m s}$
Thermal conductivity	$0.015 \text{ W/m } ^\circ\text{C}$
Density	$1.493 \text{ kg/m}^3$
$c_p$	$1.025 \text{ kJ/kg } ^\circ\text{C}$

### Solution-7

Reynolds number can be calculated from the above data,

$$Re = \frac{d v \rho}{\mu} = 14756$$

The value of Reynolds number shows that the flow is in turbulent zone. Thus the Dittus-Boelter equation (eq.4.3) should be used,

$$Nu = 0.023 Re^{0.8} Pr^n$$

$n = 0.4$  as the air is being heated,

$$\frac{h d}{k} = 0.023 (14756)^{0.8} (0.681)^{0.4} = 42.67$$

Thus  $h$  can be calculated for the known values of  $k$ , and  $d$ , which comes out to be

$$h = 25.20 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

Energy balance is required to evaluate the increase in bulk temperature in a 3 m length of the tube,

$$h(\pi d \, dL)(220 - T) = \dot{m}c_p dT$$

$$\int_{200}^T \frac{dT}{220 - T} = \frac{h\pi d}{\dot{m}c_p} \int_0^L dL$$

$$= \frac{h \pi d L}{\rho v \frac{\pi d^2}{4} c_p}$$

$$= \frac{4hL}{\rho c_p v d}$$

$$= \frac{(4)(25.2)(3)}{(1.493)(1025)(10)(0.0254)}$$

$$= 0.778$$

$$[\ln(220 - T)]_T^{200} = 0.778$$

$$\ln 20 - \ln(220 - T) = 0.778$$

$$T = 210.81 \, ^\circ\text{C}$$

Therefore the temperature of the air leaving the pipe will be at 210.81°C.

### Illustration-8

A hot oven is maintained at 180 °C having vertical door 50 cm high is exposed to the atmospheric air at 20°C. Calculate the average heat transfer coefficient at the surface of the door.

The various air properties at the average temperature  $[(180+20)/2 = 100^\circ\text{C}]$  are,

$k = 0.032 \text{ W/m } ^\circ\text{C}; \quad Pr = 0.7; \quad \text{Kinematic viscosity} = 24 \times 10^{-6} \text{ m}^2/\text{s}$

At  $T_b = 20^\circ\text{C}$ ,  $\beta = \frac{1}{293 \text{ K}}$

### Solution-8

First we have to find the Grashof number,

$$Gr_L = \frac{g\beta(T_s - T_b)L^3}{\nu^2} = \frac{9.8 \times \left(\frac{1}{293}\right) \times (180 - 20) \times 0.5^3}{(24 \times 10^{-6})^2} = 1.16 \times 10^8$$

With the help of  $Gr$  and  $Pr$ , we can estimate the  $Ra$  number,

$$Ra = GrPr = 1.16 \times 10^8 \times 0.7 = 8.12 \times 10^7$$

As  $Ra < 10^9$ , the eq.5.7 can be used,

$$\overline{Nu}_u = 0.68 + \frac{0.67Ra^{1/4}}{\left[1 + \left(\frac{0.0492}{Pr}\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.67(8.12 \times 10^7)^{1/4}}{\left[1 + \left(\frac{0.0492}{0.7}\right)^{9/16}\right]^{4/9}} = 58.8$$

$$\overline{Nu}_u = \frac{\bar{h}L}{k} = 58.8$$

Thus,

$$\bar{h} = 3.76 \frac{\text{W}}{\text{m}^2 \text{ } ^\circ\text{C}}$$

### **Illustration-9**

In the oven door described in illustration 5.1 is subjected to an upward flow of air (that is forced convection). What would be the minimum free stream velocity for which natural convection may be neglected?

### **Solution-9**

For the following condition the effect of natural convection may be neglected,

$$\frac{Gr_L}{Re_L^2} \ll 1$$

The value of  $Gr$  number calculated in the previous illustration was  $1.16 \times 10^8$

Thus,

$$\frac{1.16 \times 10^8}{\left(\frac{LU}{\nu}\right)^2} \ll 1$$

$$U \gg 0.24 \text{ m/s}$$

Therefore, the bulk velocity of the air should be far greater than 0.24 m/s.

