Chapter 8

Introduction of Trigonometry

In the Chapter

In this chapter, you will be studying the following points:

- In a right triangle ABC, right-angled at B,

$$\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}}. \quad \cos A = \frac{\text{side adjacent to angle A}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{side opposite to angle A}}{\text{side adjacent to angle A}}$$

- $\text{cosec}\, A = \frac{1}{\sin A}; \sec A = \frac{1}{\cos A}; \tan A = \frac{1}{\cot A}, \tan A = \frac{\sin A}{\cos A}$
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.
- The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°.
- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.
- $\sin (90° - A) = \cos A$, $\cos (90° - A) = \sin A$;
  $\tan (90° - A) = \cot A$, $\cot (90° - A) = \tan A$;
  $\sec (90° - A) = \text{cosec}\, A$, $\text{cosec}\, (90° - A) = \sec A$.
- $\sin^2 A + \cos^2 A = 1$,
  $\sec^2 A - \tan^2 A = 1$ for $0° < A < 90°$,
  $\text{cosec}^2 A = 1 + \cot2 A$ for $0° < A < 90°$.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 8.1

Q.1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine :

(i) sin A, cos A

(ii) sin C, cos C

Ans. Let AB = 24 cm

BC = 7 cm

Using pythagoras theorem, we have

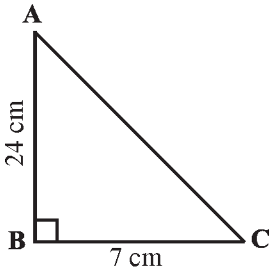
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= 576\text{ cm}^2 + 49\text{ cm}^2$

$= 625\text{ cm}^2$

So, AC= 25 cm



Now,

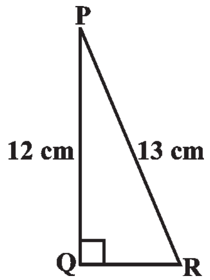
(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$

(ii) $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(iii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$

(iv) $\cos C = \frac{BC}{AC} = \frac{7}{25}$

Q.2. In Fig., find tan P – cot R.



Ans. Let PQ = 12K

and PR = 13 K

Using pythagoras theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13K)^2 = (12K)^2 + QR^2$

$\Rightarrow 168K^2 = 144K^2 + QR^2$

$\Rightarrow QR^2 = 169K^2 + 144K^2$

$\Rightarrow QR^2 = 25K^2$

So, QR = 5K

Now, $\tan P = \frac{QR}{PQ} = \frac{5K}{12K} = \frac{5}{12}$

$\cot R = \frac{QR}{PQ} = \frac{5K}{12K} = \frac{5}{12}$

Therefore,

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0.$

Q.3. If $\sin A = \frac{3}{4}$, calculate cos A and tan A.

Ans. Let us draw a right angle triangle, right angled at B.

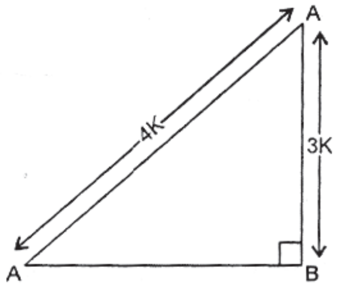
We know that :

$\sin A = \frac{3}{4} = \frac{BC}{AC}$

Let BC = 3K, AC = 4K

where K is a positive number.

Using pythagoras theorem, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow (4K)^2 = AB^2 + (3K)^2$

$\Rightarrow 16K^2 = AB^2 + 9K^2$

$\Rightarrow AB^2 = 16K^2 - 9K^2$

$\Rightarrow AB^2 = 7K^2$

$\Rightarrow AB = \sqrt{7}K$

Now, $\cos A = \frac{AB}{AC} = \frac{\sqrt{7}K}{4K} = \frac{\sqrt{7}}{4}$

and $\tan A = \frac{BC}{AB} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$

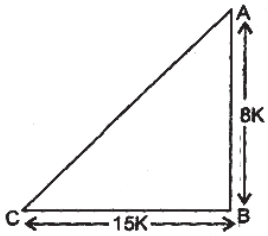
Q.4. Given 15 cot A = 8, find sin A and sec A.

Ans. Let us draw a right triangle ABC, right angled at B.

