# Chapter 10

# Circles

## In the Chapter

In this chapter, you will be studying the following points:

- The meaning of a tangent to a circle.
- The tangent to a circle is perpendicular to the radius through the point of contact.
- The lengths of the two tangents from an external point to a circle are equal.
- If the two circles touch each other, the point of contact lies on a line joining their centres.

## NCERT TEXT BOOK QUESTION (SOLVED)

### EXERCISE 10.1

**Q.1. How many tangents can a circle have?**

**Ans.** Infinite, as there are infinite number of points on the circumference but from a point on the circumference it can have only one tangent.

So, infinite tangents can be drawn to a circle.

**Q.2. Fill in the blanks :**

**(i) A tangent to a circle intersects it in ............ point (s).**

**(ii) A line intersecting a circle in two points is called a .............**

**(iii) A circle can have ............parallel tangents at the most.**

**(iv) The common point of a tangent to a circle and the circle is called .............**

**Ans.** (i) one, (ii) secant, (iii) two, (iv) point of contact.

**Q.3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :**

**(A) 12 cm** **(B) 13 cm**

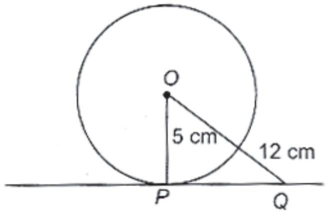
**(C) 8.5 cm** **(D) $\sqrt{119}$ cm.**

**Ans.** (d) $\sqrt{119}$ cm.

**Reason :** Radius and tangent are perpendicular to each other. Thus ∠OPQ is a right angle,

$$OQ^2 = OP^2 + PQ^2$$

$$\Rightarrow \quad PQ^2 = OQ^2 - OP^2$$



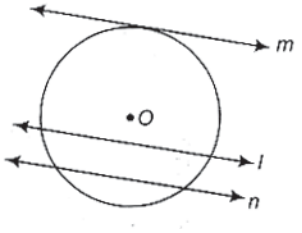

$$= 12^2 - 5^2$$

$$= 144 - 25 = 119$$

$$\Rightarrow \quad PQ = \sqrt{119}\text{ cm.}$$

**Q.4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

**Ans.** Firstly draw a circle with centre O and draw a line *l*. Now, we draw two parallel lines to *l*, such that one line *m* is tangent to the circle and another *n* is secant to the circle.



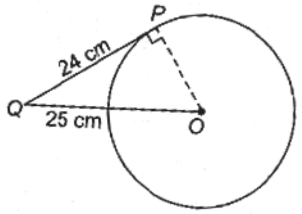
## EXERCISE 10.2

**Q.1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is**

**(A) 7 cm** **(B) 12 cm**

**(C) 15 cm** **(D) 24.5 cm**

**Ans.** (a) Given that, the length of the tangent QP = 24 cm and OQ = 25 cm. Joint OP.




We know, radius OP is perpendicular to the tangent PQ.

In right ΔOPQ

$OQ^2 = OP^2 + PQ^2$

(By Pythagores theorem)

$\Rightarrow \quad 25^2 = OP^2 + 24^2$

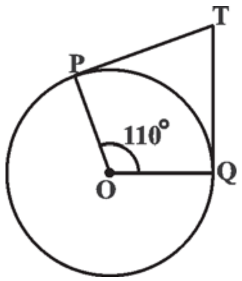
$\Rightarrow \quad OP^2 = 625 - 576 = 49$

$\Rightarrow \quad OP = 7\text{ cm}$

**Q.2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to**

**(A) 60°** **(B) 70°**

**(C) 80°** **(D) 90°**




**Ans.** It is given that,

$\angle POQ = 110°$

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPT = 90°$

and $\quad \angle OQT = 90°$

Now, in quadrilateral POQT

$\angle POQ + \angle OQT + \angle PTQ + \angle OPT = 360°$

(angle sum property of quadrilateral)

