Chapter 13

Surface Areas and Volumes

In the Chapter

In this chapter, you will be studying the following points:

- To determine the surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
- To find the volume of objects formed by combining any two of a cuboid, cone, cylinder, sphere and hemisphere.
- Given a right circular cone, which is sliced through by a plane parallel to its base, when the smaller conical portion is removed, the resulting solid is called a *Frustum of a Right Circular Cone*.
- The formulae involving the frustum of a cone are:

(i) Volume of a frustum of a cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

(ii) Curved surface area of a frustum of a cone = $\pi l(r_1 + r_2)$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

(iii) Total surface area of frustum of a cone = $\pi l(r_1 + r_2) + \pi(r_1^2 + r_2^2)$ where
h = vertical height of the frustum, l = slant height of the frustum
r_1 and r_2 are radii of the two bases (ends) of the frustum.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 13.1

Q.1. 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Ans. Given, the volume of a cube is 64 cm³. Let side of the cube is a cm, then

$$a^3 = 64 = 4^3$$

$$\Rightarrow \quad a = 4 \text{ cm}$$

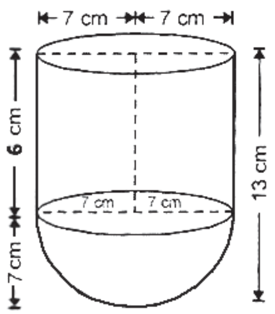
When we join end of each cube, then length of the new cuboid become (4 + 4) = 8 cm.

$\therefore$ The surface area of the resulting cuboid

$$= 2\,(lb + bh + hl)$$
$$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$$
$$= 2\,(32 + 16 + 32)$$
$$= 2\,(80)$$
$$= 160 \text{ cm}^2$$

Q.2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Ans.



Let r cm be the radius of the cylinder and h cm be the height of the cylinder, then

$r = 7$ cm,

and $h = (13 - 7)$ cm $= 6$ cm.

Let r_1 cm be the radius of the hemisphere, the

$r_1 = 7$ cm

Now,

the inner curved surface area of the vessel

= C.S.A. of hemisphere + C.S.A. of cylinder

$= (2\pi r_1^2 + 2\pi rh)$ cm^2

$= (2\pi r^2 + 2\pi rh)$ cm^2 $[r_1 = 2]$

$= [2\pi r(r + h)]$ cm^2

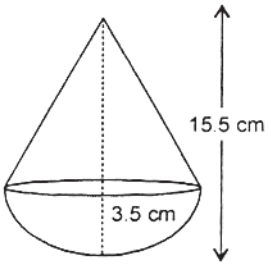
$= \left[\left(2\times\frac{22}{7}\times 7\right)(7+6)\right]$ cm^2

$= (44 \times 13)$ cm^2

$= 572$ cm^2

Q.3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Ans. Let r cm be the radius, h cm be the height and l cm be the slant height of the cone, then



$r = 3.5$ cm

$h = (15.5 - 3.5)$ cm $= 12$ cm.

Now, $l = \sqrt{r^2 + h^2}$

$\Rightarrow \quad = \sqrt{(3.5)^2 + (12)^2}$

$= \sqrt{12.25 + 144}$

$= \sqrt{156.25}$

$= 12.5$ cm.

Let r_1 cm be the radius of the hemisphere.

Then, $r_1 = 3.5$ cm.

Now,

The total surface area of the toy

= CSA of hemisphere + CSA of cone.

$= 2\pi r_1^2 + \pi rrl$

$= 2\pi r^2 + \pi rl$ $[r_1 = 1]$

$= \pi r[2r + 1]$

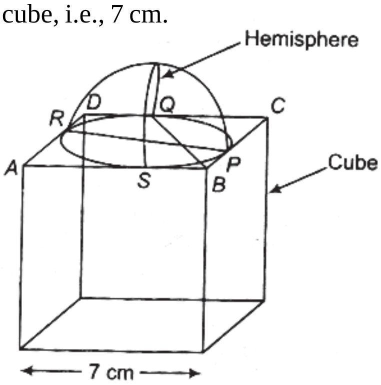
$= \frac{22}{7} \times 3.3\,[2 \times 3.5 + 12.5]$

$= 11\,(7 + 12.5)]$ cm^2

$= [11 \times 19.5]$ cm$^2 = 214.5$ cm^2

Q.4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Ans. Given, a cubical block is surmounted by a hemisphere. Therefore its diameter must be equal to the side of cube, i.e., 7 cm.



$\therefore$ Radius of hemisphere (r) $= \frac{7}{2}$ cm

Now, total curved surface area of solid

= Surfae area of five faces of the cube

+ Surface area of hemisphere except circular face PQRS

+ (Area of face ABCD – Area of circular space PQRS)

$= 5 \times (\text{side})^2 + 2\pi\,(\text{radius})^2 + [(\text{side})^2 - \pi\,(\text{radius})^2]$

$= 5 \times (7)^2 + 2 \times \frac{22}{7}\times\left(\frac{7}{2}\right)^2 + \left\{(7)^2 - \frac{22}{7}\times\left(\frac{7}{2}\right)^2\right\}$

$= 5 \times 49 + 11 \times 7 + \left(49 - \frac{77}{2}\right)$

$= 245 + 77 + 49 - \frac{77}{2}$

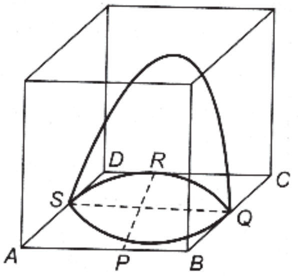
$= 294 + \frac{77}{2}$

$= \frac{665}{2} = 332.5 \text{ cm}^2$

Hemce, required area of the solid is 332.5 cm^2

Q.5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans. Given, side of the cube= diameter of the hemisphere = l



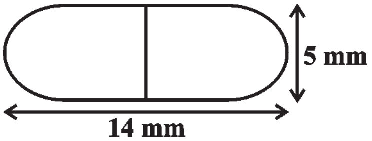
$\therefore$ Radius of a hemisphere = $\frac{l}{2}$