### **Illustration-10**

Saturated steam at 70.14 kPa is condensing on a vertical tube 0.5 m long having an outer diameter of 2.5 cm and a surface temperature of 80°C. Calculate the average heat-transfer coefficient.

### **Solution-10**

It is a problem of condensation on a vertical plate, thus eq.6.10b can be

$$h_{av} = \frac{1}{L} \int_0^L h dx = 0.943 \left[ \frac{g \lambda \rho_l (\rho_l - \rho_v) k_l^3}{\mu_l L (T_v - T_w)} \right]^{1/4}$$

used,

where, different liquid and steam properties are evaluate at average film temperature,

$$\frac{T_v + T_w}{2}$$

Using steam table, the temperature of the steam corresponding to 70.14kPa pressure is 90°C. The average film temperature will then be the average of 80 °C and 90 °C and it comes out to be 85 °C,

Using given data the different properties can be found using steam table and other relevant tables given in the standard literature. The data is tabulates below at 85°C,

Latent heat of steam ( $\lambda_v$ )	2651.9 kJ/kg	$\lambda = \lambda_v - \lambda_l$ = 2296 kJ/kg
Latent heat of water ( $\lambda_l$ )	355.9 kJ/kg	
Density of vapour ( $\rho_v$ )	1/ sp.vol.= 1/ 2.828 kg/m <sup>3</sup>	= 0.354 kg/m <sup>3</sup>
Density of water ( $\rho_l$ )	1/ sp.vol. = 1/ 0.0010325 kg/m <sup>3</sup>	= 968.5 kg/m <sup>3</sup>
Viscosity of water ( $\mu_w$ )	0.335 cP = .335 x 10 <sup>-3</sup> Pa s	
Thermal conductivity of water ( $k_l$ )	0.67	

On putting the above values in the above equation,

$$h_{av} = 1205.2 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

### **Illustration-11**

**The surface of a blackbody is at 500 K temperature. Obtain the total emissive power, the wavelength of the maximum monochromatic emissive power.**

### **Solution-11**

Using eq. 7.12, the total emissive power can be calculated,

$$E_b = \sigma T^4$$

where,  $\sigma$  ( $= 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$ ) is the Stefan-Boltzmann constant. Thus at 500 K,

$$E_b = (5.67 \times 10^{-8})(500^4) \text{ W/m}^2$$

$$E_b = 354.75 \text{ W/m}^2$$

The wavelength of the maximum monochromatic emissive power can be obtained from the Wien's law (eq. 7.11),

$$\lambda_{max} T = 2898$$

$$\lambda_{max} = \frac{2898}{500} = 5.796 \mu\text{m}$$

### **Illustration-12**

**A pipe having 10 cm of diameter is carrying saturated steam at 8 bar of absolute pressure. The pipe runs through a room. The wall of the room is at 300 °K. A portion around 1 m of the pipe insulation is damaged and exposed to the room atmosphere. Calculate the net rate of heat loss from the pipe by radiation.**

### **Solution-12**

The emissivity of the pipe surface is not given so it may be considered black. Moreover, since the room may be big compared to the surface area of the pipe, the room may also be considered to be a blackbody.

We can write  $F_{11} + F_{12} = 1$ .

The value of  $F_{11} = 0$ , as the pipe cannot see itself.

Thus  $F_{12}$ , the view factor (1-pipe, 2-room) will be 1.

The net rate heat loss due to radiation,

$$Q_{12} = A_{pipe} \epsilon_{pipe} F_{12} \sigma (T_{pipe}^4 - T_{room}^4)$$

$T_{pipe}$  can be obtained by the temperature of the steam at the prevailing pressure with the help of steam table = 450 K.

