

Statistics

In the Chapter

In this chapter, you will be studying the following points:

• The mean for grouped data can be found by :

(i) the direct method :
$$\bar{x} = \frac{\sum f_1 x_1}{\sum f_1}$$

(ii) the assumed mean method =
$$\overline{x} = a + \frac{\Sigma f_1 d_1}{\Sigma f_1}$$

(iii) the step deviation method =
$$\overline{x} = a + \left(\frac{\Sigma f_1 u_1}{\Sigma f_1}\right) \times h$$

with the assumption that the frequency of a class is centred at its mid-point, called its class mark.

The mode of grouped data can be found by using the formula :

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where symbols have their usual meanings.

- The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.
- The median for grouped data is formed by using the formula:

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

where symbols have their usual meanings

- Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.
- The median of grouped data can be obtained graphically as the x-coordiante of the point of intersection of the two ogives for this data.
- Statistic : It is the science in which we study the methods and techniques used in collection, organisation presentation, analysis and interpretation of numberical data.
- Raw Data : It is the collection of actual information used to make logical inferences
 Example : Age of 5 girls are given below :
 Bhavna : 18 yrs, Suriti : 15 yrs, Radhika : 14 yrs, Malvika : 16 yrs, Vindhya : 17 yrs.

The groups of age 14, 15, 16, 17 and 18 form a set of data.

The data, as discussed above are called raw data.

If the set of raw data is very large, we may not be able to extract any useful information out of them. Thus such raw data must be summarised to an easily manageable form.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 14.1

Q.1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses i n a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6	6-8	8 - 10	10-12	12 - 14		
Number of houses	1	2	1	5	6	2	3		
Which method did you	Which method did you use for finding the mean, and why?								

*	0		
Ans. Number of plants	Number of house (f_i)	Class mark(x)	$f_1 x_1$
0-2	1	1	1
2-4	2	3	6
4 - 6	1	5	5
6 - 8	5	7	35
8 - 10	6	9	54
10-12	2	11	22
12 - 14	3	13	39
Total	$\Sigma f_i = 20$		$\Sigma f_i x_i = 162$

Here, we have

$$\Sigma f_1 = 20$$
, $\Sigma f_1 x_1 = 162$

Now,
$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f} = \frac{162}{20} = 8.1$$

Hence, the mean number of plants per house = 8.1

We have used the direct method for finding the mean because numerical values of x_i and f_j are small. Q.2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Ans. Daily wages (in Rs)	Number of workers	Class mark	$d_i = x_i - 150$	$\boldsymbol{u}_i = \frac{x_i - 150}{20}$	$f_i \mu_i$
100 - 120	12	110	-40	-2	-24
120 - 140	14	130	-20	-1	-14
140 - 160	8	150 = A	0	0	0
160 - 180	6	170	20	1	6
180 - 200	10	190	40	2	20
Total	$\Sigma f_i = 50$				$\Sigma f_i u_i = -12$

106 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Here, we have $\Sigma f_i = 50 + \Sigma f_i \mathbf{u}_1 = 12, \mathbf{h} = 20$ and A = 150

Now,
$$\overline{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 150 + \left(\frac{-12}{50}\right) \times 20 = 150 - 4.8 = 145.20$$

Hence, the mean daily wages of the workers of the factory is Rs. 145.20

Q.3. The following distribution shows the daily pocket allowance of children of a locality	7.
The mean pocket allowance is Rs 18. Find the missing frequency <i>f</i> .	

		-			-		••			
	Daily allowa	pocket ance (in Rs)	11 - 13	13 - 15	15-	- 17	17 - 19	19-21	21 - 23	23 - 25
	Numb	er of children	7	6	9)	13	f	5	4
An	s.	Daily pocket allowance (C.I	t N [.)	o. of children (f _i)		Mid	-value x _i	$f_i x$	i	
		11 - 13		7			12	84		
		13 - 15		6			14	84		
		15 - 17		9			16	14	4	
		17 - 19		13			18	234	4	
		19-21		f			20	20	f	
		21 - 23		5			22	110	0	
		23 - 25		4			24	96	5	
				$\Sigma f_i = 44 + f$				$\Sigma f_i x_i = 75$	52 + 20f	
			_							

Here, we have $\overline{x} = 18$, $\Sigma f_i = 44 + f$ and $\Sigma f_i x_i = 752 + 20f$

We, know
$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

 $\Rightarrow \qquad 18 = \frac{752 + 20 f}{44 + f}$
 $\Rightarrow \qquad 18(44 + f) = 752 + 20 f$
 $\Rightarrow \qquad 792 + 18 f = 752 + 20 f$
 $\Rightarrow \qquad 18 f - 20 f = 752 - 792$
 $\Rightarrow \qquad -2 f = -40$
 $\Rightarrow \qquad f = 20$

Hence the missing frequency is 20

Q.4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarised as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

	Number of heart beats	65 - 68	68-71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
	per minute							
	Number of women	2	4	3	8	7	4	2
An	S.							