= {Area of open face ABCD of the cube – Area of circle PQRS + Area of remaining five faces of the cube + Area of hemisphere}

$$= \left(l^2 - \pi\frac{l^2}{4}\right) + 5t^2 + 2\pi\frac{l^2}{4}$$

$$= 6l^2 + \frac{\pi l^2}{4} = \frac{l^2}{4}(\pi + 24) \text{ sq. units.}$$

Q.6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 13.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Ans. Let r mm be radius of the hemisphere, then

$$r = \frac{5}{2} = 2.5 \text{ mm}$$

Let r mm be the radius and h mm be the height of the cylinder, then

$r_1 = 2.5$ mm

and $h = (14 - 5)$ mm $= 9$ mm

Now,

Surface area of capsule

= CSA of cylinder + CSA of two hemispherical ends

$= 2\pi r_1 h + 2(2\pi r2)$

$= (2\pi r_1 h + 4\pi r^2)$ mm^2

$= (2\pi rh + 4\pi r^2)$ mm^2 $\quad (r_1 = r)$

$= [2\pi r(h + 2r)]$ mm^2

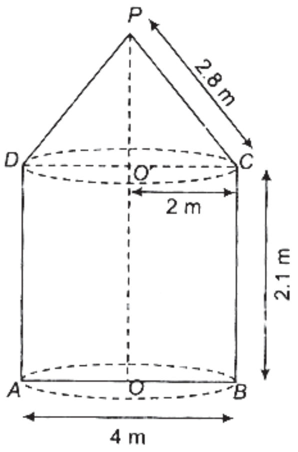
$$= \left[\left(2\times\frac{22}{7}\times 2.5\right)(9+5)\right] \text{mm}^2 = \left(\frac{44}{7}\times 2.5\times 14\right)\text{mm}^2$$

$= (44 \times 2.5 \times 2)$ mm^2

$= 220$ mm^2

Q.7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m². (Note that the base of the tent will not be covered with canvas.)

Ans. Given, a tent which is combination of a cylinder and a cone.



Also, we have slant height (l of the cone = 2.8 m

Radius of the cone, r = Radius of cylinder

$$= \frac{Diameter}{2} = \frac{4}{2} = 2m$$

and height of the cylinder, $h = 2.1$ m

$\therefore$ Required surface area of the tent = Surface area of cone + Surface area of cylinder

$= \pi rl + 2\pi rh$

$= \pi r\,(l + 2h)$

$= \dfrac{22}{7} \times 2 \times (2.8 + 2 \times 2.1)$

$= \dfrac{44}{7}\,(2.8 + 4.2)$

$= \dfrac{44}{7} \times 7 = 44\ \text{m}^2$

Hence, the cost of the canvas of the tent at the rate of Rs. 500 per m^2

= Surface area × Cost per m^2

= 44 × 500 = Rs. 22000

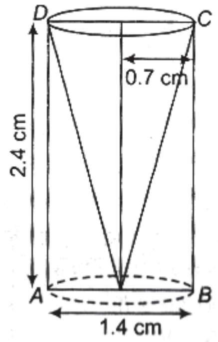
Q.8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Ans. Let r cm be the radius and h cm be the height of the cylinder, then

$r = 0.7$ cm and $h = 2.4$ cm

Let r_1 cm be the radius, l cm be the slant height and h_1 cm be the height of the cone, then

$r_1 = 0.7$ cm and $h_1 = 2.4$ cm

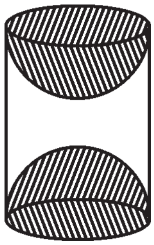


Now, $l = \sqrt{r_1^{\,2} + h_1^{\,2}}$

$= \sqrt{(0.7)^2 + (2.4)^2}$

$= \sqrt{0.49 + 5.76}$

$= \sqrt{6.25} = 2.5cm$



Hence, total surface area of remaining solid

= Surface area of conical cavity = Total surface area of cylinder

$= \pi rl + 2(\pi rh + \pi r^2) = \text{p}\, r(l + 2h + r)$

$= \dfrac{22}{7} \times 0.7\,(2.5 + 4.8 + 0.7)$

$= 2.2 \times 8$

$= 17.6 = 18\ \text{cm}^2$

Q.9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig.. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

Ans. Given,

Height of the cylinder, h = 10 cm

and radius of base of cylinder = Radius of hemisphere (r) = 3.5 cm

Now, required total surface area of the article

= 2 × Surface area of hemisphere + Lateral surface area of cylinder

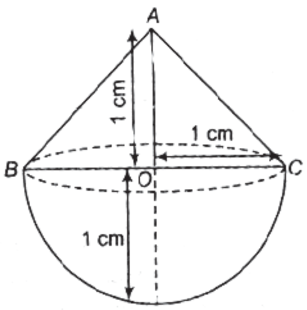
$= 2 \times (2\pi r^2) + 2\pi rh$

$= 2\,\pi r\,(2r + h) = 2 \times \dfrac{22}{7} \times 3.5 \times (2 \times 3.5 + 10)$

$= \dfrac{22}{7} \times 7 \times (7 + 10) = 22 \times 17 = 374\ \text{cm}^2$

EXERCISE 13.2

Q.1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



Ans. Given solid is a combination of a cone and a hemisphere.

Also, we have radius of the cone –r) = Radius of the hemisphere = 1 cm

and height of the cone (h) = 1 cm.

∴ Required volume of the solid = Volume of the cone + Volume of the hemisphere.

$= \dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3$

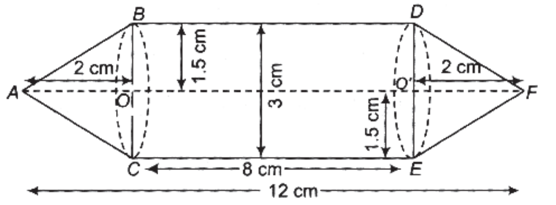
$= \dfrac{1}{3}\pi (1)^2\,(1) + \dfrac{2}{3}\pi (1)^3$

$$= \frac{\pi}{3} + \frac{2}{3}\pi$$

$$= \frac{(\pi + 2\pi)}{3}$$

$$= \frac{3\pi}{3} = \pi \text{ cm}^3$$

Q.2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)



Ans. Let r cm be the radius and h cm be the height of a cone, then

$r = 1.5$ cm, and $h = 2$ cm.

Now, volume of conical part

$$= \frac{1}{3}\pi r^2 h$$

$$= \left[\frac{1}{3}\pi \times 1.5 \times 1.5 \times 2\right] \text{cm}^2 = 1.5\,\pi$$

Let r cm be the radius and h cm be the height of the cylindrical part then $r_1 = 1.5$ cm, $h_1 = 8$ cm.

Now, volume of cylindrical part

$= \pi r_1^2 h'$

$= (\pi \times 1.5 \times 1.5 \times 8)$ cm^3

$= 18\,\pi$ cm^3

Hence, the volume of air contained in the model that Rachel made

= Volume of two conical part + Volume of cylindrical part

$= (2 \times 1.5\,\pi + 18$ p) cm^3

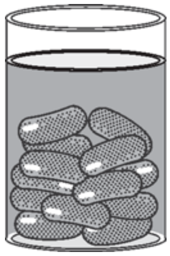
$= (3\,\pi + 18\,\pi)$ cm^3

$= (21\,\pi)$ cm^3

$$= \left(21 \times \frac{2-2}{7}\right) \text{cm}^3$$

$= 66$ cm^3

Q3. A *gulab jamun,* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns,* each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig.).