$\Rightarrow \quad 110° + 90° + \angle PTQ + 90° = 360°$

$\Rightarrow \quad \angle PTQ + 290° = 360°$

$\Rightarrow \quad \angle PTQ = 70°$

Hence, right option is (B)

**Q.3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to**

**(A) 50°** **(B) 60°**

**(C) 70°** **(D) 80°**

**Ans.** In ΔPOA and ΔPOB

PA = PB

(Tangents from externals point P)

OA = OB (radii of a circle)

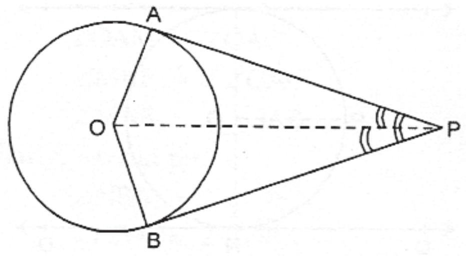
and OP = OP (common)

$\therefore \quad \Delta POA \cong \Delta POB$

(By SSS congruency)

$\Rightarrow \quad \angle OPA = \angle OPB$

$\Rightarrow \quad \angle OPA = \angle OPB = 40°$




Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OAP = 90°$

Now, in ΔOPA,

$\angle OAP + \angle OPA + \angle POA = 180°$

$\Rightarrow \quad 90° + 30° + \angle POA = 180°$

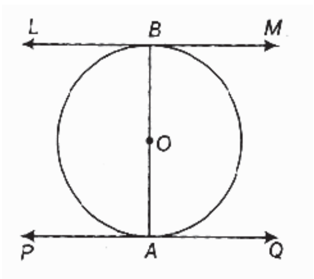
$\Rightarrow \quad 130 + \angle POA = 180°$

$\Rightarrow \quad \angle POA = 50°$

So, right option is (a).

**Q.4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

**Ans.** Let AB be a diameter of a given circle and let LM and PQ be the tangent lines drawn to the



cirdcle at points A and B, respectively. Since, the tangent at a point to a circle is perpendicular to the radius through the point.

$\therefore$ $AB \perp PQ$ and $AB \perp LM$

$\Rightarrow \quad \angle PAB = 90°$

and $\quad \angle ABM = 90°$

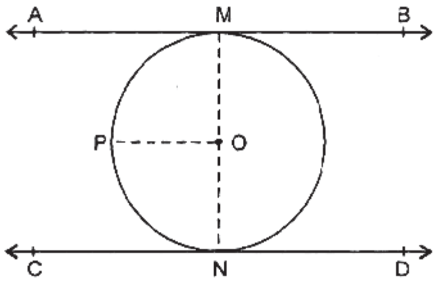
$\Rightarrow \quad \angle PAB = \angle ABM$

$\Rightarrow \quad PQ \,||\, LM$

Hence proved.

**Q.5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.**

**Ans.**




Let AMB and CND be two parallel tangents to a circle with centre O. Join OM and ON. Draw OP || AB.

Now, $\quad AM \,||\, OP$

$\Rightarrow \quad \angle AMO + \angle POM = 180°$

[Consecutive interior angles]

$\Rightarrow \quad 90 + \angle POM = 180°$

$\Rightarrow \quad \angle POM = 90°$

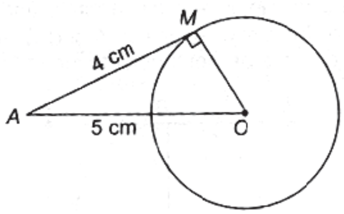
Similarly, $\quad \angle PON = 90°$

$\therefore \quad \angle POM + \angle PON = 90° + 90° = 180°$

Hence, MON is a straight line passing through O.