C.I.	f_{l}	<i>x</i> _{<i>i</i>}	$d_i (x_i - 75.5)$	$u_i = \frac{d_i}{3}$	f _i u _i
65-68	2	66.5	_9	-3	-6

Lifeskills' Complete NCERT Solutions Class-X Mathematics | 107

68-71	4	69.5	6	-2	-8
71-74	3	72.5	-3	-1	-3
74-77	8	75.5=A	0	0	0
77-80	7	78.5	3	1	7
80-83	4	81.5	6	2	8
83-86	2	84.5	9	3	6
	$\Sigma f_i = 30$				$\Sigma f_i u_i = 4$

Here, we have $\Sigma f_i = 30$, $\Sigma f_i u_i = 4$, A (assumed mean) = 75.5 and h = 3

Now,
$$\overline{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 75.5 + \frac{4}{30} \times 3 = 75.5 + \frac{4}{10} = 75.5 + 0.4 = 75.9$$

Hence, the mean heart beats per minute is 75.9.

Q.5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	53 - 55	56-58	59-61	62 - 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Ans.

C.I.	$f_{_{I}}$	<i>x</i> _{<i>i</i>}	$d_i = x_i - 57$	$u_i = \frac{d_i}{2}$	$f_i \mu_i$
50-51	15	51	6	-3	-45
53-55	110	54	-3	-1.5	-165
56-58	135	57=A	0	0	0
59-61	115	60	3	1.5	172.5
62-64	25	63	6	3	65
	$\Sigma f_i = 400$				$\Sigma f_i u_i = 37.50$

Here, we have $\Sigma f_i = 400, \ \Sigma f_i u_i = 47.50, A = 57 \text{ and } h = 2$

$$\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h = 57 + \frac{37.50}{400} \times 2 = 57 + \frac{37.50}{200} = 57 + 0.1875 = 57.1875$$

Q.6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs)	100 - 150	150-200	200 - 250	250 - 300	300 - 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Ans.

Now,

CL	f_{I}	<i>x</i> _{<i>i</i>}	$d_i = x_i - 225$	$\boldsymbol{u}_i = \frac{d_i}{50}$	$f_i \mu_i$
100-150	4	125	-100	-2	8
150-200	5	175	-50	-1	-5
200-250	12	225=A	0	0	0

2 250-300 275 50 1 2 300-350 2 325 100 2 4 $\Sigma f_i u_i = -7$ $\Sigma f_i = 25$ $\Sigma f_i = 25, \Sigma f_i u_i = -7$, A (assumed mean) = 225 and h = 50Here, we have

108 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Now,
$$\overline{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 225 + \frac{-7}{25} \times 50 = 225 - 14 = 211$$

Hene, the mean daily expenses is Rs. 211.

Q.7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO ₂ (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20 - 0.24	2

Find the means concentration of SO₂ in the air.

Ans.	C.I.	f_{I}	<i>x</i> _i	$d_i = x_i - 0.14$	$\boldsymbol{u}_i = \frac{d_i}{0.04}$	$f_i \mu_i$
	0.00 - 0.04	4	0.02	-0.12	-3	-12
	0.04 - 0.08	9	0.06	-0.08	-2	-18
	0.08 - 0.12	9	0.10	0.04	-1	-9
	0.12-0.16	2	0.14 = A	0	0	0
	0.16-0.20	4	0.18	0.04	1	4
	0.20 - 0.24	2	0.22	0.08	2	4
		$\Sigma f_i = 30$				$\Sigma f_i u_i = -31$

Here, we have $\Sigma f = 30$

 $\Sigma f_i = 30, \Sigma f_i u_i = -31$, A (assumed mean) = 0.14 and h = 0.04

 $\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 0.14 + \frac{-31}{30} \times (0.04) = 0.14 - 0.041 = 0.099 \, ppm$

Hene, the mean concentration of SO_2 in the air is 0.099 ppm.