Ans. Let r cm be the radius of hemispherical part, then

$$r = \frac{2.8}{2} = 1.4 \text{ cm.}$$

Now,

Volume of hemispherical part

$$= \left(\frac{2}{3}\pi \times 1.4 \times 1.4 \times 1.4\right) \text{cm}^3$$

$$= \left(\frac{5.488}{3}\pi\right) \text{cm}^3$$

$= 1.892\,\pi$ cm^3

Let R cm be the radius and H cm be the height of cylindrical part, then

$$\text{R} = \frac{2.8}{2} = 1.4 \text{ cm}$$

and H = 5 cm – (2 × 1.4) cm = 2.2 cm

Now,

Volume of cylindrical part

$= \pi R^2 H = (\pi \times 1.4 \times 1.4 \times 2.2)$ cm^3

$= 4.312\,\pi$ cm^3

Now,

Volume of each Gulab Jamun = Volume of Cylindrical + 2 (Volume of hemispherical part)

= [4.312 p + 2(1.829) p] cm^3

= (4.312 p + 3.658 p) cm^3 = 7.97 p

$$= 7.97 \times \frac{22}{7} = 25.05 \text{ cm}^3$$

Hence,

Volume of syrup fround in 45 Gulab Jamuns

= 45 × 30% of volume of each Gulab Jamun

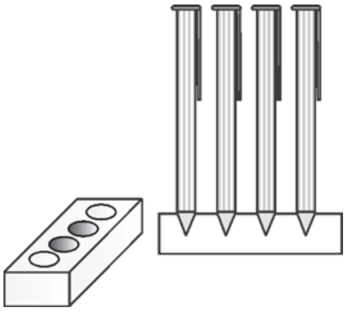
$$= \left[45 \times \frac{30}{100} \times 25.05\right] \text{cm}^3$$

$= 388.175$ cm^3 (app.)

$= 338$ cm^3 (Approx.)

Q.4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens.

The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. .



Ans. Let l, b and h be respectively the length, breadth and height of a cuboid, then

l = 15 cm

b = 10 cm

and h = 3.5 cm

Volume of cuboid

$= (l \times b \times h)$ cm^3 = $(15 \times 10 \times 3.5)$ cm^3

= 525 cm^3

Let r cm be the radius and h cm be the height of conical part, then

$r = 0.5$ cm, $h = 1.4$ cm.

Now,

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \left(\frac{1}{3}\times\frac{22}{7}\times 0.5\times 0.5\times 1.4\right)\text{ cm}^3 = \left(\frac{7.7}{21}\right)\text{cm}^3$$

Hence, the volume of wood in the entire stand.

= Volume of cuboid – 4 (volume of cone)

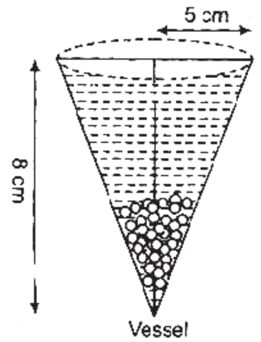
$$= \left(525 - 4\times\frac{7.7}{21}\right)\text{cm}^3$$

= (525 – 1.466) cm^3

= 523.534 cm^3

Q.5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans. Let r cm be the radius and h cm be the height of the cone. Then $r = 5$ cm and $h = 8$ cm.



Now, Volume of cone = Volume of water in cone

$$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\times\frac{22}{7}\times 5\times 5\times 8\right)\text{ cm}^3 = \frac{4400}{21}\text{ cm}^3$$

Volume of water flows

= Volume of lead shots

$$= \frac{1}{4}\text{ of the volume of water in cone}$$

$$= \frac{1}{4}\times\frac{4400}{21} = \frac{1100}{21}\text{ cm}^3$$

Let radius of the lead shot (spherical) be R cm. Then

R = 0.5 cm

Now, Volume of spherical lead shot

$$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.5)^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{5}{10}\times\frac{5}{10}\times\frac{5}{10} = \frac{11}{21}\text{ cm}^3$$

Therefore,

Required number of lead shots

$$= \frac{\textit{Volume of water flows out}}{\textit{Volume of one lead shot}}$$

$$= \frac{1100}{21} + \frac{11}{21} = 100$$

Q.6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use π = 3.14)

Ans. Given

Height of the first cylinder, h_1 = 220 cm

Radius of the first cylinder, $r_1 = \frac{24}{2} = 12$ cm

Height of the second cylinder, $h_2 = 60$ cm
and radius of the second cylinder, $r_2 = 8$ cm

$\therefore$ Volume of iron = Volume of first cylinder + Volume of second cylinder

$= \pi r_1^2 h_1 + p r_2^2 h_2$
$= \pi (r_1^2 h_1 + r_2^2 h_2)$
$= 3.14 (144 \times 220 + 64 \times 60)$
$= 3.14 (31680 + 3840) = 3.14 \times 35520$ cm^3

Also, given that
1 cm^3 of iron has approximately mass

$$= 8 \text{ g} = \frac{8}{1000} \text{ kg}$$

$\therefore$ (3.14×35520) cm^3 of iron has approximately mass

$$= 3.14 \times 35520 \times \frac{8}{1000}$$
$$= 3.14 \times 284.160$$
$$= 892.26 \text{ kg}$$

Q.7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans. Given,
Height of the cylinder (h) = 180 cm = 1.8 cm
and radius of he cylinder (r) = 60 cm = 0.6 m
Volume of water filled in a right circular cylinder

$$= \pi r^2 h$$
$$= \frac{22}{7} \times 0.6 \times 0.6 \times 1.8$$
$$= \frac{14.256}{7} \text{ m}^3$$

Also, given solid is a combination of a cone and a hemisphere

Height of the cone (h_1) = 120 cm = 1.2 m
Radius of the cone (r_1) = 60 cm = 0.6 m
and height/radius of the hemisphere (r_2) = 60 cm = 0.6m

Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3} \times \pi r_1^2 h_1 + \frac{2}{3} \pi r_2^3$$
$$= \frac{1}{3} \times \frac{22}{7} \times (0.6)^2 \times (1.2) + \frac{2}{3} \times \frac{22}{7} \times (0.6)^3$$
$$= \frac{22}{21} \times (0.6)^2 \, (1.2 + 2 \times 0.6)$$
$$= \frac{22}{21} \times 0.36 \, (1.2 + 1.2)$$
$$= \frac{22}{21} \times 0.36 \times 2.4 = \frac{19.008}{21} \text{ m}^3 = \frac{6.336}{7} \text{ m}^3$$

Hence, the volume of water left in the cylinder
= Volume of water filled in a right circular cylinder – Volume of the solid

$$= \frac{14.256}{7} - \frac{6.336}{7} = \frac{7.92}{7} = 1.131428 \text{m}^3$$
$$= 1.131 \text{m}^3$$

Q.8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Ans. Let r cm be the radius of the spherical glass. Then

$$r = \frac{8.5}{2} \text{ cm}$$

Now,

$$\text{Volume} = \frac{4}{3} \pi r^3$$
$$= \left(\frac{4}{3} \times 3.14 \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2} \right) \text{cm}^3$$
$$= \left(\frac{7713.41}{24} \right) \text{cm}^3$$
$$= 321.4 \text{ cm}^3$$

Let R cm be the radius and h cm be the height of cylindrical part.