**Q.6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.**

**Ans.** Given, OP = 5 cm and AM = 4 cm




$OM \perp AM$

( Radius is perpendicular to AM)

In right $\Delta OMA$

$OP^2 = OM^2 + MA^2$

(By Pythagoras theorem)

$\Rightarrow \quad 5^2 = OM^2 + 4^2$

$\Rightarrow \quad OM^2 = 25 - 16 = 9$

$\Rightarrow \quad OM = 3$ cm

Hence, radius of the circle is 3 cm

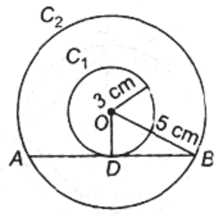
**Q.7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

**Ans.** Here, we draw two circles, $C_1$ and $C_2$ of radii $r_1 = 3$ cm and $r_2 = 5$ cm.

Now, we draw a chord AB such that it touches the circle $C_1$ at point D.

The centre of concentric circle is O.

Now, we draw a perpendicular bisector from O to AB which meets AB at D.




i.e. $\quad AD = BD$

In right $\Delta OBD$

$OB^2 = OD^2 + DB^2$

(By pythagoras theorem)

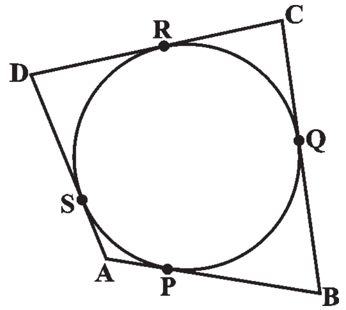
$\Rightarrow \quad 5^2 = 3^2 + DB^2$

$\Rightarrow \quad DB^2 = 25 - 9 = 16$

$\Rightarrow \quad DB = 4$ cm

$\therefore$ Length of chord = AB = 2, AD = 2 × 4 = 8 cm

**Q.8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. ). Prove that AB + CD = AD + BC**




**Ans.** Using theorem, the lengths of tangents drawn from an external point to a circle are equal.

Suppose, A is an external point, then

$AP = AS$ ... (i)

Suppose, B is an external point, then

$BP = BQ$ ...(ii)

Suppose, C is an external point, then

$CQ = RC$ ...(iii)

Suppose, D is an external point, then

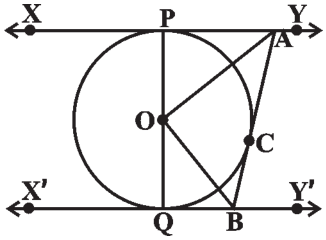
$$SD = RD \quad \text{...(iv)}$$

On adding Eqs. (i), (ii), (iii) and (iv), we get

$$(AP + BP) + (RC + RD) = (AS + BQ) + (CQ + SD)$$
$$\Rightarrow \quad AB + CD = (AS + SD) + (BQ + CQ)$$
$$\Rightarrow \quad AB + CD = AD + BC$$

Hence proved.

**Q.9. In Fig., XY and X' Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X' Y' at B. Prove that $\angle AOB = 90°$.**




**Ans.** Since, tangents drawn from an external point to a circle are equal.

$\therefore \quad AP = AC$

Thus, in $\Delta APO$ and $\Delta ACO$,

$AP = AC$

$AO = AO$ (common)

$OP = OC$ (Radius of circle)

$\therefore$ By SSS criterion of congruency, we have

$\Delta APO \cong \Delta ACO$

$\Rightarrow \quad \angle PAO = \angle OAC$

$\Rightarrow \quad \angle PAC = 2\angle CAO$

Similarly, we can prove that

$\angle CBO = \angle OBQ$

$\Rightarrow \quad \angle CBQ = 2\angle CBO$

Since XY || X'Y'

$\therefore \quad \angle PAC + \angle QBC = 180°$

(Sum of interior angles on the same sides of transversal is 180°)