Q.8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0-6	6 - 10	10-14	14 - 20	20-28	28 - 38	38-40
Number of students	11	10	7	4	4	3	1

Ans.	Number of days	Number of students (f_i)	Class mark(x)	$f_1 x_1$
	0-2	11	3	33
	2-6	10	8	80
	10-14	7	12	84
	14-20	4	17	68

20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
Total	$\Sigma f_i = 40$		$\Sigma f_i x_i = 499$

Here, we have $\Sigma f_1 = 40$, $\Sigma f_1 x_1 = 499$

Now,
$$\bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{499}{40} = 12.47$$

Hence, the mean number of day a student was absent is 12.47.

Q.9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

	Lite	racy rate (in %) 45-55	55.	- 65	65 - 75		75 - 85	85	5-95		
	Nun	ıber of cities	3	1	0	11		8		3		
Ans	s.	CL	f_{I}			x _i	0	$d_i = x_i - 70$,	$u_i = \frac{x_i}{x_i}$	$\frac{-70}{10}$	$f_i \mu_i$
	ĺ	45-55	3			50		-20			2	-6
		55-65	10			60		-10		-	1	-10
		65-75	11			70=A		0		()	0
		75-85	8			80		10		1	l	8
		85-95	3			90		20		2	2	6
			$\Sigma f_i = 35$									$\Sigma f_i u_i = -2$

Here, we have $\Sigma f_i = 35, \Sigma f_i u_i = -2$, A (assumed mean) = 70 and h = -10

Now,
$$\overline{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 70 + \frac{-2}{34} \times 10 = 70 - \frac{4}{7} = 70 - 0.57 = 69.43\%$$

Hene, the mean literacy rate is 69.43%.

EXERCISE 14.2

Q.1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15-25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Ans. Case I : Finding the mode

Here, the maximum class frequency is 23 and the class corresponding to frequency is 35-45. So, the modal class is 35-45.

Thus, we have

Modal class = 35-45 l = 35 $f_i = 23, f_0 = 21, f_2 = 14$ and h = 10

Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 35 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14}\right) \times 10$$

$$= 35 + \left(\frac{2}{46 - 35}\right) \times 10 = 35 + \frac{2}{11} \times 10 = 35 + \frac{20}{11} = 35 + 1.818 = 36.818$$

Case II : Finding the median

C.I.	f_{I}	<i>x</i> _{<i>i</i>}	$d_i = x_i - 30$	$\boldsymbol{u}_i = \frac{d_i}{10}$	$f_i \mu_i$
5-15	6	10	-20	-2	-12
15-25	11	20	-10	-1	-11
25-35	21	30=A	0	0	0
35-45	23	40	10	1	23
45-55	14	50	20	2	28
55-65	5	60	30	3	15
	$\Sigma f_i = 80$				$\Sigma f_i u_i = 43$

Now,

$$x = A + \frac{\sum f_i u_i}{\sum f_i} \times h = 30 + \frac{43}{80} \times 10 = 30 - \frac{43}{8} = 30 + 5.373 = 35.373$$

Hence, the mode of the given data is 36.818 while mean is 35.373

Interpretation : Maximum number of patients admitted in the hospital are of the age 36.818 years, while an average age of patients admitted to the hospital is 35.373 yrs.

Q.2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0 - 20	20-40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Ans. Here, the maximum class frequency is 61 and the class corresponding to frequency is 60-80. So the modal class is 60-80.

Thus, we have

Modal class = 60-80 l = 60 $f_i = 61$ $f_0 = 52$ $f_2 = 38$ and h = 20

Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 60 + \left(\frac{61 - 52}{2 \times 61 - 52 - 38}\right) \times 20$$

= $60 + \frac{9}{122 - 90} \times 20 = 60 + \frac{9}{32} \times 20 = 60 + \frac{45}{8} = 60 + 5.625 = 65.625$

Hence, the modal life-times of the components are 65.625 hrs.

Q.3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in Rs)	Number of families
1000 - 1500	24
1500 - 2000	40
2000 - 2500	33

2500 - 3000	28
3000 - 3500	30
3500 - 4000	22
4000 - 4500	16
4500 - 5000	7

Ans. Here, the maximum class frequency is 30 and the class corresponding to frequency is 3000-3500. So, the modal class = 1500-2000.