Then, $\quad R = \frac{2}{2} = 1$ cm, $h = 8$ cm

Volume $= \pi R^2 h$
$= (3.14 \times 1 \times 1 \times 8)$ cm^3
$= (25.12)$ cm^3

Quantity of water = $(321.4 + 25.12)$ cm^3
$= 346.52$ cm^3

Hence, answer is not correct.

EXERCISE 13.3

Q.1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans. Let r cm be the radius of the sphere. Then,

$$r = 4.2 \text{ cm}$$

Now,

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \left(\frac{4}{3}\times\pi\times(4.2)\times(4.2)\times 4.2\right)\text{cm}^3$$

Let R cm be the radius and h cm be the height at the cylinder. Then

$$R = 6 \text{ cm and } h = ?$$

Now, Volume $= \pi R^2 h$

$$= (\pi \times 6 \times 6 \times h)\text{ cm}^3$$

Since sphere is recast into the shape of a cylinder. So, volume remains same

i.e. $$\frac{4}{3}\pi r^3 = r R^2 h$$

$$\Rightarrow \frac{4}{3}\pi \times 4.2 \times 4.2 \times 4.2 = p \times 6 \times 6 \times h$$

$$h = \frac{4\times 4.2\times 4.2\times 4.2}{6\times 6\times 3}$$

$$= \frac{296.352}{108} = 2.74 \text{ cm}$$

Q.2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans. Let r_1, r_2 and r_3 be the radius of metallic spheres, then $r_1 = 6$ cm, $r_2 = 8$ cm, $r_3 = 10$ cm.

Let R cm be the radius of a single solid sphere.

Since, three metallic spheres are formed from a single solid sphere, so their volumes are equal.

$$\Rightarrow \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3) = \frac{4}{3}\pi(R)^3$$

$$\Rightarrow r_1^3 + r_2^3 + r_3^3 = R^3$$

$$\Rightarrow 6^3 + 8^3 + 10^3 = R^3$$

$$\Rightarrow 216 + 512 + 1000 = R^3$$

$$\Rightarrow R^3 = 1728$$

$$\Rightarrow R = 12 \text{ cm}$$

Hence, radius of sphere $= 12$ cm

Q.3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. r m be the radius of h m be the height of the well

Then, $$r = \frac{7}{2} \text{ m and } h = 20,$$

Now,

Volume of earth dug out $= \pi r^2 h$

$$= \left(\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 20\right)\text{m}^3$$

$$= 770 \text{ m}^3$$

Area of embankment

$$= 22 \text{ m} \times 14 \text{ m}$$

$$= 308 \text{ m}^2$$

$\therefore$ Height of the embankment

$$= \frac{\textit{Volume of the earth dug out}}{\textit{Area of the embankment}}$$

$$= \frac{770}{308} = 2.5 \text{ m.}$$

Q.4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans. Let r m be the radius and h m be the height of the well (cylindrical shape). Then

$$r = \frac{3}{2} \text{ m and } h = 14 \text{ m}$$

Now, Volume of the earth dug out

$$= \pi r^2 h$$

$$= \left(\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times 14\right)\text{m}^3$$

$$= \frac{2772}{28} = 99 \text{ m}^3$$

and Area of the embankment (shaded part)

$$= \pi R^2 - \pi r^2$$

$$= \pi(R^2 - r^2)$$

$$= \frac{22}{7}[(5.5)^2 - (1.5)^2]$$

$$= \frac{22}{7}(5.5+1.5)(5.5-1.5)$$

$$= \left(\frac{22}{7}\times 7\times 4\right)$$

$= 88 \text{ cm}^2$

$\therefore$ Height of the embankment

$$= \frac{\textit{Volume of the earth dug out}}{\textit{Area of the embankment}}$$

$$= \frac{99}{88} = 1.125 \text{ m}$$

Q.5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. Let R cm be the radius and H cm be the height of a container, then

$$R = \frac{12}{2} = 6 \text{ cm and H} = 15 \text{ cm}$$

Therefore, Volume of cylindrical container

$= \pi R^2 H$

$= (\pi \times 6 \times 6 \times 15)$ ck^3

$= 540$ p cm^3

Let r_1 cm be the radius and h cm be the height of a cone, then

$$r_1 = \frac{6}{2} = 3 \text{ cm and } h = 12 \text{ cm}$$

Therefore, volume of conical part

$$= \frac{1}{3}\pi r_1^2 h$$

$$= \left(\frac{1}{3}\pi \times 3\times 3\times 12\right) \text{cm}^3$$

$= 36\pi \text{ cm}^3$

Let r_2 cm be thr radius of hemispherical part, then

$r_2 = 3$ cm $\qquad (r_1 = r_2)$

Therefore, Volume of Hemispherical part

$$= \frac{2}{3}\pi r_2^3$$

$$= \left(\frac{2}{3}\pi \times 3\times 3\times 3\right) \text{cm}^3$$

$= 18\pi \text{ cm}^3$

Now, Volume of ice-cream cone with hemisphrical top

= Volume of cone

+ Volume of Hemisphere

$= (36\pi + 18$ p) cm^3

$= 54\pi \text{ cm}^3$

Therefore,

the required no. of such cones

$$= \frac{\textit{Volume of Cylindrical container}}{\textit{Volume of cone with hemispherical top}}$$

$$= \frac{540\pi}{54\pi} = 10.$$

Q.6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Ans. We know that, every coin has a shape of cylinder. Let radius and height of the coin are r_1 and h_1.

Given, diameter of a coin = 1.75 cm

$$\therefore \quad r_1 = \frac{1.75}{2} \text{ cm and } h_1 = 2 \text{ mm} = 0.2 \text{ cm}$$

Also, given that the length, breadth and height of the cuboid are 5.5 cm, 10 cm and 3.5 cm, respectively.

$\therefore$ Volume of cuboid = Length × Breadth × Height

$= 5.5 \times 10 \times 3.5 = 192.5 \text{ cm}^3$

Let 'n' be the number of coins of melting to make the cuboid.