It is given that :

15 cot A = 8

$\Rightarrow \cot A = \frac{8}{15} = \frac{AB}{BC}$



Let AB = 8K

BC = 15K

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$= (8K)^2 + (15K)^2$

$= 64K^2 + 225K^2$

$= 289K^2$

So, AC = 17 K

Now, $\sin A = \frac{BC}{AC} = \frac{15K}{17K} = \frac{15}{17}$

$\sec A = \frac{AC}{AB} = \frac{16K}{8K} = \frac{17}{8}$

Q.5. Given sec θ = $\frac{13}{12}$ calculate all other trigonometric ratios.

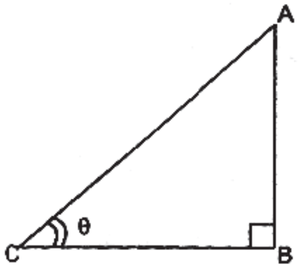
Ans. Let us draw a right angled triangle, right angled at B.

We know that :

$$\sec\theta = \frac{13}{12} = \frac{AC}{AB}$$

Let AB = 12K, AC = 13 K

Where K is a positive number.



Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (13K)^2 = (12K)^2 + BC^2$$
$$\Rightarrow 169K^2 = 144K^2 + BC^2$$
$$\Rightarrow BC^2 = 169K^2 - 144K^2$$
$$\Rightarrow BC^2 = 25K^2 \Rightarrow BC = 5K$$

Now, $$\cos\theta = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$$

$$\tan\theta = \frac{BC}{AB} = \frac{5K}{12K} = \frac{5}{12}$$

$$\cot\theta = \frac{AB}{BC} = \frac{12K}{5K} = \frac{12}{5}$$

$$\sin\theta = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$$

and $$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13K}{5K} = \frac{13}{5}$$

Q.6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Ans. Let us consider two right angle triangles right angled at Q and S respectively.

Now, $$\cos A = \frac{AS}{AR}$$

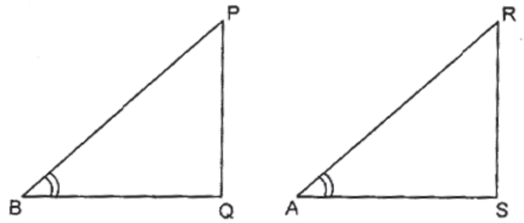
$$\cos B = \frac{BQ}{BP}$$

But cos A = cos B (given)

$$\therefore \frac{AS}{AR} = \frac{BQ}{BP}$$

$$\Rightarrow \frac{AS}{BQ} = \frac{AR}{BP}$$

Let $$\frac{AS}{BQ} = \frac{AR}{BP} = K \quad ...(i)$$



In ΔRAS,

Using Pythagoras theorem, we have

$$RS = \sqrt{AR^2 - AS^2}$$
$$= \sqrt{(K.BP)^2 - (K.BQ)^2}$$
$$= \sqrt{K^2.BP^2 - K^2.BQ^2}$$
$$= \sqrt{K^2(BP^2 - BQ^2)}$$
$$= K\sqrt{(BP^2 - BQ^2)}$$

In ΔPBQ,

Using Pythagoras theorem, we have

$$PQ = \sqrt{BP^2 - BQ^2}$$

So, $$\frac{RS}{PQ} = \frac{K.\sqrt{BP^2 - BQ^2}}{\sqrt{BP^2 - BQ^2}} = K \quad ...(ii)$$

Comparing (i) and (ii), we get

$$\frac{AS}{BQ} = \frac{AR}{BP} = \frac{RS}{PQ}$$

So, by using SSS similarity condition

Δ RSA ~ ΔPQB

∴ ∠A = ∠B

Q.7. If cot θ = $\frac{7}{8}$ evaluate :

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ **(ii)** $\cot^2\theta$

Ans. (i) Consider a right angle triangle, right angled at B and $\angle ACB = \theta$.

Here, $\cot\theta = \frac{BC}{AB} = \frac{7}{8}$

Let $BC = 7K$

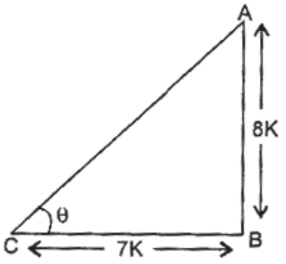
$AB = 8K$

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (8K)^2 + (7K)^2$

$= 68K^2 + 49K^2 = 113K^2$

$\Rightarrow \quad AC = \sqrt{113}K$



Now, $\sin\theta = \frac{AB}{AC} = \frac{8}{\sqrt{113}K} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{BC}{AC} = \frac{7}{\sqrt{113}K} = \frac{7}{\sqrt{113}}$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = \left(\frac{BC}{AB}\right)^2$