Thus, we have Modal class = 1500-2000

l = 1500

$$f_1 = 40, f_0 = 23, f_2 = 33$$
 and $h = 500$

Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 1500 + \left(\frac{40 - 24}{2 \times 40 - 24 - 33}\right) \times 500$$

$$= 1500 + \frac{16}{80 - 57} \times 500 = 1500 + \frac{16 \times 500}{23} = 1500 + \frac{8000}{23} = 1500 + 347.83 = 1847.83$$

Hence, the modal monthly expenditure of the families is Rs. 1847.83. **Finding mean :**

Expenditure	Number of families	Class Mark	$d_i = x_i - 3250$	$\boldsymbol{u}_i = \frac{d_i}{500}$	$f_i \mu_i$
(in Rs.)	$f_{_{I}}$	<i>x</i> _{<i>i</i>}			
1000-1500	24	1250	-2000	-4	-96
1500-2000	40	1750	-1500	-3	-120
2000-2500	33	2250	-1000	-2	-66
2500-3000	28	2750	-500	-1	-28
3000-3500	30	3250=A	0	0	0
3500-4000	22	3750	500	1	22
4000-4500	16	4250	1000	2	32
4500-5000	7	4750	1500	3	21
	$\Sigma f_i = 200$				$\Sigma f_i u_i = -235$

Now,
$$x = A + \frac{\sum f_i u_i}{\sum f_i} \times h = 3250 + \frac{-235}{200} \times 500 = 3250 - \frac{235 \times 5}{2} = 3250 - \frac{1175}{2}$$

$$= 3250 - 587.50 = 2662.50$$

Hence, the mean monthly expenditure is Rs. 2662.50.

Q.4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states / U.T.
15-20	3
20-25	8
25 - 30	9
30-35	10
35-40	3
40-45	0
45 - 50	0
50 - 55	2

112 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Here, the maximum class frequency is 10 and the class corresponding frequency is 30-35. So, the modal class = 30-35.

Thus, we have

Modal class = 30–35, l = 30, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$ and h = 5Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 30 + \left(\frac{10 - 9}{2 \times 10 - 9 - 3}\right) \times 5 = 30 + \frac{1}{8} \times 5 = 30 + \frac{5}{8} = 30 + 0.6 = 30.6$$

Finding Mean :

Number of students	Number of states/	Class Mark	$d_i = x_i - 37.5$	$u_i = \frac{d_i}{5}$	$f_i \mu_i$
per teacher	$U.T. = f_1$	<i>x</i> _{<i>i</i>}			
15-20	3	17.5	-20	-4	-12
20-25	8	22.5	-15	-3	-24
25-30	9	27.5	-10	-2	-18
30-35	10	32.5	-5	-1	-10
35-40	3	37.5=A	0	0	0
40-45	0	42.5	5	1	0
45-50	0	47.5	10	2	0
50-55	2	52.5	15	3	6
	$\Sigma f_i = 35$				$\Sigma f_i u_i = -58$

Here, we have $\Sigma f_i = 35$, $\Sigma f_i u_i = -58$, A (assumed mean) = 37.5 and h = 3

Now,
$$\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 37.5 + \frac{-58}{35} \times 5 = 36.5 - 8.3 = 29.2$$

Interpretation : Most state/U.T. have a student teacher ratio of 30.6 and on an average, this ratio is 29.2. Q.5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000-4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Find the mode of the data.

Ans. Here maximum class frequency is 18 and the class corresponding to frequency is 4000-5000. So, the modal class = 4000-5000.

Thus, we have $l = 4000, f_1 = 18, f_0 = 4, f_2 = 9$ and h = 1000

Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 4000 + \left(\frac{18 - 4}{2 \times 18 - 4 - 9}\right) \times 1000$$

$$=4000+\frac{14000}{23}=4000+608.7=4608.7$$

Hence, the mode of the data is 4608.7.

Q.6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0 - 10	10-20	20-30	30-40	40 - 50	50-60	60 - 70	70-80
Frequency	7	14	13	12	20	11	15	8

Ans. Here, maximum class frequency is 20 and the class corresponding to frequency is 40-50. So, the modal class = 40-50.

Thus, we have modal class = 40-50, l = 40, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$ and h = 10. Now, substituting these values in the formula of mode, we get

Mode =
$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11}\right) \times 10$$

= $40 + \frac{80}{17} = 40 + 4.7 = 44.7$

Hence, the mode of the data is 44.7 cars.

EXERCISE 14.3

Q.1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

Ans. Finding Median :

Monthly consumption (in units)	Number of consumers	Cumulative frequency
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68
	n = 68	

We have n = 68. So, $\left(\frac{n}{2}\right)$ th observation = 34th observation.