Then, $\quad n$ × volume of a coin = Volume of the cuboid.

$$\Rightarrow \quad n\times\frac{13.475}{28} = 192.5 \Rightarrow n = \frac{192.5\times 28}{13.475} = 400$$

Hence, 400 silver coins must be melted to form a cuboid.

Q.7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Let the radius and slant height of the heap of sand are r and l. Given, the height of the heap of sand $h = 24$ cm.

$\therefore$ Volume of the heap of sand = $\frac{1}{3}\ \pi r^2 (24) = 8\ \pi r^2$

Also, given the height and radius of cylindrical bucket are 32 cm and 18 cm, respectively.

$\therefore$ Volume of cylindrical bucket = $\pi(\text{Radius})^2 \times (\text{Height}) = \pi\,(18)^2 \times 32$

Now, according to question,

Volume of the heap of sand = Volume of the cylindrical bucket

$\Rightarrow \quad 8\pi r^2 = \pi \times 18 \times 18 \times 32$

$\Rightarrow \quad r^2 = 18 \times 18 \times 4 \Rightarrow r = 36$ cm

Now, the slant height of the conical heap of sand

$$= \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (36)^2}$$

$$= \sqrt{576 + 1296} = \sqrt{1872}$$

$$l = \sqrt{144 \times 13 +} = 12\sqrt{13} \text{ cm}$$

Q.8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Ans. Given, speed of flow of water (l) = 10 kmh^{-1} = 10 × 1000 mh^{-1}

Area of canal = 6 × 1.5 = 9 m^2

Volume of water flowing in 1 h = 10 × 1000 × 9m^3

$\therefore$ Volume of water flowing in 1/2 h

$$= \frac{10 \times 1000 \times 9}{2} = 45000\text{m}^2$$

$\therefore$ Required area for covering 8 cm height of standing water

$$= \frac{45000}{8} \times 100 = 562500\text{m}^2$$

$$= \frac{562500}{10000} \text{ hectares -= } 56.25 \text{ hectares}$$

Q.9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Ans. Let r m be the radius and h m be the height of the cylindrical tank.

Then, $r = \frac{10}{2} = 5$ cm and $h = 2$ m

Now, Volume of cylindrical tank

$= \pi r^2 h$

$= \pi \times 5^2 \times 2 = 50$ pm^3

Now, Volume of the water that flows through the pipe t hours

= Volume of cylinder of radius of 10 cm and length = (3000t m)

$\therefore \quad 50\,\pi\,\text{m}^3 = 30\,\pi\, t\,\text{m}^3$

$\Rightarrow \quad t = \frac{5}{3}$ hour

= 1 hour 40 minutes.

EXERCISE 13.4

Q.1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Ans. Let R and r be the radii of bigger and smaller ends of the frustum and h be its height. Then,

$$R = \frac{4}{2} = 2 \text{ cm}, r = \frac{2}{2} = 1 \text{ cm and } h = 14 \text{ cm}$$

Now, Volume = $\frac{\pi h}{3}\ [R^2 + R.\,r + r^2]$

$$= \frac{1}{3} \times \frac{22}{7} \times 14\,[(2)^2 + 2 \times 1\,(1)^2]$$

$$= \frac{1}{13} \times 44\,(4 + 2 + 1)$$

$$= \frac{44 \times 7}{3} = \frac{308}{2} \text{ cm}^3 = 102\frac{2}{3} \text{ cm}^3$$

Q.2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans. Let R and r be the radii of bigger and smaller ends of the frustum, and l be the slant height, then l = 4 cm

Perimeter of bigger end = 18 cm

$\Rightarrow \quad 2\pi R = 18$

$\Rightarrow \quad \pi R = 9$ cm

Perimeter of smaller end = 6 cm

$\Rightarrow \quad 2\pi R = 6$

$\Rightarrow \quad \pi R = 3$ cm

Now,

Curved surface of the fustum

$= \pi Rl + \pi rl$

$= 1(\pi R + \pi r)$

$= 4(9+3) = 4 \times 12$

$= 48 \text{ cm}^2$

Q.3. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Ans. Let R and r be respectively the radii of bigger and smaller ends of the frustum and l be the slant height then,

R = 10 cm, r = 4 cm and l = 15 cm

Now, Curved Surface Area

$$= \pi Rl + \pi rl$$

$$= \pi l(R + r)$$

$$= \frac{22}{7} \times 15(10+4) \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 15 \times 14\right) \text{cm}^2$$

$$= 22 \times 30 = 660 \text{ cm}^2$$

and area of the top fo the cap

$$= \pi r^2 = \frac{22}{7} \times 16$$

$$= \frac{352}{7} \text{ cm}^2$$

Thus,

Total area of the material used for making the cap

$$= \left(660 + \frac{352}{7}\right) \text{cm}^2$$

$$= 710\frac{2}{7} \text{ cm}^2$$

Q.4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take π = 3.14)

Ans. Let R and r be respectively the radii of bigger and smaller ends of the frustum, then

R = 20 cm, r = 8 cm

Let l and h be respectively the slant height and height of the frustum then

$$h = 16 \text{ cm}$$

and
$$l = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{16^2 + (20-8)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400} = 20 \text{ cm}$$

Now,

(i) Curved Surface area of frustum

$$= \pi l(R + r)$$

$$= 3.14 \times 20(20+8)$$

$$= (3.14 \times 20 \times 28) \text{ cm}^2$$

$$= 1758.4 \text{ cm}^2$$

(ii) Total tin required

= CSA. + area of base

$$= 1758.4 + 3.14 \times (8)^2$$

$$= 1758.4 + 200.96$$

$$= 1959.36 \text{ cm}^2$$

(iii) Cost of required tin

$$= \frac{1959.36 \times 8}{100} = \text{Rs. } 156.75$$

(iv) Volume of frustum

$$= \frac{\pi h}{3}[R^2 + R \cdot r + r^2]$$

$$= \frac{3.14 \times 15}{7 \times 3}[(20)^2 + 20 \times 8 + (8)^2]$$

$$= \frac{50.24}{3}(400 + 64 + 160) \text{cm}^3$$

$$= \frac{50.24}{3} \times 624 \text{ cm}^3$$

$$= 50.24 \times 208 \text{ cm}^3$$

$$= \frac{10449.92}{1000} \text{ Litres}$$

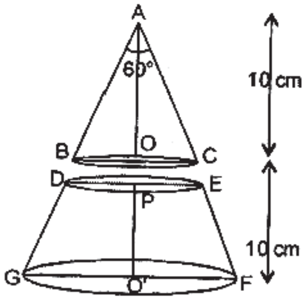
Cost of milk @ 20 per litres

$$= \text{Rs.}\left(20 \times \frac{10449.92}{1000}\right)$$

$$= \text{Rs. } 208.99 = \text{Rs. } 209$$

Q.5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans.