$$= \left(\frac{7K}{8K}\right)^2 = \frac{49}{64}.$$

Q.8. If 3 cot A = 4, check whether

$$\frac{1-\tan^2 A}{1+\tan^2 B} = \cos^2 A - \sin^2 A \text{ or not.}$$

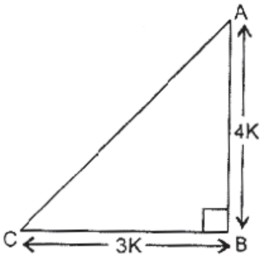
Ans. Let us draw a right triangle right angled at B.

We have,

$3\cot A = 4$

$\Rightarrow \quad \cot A = \frac{4}{3} = \frac{AB}{BC}$

Let $AB = 4K$ and $BC = 3K$



Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$= (4K)^2 + (3K)^2$

$= 16K^2 + 9K^2 = 25K^2$

So, $AC = 5K$

Now, $\tan A = \frac{BC}{AB} = \frac{3K}{4K} = \frac{3}{4}$

$\cos A = \frac{AB}{AC} = \frac{4K}{5K} = \frac{4}{5}$

and $\sin A = \frac{BC}{AC} = \frac{3K}{5K} = \frac{3}{5}$

Now, L.H.S. $= \frac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{26}{16}} = \frac{7}{25}$$

R.H.S. $= \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$$

Hence, L.H.S. = R.H.S.

or $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

Q.9. In triangle ABC, right-angled at B, if tan A $= \frac{1}{\sqrt{3}}$ find the value of:

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C

Ans. We have,

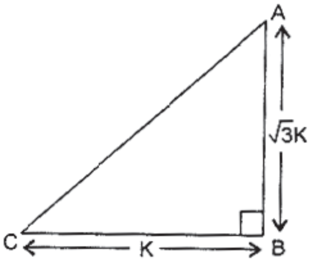
$$\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$$

Let $AB = \sqrt{3}K$, $BC = K$

Using pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$= (\sqrt{3}K)^2 + (K)^2$$
$$= 3K^2 + K^2 = 4K^2$$

So, $AC = 2K$



Now, $\sin A = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$$

and $\cos C = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

Now,

(i) sin A cos C + cos A sin A

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

(ii) cos A. cos C – sin A . sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q.10. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Ans. We have,

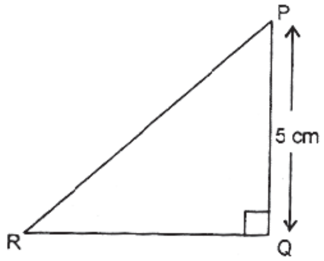
$$PR + QR = 25 \text{ cm} \quad \text{...(i)}$$

Let $PR = x$ cm

$\therefore$ $QR = (25 - x)$ cm.

Using Pythagoras theorem, we have

$$PR^2 = PQ^2 + RQ^2$$
$$\Rightarrow x^2 = (5)^2 + (25 - x)^2$$
$$\Rightarrow x^2 = 25 + 625 + x^2 - 50x$$
$$\Rightarrow 50x = 650$$
$$\Rightarrow x = 13 \text{ cm}$$



Now, $PR = x = 13$ cm

$QR = (25 - x) = 12$ cm

and $PQ = 5$ cm

Now, $\sin P = \frac{RQ}{PR} = \frac{12}{13}$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q.11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = $\frac{4}{3}$ for some angle *θ*.

Ans. (i) False, since tan A = $\frac{\text{Perpendicular}}{\text{Base}}$ and perpendicular may be longer than base.

(ii) True, since sec A = $\frac{\text{Hypotenuse}}{\text{Base}}$ and hypotenuse being the longest side may be $\frac{12}{5}$ times the base.

(iii) False, since cos A is the abbreviation used for the cosine of angle A.

(iv) False, since cot A is used as an abbreviation for 'the cotangent of the angle A'.

(v) False, since the hypotenuse is the longest side in a right triangle. As such the value of sin A is always less than 1 (or in particular equal to 1).