So, median lies in the group of 125-145

i.e. median class = 125-145

114 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Now, we have median class = 125-145, l = 125, $\frac{n}{2} = 34$, cf = 22, f = 20 and h = 20. Substituting these values in the formula of median, we get

Substituting these values in the formula of median, we get

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) + h = 125 + \left(\frac{34 - 22}{20}\right) \times 20 = 125 + 12 = 137$$
units

Finding mean :

Monthly consumption	Number of consumers	Class Mark	$d_i = x_i - 135$	$\boldsymbol{u}_i = \frac{d_i}{10}$	$f_i u_i$
(in units)	$f_{_{I}}$	x _i			
65-85	4	75	-60	-3	-12
85-105	5	95	-40	-2	-10
105-125	13	115	-20	-1	-13
125-145	20	134=A	0	0	0
145-165	14	155	20	1	14
165-185	8	175	40	2	16
185-205	4	195	60	3	12
	$\Sigma f_i = 68$				$\Sigma f_i u_i = 7$

Here, we have $\Sigma f_i = -68$, $\Sigma f_i u_i = 7$, h = 20 and A = 135Now,

$$x = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

= $x = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 135 + \frac{7}{68} \times 20 = 135 + \frac{35}{17} = 135 + 2.05 = 137.05$

Find mode:

Here, the maximum class frequency is 20 and the class corresponding to frequency is 125-145. So, the modal class = 125 - 145.

Thus, we have, Modal class = 125-145, l = 125, $f_1 = 20$, $f_0 = 13$, $f_2 = 14$, and h = 20. Now, substituting these values in the formula of mode, we get

Mode
$$= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14}\right) \times 10 = 125 + \left(\frac{7}{40 - 27}\right) \times 20$$

= $125 + \frac{140}{13} = 125 + 10.76 = 135.76$ units.

Comparison : On comparison, we find that the three measures are approximately the same in this case. **Q.2. If the median of the distribution given below is 28.5, find the values of** *x* **and** *y***.**

Class Interval	Frequency
0 - 10	5
10-20	<i>x</i>
20-30	20
30-40	15

40 - 50	у
50 - 60	5
Total	60

Ans. Forming the cumulative frequency table, we have

Class Interval	Frequency	c.f.
0 - 10	5	5
10-20	x	5+ <i>x</i>
20-30	20	25+x
30-40	15	40 + x
40 - 50	у	40 + x + y
50 - 60	5	45 + x + y
Total	60	

It is given that n = 6045 + x + y = 60

$$\Rightarrow$$

 \Rightarrow

x + y = 15

It is given that median is 28.5.

So, the median lies in the group of 20-40.

i.e., median class = 20-30.

Thus, we have median class = 20 - 30.

$$1 = 20, \frac{n}{2} = 30, c.f. = 5 + x, h = 10 and f = 20.$$

Now, substituting these values in the formula of median, we get

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \qquad 28.5 = 20 + \left(\frac{30 - (5 + x)}{20}\right) \times 10$$

$$\Rightarrow \qquad 28.5 = 20 + \left(\frac{30 - 5 + x}{20}\right) \times 10$$

$$\Rightarrow \qquad 28.5 = 20 + \left(\frac{25 - x}{20} \times 10\right)$$

$$\Rightarrow \qquad 28.5 = 20 + \frac{25 - x}{2}$$

\Rightarrow	$28.5 = \frac{40 + 25 - x}{2}$
\Rightarrow	57 = $65 - x$

$$\Rightarrow \qquad x = 65 - 57$$

$$\Rightarrow$$
 $x = 8$

Putting the value of x = 8 in (i), we get

x + y = 15 $\Rightarrow 8 + y = 15$ $\Rightarrow y = 15 - 8 = 7$ Hence, x = 8 and y = 7.

Q.3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Ans.

Age (in years)	Number of policy holders	c.f.
Below 20	2	2
Below 25	6	4
Below 30	24	18
Below 35	45	21
Below 40	78	33
Below 45	89	11
Below 50	92	3
Below 55	98	6
Below 60	100	2
		n = 100

Here, middle term is $\left(\frac{100}{2}\right)$ th or 50th observation, which lies in class 35-45.

Thus, we have Median class = 35-40, l = 35, $\frac{n}{2} = 50$, c.f. = 45, f = 33 and h = 5

Putting these values in the formula of median, we get

Median = Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 35 + \frac{50 - 45}{33} \times 5 = 35 + \frac{5 \times 5}{33} = 35 + \frac{25}{33} = 35 + 0.75 = 35.75$$

Hence, the median age is 35.75 yrs.