In Fig Cone ABC is cut out by a plane parallel to the base FG. DEFG is the frustum so obtained. Let O be the centre of the base of the cone and O' the centre of the base of the frustum.

It is given that $\angle BAC = 60°$

$\angle OAC = 30°$

In right triangle AOC, $\tan 30° = \frac{OC}{OA}$

$\Rightarrow \quad OC = OA \times \tan 30°$

$= 10 \times \frac{1}{\sqrt{3}} = \frac{10}{\sqrt{3}}$ cm.

And, $\quad PE = OC = \frac{10}{\sqrt{3}}$ cm. $\quad$ (i)

Since, $\quad \Delta AOC \sim \Delta AO'F$

(Using AA similarity condition)

$\therefore \quad \frac{AO}{AO'} = \frac{OC}{O'F}$

$\Rightarrow \quad \frac{10}{20} = \frac{\frac{10}{\sqrt{3}}}{O'F}$

$\Rightarrow \quad O'F = \frac{20}{\sqrt{3}}$

Height of the frustum = PO' = $\frac{1}{2}$ AO' = 10 cm

$\therefore$ Volume of the frustum = $\frac{\pi h}{3}$ $[R^2 + Rr + r^2]$

$= \frac{\pi \times 10}{3}$

$[OF^2 + O'F + PE + PE^2]$

$= \frac{\pi \times 10}{3}$

$$\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}} + \left(\frac{10}{\sqrt{3}}\right)^2\right]$$

(Using (i) and (ii)]

$= \frac{\pi \times 10}{3}$

$$\left[\frac{400 + 200 + 100}{3}\right]$$

$= \frac{\pi \times 10 \times 700}{9}$ cm^3

$= \frac{\pi \times 7000}{9}$ cm^3 $\quad$ ...(iii)

Radius of the wire = $\frac{1}{32}$ cm

Let h be length of the wire

$\therefore$ Volume of wire = $p \times \left(\frac{1}{32}\right)^2 \times h$ $\quad$ ...(iv)

From (iii) and (iv), we get

$$\pi \times \left(\frac{1}{32}\right)^2 \times h = \frac{\pi \times 7000}{9}$$

$$h = \frac{\pi \times 7000}{9} \times \frac{32 \times 32}{\pi} \text{ cm}$$

$$= \frac{7168000}{9}$$

= 796444.4 cm (approx)

= 7964.44 m

= 7964 m. (approx.)

EXERCISE 13.5 Optional

Q.1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm³.

Ans. Length of the cylinder = 12 cm = 120 mm

$\therefore$ Number of rounds of cover 3mm = 1

$\therefore$ Number of rounds of cover 120 mm

$$= \frac{120}{3} = 40$$

Let r cm be the radius of the cylinder. Then

$$r = \frac{10}{2} = 5 \text{ cm}$$

$\therefore$ Length of the wire in completing one round

$$= 2\pi r$$
$$= 2\pi (5) = 10\pi \text{ cm}$$

$\therefore$ Length of the wire in completing the whole surface (40 rounds)

$$= 10\pi \times 40 = 400\pi \text{ cm}$$

Radius of the copper wire = $\frac{3}{2}$ mm = $\frac{3}{20}$ cm

$\therefore$ Volume of wire $= \pi\left(\frac{3}{20}\right)^2 (400\pi)$

$= 9\pi^2 \text{ cm}^3$

$\therefore$ Mass of wire $= 9\pi^2 \times 8.88$

$= 9\left(\frac{22}{7}\right)^2 \times 8.88$

$= 789.42$ gm

Q.2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

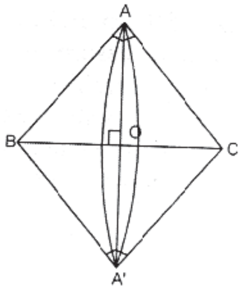
Ans. Here, ABC is a right angled triangle at A and BC is the hypotenuse

$\therefore$ $BC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ cm

As Δ ABC revolves about the hypotenuse BC. It formes two cones ABD and ACD. Since, ΔAEB and ΔABC are similar.

$$\therefore \quad \frac{AE}{CA} = \frac{AB}{BC} \Rightarrow \frac{AE}{4} = \frac{3}{5}$$

$$\Rightarrow \quad AE = \frac{12}{5} = 2.4 \text{ cm}$$



So, radius of the base of cone = AE = 2.4

Now, in right angled ΔABE

\

$BE = \sqrt{AB^2 - AE^2} = \sqrt{9-(2.4)^2} = \sqrt{3.24}$

$= 1.8$ cm

$\therefore$ Œ $= BC - BE = 5 - 1.8 = 3.2$ cm

Now, volume of cone ABD = $\frac{1}{3}\times\frac{22}{7}\times(2.4)^2\times1.8$

$= \frac{22}{21}\times10.368 = 10.86 \text{ cm}^3$

and Volume of the cone ACD = $\frac{1}{3}\times\frac{22}{7}\times(2.4)^2\times3.2$

$= \frac{405.504}{21} = 19.31 cm^3$

$\therefore$ Required volume of double cone

$= 10.86 + 19.31 = 30.17 \text{ cm}^3$

Now, surface area of cone ABD = $\pi rl = \frac{22}{7}\times 2.4\times 3$

$= \frac{158.4}{7} = 22.63 \text{ cm}^2$

and surface area of cone ACD = $\frac{22}{7}\times 2.4 \times 4$

$= \frac{211.2}{7} = 30.17 \text{ cm}^2$

$\therefore$ Required surface area of double cone

$= 22.63 + 30.17 = 52.8 \text{ cm}^2$

Q.3. A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm³ of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Ans. Given internally dimensions of cistern = 150 cm × 120 cm × 110 cm

$\therefore$ Volume of cistern = $150 \times 120 \times 110$ cm^3

= 1980000 cm^3

Volume of water = 129600 cm^3

$\therefore$ Volume of cistern to be filled

= 1980000 – 129600 = 1850400 cm^3

Let required number of bricks = n

Then water absorbed by n bricks = $n\left(\frac{1096.875}{17}\right)$ cm^3

Now, $1850400 + n\left(\frac{1096.875}{17}\right) = n\,(1096.875)$

$\Rightarrow$ $n \times 1096.875 \times \frac{16}{17} = 1850400$

$\Rightarrow$ $n = \frac{1850400 \times 17}{1096.875 \times 16}$

= 1792.4102 = 1792 (approx.)

Hence, 1792 brickes can be put in cistern.