$\therefore \quad 2\angle CAO + 2\angle CBO = 180° \quad \text{...(i)}$

$\Rightarrow \quad \angle CAO + \angle CBO = 90°$

In $\Delta AOB$,

$\Rightarrow \quad \angle CAO + \angle CBO + \angle AOB = 180°$

$\Rightarrow \quad \angle CAP + \angle CBO = 180° - \angle AOB \quad \text{...(ii)}$

$\therefore$ From Eqs. (i) and (ii), we get

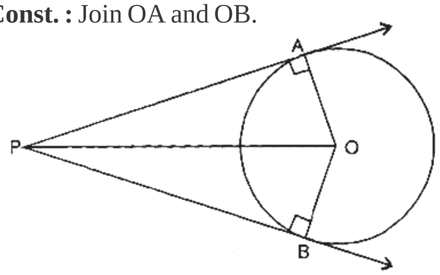
$$180° - \angle AOB = 90°$$
$$\angle AOB = 90°$$

**Q.10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.**

**Ans. Given :** PA and PB are two tangents drawn from an external point P to a circle with centre O.

**To prove :** $\angle AOB + \angle APB = 180°$

**Const. :** Join OA and OB.




**Proof :** The tangent at any point of circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OAP = 90° \quad \text{...(i)}$

and $\angle OBP = 90° \quad \text{...(ii)}$

Adding (i) and (ii), we get

$$\angle OAP + \angle OBP = 180°$$

Now in quadrilateral AOBP,

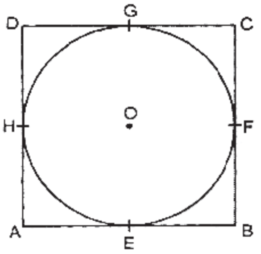
$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$

$\Rightarrow \quad 180° + \angle APB + \angle AOB = 360°$

$\Rightarrow \quad \angle APB + \angle AOB = 180°$

**Q.11. Prove that the parallelogram circumscribing a circle is a rhombus.**

**Ans.** Let ABCD be a parallelogram and a circle with centre O. Let sides AB, BC, CD and AD of the parallelogram touch the circle at E, F, G and H respectively.




Since, the length of two tangents drawn from an external point to a circle are equal.

So, $AE = AH \quad \text{...(i)}$

$BE = BF \quad \text{...(ii)}$

$CG = CF \quad \text{...(iii)}$

and $DG = DH \quad \text{...(iv)}$

Adding (i), (ii), (iii) and (iv), we get

$$AE + BE + GC + DG = AH + BF + CF + DH$$
$$\Rightarrow \quad (AE + BE) + (GC + DG) = (AH + DH) + (FG + CF)$$
$$\Rightarrow \quad AB + CD = AD + BC$$
$$\Rightarrow \quad AB + CD = AD + BC$$
$$\Rightarrow \quad 2\,AB = 2\,BC$$

[ ABCD is a || gm So, AB = CD and BC = AD]

$\Rightarrow$ AB = BC

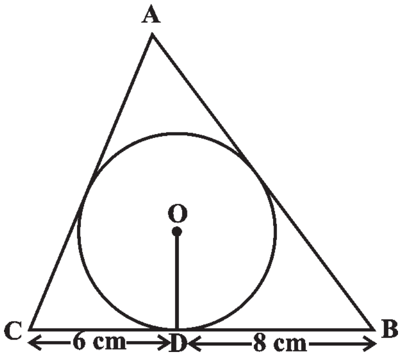
Similarly, BC = CD

and CD = AD

Thus, AB = BC = CD = DA

Hence, ABCD is rhombus.

**Q.12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. ). Find the sides AB and AC.**




**Ans.** Let O be the incentre of DABC such that OD = OE = OF = 4 cm.

Also, BD = 6cm, CD = 8 cm.

Since, length of tangents drawn fom an external point are equal.

So, BD = BF = 6 cm

and, CD = CE = 8 cm

Let the length of tangents drawn from first vertex be $x$.

$\Rightarrow$ AF = AE = $x$

[Tangents from external point A]

Now, sides of triangle are

AB = $x$ + 6 = c.