EXERCISE 8.2

Q.1. Evaluate the following :

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) 2 tan² 45° + cos² 30° – sin² 60°

(iii) $\frac{\cos 45°}{\sec 30° + \cos 60° + \cot 45°}$

(iv) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Ans. (i) sin 60° cos 30° + sin 30° cos 60°

$$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1$$

(ii) 2 tan² 45° + cos² 30° – sin² 60°

$$= 2(1)^2+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2 = 2(1)+\frac{3}{4}-\frac{3}{4} = 2$$

(iii) $\frac{\cos 45°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+\frac{2}{1}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{(2+2\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}\times\frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$$

(Multiply by conjugate of $2\sqrt{2}+2\sqrt{6}$ in numerator and denominator both to make denominator free from radical sign.)

$$= \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2} = \frac{2\sqrt{6}-2(3\sqrt{2})}{8-24}$$

$$(\sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2})$$

$$= \frac{-2(3\sqrt{2})+2\sqrt{6}}{-16} = \frac{-2(3\sqrt{2}-\sqrt{6})}{-16}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2}+\frac{1}{1}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}}\times\frac{4-3\sqrt{3}}{4-3\sqrt{3}}$$

(Multiply by conjugate of 4 + 3√3 in numerator and denominator both)

$$= \frac{12\sqrt{3}-27-16+12\sqrt{3}}{16-27} = \frac{24\sqrt{3}-43}{-11}$$

$$= \frac{-(43-24\sqrt{3})}{-11} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + 4\left(\frac{4}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{(15+64-12)}{12}}{\left(\frac{1+3}{4}\right)} = \frac{67}{12} \times \frac{4}{4} = \frac{67}{12}$$

Q.2. Choose the correct option and justify your choice :

(i) $\dfrac{2\tan 30°}{1+\tan^2 30°} =$

(A) sin 60° (B) cos 60°
(C) tan 60° (D) sin 30°

(ii) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} =$

(A) tan 90° (B) 1
(C) sin 45° (D) 0

(iii) sin 2A = 2 sin A is true when A =

(A) 0° (B) 30°
(C) 45° (D) 60°

(iv) $\dfrac{2\tan 30°}{1-\tan^2 30°}$

(A) cos 60° (B) sin 60°
(C) tan 60° (D) sin 30°

Ans. (i) (a) $\dfrac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\left(1+\frac{1}{3}\right)}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\left(1+\frac{1}{3}\right)} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60°$$

(ii) (d) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°}$

$$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

(iii) (a) sin 2A = 2 sin A

When A = 0°, sin (2 × 0°) = 2 sin (0°) ⇒ sin (0°) = 2(0) 0 = 0. True.

(iv) (c) $\dfrac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

Q.3. If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$; 0° < A + B < 90°; A > B, find A and B.

Ans. We have tan(A+B) = $\sqrt{3}$ ⇒ tan (A + b) = tan 60°.

∴ A + B = 60° ⇒ A = 60° – B

and tan (A – B) = $\frac{1}{\sqrt{3}}$ ⇒ tan (A– B) = tan 30°

∴ A – B = 30°

⇒ 60° – B – B = 30°

[put the value of A from Eq. (i)]

⇒ 2B = 60° – 30°

⇒ 2B = 30°

⇒ B = $\frac{30°}{2}$

⇒ B = 15°

Now, from Eqs. (i) and (ii), we get

A = 60° – 15° = 45°

Hence, A = 45° and B = 15°

Q.4. State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B.

(ii) The value of sin θ increases as θ increases.

(iii) The value of cos θ increases as θ increases.

(iv) sin θ = cos θ for all values of θ.

(v) cot A is not defined for A = 0°.

Ans. (i) False, bacause

when A = 60° and B = 30°

$$\sin(A+B) = \sin(60° + 30°) = \sin 90° = 1$$

and $\sin A + \sin B = \sin 60° + \sin 30°$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

So, sin (A + B) ≠ sin A + sin B

(ii) True, as it is clear from the ready reference table.

(iii) False as we observe from the ready reference table that the value of cosθ decreases as θ increases.

(iv) Table θ = 60°

Then $\sin\theta = \sin 60° = \frac{\sqrt{3}}{2}$

and $\cos\theta = \cos 60° = \frac{1}{2}$

So, sin θ ≠ cos θ

Hence, the statement is false.

(v) True as it is clear from the ready reference table.