Q.4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Ans. Given, area of the valley = 97280 km^2

and rainfall = 10 cm = $\frac{10}{100 \times 1000}$ km

$\therefore$ Volume of rainfall = $\frac{97280 \times 10}{100 \times 1000}$ = 9.728 km^3

Also, given length of the river = 1072 km

breadth of the river = 75 m = $\frac{75}{1000}$ km

and height (deep) of the river = 3 m = $\frac{3}{1000}$ km

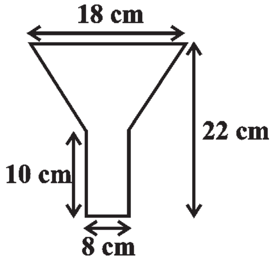
$\therefore$ Volume of one river = $1972 \times \frac{75}{1000} \times \frac{3}{1000}$

= 0.2412 km^3

So, volume of three rivers = 3×0.2412 = 0.7236 km^3

From Eqs. (i) and (ii), it is clear that total rainfall is not approximately equivalent to normal water of three rivers.

Q.5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig.).



Ans. Given oil funnel is a combination of a cylinder and a frustum of a cone.

Also, given height of cylindrical portion h = 10 cm

and radius $r_2 = \frac{8}{2} = 4$ cm

Diameter of top of the funnel = 18 cm

$\therefore$ Radius of top of the funnel $r_1 = \frac{18}{2} = 9$ cm

Now, height of the frustum of cone= 22 – 10 = 12 cm

$\therefore$ Slant heightof the frustum $l = \sqrt{12^2 + (9-4)^2}$

$= \sqrt{144 + 25} = \sqrt{169}$

l = 13 cm

Required area of the the tin sheet = Curved surface area of cylindrical portion + Curved surface area of the frustum

$= 2\pi r^2 h + \pi(r_1 + r_2)l$

$= \frac{22}{7}\,[2 \times 4 \times 10 + (9+4)\,13]$

$= \frac{22}{7}\,(80 + 169)$

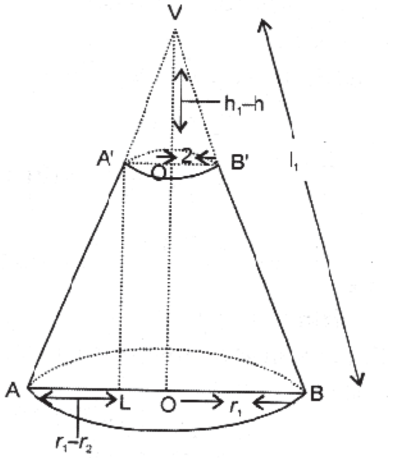
$= \frac{22}{7} \times 249$

$= \frac{5478}{7} = 782\frac{4}{7}$ cm^2

Q.6. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. Let h be the height, l be the slant height and r_1 and r_2 be the radii of the bases ($r_1 > r_2$) of the frustum fo a cone. We complete the conical part OCD.

The frustum of the right circular cone can be viewed as the difference of the two right circular



cones OAB and OCD. Let slant height of the cone OAB be l_1 and its height be h_1 i.e., OB = OA = l_1 and OP = h_1

Then, in ΔACE

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

Slant height of the cone OCD = $l_1 - l$

ΔOQD ~ Δ OPB (AA similarity criterion)

$$\frac{OD}{OB} = \frac{DQ}{PB} \Rightarrow \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad 1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\Rightarrow \quad l_1 = \frac{lr_1}{r_1 - r_2} \qquad ...(i)$$

$$\therefore \quad l_1 - l = \frac{lr_1}{r_1 - r_2} - l - \frac{lr_2}{r_1 - r_2} \qquad ...(ii)$$

Hence, curved surface area of the frustum of cone

= curved surface area of he cone OAB

– curved surfaced area of the cone OCD

$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$

$$= \pi r_1 \frac{lr_1}{r_1 - r_2} - \pi r_2 - \frac{lr_2}{r_1 - r_2}$$

[From Eqs. (i) and (ii)]

$$= \pi l \left(\frac{r_1^2 - r_2^2}{r_1 - r_2} \right) = \pi l (r_1 + r_2)$$

Therefore, curved surface area of the frustum of cone $= \pi l (r_1 + r_2)$

where, $\quad l = \sqrt{h^2 + (r_1 - r_2)^2}$

If A_1 and A_2 are the surface area ($A_1 > A_2$) of two circular bases, then

$$A_1 = \pi r_1^2 \text{ and } A_2 = \pi r_2^2$$

where, $\quad l = \sqrt{h^2 + (r_1 - r_2)^2}$

Q.7. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. Let V be the volume of the frustum of cone. Then,

V = Volume of cone VAB – Volume of cone VA'B'

$$\Rightarrow \quad V = \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$\Rightarrow \quad V = \frac{\pi}{3}[\, h_1 r_1^2 - (h_1 - h)\, r_2^2]$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{ \left(\frac{hr_1^3}{r_1 - r_2} \right) - \left(\frac{hr_2^3}{r_1 - r_2} \right) \right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{ \frac{h}{r_1 - r_2}(r_1^3 - r_2^3) \right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{ \frac{h}{r_1 - r_2}(r_1 - r_2)(r_1^2 + r_1 r_2 + r_2^2) \right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3} h (r_1^2 + r_1 r_2 + r_2^2)$$

Thus, the volume of the frustum of the cone is given by

$$\Rightarrow \quad V = \frac{1}{3}\pi (r_1^2 + r_1 r_2 + r_2^2) h.$$

Additional Questions

Q.1. Three metallic solid cubes whose edges are 3 cm and 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the single cube.

Ans. Volume of first cube = $(3)^3 = 27$ cm^3

Volume of second cube = $(4)^3 = 64$ cm^3

Volume of third cube = $(5)^3 = 125$ cm^3

Volume of single cube

$= 27 + 64 + 125$

$= 216$ cm^3

Let edge of the single cube = a cm

$\therefore \quad a^3 = 216 = (6)^3 \Rightarrow a = 6.$

Q.2. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid ?

Ans. Given volume of one cube = 64 cm^3

Let its side = a cm

$\therefore \quad a^3 = 64 \Rightarrow a = 4$ cm

Now for joined cuboid,

$l = 8$ cm

$b = 4$ cm

$h = 4$ cm

New surface area $= 2(lb + bh + hl)$

$= 2(8 \times 4 + 4 \times 4 + 8 \times 4)$

$= 2(32 + 16 + 32)$

$= 2 \times 80$

$= 160$ cm^2

Q.3. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

Ans. Volume of cube = $(7)^3 = 343$ cm^3

For concial cavity, $h = 7$ cm, $r = 3$ cm

Volume of cavity $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$

$= 66$ cm^3

Volume of the remaining solid

$= 343 - 66 = 277$ cm^3

Q.4. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm?