Q.4. The lengths of 40 leaves of a plant are measured correct to the nearest mill imetre, and the data obtained is represented in the following table :

Length (in mm)	Number of leaves
118 - 126	3

_		
	127 - 135	5
	136 - 144	9
	145 - 153	12
	154 - 162	5
	163 - 171	4
	172 - 180	2

Find the median length of the leaves.

(Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, ..., 171.5 - 180.5.) Ans. We shall first convert the given data to continuous classes. Then the data become

Length (in mm)	Number of leaves	Cumulative frequency
117.5 - 126.5	3	3
126.5 - 135.5	5	8
135.5 - 144.5	9	17
144.5 - 153.5	12	29
153.5 - 162.5	5	34
162.5 - 171.5	4	38
171.5 - 180.5	2	40
		n=40

We have n = 40. So, $\left(\frac{n}{2}\right)$ the observation = 20th observation

So, median lies in the group of 144.5 - 153.5i.e., median class = 144.5 - 153.5Thus, we have

Median class = 144.5 – 153.5,
$$l = 144.5$$
, $\frac{n}{2} = 20$, $cf = 17$, $f = 12$ and $h = 10$

Substituting these values in the formula of median we get

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

= $144.5 + \frac{3}{12} \times 9 = 144.5 + \frac{9}{4}$
= $144.5 + 2.25 = 146.75$ mm

Hence, the median length of the leaves is 146.75 mm. Q.5. The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60
3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp.

118 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Life time (in hours)	Number of lamps	<i>c.f.</i>
1500 - 2000	14	14
2000 - 2500	56	70
2500 - 3000	60	130
3000 - 3500	86	216
3500 - 4000	74	290
4000 - 4500	62	352
4500 - 5000	48	400
		n=400

Ans. Forming the cumulative frequency table, we get

We have n = 400, so, $\left(\frac{n}{2}\right)$ the observation = 200th observation.

So, medianlies in the group of 3000-3500.

i.e., Median class = 3000 - 3500

Now, we have Median class
$$= 3000 - 3500$$

$$l = 3000, \frac{n}{2} = 200, cf = 130, f = 86 \text{ and } h = 500$$

Substituting these values in the formula of Median, we get

Median =
$$l + \left[\frac{n}{2} - cf}{f}\right] \times h$$

= $3000 + \left[\frac{200 - 130}{86}\right] \times 500 = 3000 + \frac{70}{86} \times 500 = 3000 + \frac{35000}{86}$
= $3000 + 40697 = 340697$

Hence, the median life time of a lamps = 3406.97.

Q.6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 - 4	4 - 7	7 - 10	10-13	13 - 16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Ans.

Number of letters	Number of surnames	Cumulative frequency		
1-4	6	6		
4-7	30	36		
7 - 10	40	76		
10-13	16	92		
13-16	4	96		
16-19	4	100		
		<i>x</i> = 100		

We have n = 100, so $\left(\frac{n}{2}\right)$ th observation = 50 the observation. So, median lies in the group of 7 – 10. i.e., Median class = 7 – 10 Now, we have Median class = 7 – 10, i = 7, $\frac{n}{2} = 50$, cf = 36, f = 40, and h = 3.

Now, substituting these values in the formula of median, we get

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 7 + \left(\frac{50 - 36}{40}\right) \times 3 = 7 + \frac{14 \times 3}{40} = 7 + \frac{21}{20} = 7 + 1.05 = 8.05$$

Hence the median number of letters in the surnames is 8.05. (ii) Modal class is (7-10) (because it has maximum frequency $f_m = 40$) (Lower modal class) l = 7, $f_m = 40$, $f_1 = 30$, $f_2 = 16$, (class width) h = 3

Mode =
$$l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h = 7 + \left(\frac{40 - 30}{80 - 30 - 16}\right) \times 3$$

= $7 + \frac{30}{34} = 7 + 0.88 = 7.88$

Hence, the modal size of the surnames is 7.88.

(ii)	Number of letters	f_{I}	Class marks	$u_i = \frac{x_i - a}{h} = \frac{x_i - 8.5}{3}$	$f_{i} \times u_{i}$
			x_1		
	1-4	6	2.5	-2	-12
	4-7	30	5.5	-1	-30
	7-10	40	8.5 = a	0	0
	10-13	16	11.5	1	16
	13-16	4	14.5	2	8
	16-19	4	17.5	3	12
	Total	N = 100			- 6

Here, (total observation) N = 100, (Assumed mean) a = 8.5.