EXERCISE 8.3

Q.1. Evaluate :

(i) $\frac{\sin 18°}{\cot 72°}$

(ii) $\frac{\tan 26°}{\cot 64°}$

(iii) cos 48° – sin 42°

(iv) cosec 31° – sec 59°

Ans. (i) $\frac{\sin 18°}{\cot 72°} = \frac{\sin(90°-72°)}{\cos 72°}$

$$= \frac{\cos 72°}{\cot 72°} = 1.$$

[sin (90° –θ) = cos θ]

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90°-64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

[tan (90° – θ) = cos θ]

(iii) cos 48° – sin 42°

= cos (90° – 42°) – sin 42°

= sin 42° – sin 42° = 0

(iv) cosec 31° – sec 59°

= cosec (90° – 59°) – sec 59°

= sec 59° – sec 59° = 0

Q.2. Show that :

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Ans.(i) tan 48° tan 23° tan 42° tan 67° = 1

LHS

= tan 48°. tan 23°. tan 42°. tan 67°

= tan 48°. tan 42°. tan 23°. tan 67°

= tan (90 – 42). tan 42°. tan (90–67). tan 67°

= cot 42°. tan 42°. cot 67°. tan 67°

= 1 × 1 = 1 = RHS

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

LHS

= cos 38°. cos 52° – sin 38°. sin 52°

= cos (90–52). cos 52° – sin (90–52°). sin 52°

= sin 52°. cos 52° – cos 52°. sin 52°

= 0 = RHS.

Q.3. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Ans. We have,

tan 2A = cot (A – 18°)

⇒ cot (90° –2A) = cot (A – 18°)

⇒ 90° – 2A = A–18°

⇒ –3A = – 108°

⇒ A = 36°

Q.4. If tan A = cot B, prove that A + B = 90°.

Ans. We have,

tan A = cot B

⇒ cot (90° – A) = cot B

⇒ 90° – A = B

⇒ 90° = A + B or A + B = 90°

Q.5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.

Ans. We have,

$$\sec 4A = \text{cosec}(A - 20°)$$

$$\Rightarrow \text{cosec}(90° - 4A) = \text{cosec}(A - 20°)$$

$$\Rightarrow 90° - 4A = A - 20°$$

$$\Rightarrow -5A = -110°$$

$$\Rightarrow A = 22°.$$

Q.6. If A, B and C are interior angles of a triangle ΔABC, then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Ans. In ΔABC, we have

$$A + B + C = 180°$$

$$\Rightarrow B + C = 180° - A$$

$$\Rightarrow \frac{B+C}{2} = 90° - \frac{A}{2}$$

[dividing both side by 2]

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Q.7. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67° + \cos 75°$

$= \sin(90° - 23°) + \cos(90° - 15°)$

$= \cos 23° + \sin 15°.$

EXERCISE 8.4

Q.1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Ans. $\cot^2 A + 1 = \text{cosec}^2 A$

$$\Rightarrow \text{cosec } A = \pm\sqrt{\cot^2 A + 1}$$

This gives $\text{cosec } A = \sqrt{\cot^2 A + 1}$

Hence, $$\sin A = \frac{1}{\text{cosec} A} = \frac{1}{\sqrt{\cot^2 A + 1}}$$

$$\sec A = \frac{1}{\cos A} = \frac{\text{cosec} A}{\cot A}$$

$$= \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

and $$\tan A = \frac{1}{\cot A}$$

Q.2. Write all the other trigonometric ratios of ∠ A in terms of sec A.

Ans. $$\sin A = \frac{1}{\text{cosec} A} = \frac{1}{\sqrt{\text{cosec}^2 A}}$$

$$= \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$= \frac{1}{\sqrt{1 + \frac{1}{\tan^2 A}}}$$

$$= \frac{1}{\sqrt{\frac{\tan^2 A + 1}{\tan^2 A}}}$$

$$= \frac{\tan A}{\sqrt{\tan^2 A + 1}} = \frac{\tan A}{\sec A}$$

$$\sin A = \frac{\sqrt{\tan^2 A}}{\sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \sqrt{\tan^2 A} = \sqrt{\sec^2 A - 1}$$

$$\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

and $$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

Q.3. Evaluate :

(i) $$\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $$\frac{\sin^2(90° - 27°) + \sin^2 27°}{\cos^2(90° - 73°) + \cos^2 73°}$$

$= \dfrac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73}$

[Using $\sin^2 A + \cos^2 A = 1$]

$= \dfrac{1}{1} = 1$

(ii) $\sin(90° - 65°).\cos 65° + \cos(90° - 65°).\sin 65°$
$= \cos 65°.\cos 65° + \sin 65° \sin 65°$
$= \cos^2 65° + \sin^2 65°$ [Using $\sin^2 A + \cos^2 A = 1$]
$= 1.$

Q.4. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9
(C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