BC = 6 + 8 = 15 = $a$

and AC = $x$ + 8 = $b$

We know that : $S = \dfrac{a+b+c}{2}$

$\Rightarrow \quad S = \dfrac{14+x+8+x+6}{2}$

$\Rightarrow \quad S = \dfrac{2x+28}{2} = x = 14$

Therefore,

Area of $\Delta$ABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

(Using Heron's formula)

$= \sqrt{(x-14)(x+14-14)(x+15-x-8)(x+14-x-6)}$

$= \sqrt{x(x+14)(6)(8)}$

$= \sqrt{48x(x+14)}$ ...(i)

Also, area of DABC,

= Area of $\Delta$BOC + Area of $\Delta$AOC + Area of $\Delta$AOB

$= \dfrac{1}{2} \times \text{BC} \times \text{OD} + \dfrac{1}{2} \times \text{AC} \times \text{OE} + \dfrac{1}{2} \times \text{AB} \times \text{OF}$

$= \dfrac{1}{2} \times 14 \times 4 + \dfrac{1}{2} \times (x+8) \times 4 + \dfrac{1}{2} \times (x+6) \times 4$

$= 28 + 2(x+8) + 2(x+6)$

$= 28 + 2x + 16 + 2x + 12$

$= 4x + 56$ ...(ii)

Comparing (i) and (ii), we get

$\sqrt{48x(x+14)} = 4x + 56$

Squaring both sides, we get

$48x(x+14) = (4x+56)^2$

$\Rightarrow \quad 48x(x+14) = [4(x+14)]^2$

$\Rightarrow \quad 48x(x+14) = 16(x+14)^2$

$\Rightarrow \quad 3x(x+14) = (x = 14)^2$

$\Rightarrow \quad 3x(x+14) = (x+14)^2$

$\Rightarrow \quad 3x(x+14) - (x+14)^2 = 0$

$\Rightarrow \quad (x+14)\ [(3x-(x+14)] = 0$

$\Rightarrow \quad 3x - x - 14 = 0$

$\Rightarrow \quad 2x - 14 = 0$

$\Rightarrow \quad 2x = 14$

$\Rightarrow \quad x = 7$

Hence, AB = $x$ + 6

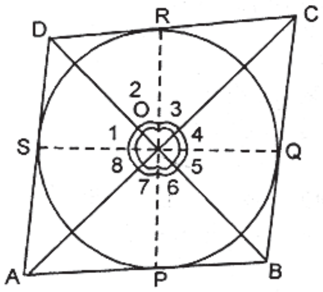
= 7 + 6 = 13 cm

and AC = $x$ + 8

= 7 + 8 = 15 cm

**Q.13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

**Ans.** Given : A circle with centre O touches the sides AB, BC, CD and DA of a quadrilatral ABCD at the points P, Q, R andS respectively.



**To prove :** $\angle AOB + \angle COD = 180°$
$\angle AOD + \angle BOC = 180°$

**Const. :** Join OP, OQ, OR and OS.

**Proof :** Since, the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since sum of all the angles subtended at a point is 360°.

$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \quad 2(\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7) = 360°$

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

$\Rightarrow \quad \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

$\Rightarrow \quad (\angle 6 + \angle 7) + (\angle 2 + \angle 3) = 180°$

$\Rightarrow \quad \angle AOB + \angle COD = 180°$

Similarly, we can prove

$\angle AOD + \angle BOC = 180°$

## Additional Questions

**Q.1. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.**

**Ans.** We know that if a chord AB makes an angle of 60° at the centre, then the angle between the tangents at A and B is complementary.

Thus angle = 120°.

Hence our statement is false.

**Q.2. The length of tangent from an external point P on a circle with centre O is always less than OP.**

**Ans.** We know that the length of the tangent from an external point P on a circle with centre O is always less than OP.