(Class width) h = 3 and $\sum_{i=1}^{6} f_i u_i = -6$ Now, by step deviation method

Mean
$$= a + h \times \frac{1}{N} \times \sum_{i=1}^{6} f_i u_i$$

= $8.5 + 3 \times \frac{1}{100} \times (-6)$
= $8.5 - \frac{18}{100} = 8.5 - 0.18 = 8.32$

Hence, the mean number of letters in the surname is 8.32

120 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

Q.7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45 - 50	50 - 55	55 - 60	60-65	65 - 70	70-75
Number of students	2	3	8	6	6	3	2

Weight (in k.g)	No. of students (f)	c.f.
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
		x = 30

We have n = 30, So, $\left(\frac{n}{2}\right)$ the observation = 15 observation

So, median lies in the group of 55-60

median class = 55-60i.e.,

Now, we have median class = 55-60

$$l = 55, \frac{n}{2} = 15, cf = 13, f = 6 \text{ and } h = 5$$

1

Substituting these values in the formula of Median, we get

Median =
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 55 + \left(\frac{15 - 3}{6}\right) \times 5 = 55 + \frac{2}{6} \times 5 = 55 + \frac{5}{3} = 55 + 1.67 = 56.67$$

Hence, the median weight of the students = 56.67 kg.

EXERCISE 14.4

Q.1. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive. Ans. Cf distribution table.

Daily Income in (Rs.)	Number of workers
Less then 120	12
Less then 140	26
Less than 160	34
Less than 180	40
Less than 200	50



Now, we plot the points (120, 12), (140, 26, (160, 34), (180, 40), (200, 50).

Q.2. During the medical check-up of 35 students of a class , their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Ans. Here,
$$\frac{n}{2} = \frac{35}{2} = 17.5$$

Locate 17.5 on the *y*-axis. From this point, draw a line parallel to the *x*-axis cutting the curve at a point. From this point, draw a perpendicular to the *x*-axis. The point of intersection of this perpendicular with the *x*-axis determines the median of the given data as 46.5 kg.

Median weight by using the formula :

Weight (in kg)	Number of students	Cumulative frequency
Less than 38	0	0
Less than 40	3	3
Less than 42	5	5
Less than 44	9	9
Less than 46	14	14
Less than 48	28	28
Less than 50	32	32
Less than 52	35	35
		n = 35



Verification : We find that the median weight obtained from the graph is the same on the median weight obtained by using the formula.

().3	. The following	table gives	production vi	vield per hectare o	of wheat of 100) farms of a village
`	·•~	· I he lone whig	cubic Sires	production y	nei per neemi e	n multur or rou	i ui mo oi u vimugu

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70-75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive. Ans. More than type distribution

111010101010	JPC ansure
More than 50	100
More than 55	98
More than 60	90
More than 65	78
More than 70	54
More than 75	16



Additional Questions

Q.1.The median of an ungrouped data and the median calculated when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.

Ans. Not always, because for calculating median of a grouped data, the formula used is based on the assumption that the observations in the classes are uniformly distributed (or equally spaced).

Q.2. Will the median class and modal class of grouped data always be different ? Justify your answer.

Ans. Not always. It depends on the data.

Q.3. It is true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

Ans. No, it is not always the case. The values of these three measures can be the same. It depends on the type of data.

Q.4. In calculating the median of grouped data, grouped in classes of equal width, we may use the formula :

$$c = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

3

where a is the assumed mean. a must be one of

the mid-points of the classes. Is the last statement correct? Justify your answer.

Ans.Not necessary, the mean of the data does not depend on the choice of a assumed mean.

Q.5. Given below is a cumulative frequency distribution showing the marks secured by 50 students of a class :

Marks	No of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the frequency distribution table for data :

Ans.	Marks	No of Students	C.I.	f_i
	Below 20	17	0-20	17
	Below 40	22	20-40	5
	Below 60	29	40-60	7
	Below 80	37	60-80	8
	Below 100	50	80-100	13

The above table is the required frequency distribution table.

Q.6. The monthly income of 100 families are given as below :

Income (in Rs.)	Number of families
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income

Ans. Here the maximum class frequency is 41 and the class corresponding to this frequency is 10000-15000.