(A) 0 (B) 1
(C) 2 (D) –1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) sec A (B) sin A
(C) cosec A (D) cos A

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ (B) –1
(C) $\cot^2 A$ (D) $\tan^2 A$

Ans. (i) $9\sec^2 A - 9\tan^2 A$
$= 9(\tan^2 A + 1) - 9\tan^2 A$
$= 9\tan^2 A + 9 - 9\tan^2 A$
$= 9$

Hence, correct option is (B)

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{\{(\sin\theta + \cos\theta) + 1\}\{(\sin\theta + \cos\theta) - 1\}}{\cos\theta.\sin\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\cos\theta.\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos - 1}{\sin\theta.\cos\theta}$$

$$= \frac{1 + 2\sin\theta.\cos - 1}{\sin\theta.\cos\theta} = \frac{2\sin\theta.\cos\theta}{\sin\theta.\cos\theta} = 2$$

Hence, correct option is (C)

(iii) $(\sec A + \tan A)(1 - \sin A) =$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)}{\cos A}(1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

Hence, correct option is (D)

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{\sec^2 A}{\text{cosec}^2 A}$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence, correct option is (D).

Q.5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

(ii) $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2\sec A.$

(iii) $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\ \text{cosec}\,\theta.$

[**Hint :** Write the expression in terms of sin θ and cos θ]

(iv) $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$

[**Hint :** Simplify LHS and RHS separately]

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$ = cosec A + cot A, = using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$ = sec A + tan A.

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$ = tan θ

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) (cosec A – sin A) (sec A – cos A)

$$\frac{1}{\tan A+\cot A}$$

[**Hint :** Simplify LHS and RHS separately]

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2$

Ans. (i) $(\text{cosec }\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

L.H.S. $= (\text{cosec }\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

Hence, L.H.S. = R.H.S.

(ii) $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}$ = 2 secA.

$$= \frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$$

$$= \frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{(\cos^2 A+\sin^2 A)+2\sin A+1}{(1+\sin A)\cos A}$$

$$= \frac{1+2\sin A+1}{(1+\sin A)\cos A} = \frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A} = \frac{2}{\cos A}$$

$= 2\sec A =$ R.H.S.

Hence, L.H.S. = R.H.S.

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\ \text{cosec}\theta.$

[**Hint :** Write the expression in terms of sin θ and cos θ]

L.H.S. $= \dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta-\cos^2\theta}{\cos\theta.\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta.\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1+\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1}{\sin\theta.\cos\theta} + \frac{\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

= sec θ. cosec θ + 1 = R.H.S.
Hence, L.H.S. = R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

[**Hint :** Simplify LHS and RHS separately]

L.H.S. $= \frac{1+\sec A}{\sec A}$

$$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

= 1 + cos A

R.H.S. $= \frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$

$$= \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$$

= 1 + cos A

Hence, L.H.S. = R.H.S.

Alternative Method

R.H.S. $= \frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$

$$= \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)}$$

$= 1 + \cos A = 1 + \frac{1}{\sec A}$

$$= \frac{\sec A+1}{\sec A} = \frac{1+\sec A}{\sec A}$$

= R.H.S.

Hence, L.H.S. = R.H.S.

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$ = cosec A + cot A, = using the identity cosec² A = 1 + cot² A.

L.H.S. $= \frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing N' and D' by sin A, we get

$$= \frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}}$$

$$= \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

$$= \frac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$$

$$= \frac{\cot A+\operatorname{cosec} A-1}{\cot A-\operatorname{cosec} A+1}$$

$$= \frac{(\cot A+\operatorname{cosec} A)-(\operatorname{cosec}^2 A-\cot^2 A)}{\cot A-\operatorname{cosec} A+1}$$

$$= \frac{(\cot A+\operatorname{cosec} A)-(\operatorname{cosec} A-\cot A)(\operatorname{cosec} A - \cot A)}{\cot A-\operatorname{cosec} A+1}$$

$$= \frac{(\cot A+\operatorname{cosec} A)[1-(\operatorname{cosec} A+\cot A)]}{(\cot A-\operatorname{cosec} A+1)}$$

= cot A + cosec A = R.H.S.
Hence, L.H.S. = R.H.S.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A.

L.H.S. $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= sec A + tan A = R.H.S.
Hence, L.H.S. = R.H.S.