Hence the statement is true.

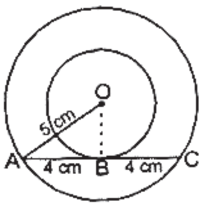
**Q.3. The angle between two tangents to a circle may be 0°.**

**Ans.** We know that two tangents to a circle can be parallel. Thus the angle between two tangents to a circle can be 0°.

Hence the statement is true.

**Q.4. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.**

**Ans.** Two concentric circles with centre O.




OA = 5 cm, AC = 8 cm, Ac = 8 cm.

We know that perpendicular from the centre of a circle bisects a chord.

$\therefore \quad AB = BC = 4$ cm

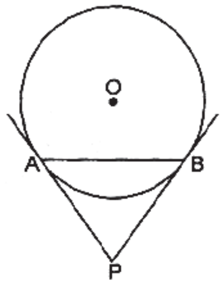
Now $\quad OB^2 = OA^2 - AB^2$

$\Rightarrow \quad OB^2 = (5)^2 - (4)^2$

$\Rightarrow \quad OB = \sqrt{25-16} = \sqrt{9} = 3.$

Hence radius for the inner circle = 3 cm

**Q.5. Prove that the tangents drawn at the end, of a chord of a circle make equal angles with the chord.**




**Ans.** Given : A circle with centre O. AB is a chord. At A and B tangents AP and BP are drawn.

**To prove :** $\angle PAB = \angle PBA$

**Proof :** We know that tangents drawn from an external point to a circle are equal.

$\therefore \quad PA = PB$

$\therefore$ In $\Delta PAB$, $\angle PAB = \angle PBA$

(isosceles $\Delta$ theorem)

**Q.6. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.**

**Ans.** Given : A circle in which AOB is a diameter. O being the cente of the circle.

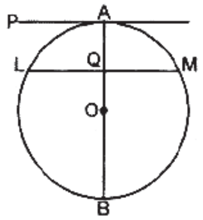
PA | | LM (a chord)

**To prove :** AB bisect LM at Q.

**Proof :** PA is tangent to the circle and OA is a radius through the point of contact.

$\therefore \quad OA \perp AP$

Now $\quad LM \parallel PA \Rightarrow OQ \perp LM$




We know that perpenedicular from the centre bisects the chord.

$\therefore \quad LQ = QM$

Similarly the result holds good for all chords parallel to the tangent.

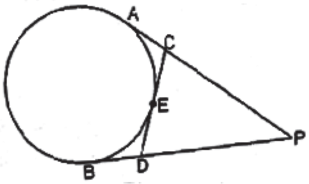
**Q.7. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangenet is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of the triangle PCD.**

**Ans.** We know that

PA = PB = 10 cm

AC = CE,

BD = DE




Perimeter of DPCD

= PC + CD + PD

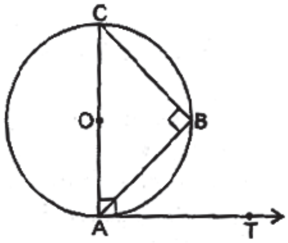
= (PA – AC) + (CE + ED) + PB – BD

= PA (–AC + CE) + (ED – BD) + PB

= PA + O + PB

= 10 + 10 = 20 cm

**Q.8. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A shown in the figure. Prove that $\angle$BAT = $\angle$ACB.**




**Ans.** OT $\perp$ AT

$\therefore$ $\angle CAT = 90°$

$\angle ABC = 90°$ ...(Angle in a semicircle)

Now $\angle ACB + \angle ABC + \angle CAB = 180°$

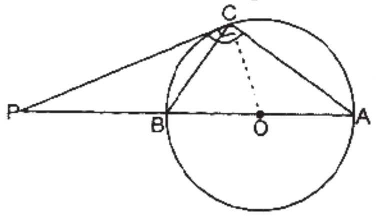
$\Rightarrow$ $\angle ACB + 90° + 90° - \angle BAT = 180°$

$\Rightarrow$ $\angle ACB - \angle BAT = 0$

Hence $\angle ACB = \angle BAT$.