So, the modal class is 10000 – 15000 l = 10000 $f_1 = 41$ $f_0 = 26$ $f_2 = 16$ h = 5000Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ $= 10000 + \left(\frac{41 - 26}{2 \times 41 - 26 - 16}\right) \times 5000$

$$= 10000 + \frac{15}{82 - 42} \times 5000$$
$$= 10000 + \frac{15}{40} \times 5000$$
$$= 10000 + 15 \times 125$$
$$= 10000 + 1875$$
$$= 11875$$
Therefore, the model inco

Therefore, the modal income is Rs. 11,875. **Q.7. Find the mode of the following data :**

Marks	No. of students
0-10	3
10-20	12
20-30	32
30-40	20
40-50	6

Ans.

Here, the maximum class frequency is 32 and ther class corresponding to this frequency is 20-30. So, the modal class is 20-30.

$$l = 20 \qquad f_{1} = 32 \\ f_{0} = 12 \qquad f_{2} = 20 \\ h = 10 \end{cases}$$

Mode = $l + \left(\frac{f_{1} - f_{0}}{2f_{1} - f_{0} - f_{2}}\right) \times h$
= $20 + \left(\frac{32 - 12}{2 \times 32 - 12 - 20}\right) \times 10$
= $20 + \left(\frac{20}{64 - 32}\right) \times 10$
= $20 + \frac{30}{32} \times 10$
 $5 \times 5 \qquad 25$

$$= 20 + \frac{5 \times 5}{4} = 20 + \frac{25}{4}$$

$$=20+6.125=26.125$$

Marks	Frequency
0-10	5
10-30	15
30-60	30
60-80	8
80-100	2

Ans. Here, N = 60 (even)

: median class is in which 30th and 31st observation lie.

∴ median class is 30-60. 1 - 30Now,

$$l = -30$$

$$\frac{N}{2} = 30 \qquad Cf = 20$$

$$f = 30 \qquad h = 30$$

$$Median = l + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$$

$$= 30 + \left(\frac{30 - 20}{30}\right) \times 30$$

=30+10=40

O.9. (a) The median of the following observations given in order 16, 18, 20, 24 -x, 22 + 2x, 28, 30, 32, is 24. Find the value of value of x.

(b) The mean of 10 observations is 15.3. If two

Ans. (a) n = 8

 \therefore Median numbers are $\frac{8}{2}$ and $\frac{8}{2}$ + 1 i.e. 4th and

5th items.

Median =
$$\frac{24 - x + 22 + 2x}{2} = \frac{46 - x}{2}$$

 $\therefore \quad \frac{46 - x}{2} = 24 \Rightarrow x = 2.$
(b) Total of 10 observations
 $= 10 \times 15.3 = 153.$
New sum $= 153 - 6 - 9 + 8 + 14$
 $= 153 - 15 + 22$
 $= 160$
New Mean $= \frac{160}{10} = 16.$

Multiple Choice Questions

Q.1. Less than ogive and more than ogive of a grouped cumulative frequency distribution intersect at (140,70). The median of the distribution is : (a) 105 (b) 140

$$\begin{array}{c} (a) & 105 \\ (c) & 70 \\ (d) & 210 \\ \end{array}$$

Q.2. If the less than type ogive and 'more than' type ogive intersect each other at (20.5, 15.5), then the median of the given data is : (a) 360(h) 20.5

(a)
$$50.0$$
 (b) 20.3 (c) 15.5 (d) 5.5

Ans. (b)

Q.3. In an arranged series of an even number of 2n terms the median is :

> (a) nth terms (b) $(n+1)^{th}$ term

(c) Mean of $(n)^{th}$ term and $(n+1)^{th}$ term

(d)
$$\left(\frac{n+1}{2}\right)^{th}$$
 term

Ans. (c)

Q.4. The mode of a frequency distribution can be determined graphically from :

(a) histogram (b) frequency polygon (c) frequency curve (D) ogive

Ans. (a)

- Q.5. Mode is the value of the variable which has : (a) maximum frequency

 - (b) minimum frequency
 - (c) mean frequency
 - (d) middle most frequency
- Ans. (a)
- Q.6. The class mark of the class 10-25 is :
 - (a) 17 (b) 18 (d) 15
 - (c) 17.5

Ans. (c)

- Q.7. The abscissa of the point of intersection of the "less than type" and of the "more than type" cumulative frequency curve of a ground data is:
 - (b) median (a) mean
 - (c) mode

(d) half of the total frequency

Ans. (c)

- Q.8. The mean of first five prime numbers is :
 - (a) 5.6 (b) 5.4

(d) 3.6 (c) 5

Ans. (a)