Ans. Volume of solid cube = $44(3)^3$ cm^3

For spherical lead shots, $r = 2$ cm

Volume of one lead short = $\frac{4}{3}\pi (2)^3$ cm^3

Number of lead shots

$$= \frac{44 \times 44 \times 44}{\frac{4}{3} \times \frac{22}{7} \times 8}$$

$$= \frac{22 \times 44 \times 21}{8}$$

$= 11 \times 11 \times 21$

$= 2541$ (lead shots)

Q.5. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic cm oif iron weight 7.5 g, find the weight of the box.

Ans. External dimesnions,

$l = 36$ cm, $b = 25$ cm, $h = 16.5$ cm

Internal dimensions

$l = 36 - 3 = 33$ cm

$b = 25 - 3 = 22$ cm

$h = 16.5 - 1.5 = 15$ cm

Volume of iron $= 36 \times 25 \times 16.5 - 33 \times 22 \times 15$

$= 14850 - 10890$

$= 3960$ cm^3

Weight of the iron $= 3960 \times 7.5$

$= 29700$ g

$= 29.7$ kg.

Q.6. 16 glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box.

Ans. Volume of cuboidal Box

$= 16 \times 8 \times 8$ cm^3

Volume of one sphere $= \frac{4}{3} \times \pi \times (2)^3$

Volume of 16 glass spheres

$= 16 \times \frac{4}{3} \times \frac{22}{7} \times 8$ cm^3

Volume of water

$= 16 \times 8 \times 8 - 16 \times \frac{4}{3} \times \frac{22}{7} \times 8$

$= 16 \times 8 \left(8 - \frac{88}{21}\right)$

$= 16 \times 8 \left(\frac{168 - 88}{21}\right)$

$= 128 \times \frac{80}{21} = \frac{10240}{21} = 487.6 \text{ cm}^3$

Q.7. A milk container of height 16 cm is made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs. 22 per litre which the container can hold.

Ans. For milk container :

R = 20 cm, r = 8 cm, h = 16 cm

Volume of container

$= \frac{1}{3} \times p\,(R^2 + r^2 + R^r) \times h$

$= \frac{1}{3} \times p\,[(20)^2 + (8)^2 + 20 \times 8] \times 16$

$= \frac{1}{3} \times p \times 624 \times 16 \text{ cm}^3$

$= \frac{1}{3} \times \frac{624 \times 16}{1000}$ litres

Cost of milk $= \frac{1}{3} \times \frac{624 \times 16}{1000} \times 22$

$= \frac{4832256}{21000}$

= Rs. 230.11

Q.8. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylinderical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Ans. For Bowl : r = 9 cm

Volume of bowl $= \frac{2}{3} \pi (9)^3 \text{ cm}^3$

For bottle : Let the numbr of bottles be n

r = 1.5 cm, h = 4 cm

Volume of cylinderical bottle

$= \pi r^2 h$

$= \pi (1.5)^2 \times 4$

Number of bottles = $\dfrac{\frac{2}{3}\pi (9)^3}{\pi (1.5)^2 \times 4}$

$= \frac{2 \times 100}{3 \times 225} = \frac{729}{4}$

= 54 bottles

Q.9. The rain water from a roof of dimensions 22 m × 20 m drains into a cylindercal vessel having diameter of base 2m and height 3.5 m. If the rain water collected from the roof just fills the cylinderical vessel, then find the rainfall in cm.

Ans. For cylinderical vessel

r = 1m, h = 3.5 m

Volume $= \pi r^2 h$

$= \pi (1)^2 \times 3.5 = 3.5 \text{ p m}^3$

Let the rainf all = h m

$\therefore$ Volume of water on the roof

$= 22 \times 20 \times h \text{ m}^3$

$\therefore \quad 22 \times 20 \times h = 3.5 \times \frac{22}{7}$

$\Rightarrow \quad h = \frac{0.5}{20} = \frac{5}{200}$ m

$\Rightarrow \quad h = \frac{5}{200} \times 100$ cm

Hence, h = 2.5 cm

Q.10. A pen stand made of wood is in the shape of a cuboid with four conical depressiosn and a cubical depression to hold the pens and pins, respectively. The dimesnions of the cuboid are 10 cm, 5 cm and 4 cm. The radius of each of the conical depressions is 0.5 cm and depth is 2.1 cm. The edge of the cubicle depression is 3 cm. Find the volume of wood is the entire stand.

Ans. Volume of cuboid = $10 \times 5 \times 4 = 200 \text{ cm}^3$

Volume of 4 depressions

$= 4 \times \frac{1}{3} \pi r^2 h + a^3$

$= \frac{4}{3} \times p\,(0.5)^2 (2.1) + (3)^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{25}{100} \times \frac{21}{10} + 27$

$= \frac{11}{5} + 27 = 2.2 + 27 = 29.2 \text{ cm}^3$

Volume of wood in the entire stand

$= 200 - 29.2$

$= 170.8 \text{ cm}^3$

Multiple Choice Questions

Q.1. The ratio of the colume of two spheres is 8 : 27. If *r* and R are the radii of spheres respectively, then (R – *r*) : *r* is :

(a) 1 : 2 (b) 1 : 3
(c) 2 : 3 (d) 4 : 9

Ans. (a)

Q.2. The volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm is :

(a) 9.7 (b) 77.6
(c) 58.2 (d) 19.4

Ans. (d)

Q.3. A cylindrical pencil sharpened at one edge is the combination of :

(a) a cone and a cylinder
(b) frustum of a cone and cylinder
(c) a hemisphere and a cylinder
(d) two cylinders

Ans. (a)

Q.4. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is :

(a) 4.2 (b) 2.1
(c) 8.4 (d) 1.05

Ans. (b)

Q.5. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. if the sphere is completely submerged, then the water level rises (in cm) by :

(a) 3 (b) 4
(c) 5 (d) 6

Ans. (a)

Q.6. A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice-cream cones of radius 2 cm and height 7 cm up to its brim. How many children will get ice cream cones?

(a) 163 (b) 263
(c) 363 (d) 463

Ans. (c)

Q.7. If each edge of a cube is increased by 50%, the percentage increase in the surface area is :

(a) 25% (b) 50%
(c) 75% (d) 125%

Ans. (d)

Q.8. Two cubes each of volume 8 cm^3 are joined end to end, then the surface area of the resulting cuboid is :

(a) 80 cm^2 (b) 64 cm^2
(c) 40 cm^2 (d) 8 cm^2

Ans. (c)

Q.9. The radius of a sphere is *r* cm. it is divided into two equal parts. The whole surface of two parts will be :

(a) $8\pi r^2$ cm^2 (b) $4\pi r^2$ cm^2
(c) $2\pi r^2$ cm^2 (d) $6\pi r^2$ cm^2

Ans. (d)

Q.10. The radius of the largesr right circular cone that be cut out from a cube of edge 4.2 cm is :

(a) 4.2 cm (b) 8.4 cm
(c) 1.05 cm (d) 2.1 cm

Ans. (d)