**Q.9. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle$PCA = 110°, find $\angle$CBA (see figure),**

**[Hint : Join C with centre]**




**Ans.** $\angle PCA = 110°$ (Given)

$\angle ACB = 90°$ (Angle in a semi-circle)

$\therefore$ $\angle BCP = 110° - 90° = 20°$

$\angle OCB + \angle BCP = 90°$ [tangent $\perp$ radius]

$\Rightarrow$ $\angle OCB + 20° = 90°$

$\Rightarrow$ $\angle OCB = 70°$

In $\Delta OCB$, OC = OB ... (radiis of the same circle)

$\therefore$ $\angle CBA = \angle OCB$

Hence $\angle CBA = 70°$.

**Q.10. To circles with centres X and Y touch externally at P. If tangents AT and BT meet the common tangent at T, then prove that AT = BT.**

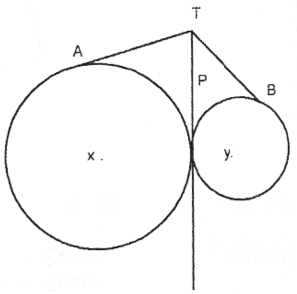
**Ans.** $\therefore$ AT and PT are tangents from external point

$\therefore$ AT = TP ...(i)

Similarly TP = TB ...(ii)

From (i) and (ii), we get

Hence, AT = BT Proved.




## Multiple Choice Questions

**Q.1. If angle between two radii of circle is 130°m the angle between the tangents at the ends of the radii is :**

(a) 90° (b) 50°

(c) 70° (d) 40°

**Ans.** (b)

**Q.2. If two tangents inclined at an angle of 90°, are drawn to a circle or radius 3 cm, then the length of each tangents is :**

(a) 2 cm (b) 3 cm

(d) 4 cm (d) 1 cm

**Ans.** (b)

**Q.3. If the radii of two concentric circles are 6 cm and 10 cm, then the length of each chord of one circle which is tangent to the other circle is :**

(a) 8 cm (b) 16 cm

(c) 10 cm (d) 6 cm

**Ans.** (b)

**Q.4. If radii of the two conentric circles are 15 cm and 17 cm, then the length of each chord of one circle which is tangent to other is :**

(a) 8 cm (b) 17 cm
(c) 30 cm (d) 16 cm

**Ans.** (d)

**Q.5. To draw a pair of tangents to a circle which are inclined to each other at an angle of 100°, it is required to draw tangents at end points of those two radii of the circle, the angle between which should be :**

(a) 100° (b) 50°
(c) 80° (d) 200°

**Ans.** (c)

**Q.6. TP and TQ are two tangents to a circle with centre O, such that ∠POQ = 110°, then ∠PTQ is:**

(a) 30° (b) 45°
(c) 70° (d) 110°

**Ans.** (c)

**Q.7. Two parallel lines touch the circle at point A and B separately. If the area of the circle is 25 $cm^2$, then AB is equal to :**

(a) 8 cm (b) 5 cm
(c) 10 cm (d) 25 cm

**Ans.** (c)

**Q.8. To draw two tangents to a circle from an external point, inclined at an angle of 50°m the tangents at the ends of two radii have to be drawn, the angle between which is :**

(a) 150° (b) 40°
(c) 50° (d) 130°

**Ans.** (d)

**Q.9. A quadrilateral PQRS is drawn to circumscribe a circle. If PQ, QR, RS (in cm) are 5, 9, 8 respectively, then PS (in cm) equals:**

(a) 7 (b) 6
(c) 5 (d) 4

**Ans.** (d)

**Q.10. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other (in cm) is :**

(a) 3 (b) 6
(c) 9 (d) 1

**Ans.** (b)