$$\sigma (= 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4)$$

On putting the value,

$$Q_{12} = \{\pi(0.1)(1)\}(1)(1)(5.67 \times 10^{-8})\{450^4 - 300^4\}$$

$$Q_{12} = 586 \text{ W}$$

### Illustration-13

A heat transfer fluid is leaving a reactor at a rate of 167 kg/s at 85°C. The fluid is to be cooled to 50°C before it can be recycled to the reactor. Water is available at 30°C to cool the fluid in a 1-2 pass heat exchanger having heat transfer area of 15 m<sup>2</sup>. The water, which is being used to cool the fluid, must not be heated to above 38°C at the exit of the heat exchanger. The overall heat transfer co-efficient of 400 Kcal/hm<sup>2</sup>°C can be used for the heat exchanger. The water flows through the shell and the oil flows through the tubes. The specific heat of the fluid may be taken as 0.454 kcal/kg°C. Find out whether the heat exchanger would be suitable for the given heat duty?

### Solution-13

It is given,

$$m_f = 667 \text{ kg/s} \approx 10,000 \text{ kg/h}$$

$$C_{pf} = 0.454 \text{ kcal/kg} \cdot ^\circ\text{C}$$

$$T_{f1} = 85^\circ\text{C} \quad T_{f2} = 50^\circ\text{C}$$

f : hot stream (fluid)

c : cold stream (water)

Energy balance across the heat exchanger will be,

$$\dot{m}_w(1)(38 - 30) = 10,000(0.454)(85 - 50) = 19,862 \text{ kg/h}$$

$$\dot{m}_f c_{pf} \text{ for hot stream} = (10,000)(0.454) = 4,540 \text{ kcal/h}^\circ\text{C}$$

$$\dot{m}_w c_{pw} \text{ for cold stream} = (19,862)(1) = 19,862 \text{ kcal/h}^\circ\text{C}$$

Thus the minimum stream will be the hot stream.

$$C_r = \frac{(\dot{m}c_p)_{\min}}{(\dot{m}c_p)_{\max}} = \frac{4,540}{19,862} = 0.2286$$

$$\eta = \frac{T_{f1} - T_{f2}}{T_{f1} - T_{c2}} = \frac{85 - 50}{85 - 30} = 0.636$$

Putting the values in the eq. 8.19,



$$NTU = -(1 + C_R^2)^{-1/2} \ln \left[ \frac{2/\eta - 1 - C_R - (1 + C_R^2)^{1/2}}{2/\eta - 1 - C_R + (1 + C_R^2)^{1/2}} \right]$$

$$NTU = 1.1652$$

$$NTU = UA/(mC_p)_{min}$$

$$\frac{UA}{(mC_p)_{min}} = 1.1652$$

$$A = \frac{1.1652 \times 4540}{400} = 13.2 \text{ m}^2$$

The area 13.2 m<sup>2</sup> found is less than the available area (15 m<sup>2</sup>). Therefore, the given heat exchanger will perform the required heat duty.

#### Illustration-14

A triple effect forward feed evaporator is used to concentrate a liquid which has marginal elevation in boiling point. The temperature of the stream to the first effect is 105°C, and the boiling point of the solution within third effect is 45°C. The overall heat transfer coefficients are,

2,200 W/m<sup>2</sup>: in the I-effect,  
1,800 W/m<sup>2</sup>: in the II-effect,  
1,500 W/m<sup>2</sup>: in the III-effect.

Find out at what temperatures the fluid boils in the I and II effects.

#### Solution-14

**Assumptions:-**

1. We may assume that there is no elevation in boiling point in the evaporators.
2. Area of all the three evaporators are same ( $A_I = A_{II} = A_{III} = A$ ) Total temperature drop = (105-45) °C = 60 °C Using eq. 9.5, the temperature drop across I-effect,

$$\Delta T_I = \frac{\frac{1}{2200}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 15.2 \text{ } ^\circ\text{C}$$

Similarly, the temperature drop across II-effect,

$$\Delta T_{II} = \frac{\frac{1}{1800}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 18.6 \text{ } ^\circ\text{C}$$

And the temperature drop across III-effect,

$$\Delta T_{III} = \frac{\frac{1}{1500}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 22.3 \text{ } ^\circ\text{C}$$

Therefore, the boiling point in the first effect will be = (105 – 15.2) °C = 89.8 °C  
Similarly, the boiling point in the second effect will be = (89.8 – 18.6) °C = 71.2 °C.