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

L.H.S. $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta}{\cos\theta} \times \left[\dfrac{[1-2(1-\cos^2\theta)]}{2\cos^2\theta - 1}\right]$

$= \tan\theta \,.\, \left[\dfrac{1-2+2\cos^2\theta}{2\cos^2\theta - 1}\right]$

$= \tan\theta \,.\, \left[\dfrac{2\cos^2\theta - 1}{2\cos^2\theta - 1}\right]$

$= \tan\theta =$ R.H.S.

Hence, L.H.S. = R.H.S.

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S.

$= (\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= (\sin^2 A + \text{cosec}^2 A + 2 \sin A. \text{cosec } A$

$+ \cos^2 A + \sec^2 A + 2 \cos A. \sec A$

$= (\sin^2 A + \cos^2 A) + 2 \sin A. \text{cosec } A.$

$+ 2 \cos A. \sec A + \text{cosec}^2 A + \sec^2 A$

$= 1 + 2 + 2 + (\cot^2 A + 1) + (\tan^2 A + 1)$

$= 5 + 1 + 1 + \cot^2 A + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A =$ R.H.S.

Hence, L.H.S. = R.H.S.

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A)$

$= \dfrac{1}{\tan A + \cot A}$

[**Hint :** Simplify LHS and RHS separately]

L.H.S. $= (\text{cosec } A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A} = \cos A. \sin A.$

R.H.S. $= \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$= \dfrac{1}{\dfrac{\sin^2 A}{\cos A} + \dfrac{\cos^2 A}{\sin A}} = \dfrac{\cos A.\sin A}{\sin^2 A + \cos^2 A}$

$= \cos A. \sin A.$

Hence, L.H.S. = R.H.S.

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2$

L.H.S. $= \dfrac{1+\tan^2 A}{1+\cot^2 A}$

$= \dfrac{\sec^2 A}{\text{cosec}^2 A} = \dfrac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}}$

$= \dfrac{1}{\cos^2 A} \times \dfrac{\sin^2 A}{1}$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$

R.H.S. $= \left(\dfrac{1-\tan A}{1+\tan A}\right)^2$

$= \left(\dfrac{1-\dfrac{\sin A}{\cos A}}{1-\dfrac{\cos A}{\sin A}}\right)^2 = \left(\dfrac{\dfrac{\cos A - \sin A}{\cos A}}{\dfrac{\sin A - \cos A}{\sin A}}\right)$

$= \left(\dfrac{\cos A - \sin A}{\cos A} \times \dfrac{\sin A}{\sin A - \cos A}\right)^2$

$= \left(\dfrac{\cos A - \sin A}{\sin A - \cos A} \times \dfrac{\sin A}{\cos A}\right)^2$

$= \left\{\dfrac{-(\sin A - \cos A)}{\sin A - \cos A} \times \dfrac{\sin A}{\cos A}\right\}^2$

$= \left(-\dfrac{\sin A}{\cos A}\right)^2 = \dfrac{\sin^2 A}{\cos^2 A}$

$= \tan^2 A$, Hence, L.H.S. = R.H.S.

Additional Questions

Q.1. Simplify $(1 + \tan^2 \theta)(1-\sin\theta)(1+\sin\theta)$

Ans. $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= \sec^2\theta(1-\sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= 1.$

Q.2. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

Ans. $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2\sin^2\theta - (1-\sin^2\theta) = 2$

$\Rightarrow 2\sin^2\theta - 1 + \sin^2\theta = 2$

$\Rightarrow 3\sin^2\theta - 1 = 2$

$\Rightarrow \sin^2\theta = \frac{3}{3}$

$\Rightarrow \sin\theta = 1 \Rightarrow \theta = 90°.$

Q.3. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

Ans. LHS $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$= (\sec^2\theta - 1)(\sec^2\theta)$

$= \sec^4\theta - \sec^2\theta$

= RHS.

Q.4. Show that $\frac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$ =1.

Ans. L.H.S. $= \frac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$

$= \frac{\cos^2(90-(45°+\theta))+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(90°-(60°-\theta))}$

$= \frac{\sin^2(45°-\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\cot(60°+\theta)}$

$= \frac{1}{1}$

$= 1 =$ R.H.S.

Q.5. Express cot 85° + cos75° in terms of trigonometric ratio of angles between 0° and 45°.

Ans. We have : $\cot 85° + \cos 75°$

$= \cot(90° - 05°) + \cos(90° - 15°)$

$= \tan 5° + \sin 15°$

[$\cot(90°-\theta) = \tan\theta$ and $\cos(90°-\theta) = \sin\theta$]

which is the required result.

Q.6. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Ans. We have : $\sin A = \frac{\sqrt{3}}{2} = \sin 60°.$

$\Rightarrow A = 60°$

$\therefore 2\cot^2 A - 1$

$2\cot^2 60° - 1$

$2\left(\frac{1}{\sqrt{3}}\right)^2 - 1$

$2 \times \frac{1}{3} - 1$

$\frac{2}{3} - 1 = -\frac{1}{3}$

Q.7. Given that $\sin\theta + 2\cos\theta = 1$, then prove that $2\sin\theta - \cos\theta = 2$.

Ans. Given $\sin\theta + 2\cos\theta = 1$

Squaring on both sides, we get

$(\sin\theta + 2\cos\theta)^2 = 1$

$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta \, . \cos\theta = 1$

$\Rightarrow (1-\cos^2\theta) + 4(1-\sin^2\theta) + 4\sin\theta \, . \cos\theta = 1$

$(\sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \cos^2\theta - 4\sin^2\theta + 4\sin\theta \, . \cos\theta = -4$

$\Rightarrow 4\sin^2\theta + \cos^2\theta - 4\sin\theta \, . \cos\theta = 4$

$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4$

$[a^2 + b^2 - 2ab = (a-b)^2]$

$\Rightarrow 2\sin^2 - \cos\theta = 2$

Hence Proved.

Q.8. $\tan\theta + \tan(90° - \theta) = \sec\theta \sec(90° - \theta)$.

Ans. LHS $= \tan\theta + \tan(90° - \theta)$

$= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \sec\theta \operatorname{cosec}\theta$

$= \sec\theta \sec(90° - \theta)$

= RHS (Proved).

Q.9. If $\tan\theta + \sec\theta = l$, then prove that $\sec\theta = \frac{l^2+1}{2l}$.

Ans. $\tan\theta + \sec\theta = l$

$\Rightarrow \tan\theta = l - \sec\theta$

Squaring both sides, we get

$\tan^2\theta = l^2 + \sec^2\theta - 2l\sec\theta$

$\Rightarrow \sec^2\theta - 1 = l^2 + \sec^2\theta - 2l\sec\theta$

$\Rightarrow -1 = l^2 + 2^l\sec\theta$

$\Rightarrow 2^l\sec\theta = l^2 + 1$

$\Rightarrow \sec\theta = \frac{l^2+1}{2l}$

Q.10. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\frac{1}{2}$.

Ans. $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta}$$

$\Rightarrow \sin^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow (1+\tan^2\theta) + \tan^2\theta = 3\tan\theta$

$(\sec^2\theta = 1 + \tan^2\theta)$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow 1 + 2\tan^2\theta = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow \tan\theta(\tan\theta - 1) - (\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - 1)(2\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = 1, \frac{1}{2}$. Proved.

Multiple Choice Questions

Q.1. The maximum value of $\sin\theta$ is :

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

Ans. (c)

Q.2. If $\tan x = \sin 45° \cos 45° + \sin 30°$ than x equal to :

(a) 45° (b) 90°

(c) 30° (d) $\frac{1}{2}$

Ans. (a)

Q.3. The value of $\sin^2 60° - \sin^2 30°$ is:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{4}$

Ans. (a)

Q.4. If $A + B = 90°$, $\cot B = \frac{3}{4}$, then $\tan A$ is equal to:

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

Ans. (c)

Q.5. In right triangle ABC, AB = $6\sqrt{3}$ cm, BC = 6 cm and AC = 12 cm. $\angle$A is given A.

(a) 90° (b) 45°

(c) 30° (d) 60°

Ans. (c)

Q.6. If $x = a\cos\theta$, $y = b^2x^2 + a^2y^2 - a^2b^2$ is equal to :

(a) 1 (b) –1

(c) 0 (d) 2 ab

Ans. (c)

Q.7. If A is an acute angle of ΔABC, right angled at B, then the value of sinA + cos A is :

(a) equal to one (b) greater than one

(c) less than one (d) equal to two

Ans. (b)

Q.8. If sec 2A = cosec (A – 27°) where 2A is an acute angle, then the measure of $\angle$A is :

(a) 35° (b) 37°

(c) 39° (d) 21°

Ans. (c)

Q.9. If $\tan\theta + \cot\theta = 5$, then the value of $\tan^2\theta + \cot^2\theta$ is:

(a) 23 (b) 25

(c) 27 (d) 1

Ans. (a)